Transport equations and features of the long-wavelength oscillation of the quark-gluon plasma

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We formulate the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy of kinetic equations for a quark-gluon plasma. Assuming a Bose-Einstein distribution for the equilibrium distribution of hot gluons and ghosts and the Fermi distribution for the equilibrium distribution of hot quarks, we solve those kinetic equations in the mean-field limit and obtain the dispersion relation in the order g^2. We find that, contrary to this above assumption, the state of noninteracting quarks and gluons is unstable; i.e., the damping constant of the color oscillations is negative. We argue that the nonperturbative effects at the scale $\sim g^2T$ make the perturbative approximation to the equilibrium distribution of hot quarks and gluons inconsistent with the kinetic equations already at the lowest, nontrivial order g^2.

I. INTRODUCTION

Heavy-ion collisions at ultrarelativistic energies offer a unique opportunity to explore the large-scale properties of quantum chromodynamics (QCD), in particular the phase structure at high temperatures and densities. It is expected that at sufficiently high beam energies the produced matter will initially take the form of a plasma of unconfined quarks and gluons. This plasma would immediately begin a complex evolution, culminating in its decay into ordinary hadrons. This poses several new and difficult questions about the formation conditions of the quark-gluon plasma and the existence of any relics of plasma formation in the final decay products, as well as about the long-wavelength properties of the plasma and, in particular, about the dynamics of the plasmon excitations. The latter problem is the main subject of this paper.

The thermal properties of gluons have recently been the subject of both much controversy and intense investigation. When one applies the techniques from the Abelian theories to the non-Abelian case of QCD, one finds a serious disagreement concerning the damping of color oscillations in a quark-gluon plasma. The quark-gluon plasma has been investigated in perturbative QCD using the gluon propagator, the quantity which is explicitly gauge dependent. It turns out that, in the one-loop approximation, independently of the chosen gauge, the oscillation frequency of the long-wavelength excitations of the plasma is the same and equals $\omega=\frac{1}{3}gTN^{1/2}$. On the contrary, the damping constant depends on the chosen gauge and equals $\gamma=g^2NT/24\pi$ (the damped plasma) both in the Coulomb gauge and in the axial gauge[1] ($A^0=0$) and becomes $\gamma=-5g^2NT/24\pi$ and $\gamma=-11g^2/NT24\pi$ (the unstable plasma) in the covariant gauges,[2] $\partial_\mu A^\mu=0$ (the Lorentz gauge), and $D_\mu A^\mu=0$ (the background gauge), respectively. Those surprising results could have been attributed to a wrong method chosen by the authors of Ref. 2. However, the results obtained in this paper and the results of Hansson and Zahed[3] seem to contradict such a statement. Using the background gauge, Hansson and Zahed have constructed the gauge-independent, finite-temperature effective action for gluons.[4] This effective action contains all the information about dynamics of the plasma in the first nontrivial order in $\hbar$. In the one-loop approximation and keeping only terms quadratic in the background fields, Hansson and Zahed[3] restore the perturbative results for the damping constant and the frequency of long-wavelength oscillations in a gluon plasma.[2] Again, γ is negative and explicitly gauge-parameter dependent. Also the calculation of Nadkarni,[5] using the method of a gauge-invariant propagator extracted from physical processes, gives a negative value of the damping constant $\gamma=-11g^2NT/24\pi$. This shows that there is at present a severe problem in our understanding of the nature of the plasmon within finite-temperature QCD.

The main challenge in the field of the quark-gluon plasma is to specify its nonequilibrium properties and, in particular, to describe the way the equilibrium in the QCD plasma is reached. The relativistic quantum kinetic theory and resulting from it the relativistic transport equations seem to be well suited for that purpose.[6,7] We do not address here the problem of the gauge dependence[8] of γ or the problem of a correct definition[5] of γ. Instead, our aim here is to formulate the relativistic kinetic theory on a quantum-field-theoretical basis by developing the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy and truncating it at the mean-field level. This field is largely unexplored in spite of large interest in developing the kinetic equations of quarks and gluons. (For a recent review of the subject, see Ref. 9 and references quoted therein.)

Calculating the damping constant of color oscillations for the non-Abelian, finite-temperature field theory, one faces essentially three problems: (i) the gauge-invariant definition of γ; (ii) the self-consistent calculation of γ; (iii) the nonperturbative calculation of γ.

In the scheme of the relativistic transport equations proposed in this paper, one can address the last two points. The mean-field equations, derived in Sec. II E in the covariant gauge, consist of a nonperturbative, self-consistent set of equations. These equations are completely general and can be used to study dynamics of the QCD plasma far from the equilibrium. Formally, they are gauge dependent (see Sec. II E); however, these equations contain essentially the same information about the dynamics of the system as the equations explicitly gauge covariant, being considerably less complicated.[7] For those transport equations, in the vicinity of the equilibrium, one can obtain the dispersion relation (see Sec. III B). Solutions of the dispersion relations for the frequency and the damping constant of the plasmon are the same (see Sec. III C) as obtained in Ref. 2 provided one takes the equilibrium distribution for the noninteracting gas of quarks and gluons.

The dependence of the frequency and the damping constant of the plasmon on the chosen equilibrium distribution function (see Sec. III C) can be seen clearly in the framework of the relativistic transport equations, which are developed in this work. This feature is important due to the expected instability of the perturbative QCD vacuum at the confining scale $\sim g^2T$ (Ref. 10). Equilibrium distribution functions at high temperatures which are known only for free gluons and quarks can undergo important changes, when the interactions between particles are taken into the consideration. Indeed, the QCD lattice simulations show important deviations from the free quark and gluons approximation of the plasma.[11] Even at high temperatures the magnetic sector of the gauge action is nonperturbative, due to infrared divergences.[12] This nonperturbative action has been studied by Polonyi and classical monopole solutions have been shown to be important.[13]

This paper is organized as follows. In Sec. II the gluon sector of the theory is analyzed. After the first three introductory sections (Secs. II A–II C), we discuss in Sec. II D the mean-field approximation to the BBGKY hierarchy of kinetic equations. In the next section (Sec. II E) we present the derivation of the hierarchy of relativistic transport equations from QCD field equations. This hierarchy of transport equations is closed at the mean-field level. The mean-field transport equations are then linearized in the vicinity of equilibrium (Sec. II F). The linearized version of those equations provides a starting point in the derivation of the perturbative dispersion relation (Sec. III). The dispersion relation is written down in Sec. III B and in Sec. III C the dispersion relation is analyzed in the limit $\omega \gg |\mathbf{k}|$. In Sec. IV A inclusion of quarks at high temperatures will be discussed. For such a generalized kinetic theory, the dispersion relations will be given (see Sec. IV D) in the limits $T \gg \mu$, $\mu \gg T$. Finally, in Sec. V we summarize the most important results of our calculations.

II. GLUONS IN THE MEAN-FIELD APPROXIMATION

A. QCD Lagrangian

An effective QCD Lagrangian for hot gluonic matter consists of three terms:

$$L = L_0 + L_{\rm fix} + L_{\rm FP} \ . \qquad (1)$$

Non-Abelian, self-interacting fields for gluons, A_μ^a ($\mu = 0, \ldots, 3, a = 1, \ldots, N^2-1$) are described by the Lagrangian L_0:

$$L_0 = -\tfrac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \ ,$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{abc} A_{\mu a} A_{\nu c}$ and $i f_{abc}$ are the structure constants of the SU(N) group. L_0 possesses the local SU(N) gauge symmetry. $L_{\rm fix}$ is a gauge-fixing term which is introduced in order to eliminate the unphysical degrees of freedom of the fields A. Throughout this paper we employ the covariant gauge, for which

$$L_{\rm fix} = -\frac{1}{2\alpha} (\partial^\mu A_\mu^a)^2 \ .$$

Moreover, in the covariant gauge one has to introduce Faddeev-Popov ghosts, the unphysical fermionic fields $c^\dagger, c$ interacting with gluons. They are described by the Lagrangian $F_{\rm FP}$:

$$L_{\rm FP} = c_a^\dagger \partial_\mu (\delta^{ab} \partial^\mu - g f_{abc} A_c^\mu) c_b \ .$$

In the perturbative calculation, this theory contains three- and four-gluon interactions, as well as the interaction of the field A with the ghosts.

