Limit on possible energy-dependent velocities for massless particles

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A basic tenet of special relativity is that all massless particles travel at a constant, energy-independent velocity. Astrophysical data, including observation of the Crab pulsar at ~ 100 MeV and the recent detection of the pulsar in Hercules X-1 at energies ≥ 100 TeV, are used to place new experimental constraints on energy-dependent deviations from constant velocity for massless particles. Previous experiments reached energies ~ 10 GeV; this analysis improves the previous constraints by 7 orders of magnitude.

The principle that a massless particle travels at a constant velocity is fundamental to the theory of special relativity and is one of the few essential postulates on which the entire theory can be based.[1] In fact, most past experimental tests of special relativity[2] depend critically on this postulate. In principle, the speed of a massless particle can depend on many parameters, including the energy of the particle. The energy independence of the speed of a massless particle, tested in the past to energies up to ~ 10 GeV (Ref. 3), is examined here by using astrophysical data, including observation of the Crab pulsar at ~ 100 MeV and a recent observation of Hercules X-1 at energies >100 TeV (Ref. 4), to constrain possible violations of this principle.

The CYGNUS air-shower experiment recently observed pulsed radiation from Hercules X-1 (Ref. 4); the radiation was pulsed at (nearly) the 1.24-sec pulsar period. The probability that these observations represented random statistical fluctuations of the background was conservatively estimated to be 2×10^{-5}.

The observed radiation from Hercules X-1 must be neutral in order to traverse the galactic magnetic fields and still point back to the source; it must also be long lived to reach Earth (for Hercules X-1, $L=5$ kpc so $t=L/c=5.1\times10^{11}$ sec). In addition, the particle must be relatively light to maintain phase coherence over such a long distance. The rough upper limit on the mass is 60 MeV/c^2, using a maximum phase dispersion of 0.05 cycles corresponding to 0.07 sec. The only neutral, light particles known that fit this limit are the photon and neutrino, both known to be very light or massless. Thus, the carrier of this radiation must be nearly massless.

The mass limit can be extended even further by noting that the Hercules X-1 pulsar was also detected at lower energies (~ 1 TeV) at nearly the same time by two other experiments,[5] each with a statistical significance of about 1%; these data themselves place an upper limit on the mass of about 2 MeV, using a maximum time dispersion of about 1 sec relative to the 1.24-sec period. This further mass constraint need not be made for the results discussed below.

The data observed from Hercules X-1 by the CYGNUS experiment are summarized in Table I. The event energies, as given in Ref. 4, are computed from the observed shower size and the assumption that the events are initiated by a proton; if the assumption of a photon primary was made, the energies would be increased by about 10% for the first six events and reduced by about 40% in the last five events, the difference being due to the fact that the two bursts were observed with Hercules X-1 located at rather different zenith angles. The uncertainties in primary energies, about a factor of 2, are mostly

TABLE I. Event energies and times for the events in phase from Hercules X-1. The times of the events are given in seconds UTC (coordinated universal time) on JD 2 446 635.5. The reconstructed energies (E_0^p) assume a proton primary and have an uncertainty of about a factor of 2; the phases are given relative to the first event at the period 1.235 68 sec.

UTC (sec)	E_0^p (TeV)	phase (cycles)
2 920.988	480	0.000
3 790.129	280	0.012
4 005.211	780	0.986
4 330.355	540	0.985
4 486.180	320	0.027
4 608.578	570	0.032
16 417.531	100	0.997
17 419.969	850	0.035
17 482.938	1580	0.982
17 610.305	205	0.031
17 683.188	2590	0.999

due to fluctuations in shower development through the atmosphere.

The data in Table I have been selected to include only those events in the main phase peak, thus eliminating contamination from background cosmic-ray showers; there are ~0.3 events background expected in this sample. Simple inspection of this table shows that there is no evidence of an energy-dependent velocity in the data. Such an energy dependence would show up as the high-energy events preceding the low-energy events in phase. The data span an energy range of 100–2000 TeV and yet all of the events fall within ~0.07 sec of each other relative to the 1.24-sec period. Therefore, the maximum relative time difference between events is

$$\Delta t/t \simeq \Delta v/v < 2\times 10^{-13} \tag{1}$$

over the energy range 100 TeV $< E <$ 2000 TeV. This is an improvement over published limits,[3] at energies ~10 GeV, by more than 7 orders of magnitude. Although a more sophisticated analysis might yield a slightly improved limit, it appears unwarranted because of the limited statistics of the data.

