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Comment on anomaly cancellation in the standard model

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We review the arguments of anomaly cancellation in the standard model and point out that charge quantization is implied without reference to grand unification. In the absence of any reference to Higgs particles, anomaly cancellation, including the mixed gravitational anomaly, among the standard-model fermions yields two possible hypercharge assignments, one of which is the standard one. The uniqueness of the hypercharge assignments is recovered by extending the standard-model fermions by a right-handed neutrino and enlarging the gauge group to $SU(2)_L \times SU(2)_R \times U(1)$.

We point out that charge quantization is a consequence of the standard model, not of grand unification. This trivial point seems to have been underemphasized by the advocates of grand unification. Some of the points raised below have appeared in the paper of Geng and Marshak,[1] about which this paper comments. We also extend this analysis to a theory without Higgs particles.

Start with the fermions of the standard model, namely, one lepton doublet $L(y_1)$, one antilepton singlet $\bar{l}(y_2)$, one quark doublet $Q(y_3)$, two antiquark weak singlets $\bar{\mathbf{q}}(y_4)$ and $\bar{\mathbf{q}}'(y_5)$, where we have indicated the hypercharge assignments in parentheses. In our notation, all fields are taken to be left handed. Since we have as many quarks as antiquarks, the color group is automatically vectorlike.

The cancellation of the Adler-Bell-Jackiw anomalies implies three equations, the first two coming from the mixed triangle graphs with U(1) and $SU(2)^2$ and $SU(3)^2$, respectively, the third arising from the pure $U(1)^3$ triangle. They are

$$y_1+3y_3=0 , \tag{1a}$$

$$2y_3+y_4+y_5=0 , \tag{1b}$$

$$2y_1^3+y_2^3+3(2y_3^3+y_4^3+y_5^3)=0 . \tag{1c}$$

These equations leave one undetermined hypercharge assignment (one y_i can be chosen arbitrarily).

We will now proceed in two different directions. The first follows the standard model with one Higgs field. The second examines anomaly cancellation without using the additional constraints imposed by the Higgs particle.

Let us introduce a Higgs field H with hypercharge Y. It couples to the fermions via the Yukawa couplings (all indices have been suppressed)

$$\mathcal{L}_Y=L\bar{l}H+Q\bar{q}H+Q\bar{q}\,'H^* . \tag{2}$$

These imply the further relations

$$y_1+y_2=Y , \tag{3a}$$

$$y_3+y_4=Y , \tag{3b}$$

$$y_3+y_5=-Y . \tag{3c}$$

Note that the sum of the last two equations is the same as the second anomaly-cancellation equation. This still leaves two additional equations, one to fix Y, the other to completely fix the hypercharges. Thus, these charges are quantized in the standard model.

It is interesting to note that none of these conclusions are changed by supersymmetrizing the standard model. The reason is that even though one has to introduce an extra Higgs (super)field, with hypercharge Y', it involves only the third of the Yukawa couplings which was redundant with the anomaly condition. Thus, the last equation serves to determine Y' and the hypercharge values are all fixed. Since $Y+Y'=0$ the anomaly of the superpartners of the Higgs fields cancels out automatically.

Following Geng and Marshak,[1] let us invoke cancellation of the triangle anomaly involving two gravitons and the U(1) current. The new constraint equation reads

$$2y_1+y_2+3(2y_3+y_4+y_5)=0 , \tag{4}$$

since the gravitons couple universally through the energy-momentum tensor. The quantity in parentheses vanishes, leaving one relation between y_1 and y_2 only. It might be argued that the use of the mixed gravitational anomaly is unwarranted; on the other hand, one does believe that gravitons exist and that their contributions to the anomaly in the hypercharge current should be considered, although we do not yet know how to do quantum gravity without introducing extra fields, as in string theory. Note that with the standard model Higgs field, this equation is automatically satisfied. This remarkable fact means that whatever fermions exist beyond the standard model, they must satisfy all of the anomaly constraints by themselves. This certainly speaks well for the consistency of the standard model.

Let us explore to what extent the Higgs field is needed to fix the standard-model hypercharge assignments. In this case, we no longer use Eqs. (3). We find that even the use of the mixed gravitational anomaly constraint (4), does not uniquely determine the hypercharge assignments. Rather, we find two solutions: one yielding the bizarre hypercharges

$$y_1=y_2=y_3=y_4+y_5=0\ , \tag{5}$$

the other case being the standard hypercharge assignments. Thus, extra information is needed to distinguish between these two cases—anomalies alone will not do it.

It might be instructive to devise an extension of the standard model where anomalies alone determine the hypercharges. We find that it is not possible to find such a model by keeping with the fermions of the standard model. Adding a U(1) gauge will not improve the situation and using an SU(2) family gauge symmetry runs into the Witten anomaly.

We must therefore add other fermions. Adding a right-handed neutrino without any extra gauge symmetries allows a continuous range of hypercharges.[2,3] However, we find that extending the gauge group by one right-handed SU(2) to $SU(2)_L\times SU(2)_R\times U(1)$ restores the unique quantization of the hypercharges, using anomalies alone.

The model now contains the usual lepton doublet $L(y_1)$ and quark doublet $Q(y_3)$, but the right-handed particles are assembled as doublets of the other SU(2), $\bar{R}(y_2)$, and $\bar{P}(y_4)$, all fields left handed. The anomaly equations now read

$$y_1+3y_3=0\ , \tag{6a}$$

$$2y_3+2y_4=0\ , \tag{6b}$$

$$2y_1^3+2y_2^3+3(2y_3^3+2y_4^3)=0\ , \tag{6c}$$

as well as

$$y_2+3y_4=0\ , \tag{6d}$$

coming from the extra SU(2). The mixed gravitational anomaly and the cubic U(1) anomalies are automatically satisfied. The unique solution is (with y_1 set to unity)

$$y_1=1,\quad y_2=-\tfrac{1}{3},\quad y_3=-1,\quad y_4=\tfrac{1}{3}\ .$$

These are the usual (L-B) assignments of SO(10)-inspired theories.[4] We emphasize that this assignment does not guarantee that the electric charges come out right. It is a challenge to the practitioners of alternative non-Higgs symmetry-breaking schemes to obtain in a consistent way the alignment of their various condensates with the condensates generated by QCD for $\langle Q\bar{q}\rangle$ and $\langle Q\bar{q}'\rangle$.

Finally, we remark that the correct electric charges are obtained with

$$Q=I_3+Y/2 \tag{7}$$

for the standard model and

$$Q=I_{3L}+I_{3R}+Y/2 \tag{8}$$

for the left-right-symmetric model. In the simplest implementation of the symmetry breaking via technicolor, we find that the uniqueness of the hypercharge assignments is lost in both cases, even if we demand the correct alignment of the condensates so that the electromagnetic U(1) is not broken by QCD. We regard this as an argument in favor of the Higgs versus technicolor theories as we know them.

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[1] C. Q. Geng and R. E. Marshak, Phys. Rev. D **39**, 693 (1989).
[2] R. Foot, G. C. Joshi, H. Lew, and R. R. Volkas, Melbourne Report No. UM-P89-39 (unpublished).
[3] C. B. Thorn (private communication).
[4] For a review of left-right-symmetric theories, see R. N. Mohapatra, *Unification and Supersymmetry* (Springer, Berlin, 1986).