B. Statistical average

The effective QCD Lagrangian is a function of quantum fields A and c. Solving field equations for A and c in a system as complicated as the quark-gluon plasma is an impossible task to perform. On the other hand, a statistical description of the hot plasma seems to be physically more appropriate and, moreover, it offers the possibility of simplifying the original quantum-field-theoretical problem. In this kind of description, one has to introduce suitable statistical function, e.g., the statistical averages of the quantum operators

$$\langle O \rangle = {\rm Tr} \bar{\rho} O \ ,$$

which are built from the quantum operators A, c, and the density operator $\bar{\rho}$. Such a statistical formalism can be employed, as will be explained in Sec. II E, even though, in general, $\bar{\rho}$ is unknown. The resulting equations for the averages of quantum operators can be treated as functional identities which yield the solution describing the system.

In this paper we follow a different strategy. We shall assume a precise form of the density operator, the grand canonical density operator

$$\bar{\rho} = \frac{e^{-\beta(H + \mu_A Q^A)}}{{\rm Tr} e^{-\beta(H + \mu_A Q^A)}} \ ,$$

where H is the Hamilton operator and Q^A are the charge operators commuting with H. In the case of gluons we take $\mu_A=0$. For the quarks in the symmetric nuclear matter, we assume $\mu=\mu_A Q^A=\frac{1}{3}\mu_b$, because we consider only quarks u and d. In the limit $T\to 0$, the statistical averages become $\langle O\rangle=\langle 0|O|0\rangle$, where $|0\rangle$ is the state of minimal energy. In this limit, the averages $\langle O\rangle$ reduce to the averages of the usual, zero-temperature field theory. One should mention that this way of introducing the temperature destroys the Lorentz covariance of the theory and, consequently, we work in the plasma rest frame. The statistical averages, calculated with the help of this operator, are expected to yield a good approximation of the equilibrium state. This is required for a proper description of the near-equilibrium dynamics of the quark-gluon plasma. However, it should be stressed that the kinetic equations of Sec. II E are completely general and their form does not depend on any particular choice of the statistical operator.

C. The field equations

In Sec. II A we wrote the effective Lagrangian L of self-interacting gluons. Using Eq. (1) and differentiating it with respect to the components of fields A or c we obtain field equations, which will be used below to derive kinetic equations. However, before we write these field equations explicitly, we would like to make a comment about the way we obtain kinetic equations in the mean-field approximation. In this approximation, one restricts the number of possible statistical averages of quantum fields by selecting only those which involve two quantum fields. It is then assumed that these statistical averages are symmetric; i.e., one neglects the statistical average of the commutator of two fields : $\langle [A^a_\mu, A^b_\nu]\rangle=0$. This follows from the symmetry in Lorentz and color indices, which is assumed for the equilibrium distributions. Hence, in the equilibrium, the average of two fields will take the form $\langle A^a_\mu(x)A^b_\nu(y)\rangle=\delta^{ab}\mathcal{A}_{\mu\nu}(x-y)$, where $\mathcal{A}_{\mu\nu}(x-y)$ is a symmetric matrix in Lorentz indices ($\mathcal{A}_{\mu\nu}=\mathcal{A}_{\nu\mu}$) which due to the translational invariance of the equilibrium distributions depends only on the difference $x-y$. Consequently, one neglects the genuine noncommutability of quantum fields in the Lagrangian (1). In this way one obtains

$$\left[g_{\mu\nu}\Box+\left[\frac{1}{\alpha}-1\right]\partial_\mu\partial_\nu\right]A^\mu_a(x)=-gf_{abc}\partial_\mu A^\mu_b(x)A_{\nu c}(x)-2gf_{abc}A^\mu_b(x)\partial_\mu A_{\nu c}(x)$$
$$+gf_{abc}A^\mu_b(x)\partial_\nu A_{\mu c}(x)-g^2f_{abc}f_{cde}A^\mu_b(x)A_{\mu d}(x)A_{\nu e}(x)-gf_{abc}\partial_\nu c^\dagger_b(x)c_c(x)\ , \quad (2)$$

$$\Box c_a(x)-gf_{abc}\partial_\mu(A^\mu_c(x)c_b(x))=0\ , \quad (3)$$

$$\Box c^\dagger_b(x)+gf_{abc}A^\mu_c(x)\partial_\mu g c^\dagger_a(x)=0\ . \quad (4)$$

Notice that Eq. (2), besides the first power in A, contains also the products of two and three fields. Similar features can be seen in the Eqs. (3) and (4). These coupling terms lead in the statistical description to the existence of an infinite hierarchy of coupled integrodifferential equations.

D. The BBGKY hierarchy and its truncation

As mentioned above, Eq. (2) after averaging takes the form

$$\langle A\rangle=\mathcal{F}[\langle AA\rangle,\langle AAA\rangle,\langle c^\dagger c\rangle]\ . \quad (5)$$

On the right-hand side, this equation contains three unknown functions: $\langle AA\rangle,\langle AAA\rangle$, and $\langle c^\dagger c\rangle$. In order to solve Eq. (5) one should construct, in addition, the evolution equations also for these functions. It turns out that these additional equations[14] contain still higher-order statistical averages:

$$\langle AA\rangle=\mathcal{F}_2(\langle AAA\rangle,\langle AAAA\rangle,\langle Ac^\dagger c\rangle\ ,\langle c^\dagger cA\rangle)\ ,$$
$$\langle c^\dagger c\rangle=\mathcal{G}(\langle c^\dagger cA\rangle,\langle Ac^\dagger c\rangle)\ ,$$

$$\langle AAA\rangle=\mathcal{F}_3(\langle AAAA\rangle,\langle AAAAA\rangle\ ,$$
$$\langle AAc^\dagger c\rangle,\langle Ac^\dagger cA\rangle\ ,\langle c^\dagger cAA\rangle)\ .$$

One can continue the above procedure infinitely, adding more and more equations for the averages appearing on the right-hand side of the equations obtained till then. In this way one gets an infinite set of equations for infinitely many functions: $\langle A\rangle,\langle AA\rangle,\langle c^\dagger c\rangle,\langle AAA\rangle,\langle Ac^\dagger c\rangle$, $\langle c^\dagger cA\rangle,\langle AAAA\rangle,\ldots$. This set of equations is called the BBGKY hierarchy. If we were able to solve equations of this hierarchy, i.e., knowing all correlations of a system, we would be able to describe dynamics of the system completely. The solution of this problem would reduce the whole macroscopic thermodynamics of the system to its microscopic characteristics. Unfortunately, from the mathematical point of view, this is a very complicated set of equations. In four dimensions one cannot even say if there exists a solution, and, if so, then whether the solution is unique. Consequently, we have no answer to the fundamental question of statistical physics: Is the macroscopic behavior of the system uniquely determined by microscopic, dynamical laws?

Hence, in order to obtain any solution we have to propose a truncation scheme which cuts the kinetic equations of the BBGKY hierarchy. One may achieve this by approximating the averages of many fields by the corresponding products of the averages of a smaller number of fields. This procedure becomes clear when we rewrite the averages of field operators using the correlation functions $\mathcal{K}_n$ (Ref. 14):

$$\langle A(1)A(2)\rangle = \mathcal{K}_2(1,2)+\langle A(1)\rangle\langle A(2)\rangle ,$$

$$\langle A(1)A(2)A(3)\rangle = \mathcal{K}_3(1,2,3)+\sum_{\text{sym}}\mathcal{K}_2(1,2)\langle A(3)\rangle + \langle A(1)\rangle\langle A(2)\rangle\langle A(3)\rangle ,$$

$$\langle A(1)A(2)A(3)A(4)\rangle = \mathcal{K}_4(1,2,3,4)+\sum_{\text{sym}}\mathcal{K}_3(1,2,3)\rangle A(4)\rangle + \sum_{\text{sym}}\mathcal{K}_2(1,2)\mathcal{K}_2(3,4) + \sum_{\text{sym}}\mathcal{K}_2(1,2)\langle A(3)\rangle\langle A(4)\rangle + \langle A(1)\rangle\langle A(2)\rangle\langle A(3)\rangle\langle A(4)\rangle .$$

Numbers 1–4 in the above equations stand for the color and Lorentz indices as well as for the coordinates. The functions $\mathcal{K}_n$ stand for the n-particle correlation functions for the fields A. Truncating the hierarchy at the level of the two-particle correlation functions means putting $\mathcal{K}_n=0$ for $n>2$. If, in addition, one neglects the higher-order correlations between the gauge and ghost fields, then one gets a closed set of equations for a finite number of unknown functions. Physically, one expects that the first two correlation functions are essentially sufficient for the description of the system. On the other hand, neglecting the two-particle correlation functions corresponds to neglecting quantum fluctuations, i.e., to the classical limit.