This result can be compared with other limits which can be derived from astrophysical data. For example, the neutrinos from SN 1987A (Ref. 6), which traveled about 50 kpc, were detected at Earth with a time spread of about 10 sec, most of which is expected to be due to the emission time at the source; this yields a limit of about $\Delta t/t < 2\times 10^{-12}$ over the much smaller energy range 10–30 MeV. Observation of narrow phase structure (over a phase interval corresponding to about 5 msec) from the 33-msec Crab pulsar, at energies from 50 to 500 MeV and a distance of 2 kpc (Ref. 7), yields a limit of about 2.5×10^{-14}, but again at much lower energies.

The energy dependence of the velocity of a massless particle can be phenomenologically parametrized in a model-independent way as

$$v(E)=c[1+(\alpha E/\hbar c)^{\beta}] \tag{2}$$

with α having units of length and β representing how the velocity increases with energy. Such a parametrization is necessary to make relative comparisons of experimental data; it is also likely that the expression for an energy-dependent velocity from theoretical models can be approximated in this form (see below for two examples), at least for $(\alpha E/\hbar c) \ll 1$ as is the case for all data considered here. Then, the above data can be used to limit this length parameter as a function of the exponent. The results are shown in Fig. 1 for a range of exponents.

A general feature of many theoretical extensions of special relativity,[8,9] is the introduction of a characteristic length scale, denoted l_0, associated with space. While these extensions still maintain a lack of a preferred reference frame, they predict a specific energy dependence of the velocity of massless particles. Within one of these extensions, which views space as a discrete lattice of points of characteristic spacing l_0, the velocity of a massless particle is (adopting the framework of Ref. 9)

$$v(E)=c\cosh[\operatorname{arcsinh}(l_0E/\hbar c)] \tag{3}$$

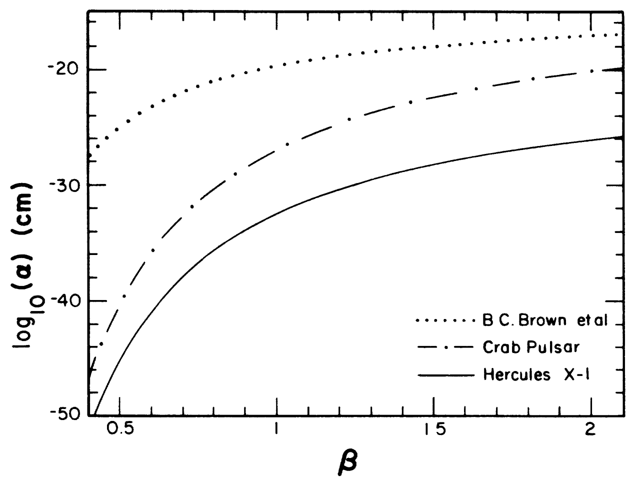

FIG. 1. Limit on the length parameter, α, as a function of the value of the exponent, β. The regions above the lines are excluded by the various data.

$$\simeq c[1+\tfrac{1}{2}(l_0E/\hbar c)^2] \quad \text{for} \quad E \ll \hbar c/l_0 \,. \tag{4}$$

Note that the expression for the energy-dependent velocity coming from a very different extension to relativity in which an additional term (of the form $l_0\delta^4\psi/\delta x^4$) is added to the electromagnetic wave equation,[8] is $v(E)\simeq c[1+\frac{3}{2}(l_0E/\hbar c)^2]$ and is only slightly different from Eq. (4). These two extensions correspond to Eq. (2) with the exponent $\beta=2$ and $\alpha=\sqrt{3/2}l_0$ and $\alpha=\sqrt{1/2}l_0$, respectively. Thus, the results below can be viewed as placing upper limits on either the presence of additional terms in the electromagnetic wave equation or the characteristic length scale between different points in space.