E. The hierarchy of kinetic equations

Kinetic equations are obtained from the field equations (2)–(4) by performing statistical averaging and taking the Fourier transform. In addition, we put $\alpha=1$; i.e., we choose the Feynman gauge. In this gauge the left-hand side of Eq. (2) takes the form of the dispersion relation for free gluons and the lowest-order equation for fields A becomes

$$\begin{aligned}-k^2 g_{\mu\nu}\langle A_a^\mu(k)\rangle = {} & igf_{abc}\int\frac{d^4l}{(2\pi)^4}l^\mu\langle A_{\mu b}(l)A_{\nu c}(k-1)\rangle + 2igf_{abc}\int\frac{d^4l}{(2\pi)^4}l^\mu\langle A_{\mu b}(k-l)A_{\nu c}(l)\rangle \\ & -igf_{abc}\int\frac{d^4l}{(2\pi)^4}l^\nu\langle A_{\mu b}(k-l)A_c^\nu(l)\rangle \\ & -g^2 f_{abc}f_{cde}\int\frac{d^4l_1}{(2\pi)^4}\int\frac{d^4l_2}{(2\pi)^4}\langle A_{\mu b}(k-l_1)A_d^\mu(l_1-l_2)A_{\nu e}(l_2)\rangle \\ & +igf_{abc}\int\frac{d^4l}{(2\pi4)^4}l_\nu\langle c_b^\dagger(l)c_c(k-l)\rangle ,\end{aligned} \tag{6}$$

where

$$A_{\mu a}(k)=\int d^4x\,\exp(ikx)A_{\mu a}(x) . \tag{6a}$$

The corresponding equations for the ghost fields can be obtained acting with operators $\Box_x$ and $\Box_y$ on the product $\langle c^\dagger(y)c(x)\rangle$ and taking the Fourier transform

$$\begin{aligned} k^2\langle c_b^\dagger(p)c_a(k)\rangle &= igf_{adc}k^\mu\int\frac{d^4l}{(2\pi)^4}\langle c_b^\dagger(p)c_d(k-l)A_{\mu c}(l)\rangle ,\\ p^2\langle c_b^\dagger(p)c_a(k)\rangle &= -igf_{dbc}\int\frac{d^4l}{(2\pi)^4}l^\mu\langle A_{\mu c}(p-l)c_d^\dagger(l)c_a(k)\rangle .\end{aligned} \tag{7}$$

Operators $c_b^\dagger(p)$ and $c_a(k)$ are defined similarly as $A_{\mu a}(k)$ [Eq. (6a)].

In the same way, one can derive equations for the average of the two gluon fields:

$$\begin{aligned} k^2\langle A_{\nu a}(k)A_{\sigma h}(p)\rangle = {} & -igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\mu\langle A_b^\mu(l)A_{\nu c}(k-l)A_{\sigma h}(p)\rangle - 2igf_{abc}\int\frac{d^4l}{(2\pi)^4}l^\mu\langle A_b^\mu(k-l)A_{\nu c}(l)A_{\sigma h}(p)\rangle \\ & +igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\nu\langle A_{\mu b}(k-l)A_c^\mu(l)A_{\sigma h}(p)\rangle \\ & +g^2 f_{abc}f_{cde}\int\frac{d^4l_1}{(2\pi)^4}\int\frac{d^4l_2}{(2\pi)^4}\langle A_{\mu b}(k-l_1)A_d^\mu(l_1-l_2)A_{\nu e}(l_2)A_{\sigma h}(p)\rangle \\ & +igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\nu\langle c_b^\dagger(l)c_c(k-l)A_{\sigma h}(p)\rangle ,\end{aligned}$$

$$p^2\langle A_{\nu a}(k)A_{\sigma h}(p)\rangle=-igf_{hbc}\int\frac{d^4l}{(2\pi)^4}l_\mu\langle A_{\nu a}(k)A_b^\mu(l)A_{\sigma c}(p-l)\rangle-2igf_{hbc}\int\frac{d^4l}{(2\pi)^4}l_\mu\langle A_{\nu a}(k)A_b^\mu(p-l)A_{\sigma c}(l)\rangle$$
$$+igf_{hbc}\int\frac{d^4l}{(2\pi)^4}l_\sigma\langle A_{\nu a}(k)A_{\mu b}(p-l)A_c^\mu(l)\rangle$$
$$+g^2f_{hbc}f_{cde}\int\frac{d^4l_1}{(2\pi)^4}\int\frac{d^4l_2}{(2\pi)^4}\langle A_{\nu a}(k)A_{\mu b}(p-l_1)A_d^\mu(l_1-l_2)A_{\sigma e}(l_2)\rangle$$
$$+igf_{hbc}\int\frac{d^4l}{(2\pi)^4}l_\sigma\langle A_{\nu a}(k)c_b^\dagger(l)c_c(p-l)\rangle\ . \qquad (8)$$

If we use the truncation scheme described in Sec. II D, i.e., if we neglect the correlations of three and more gluon fields, and if, in addition, we apply the Vlasov approximation to the statistical averages containing fields A and c,

$$\langle c^\dagger cA\rangle=\langle Ac^\dagger c\rangle=\langle c^\dagger c\rangle\langle A\rangle\ ,$$

then we obtain three equations for the three unknown functions $\langle A\rangle,\langle AA\rangle$, $\langle c^\dagger c\rangle$:

$$-k^2g_{\mu\nu}\langle A_a^\mu(k)\rangle=igf_{abc}\int\frac{d^4l}{(2\pi)^4}l^\mu\langle A_{\mu b}(l)A_{\nu c}(k-l)\rangle+2igf_{abc}\int\frac{d^4l}{(2\pi)^4}l^\mu\langle A_{\mu b}(k-l)A_{\nu c}(l)\rangle$$
$$-igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\nu\langle A_{\mu b}(k-l)A_c^\mu(l)\rangle$$
$$-g^2f_{abc}f_{cde}\int\frac{d^4l_1}{(2\pi)^4}\int\frac{d^4l_2}{(2\pi)^4}\Big[\sum_{\rm sym}\langle A_{\mu b}(k-l_1)A_d^\mu(l_1-l_2)\rangle\langle A_{\nu e}(l_2)\rangle$$
$$-2\langle A_{\mu b}(k-l_1)\rangle\langle A_d^\mu(l_1-l_2)\rangle\langle A_{\nu e}(l_2)\rangle\Big]$$
$$+igf_{abc}\int\frac{d^4l}{(2\pi 4)^4}l_\nu\langle c_b^\dagger(l)c_c(k-l)\rangle\ , \qquad (9)$$

$$k^2\langle c_b^\dagger(p)c_a(k)\rangle=igf_{adc}k^\mu\int\frac{d^4l}{(2\pi)^4}\langle c_b^\dagger(p)c_d(k-l)\rangle\langle A_{\mu c}(l)\rangle\ ,$$
$$p^2\langle c_b^\dagger(p)c_a(k)\rangle=-igf_{dbc}\int\frac{d^4l}{(2\pi)^4}l^\mu\langle A_{\mu c}(p-l)\rangle\langle c_d^\dagger(l)c_a(k)\rangle\ , \qquad (10)$$