In a recent report,[10] it was speculated that the 5–10-sec time spread of the neutrino bursts observed in IMB and Kamiokande-II from SN 1987A (Ref. 6) was due to an increase of a massless particle's velocity with energy. Using laboratory data,[11] an approximate upper limit on l_0 was determined to be $l_0 < (1.1-2)\times 10^{-18}$ cm. The assumption that the time dispersion between the neutrino events from SN 1987A is due to this energy-dependent velocity is used to limit l_0 from below as $l_0 > 6\times 10^{-19}$ cm. Thus, the limits placed on l_0,

$$6\times 10^{-19}\ \text{cm} < l_0 < (1.1-2)\times 10^{-18}\ \text{cm}\ , \tag{5}$$

imply the need to extend special relativity.

In this well-defined theoretical framework the time dispersion between events is $\propto E^2$, very different from the time dispersion due to a finite particle mass ($\propto E^{-2}$), suggesting the dispersion would be most evident at high energies.

From the exact expression for the time required to traverse the 5-kpc distance to Hercules X-1 in Eq. (3), and the lowest value of l_0 suggested in Eq. (5), the time for a massless particle to reach Earth would be 1.6×10^{11} sec for 100 TeV and 1.7×10^{10} sec for 1000 TeV. Clearly, all phase coherence would be lost with such large energy-dependent time shifts. Even much lower energy data, such as the observation of the 33-msec Crab pulsar at $E > 50$ MeV (Ref. 7), excludes such large values of l_0. Thus, the phase coherence can be used to give an upper limit on the value of l_0. The equation for the approximate time delay between two events of different energies using Eq. (4), written in convenient units for this analysis with energy expressed in TeV, is

$$\Delta t_{12}(\text{sec}) \simeq -1.3\times 10^{44}(L/1\,\text{kpc}) \times (l_0/1\ \text{cm})^2(E_1^2-E_2^2)\ . \quad (6)$$

The best limit can be found by noting that all of the Hercules X-1 data came within about 0.07 sec and have an energy spread of about 1000 TeV (from ~ 100 to > 1000 TeV including the maximum uncertainty in the primary energies). Using this and the 5-kpc distance to Hercules X-1 yields an upper limit of $l_0 < 1\times 10^{-26}$ cm. This upper limit is in stark contradiction with the lower bound in Eq. (5) and indicates that there is no allowable range for l_0. This is an improvement over the laboratory upper limit in Eq. (5) by about 8 orders of magnitude. Data from the Crab pulsar place an upper limit of $l_0 < 1\times 10^{-20}$ cm, also implying no allowable range for l_0.

Since the sensitivity to l_0 is proportional to $(\Delta t/L)^{1/2}/E$, it is possible that other astrophysical data might be even more sensitive to an effect such as this, though none was found. For example, Cygnus X-3 has been observed[12] in air-shower experiments at higher energies ($\sim 20\,000$ TeV); however, because of the longer period involved (a 4.8-h period associated with the orbital motion of Cygnus X-3 instead of the 1.24-sec period associated with the pulsar rotation in Hercules X-1), these data are less sensitive to the effects of an energy-dependent velocity. It is unlikely that sources much further away than Hercules X-1 will be observed with gamma rays at much higher energies because of attenuation in the blackbody radiation. The remaining possibility for improvement in the future is observation of sources with shorter periodicities. It is unfortunate that the length scale of most interest, the Planck length ($\sim 1\times 10^{-33}$ cm), is still 7 orders of magnitude lower than the sensitivity of the Hercules X-1 data, but reaching this goal is of fundamental importance.

Special relativity is built on the principle that the velocity of massless particles is a constant. Using astrophysical data, no evidence for an energy-dependent velocity for massless particle over the energy range ~ 10 MeV ~ 2000 TeV was found. The observation of pulsed emission from Hercules X-1 at energies of 100–2000 TeV improves the existing bounds on an energy-dependent velocity by about 7 orders of magnitude. Limits on the existence of a natural length scale in space, often introduced as extensions to special relativity, are improved by about 8 orders of magnitude using the same data.

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