$$k^2\langle A_{\nu a}(k)A_{\sigma h}(p)\rangle=-igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\mu\Big[\sum_{\rm sym}\langle A_b^\mu(l)A_{\nu c}(k-l)\rangle\langle A_{\sigma h}(p)\rangle-2\langle A_b^\mu(l)\rangle\langle A_{\nu c}(k-l)\rangle\langle A_{\sigma h}(p)\rangle\Big]$$
$$-2igf_{abc}\int\frac{d^4l}{(2\pi)^4}l^\mu\Big[\sum_{\rm sym}\langle A_b^\mu(k-l)A_{\nu c}(l)\rangle\langle A_{\sigma h}(p)\rangle-2\langle A_b^\mu(k-l)\rangle\langle A_{\nu c}(l)\rangle\langle A_{\sigma h}(p)\rangle\Big]$$
$$+igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\nu\Big[\sum_{\rm sym}\langle A_{\mu b}(k-l)A_c^\mu(l)\rangle\langle A_{\sigma h}(p)\rangle-2\langle A_{\mu b}(k-l)\rangle\langle A_c^\mu(l)\rangle\langle A_{\sigma h}(p)\rangle\Big]$$
$$+g^2f_{abc}f_{cde}\int\frac{d^4l_1}{(2\pi)^4}\int\frac{d^4l_2}{(2\pi)^4}\Big[\sum_{\rm sym}\langle A_{\mu b}(k-l_1)A_d^\mu(l_1-l_2)\rangle\langle A_{\nu e}(l_2)A_{\sigma h}(p)\rangle$$
$$-2\langle A_{\mu b}(k-l_1)\rangle\langle A_d^\mu(l_1-l_2)\rangle\langle A_{\nu e}(l_2)\rangle\langle A_{\sigma h}(p)\rangle\Big]$$
$$+igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\nu\langle c_b^\dagger(l)c_c(k-l)\rangle\langle A_{\sigma h}(p)\rangle\ ,$$

$$
\begin{aligned}
p^2\langle A_{\nu a}(k)A_{\sigma h}(p)\rangle = & -igf_{hbc}\int\frac{d^4l}{(2\pi)^4}l_\mu\left[\sum_{\text{sym}}\langle A_{\nu a}(k)A_b^\mu(l)\rangle\langle A_{\sigma c}(p-l)\rangle-2\langle A_{\nu a}(k)\rangle\langle A_b^\mu(l)\rangle\langle A_{\sigma c}(p-l)\rangle\right]\\
& -2igf_{hbc}\int\frac{d^4l}{(2\pi)^4}l_\mu\left[\sum_{\text{sym}}\langle A_{\nu a}(k)A_b^\mu(p-l)\rangle\langle A_{\sigma c}(l)\rangle-2\langle A_{\nu a}(k)\rangle\langle A_b^\mu(p-l)\rangle\langle A_{\sigma c}(l)\rangle\right]\\
& +igf_{hbc}\int\frac{d^4l}{(2\pi)^4}l_\sigma\left[\sum_{\text{sym}}\langle A_{\nu a}(k)A_{\mu b}(p-l)\rangle\langle A_c^\mu(l)\rangle-2\langle A_{\nu a}(k)\rangle\langle A_{\mu b}(p-l)\rangle\langle A_c^\mu(l)\rangle\right]\\
& +g^2f_{hbc}f_{cde}\int\frac{d^4l_1}{(2\pi)^4}\int\frac{d^4l_2}{(2\pi)^4}\left[\sum_{\text{sym}}\langle A_{\nu a}(k)A_{\mu b}(p-l_1)\rangle\langle A_d^\mu(l_1-l_2)A_{\sigma e}(l_2)\rangle\right.\\
& \qquad\left.-2\langle A_{\nu a}(k)\rangle\langle A_{\mu b}(p-l_1)\rangle\langle A_d^\mu(l_1-l_2)\rangle\langle A_{\sigma e}(l_2)\rangle\right]\\
& +igf_{hbc}\int\frac{d^4l}{(2\pi)^4}l_\sigma\langle A_{\nu a}(k)\rangle\langle c_b^\dagger(l)c_c(p-l)\rangle\ .
\end{aligned}
\tag{11}
$$

In the equilibrium we assume that (i) $\langle A\rangle=0$, i.e., no classical fields are present, and (ii) the equilibrium distributions depend only on the relative coordinates, i.e.,

$$\langle A(x)A(y)\rangle=\mathcal{A}(x-y),\quad \langle c^\dagger(x)c(y)\rangle=\mathcal{C}(x-y)\ .$$

In this case, Eqs. (9) and (10) are satisfied identically and Eq. (11) reduces to

$$k^2\mathcal{A}_{\nu\sigma}(k)=-g^2N\mathcal{A}_{\nu\sigma}(k)\int\frac{d^4l}{(2\pi)^4}\mathcal{A}_\mu^\mu(k-l)+g^2N\mathcal{A}_{\mu\nu}(k)\int\frac{d^4l}{(2\pi)^4}\mathcal{A}_\sigma^\mu(k-l)\ . \tag{12}$$

The above equation introduces a correction to the free mass-shell equation $k^2=0$ for the fields A.

F. Perturbation of the equilibrium distribution

Let us suppose that by some unknown mechanism the system has reached an equilibrium state. In this case, if we would know the equilibrium distribution functions $\mathcal{A}$ and $\mathcal{C}$, then we could use the above transport equations to study the behavior of hot gluons near the equilibrium. Let δA be a perturbation of the field A, induced by the coupling to some exterior, physical system. Assuming that this perturbation is small, we can restrict ourselves to terms linear in δA. Then, using the features of the equilibrium distribution functions (Sec. II E), one obtains

$$
\begin{aligned}
-k^2Q_{\nu a}(k)= & igf_{abc}\int\frac{d^4l}{(2\pi)^4}l^\mu AQ_{\mu\nu bc}(l,k-l)+2igf_{abc}\int\frac{d^4l}{(2\pi)^4}l^\mu AQ_{\mu\nu bc}(k-l,l)\\
& -igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\nu AQ_{\mu bc}^\mu(k-l,l)+g^2N\int\frac{d^4l}{(2\pi)^4}\mathcal{A}_\mu^\mu(k-l)Q_{\nu a}(k)\\
& -g^2N\int\frac{d^4l}{(2\pi)^4}\mathcal{A}_\nu^\mu(k-l)Q_{\mu a}(k)+igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\nu\langle c_b^\dagger(l)c_c(k-l)\rangle\ ,
\end{aligned}
\tag{13}
$$

where $\langle\delta A_a^\mu\rangle=Q_a^\mu$ and $AQ_{\mu\nu ab}(k,p)=\langle A_{\mu a}(k)\delta A_{\nu b}(p)\rangle+\langle\delta A_{\mu a}(k)A_{\nu b}(p)\rangle$. In deriving this equation we have assumed that the equilibrium distribution functions $\langle A_aA_b\rangle$ are δ matrices in color indices. The functions $\langle c^\dagger c\rangle$ and AQ, occurring on the right-hand side of Eq. (13), are given by the equations

$$
\begin{aligned}
& k^2\langle c_b^\dagger(p)c_a(k)\rangle=igf_{abc}k_\mu\int\frac{d^4l}{(2\pi)^4}\langle c_b^\dagger(p)c_d(k-l)\rangle_RQ_c^\mu(l)\ ,\\
& p^2\langle c_b^\dagger(p)c_a(k)\rangle=-igf_{abc}\int\frac{d^4l}{(2\pi)^4}l_\mu\langle c_d^\dagger(l)c_a(k)\rangle_RQ_c^\mu(p-l)\ ,
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
k^2AQ_{\nu\sigma ah}(k,p)= & -igf_{ahc}[(p-k)_\lambda\mathcal{A}_{\nu\sigma}(p)+(2k+p)^\mu\mathcal{A}_{\mu\sigma}(p)g_{\nu\lambda}-(2p+k)_\nu\mathcal{A}_{\sigma\lambda}(p)]Q_c^\lambda(k+p)\\
& +g^2N\int\frac{d^4l}{(2\pi)^4}\mathcal{A}_\mu^\mu(k-l)AQ_{\nu\sigma ah}(k,p)-g^2N\int\frac{d^4l}{(2\pi)^4}\mathcal{A}_\nu^\mu(k-l)AQ_{\mu\sigma ah}(k,p)\\
& -g^2f_{abc}f_{cdh}\mathcal{A}_{\nu\sigma}(p)\int\frac{d^4l}{(2\pi)^4}AQ_{\mu bd}^\mu(k-l,p+l)-g^2f_{abc}f_{chd}\mathcal{A}_\sigma^\mu(p)\int\frac{d^4l}{(2\pi)^4}AQ_{\mu\nu bd}(k-l,p+l)\\
& -g^2f_{ahc}f_{cbd}\mathcal{A}_\sigma^\mu(p)\int\frac{d^4l}{(2\pi)^4}AQ_{\mu\nu bd}(k-l,p+l)\ ,
\end{aligned}
$$

$$p^2 AQ_{\nu\sigma ah}(k,p)=igf_{ahc}[(k-p)_\lambda \mathcal{A}_{\nu\sigma}(k)+(2p+k)^\mu \mathcal{A}_{\mu\nu}(k)g_{\sigma\lambda}-(2k+p)_\sigma \mathcal{A}_{\nu\lambda}(k)]Q_c^\lambda(k+p)$$
$$-g^2N\int \frac{d^4l}{(2\pi)^4}\mathcal{A}_\mu^\mu(p-l)AQ_{\nu\sigma ah}(k,p)+g^2N\int \frac{d^4l}{(2\pi)^4}\mathcal{A}_\sigma^\mu(p-l)AQ_{\nu\mu ah}(k,p)$$
$$+g^2f_{hbc}f_{cda}\mathcal{A}_{\nu\sigma}(k)\int \frac{d^4l}{(2\pi)^4}AQ_{\mu bd}^\mu(p-l,k+l)+g^2f_{hbc}f_{cad}\mathcal{A}_\nu^\mu(k)\int \frac{d^4l}{(2\pi)^4}AQ_{\mu\sigma bd}(p-l,k+l)$$
$$+g^2f_{hac}f_{cbd}\mathcal{A}_\nu^\mu(k)\int \frac{d^4l}{(2\pi)^4}AQ_{\mu\nu bd}(p-l,k+l)\ , \tag{15}$$

where $\langle c^\dagger c\rangle_R=\mathcal{C}$ are the equilibrium distribution functions for fields c and

$$\mathcal{A}_{\mu\nu}(k)=\int d^4x\, \exp(ikx)\mathcal{A}_{\mu\nu}(x)\ . \tag{15a}$$

It should be noticed that due to Eq. (8) the zero-order terms in Q [Eq. (15)] cancel. Thus, we have obtained the set of integral equations, which is complete, provided we know the equilibrium distribution functions $\mathcal{A}$ and $\mathcal{C}$.

One has also to find a method of solving Eq. (15). One method would be to approximate the solution perturbatively, i.e., expanding the solution in the power series of g. Then, the correlation function AQ has the form $AQ=g\sum_{n=0}^{\infty}(g^2)^n AQ^{(n)}$. One can evaluate $AQ^{(n+1)}$ by inserting $AQ^{(n)}$ and $\mathcal{A}_{\mu\nu}$, which are calculated in the order $(g^2)^n$, into the right-hand side of Eq. (15).

III. THE DISPERSION RELATION IN THE ORDER g^2

A. Equilibrium distribution functions

To obtain the dispersion relation in order g^2 from Eq. (13), one has to insert the equilibrium distribution functions for free fields into the right-hand of Eq. (13), as well as into Eqs. (14) and (15). In addition, one should keep only terms proportional to g in Eqs. (15).

The equilibrium distribution functions are defined as the statistical averages of two fields, A or c. Substituting for the field operator $A_a^\mu(x)$ in $\langle A_a^\mu(x)A_b^\nu(y)\rangle$ its expansion in the creation and annihilation operators,

$$A_a^\mu(x)=\frac{1}{(2\pi)^3}\int \frac{d^3k}{2\omega(\mathbf{k})}\sum_{s=0}^{3}[\epsilon^\mu(\mathbf{k},s)a_a(\mathbf{k},s)e^{-ikx}+\epsilon^{\mu*}(\mathbf{k},s)a_a^\dagger(\mathbf{k},s)e^{ikx}]\ ,$$

one obtains, in the Feynman gauge ($\alpha=1$), the free distribution function

$$\mathcal{A}_{\mu\nu}(k)=-\pi g_{\mu\nu}\delta(k^2)[n_b(|k_0|)+\Theta(-k_0)]\ ,$$

where $n_b(k)$ is the Bose-Einstein distribution function. Similarly, for the ghost distribution function, one finds

$$\mathcal{C}(k)=\pi\delta(k^2)[n_b(|k_0|)+\Theta(-k_0)]\ .$$

The ghosts are described by the boson distribution function[15] because they represent another way of writing the functional determinant depending only on the fields A.

B. Dispersion relation

Having defined the equilibrium distribution functions, one can solve Eqs. (14) and (15). We shall describe the procedure on the example of the equations for AQ. On the left-hand side of Eqs. (15) stands a singular operator (p^2 or k^2), acting on the function $AQ(k,p)$. Dividing the right-hand side of the first of Eqs. (15) by k^2, one obtains the solution for AQ up to the addition of an arbitrary solution of the homogeneous equation: $k^2AQ(k,p)=0$. This arbitrariness can be removed using the second of Eqs. (15). In this way, one can find the function AQ uniquely. However, there is still the problem how to avoid the zeros of k^2, which occur in the denominator. One has to turn around those singularities in such a way that the retarded response of field distributions to the small perturbation of the current appears in Eq. (13). We shall come back again to this question in Appendix A 1.

Finally, inserting expressions for AQ and $c^\dagger c$ into (13), one obtains the dispersion relation

$$k^2Q_\nu^a(k)-\Pi_{\nu\mu}(k)Q^{\mu a}(k)=0\ . \tag{16}$$

The polarization tensor Π, which occurs in this equation, is the correction of order g^2 to the zero-order dispersion relation: $k^2Q_\nu^a(k)=0$. This tensor has the form

$$\Pi_{\mu\nu}(k)=ig^2N\int \frac{d^4l}{(2\pi)^3}\frac{-4l_\mu l_\nu+k_\mu k_\nu-k^2g_{\mu\nu}-\frac{1}{2}l^2g_{\mu\nu}+2klg_{\mu\nu}-2l_\mu k_\nu-2l_\nu k_\mu}{(l+k)^2}\delta(l^2)[n_b(|l_0|)+\Theta(-l_0)] \tag{17}$$

and satisfies the transversality condition

$$k^\mu\Pi_{\mu\nu}=0\ , \tag{18}$$

on the mass shell ($k^2=0$). This property of the tensor cancels the quadratic divergences, which seem to occur in (17). If in Eq. (17) one takes the distributions for $T=0$, then, using Eq. (18), one can write

$$\Pi_{\mu\nu}(k)=(k^2g_{\mu\nu}-k_\mu k_\nu)\Pi_{\text{reg}}(k)\ ,$$

where $\Pi_{\text{reg}}(k)=\frac{1}{3}[\Pi_\mu^\mu(k)-\Pi_\mu^\mu(0)]$. The tensor for $T=0$ is called the "vacuum"-polarization tensor, because it describes an influence of the vacuum fluctuations on the propagation of the oscillation. We shall deal now with the part of the tensor, which is connected with the thermal excitation, i.e., with the "matter part" of the polarization tensor. For large temperatures this term will dominate in the dispersion relation (16). For finite temperatures, the tensor Π still satisfies the transversality condition (18), even though expressions for Π are not Lorentz covariant. Let us now separate the polarization tensor into longitudinal and transversal parts:

$$\Pi_{ij}=\Pi_T\left[\delta_{ij}-\frac{k_ik_j}{\mathbf{k}^2}\right]+\Pi_L\frac{k_ik_j}{\mathbf{k}^2}\ .$$

Using Eq. (18), one can write the identities

$$k_0\Pi_{00}=k_i\Pi_{i0},\quad k_0^2\Pi_{00}=\mathbf{k}^2\Pi_L\ .$$

The tensor Π has two eigenvalues. Consequently, one obtains two dispersion relations: One for the longitudinal oscillations

$$k_0^2-\mathbf{k}^2-\Pi_L(k)=0 \tag{19}$$

and another one for the transversal oscillations

$$k_0^2-\mathbf{k}^2-\Pi_T(k)=0\ , \tag{20}$$

where $\Pi_L=(k_0^2/\mathbf{k}^2)\Pi_{00}$ and $\Pi_T=\frac{1}{2}(\Pi_{ii}-\Pi_L)$.

C. Dispersion relation in the limit $\omega \gg |\mathbf{k}|$

After performing the integration on l_0 and the angles in Eq. (17) and leaving only the terms containing the distribution $n_b(l)$, one obtains

$$\Pi_L=\frac{-g^2Nk_0^2}{k^2}H_b\ , \tag{21}$$

$$\Pi_T=\frac{g^2N}{2}(H_b+G_b)-\frac{\Pi_L}{2}\ , \tag{22}$$

where

$$H_b=\int_0^\infty\frac{dl}{2\pi^2}\left[2l\left[1-\frac{k_0}{k}\ln\frac{k_+}{k_-}\right]+\frac{l^2}{k}L(l)+\frac{k_0^2}{4}L(l)+lk_0M(l)\right]n_b(l)\ ,$$

$$G_b=\int_0^\infty\frac{dl}{2\pi^2}\left[4l+\frac{5(k_0^2-k^2)}{k^2}L(l)\right]n_b(l)-\frac{\Pi_L}{2}\ ,$$

$$k=|\mathbf{k}|,\quad k_\pm=\frac{k_0\pm k}{2}\ ,$$

$$L(l)=\ln\frac{(l+k_+)(l-k_-)}{(l+k_-)(l-k_+)}\ ,$$

$$M(l)=\ln\frac{(l+k_+)(l-k_+)}{(l+k_-)(l-k_-)}\ .$$

These are precisely the expressions obtained in the perturbative calculations.[16] We shall calculate those integrals at large temperatures in Appendix A 1. Having the dispersion relations (19) and (20), one has to unfold them, i.e., to express k_0 by $|\mathbf{k}|$. We shall do this in the case when the real part of k_0 is much greater than its imaginary part ($k_0=\omega-i\gamma$, $\omega\gg\gamma$). In this case, if the dispersion relation has the form

$$D(k_0,|\mathbf{k}|)=0\ ,$$

then the oscillation frequency ω is the solution of the equation

$$\mathrm{Re}D(\omega,|\mathbf{k}|)=0\ ,$$

and the damping constant is given by

$$\gamma=\frac{\mathrm{Im}D(\omega,|\mathbf{k}|)}{\partial_\omega\mathrm{Re}D(\omega,|\mathbf{k}|)}\ .$$

In the limit $\omega\gg|\mathbf{k}|$, using those relations, one obtains

$$\omega^2=\mathbf{k}^2+\frac{g^2NT^2}{3}\sum_{n=1}^{\infty}\frac{1}{2n+1}\left[\frac{\mathbf{k}}{k_0}\right]^{2n-2}$$

for the longitudinal oscillations and

$$\omega^2=\mathbf{k}^2+\frac{g^2NT^2}{9}+\frac{g^2NT^2}{3}\sum_{n=1}^{\infty}\frac{1}{(2n+1)(2n+3)}\left[\frac{\mathbf{k}}{k_0}\right]^{2n}$$

for the transversal oscillations. Similarly, for the damping constant γ one derives

$$\gamma=-\left[\frac{g^2NT\omega}{4\pi}\sum_{n=0}^{\infty}\frac{2n+5}{(2n+1)(2n+3)}\left[\frac{\mathbf{k}}{k_0}\right]^{2n}\right]\times\frac{1}{\partial_\omega\mathrm{Re}D(\omega,|\mathbf{k}|)} \tag{23}$$

in the case of the longitudinal oscillations and

$$\gamma=-\frac{g^2NT\omega}{16\pi}\left[2+2\sum_{n=0}^{\infty}\frac{6n+7}{(2n+1)(2n+3)}\left[\frac{\mathbf{k}}{k_0}\right]^{2n}-3\frac{|\mathbf{k}|}{\omega}\sum_{n=0}^{\infty}\frac{1}{2n+1}\left[\frac{|\mathbf{k}|}{k_0}\right]^{2n+1}\right]\times\frac{1}{\partial_\omega\mathrm{Re}D(\omega,|\mathbf{k}|)}$$

for the transversal ones.

Following Weldon,[13] an existence of the solution of the dispersion relations (19) and (20) in the limit $T\gg\omega\gg|\mathbf{k}|$ suggests the existence of longitudinal "electric" and transverse "electric" and "magnetic" oscillations with the above-calculated frequencies. However, because of the negative sign of the damping constant γ, those oscillations would have the growing amplitude. This instability is due to the energy transfer from the heat bath into the unstable modes and leads eventually to the destruction of the assumed colored equilibrium state as defined in Secs. II E and III A. Hence, contrary to our earlier assumption, that state is not an equilibrium one. In the limit $|\mathbf{k}|\to 0$ the above relations reduce to

$$\omega_L=\omega_T=\omega_p=\frac{gT}{3}N^{1/2},\quad \gamma_L=\gamma_T=-\frac{5g^2NT}{24\pi}\ ;$$

i.e., we reproduce the results of Ref. 2.

The above result on γ puts doubts on the validity of the perturbative approximation for the equilibrium state. Clearly, the zero-order approximation of the distribution function leads to instability. This conclusion agrees with the results of Nadkarni,[5] which indicates the instability of the perturbative QCD vacuum. Additional arguments were given recently by Weiss[10] who noticed that QCD at $T \rightarrow \infty$ reduces to a three-dimensional gauge theory with an adjoint scalar field. Three-dimensional QCD is a confining theory and the characteristic scale of confinement is of order $\sim g^2T$. Hence, at this scale one may expect the nonperturbative confining properties in the QCD which will modify the equilibrium distribution. Further evidence comes from the presence of infrared divergences in the long-wavelength limit, at the scale $\sim g^2T$ (Ref. 17). According to DeTar, the free distribution function for gluons at this momentum scale is modified in the nontrivial way, even though for larger momenta it constitutes the good perturbative approximation.[17]

At this point one should discuss details of the calculation of ω and γ. Both quantities are given by the integrals of a function containing the factor n_b, i.e., the free boson distribution function. The real parts of the polarization functions Π_L and Π_T are given by integrals of the form

$$g^2N\int_0^{\infty} dl\, n_b(l)P_1(l,k_0,k) .$$

Consequently, any nonperturbative modification of the distribution function in the nonperturbative domain "$\sim g^2T$" would give a correction to the real part of the polarization tensor of the higher order g^4T^2. Hence the plasmon frequency ω would not change. However, those nonperturbative corrections could easily change the result for γ, which is given by an integral over a finite interval in the nonperturbative domain "$\sim g^2T$":

$$g^2N\int_{k_-}^{k_+} dl\, n_b(l)P_2(l,k_0,k) ,$$

where both k_+ and k_- are of the order g^2T (see also Appendix A 1).

IV. QUARKS

A. Lagrangian and the field equations

The introduction of the massless quarks into the theory changes the effective Lagrangian (1). The total Lagrangian now contains a new term

$$\mathcal{L}_q=\sum_{A=1}^{N_f}\left[\frac{i}{2}\overline{\Psi}_A(x)\gamma^{\mu}\partial_{\mu}\Psi_A(x)-\frac{i}{2}\partial_{\mu}\overline{\Psi}_A(x)\gamma^{\mu}\Psi_A(x)+\tfrac{1}{2}g\overline{\Psi}_A(x)\lambda_a\gamma_{\mu}\Psi_A(x)A_a^{\gamma}(x)\right],$$

where λ_a denotes the Gell-Mann matrices and N_f the number of quark flavors. The addition of the quarks to the theory also changes the field equations. A new term appears now on the right-hand side of Eq. (2), denoting the quark current:

$$-\frac{1}{2}g\sum_{A=1}^{N_f}\overline{\Psi}_A(x)\lambda_a\gamma_{\nu}\Psi_A(x) . \tag{24}$$

The new equations for the fields $\overline{\Psi}$ and Ψ are

$$\begin{aligned} &i\gamma^{\mu}\partial_{\mu}\Psi(x)=-g\tfrac{1}{2}\lambda_a\gamma_{\mu}\Psi(x)A_a^{\mu}(x) ,\\ &i\partial_{\mu}\overline{\Psi}(x)\gamma^{\mu}=g\tfrac{1}{2}\overline{\Psi}(x)\lambda_a\gamma_{\mu}A_a^{\mu}(x) . \end{aligned} \tag{25}$$

After taking the statistical average in the field equations, one obtains a new term on the right-hand side of Eq. (2), which results from the quark current (24):

$$\tfrac{1}{2}g\,\mathrm{Tr}\lambda_a\gamma_{\nu}\sum_{A=1}^{N_f}\langle\Psi_A(x)\overline{\Psi}_A(x)\rangle .$$

Hence, in order to close the BBGKY hierarchy, one should construct equations for the function $\langle\Psi(x)\overline{\Psi}(x)\rangle$.

B. BBGKY hierarchy equations for quarks

Transport equations for $\langle\Psi(x)\overline{\Psi}(y)\rangle$ can be derived from the field equations for quarks. Acting on $\langle\Psi(x)\overline{\Psi}(y)\rangle$ with $i\gamma_{\mu}\vec{\partial}^{\mu}{}_x$ to the right and with $i\gamma_{\mu}\overleftarrow{\partial}^{\mu}{}_y$ to the left, one obtains

$$i\gamma_{\mu}\vec{\partial}^{\mu}{}_x\langle\Psi(x)\overline{\Psi}(y)\rangle=-\tfrac{1}{2}\lambda_a\gamma_{\mu}\langle A_a^{\mu}(x)\Psi(x)\overline{\Psi}(y)\rangle , \tag{26}$$

$$i\langle\Psi(x)\overline{\Psi}(y)\rangle\gamma_{\mu}\overleftarrow{\partial}^{\mu}{}_y=\tfrac{1}{2}\lambda_a\langle\Psi(x)\overline{\Psi}(y)A_a^{\mu}(y)\rangle\gamma_{\mu} . \tag{27}$$

On the right-hand side of those equations one finds unknown averages of three fields. The evolution equations for those averages contain unknown averages of still higher order. This procedure gives rise to a new class of equations within the BBGKY hierarchy. Equations (26) and (27) are similar to Eqs. (7) for ghost fields. We shall also use the Vlasov approximation for statistical averages containing fields A and Ψ:

$$\langle A\Psi\overline{\Psi}\rangle=\langle\Psi\overline{\Psi}A\rangle=\langle\Psi\overline{\Psi}\rangle\langle A\rangle .$$

In this approximation, Eqs. (26) and (27) take the form

$$i\gamma_{\mu}\partial_x^{\mu}\langle\Psi(x)\overline{\Psi}(y)\rangle=-\tfrac{1}{2}\lambda_a\gamma_{\mu}\langle\Psi(x)\overline{\Psi}(y)\rangle\langle A_a^{\mu}(x)\rangle , \tag{28}$$

$$i\langle\Psi(x)\overline{\Psi}(y)\rangle\gamma_{\mu}\overleftarrow{\partial}_y^{\mu}=\tfrac{1}{2}\lambda_a\langle\Psi(x)\overline{\Psi}(y)\rangle\gamma_{\mu}\langle A_a^{\mu}(y)\rangle \tag{29}$$

Let us now assume that in equilibrium the structure of $\langle\Psi(x)\overline{\Psi}(y)\rangle$ in color indices is proportional to the unit matrix of dimension $N\times N$. Then, the quark current, as given by Eq. (24) and as occurs on the right-hand side of Eq. (2) is equal zero. Hence, an introduction of quarks does not change conclusions of Sec. II E, i.e., the transport equations for the statistical averages: $\langle A\rangle$, $\langle c^{\dagger}c\rangle$, and $\langle AA\rangle$ remain unchanged. Consequently, also the results of Sec. II F, for the near-equilibrium behavior of hot gluons do not change.

C. Dispersion relation

Let us apply a small perturbative δA to the equilibrium field distributions and look how the quark current [Eq. (24)], modifies propagation of the perturbation. Similarly as for the ghost fields, one can write down equations for the function $\langle\Psi\overline{\Psi}\rangle$ in the approximation of small $Q=\langle\delta A\rangle$, i.e., leaving only terms linear in Q. Hence, Eqs. (28) and (29) take the form

$$i\gamma_{\mu}\partial_x^{\mu}\langle\Psi(x)\overline{\Psi}(y)\rangle=-\tfrac{1}{2}\lambda_a\gamma_{\mu}\langle\Psi(x)\overline{\Psi}(y)\rangle_R\langle Q_a^{\mu}(x)\rangle ,$$

$$i\langle\Psi(x)\overline{\Psi}(y)\rangle\gamma_\mu\overleftarrow{\partial}_y^\mu=\tfrac{1}{2}\lambda_a\langle\Psi(x)\overline{\Psi}(y)\rangle_R\gamma_\mu\langle Q_a^\mu(y)\rangle ,$$

where $\langle\Psi\overline{\Psi}\rangle_R$ denotes the equilibrium distribution function for the quark fields. In order to find the dispersion relation to order g^2, it is enough to take for $\langle\Psi\overline{\Psi}\rangle_R$ the distribution function for the free quark fields.

Because of the translational invariance of the system in equilibrium, the distribution function $\langle\overline{\Psi}(y)\Psi(x)\rangle$ depends only on the difference $x-y$. The calculation of the equilibrium distribution function for quarks is more complicated than for gluons, because the function $\langle\overline{\Psi}\Psi(x-y)\rangle_R$ possess the structure of the $N\times N$ matrix in color indices and the 4×4 matrix in spinor indices. The structure in color indices was discussed in Sec. IV B. The structure in the spinor indices can be found from Eqs. (26) and (27) with $g=0$. In momentum space they take the form

$$\gamma_\mu p^\mu\langle\overline{\Psi}\Psi(p)\rangle_R=0,\quad \langle\overline{\Psi}\Psi(p)\rangle_R\gamma_\mu p^\mu=0 .$$

As seen from the above equations, the equilibrium distribution function for free quarks has the structure

$$\langle\overline{\Psi}\Psi(k)\rangle_R=\gamma_\mu k^\mu f(k) ,$$

where $f(k)$ is a function containing the factor $\delta(k^2)$. One can calculate this function, by inserting into $\langle\mathrm{Tr}\overline{\Psi}(y)\Psi(x)\rangle_R$ the expansion of Ψ and $\overline{\Psi}$ in the creation and annihilation operators. Finally, performing the Fourier transformation, one obtains

$$\langle\overline{\Psi}\Psi(k)\rangle=\gamma_\mu p^\mu[n_f(|k_0-\mu|)+\Theta(-k_0)]\delta(k^2) , \tag{30}$$

where n_f denotes the Fermi-Dirac distribution function.

The dispersion relation (16) for the above equilibrium distribution function (30) can be obtained following the reasoning of Sec. III B. However, the polarization tensor Π is now a sum of the two terms—the gluon term, given by Eq. (17) and the quark term Π^q:

$$\Pi^q_{\mu\nu}(k)=-4ig^2N_f\int\frac{d^4l}{(2\pi)^3}\frac{(k+l)_\mu l_\nu+(k+l)_\nu l_\mu-(k+l)lg_{\mu\nu}}{(k+l)^2}[n_f(|l_0-\mu|)+\Theta(-l_0)]\delta(l^2) . \tag{31}$$

For $T\gg\mu$, this tensor has the same form as the polarization tensor calculated to order g^2 by Weldon.[16] The tensor Π^q is transverse, which assures the cancellation of quadratic divergences. The logarithmic divergences must be included in the renormalization of the factor g^2. The terms containing $\Theta(-l_0)$ in Eq. (31) can be removed, because we are interested only in the dominant terms at high temperatures.

D. Dispersion relation in the limit $\omega\gg|\mathbf{k}|$

We separate the longitudinal and the transverse parts of the tensor Π^q, similarly as we did for gluons (Sec. II B). The longitudinal part can be written in the form

$$\Pi^q_L=-\frac{g^2N_fk_0}{k^2}H_f$$

and the transversal part has the form

$$\Pi^q_T=-\frac{g^2N_fk_0}{2}(H_f+G_f)-\frac{\Pi^q_L}{2} ,$$

where

$$H_f=\int_0^\infty\frac{dl}{4\pi^2}\left[\left[2l+\frac{4l^2-k_0^2-k^2}{4k}L(l)\right][n_f(l-\mu)+n_f(l+\nu)]+\frac{k_0^2-2lk_0}{k}\ln\frac{(l-k_-)k_+}{(l-k_+)k_-}n_f(l-\mu)\right.$$
$$\left.+\frac{k_0^2+2lk_0}{k}\ln\frac{(l+k_+)k_-}{(l+k_-)k_+}n_f(1+\mu)\right] ,$$

$$G_F=\int_0^\infty\frac{dl}{4\pi^2}\left[2\ln[n_f(l-\mu)+n_f(l+\mu)]+\frac{k_0^2-k^2}{k}\left[\ln\frac{(l-k_-)k_+}{(l-k_+)k_-}n_f(l-\mu)+\ln\frac{(l+k_+)k_-}{(l+k_-)k_+}n_f(l+\mu)\right]\right] .$$

We now study the large temperature limit $T\gg\mu,k_0,|\mathbf{k}|$. One can calculate the real and imaginary parts of the frequency similarly as for the gluons alone. It turns out that the addition of quarks changes only the multiplication factor in the real part of the dispersion relation. Hence, in the expressions for ω (see Sec. III C) one has to change the factor g^2N into $g^2(N+N_f/2)$. In the imaginary part of the dispersion relation the quark term appears with a lower power of the temperature $[\mathrm{Im}\Pi^q=O_T(1)]$. Consequently, the expressions for γ, as written down in Sec. III C remain unchanged. Also the conclusion about the instability of the quark-gluon plasma remain unchanged.

One can also study the system in another part of the diagram μ-T. Suppose that the system is in the region of large baryon densities: i.e., $\mu\gg T\gg k_0,|\mathbf{k}|$. The real part of the dispersion relation changes and for the longitudinal oscillations one obtains

$$\omega^2 = \frac{\mathbf{k}^2}{1+\frac{7g^2 N_f \mu^2}{12\pi^2 \mathbf{k}^2}} . \tag{32}$$

Similarly, for the transversal oscillations one finds

$$\omega^2 = \frac{\mathbf{k}^2 + \frac{g^2 N_f \mu^2}{24\pi^2}}{1-\frac{7g^2 N_f \mu^2}{24\pi^2 k^2}} .$$

On the contrary, the imaginary part of the dispersion relation does not change. As one can see from the above expressions, there are no transverse oscillations in the long-wavelength limit, which would be consistent with the condition $\omega \gg \gamma$. However, there are still the unstable, longitudinal modes with the frequencies given by Eq. (32) and with the negative damping constant given by Eq. (23).

V. CONCLUSION

In this paper we have derived in an alternative way the dispersion relation for gluons and quarks in the covariant Lorentz gauge. The equations of the BBGKY hierarchy have been derived for the functions $\langle A \rangle$, $\langle AA \rangle$, $\langle c^\dagger c \rangle$, $\langle \Psi\overline{\Psi} \rangle, \ldots$, which are not gauge covariant. The main argument for constructing equations in this way is their simplicity. The analogous equations for the gauge-covariant functions[6] are much more complicated and, thus, already the equation for

$$\Gamma_{\mu\nu}\left[x-\frac{y}{2},x+\frac{y}{2}\right] = \langle \{\exp[\tfrac{1}{2}y\mathcal{D}(x)]F^{\lambda}_{\mu}(x)\} \otimes \{\exp[-\tfrac{1}{2}y\mathcal{D}(x)]F_{\lambda\nu}(x)\}$$

contains the averages of fields A to all orders, in addition to the covariant quantities $F_{\mu\nu}$. Hence, one cannot directly close the equations of such a theory using only the covariant averages. The same problems arise when dealing with the covariant equation for quarks. On the other hand, the alternative approach based on gauge-dependent averages is simpler and contains essentially the same physical information. From the gauge-dependent averages one can, using their moments, calculate physical quantities such as the currents and the energy-momentum tensor.[7] Recently, Elze[18] has proposed an alternative approach which emphasizes the gauge covariance and employs a simpler Wigner function than used in Ref. 2. In principle, in his approach one may hope to go beyond the classical approximation but, unfortunately, no results concerning dispersion relation have been derived as yet.

In this paper we have constructed the BBKGY hierarchy for gluons and quarks. In order to close those equations, we have used the well-known mean-field approximation; i.e., we have neglected the correlation functions for three and more gluon fields, as well as for the coupling of gluons with quarks and ghosts. Within this approximation, we have obtained a well-defined, self-consistent set of equations. One should notice that those equations do not result from the perturbation approximation in powers of g.

We have used the derived transport equations for the calculation of the dispersion relation in the order g^2 for gluons (Sec. III B) and quarks (Sec. IV C). When solving the dispersion relation, we have explicitly chosen Bose-Einstein distribution function for gluons and ghosts and Fermi distribution function for quarks; i.e., the equilibrium distribution functions for the noninteracting gas of quarks and gluons. Calculated in this way, the frequency and the damping constant of the long-wavelength excitations of the plasma agree with the g^2 perturbative calculation.[2] Hence, contrary to our earlier assumption, the equilibrium of the quark-gluon plasma cannot be approximated by the state of noninteracting hot gluons and quarks. The perturbative approximation to the equilibrium distribution of quarks and gluons is inconsistent with the kinetic equation in the mean-field limit. This introduces an additional difficulty in the calculation of the plasmon decay rate on the top of the known difficulty with the gauge dependence of the response function. In this context application of the above proposed method is advantageous because, contrary to the method of the imaginary-time finite-temperature QCD (Refs. 1–3), one can easily generalize the relativistic transport equations and dispersion relations also for the case of other distributions.

It has been suggested that the interacting gluons, even at high temperature, may take exotic forms of ordered condensates of color-singlet states.[19,13] If this conjecture is true then the description of the quark-gluon plasma solely in terms of quark and gluonic excitations is inadequate in the infrared region. Consequently the equilibrium distribution functions should be modified and the relevant quantities should be looked for nonperturbatively.

Actually, the transitions in various regions of $(k_0,\mathbf{k})$-space between different possible regimes in the condensate remain unknown. Recently, DeTar and Polonyi[17,13] have suggested nonperturbative effects in the long-wavelength limit, i.e., at the scale $\sim g^2 T$, due to the existence of the composite color-neutral objects. According to this idea, the free distribution function at this momentum scale is modified in the nontrivial way, even though, for large momenta it constitutes the good perturbative approximation. As discussed in Sec. III C, the real part of the polarization tensor is given by an integral of the distribution function from 0 to ∞. Hence, in the leading order the corrections to the distribution function in the finite nonperturbative domain "$\sim g^2 T$" do not change the plasmon frequency ω. On the contrary, the imaginary part of the polarization tensor, which determines the damping constant is given by an integral in the domain "$\sim g^2T$" and is very sensitive to such nonperturbative infrared corrections.

Hence, we believe that no quantitative conclusions about the sign and magnitude of the plasmon decay rate can be made before the nonperturbative structure of the equilibrium distribution function is understood and the nonperturbative calculation of the damping rate from the mean-field equations is done. Solving this problem would largely help in understanding puzzling features of the collective behavior of the quark-gluon plasma using the finite-temperature QCD.

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APPENDIX

1. Calculation of the integrals for large temperatures

In Secs. III C and IV D we have obtained integrals containing the distribution functions n_b and n_f. Now we shall work on the large temperatures expansion of those expressions. In this limit, i.e., for $T \gg \mu$, we can assume $n_f(l-\mu)=n_f(l+\mu)=n_f(l)$. The real parts of those integrals were calculated in Ref. 16. In order to obtain the temperature expansion of such expressions one has to expand the distribution function as

$$n_i(l)=\sum_{m=1}^{\infty} \sigma_i(m) e^{-m\beta l} ,$$

where $\sigma_f(m)=(-1)^{m+1}$, $\sigma_b(m)=1$.

Finally, for $k_0>k$, one obtains the leading-order terms

$$\mathrm{Re}H_b=\frac{T^2}{3}\left[1-\frac{k_0}{2k}\ln\frac{k_+}{k_-}\right]+\text{const} ,$$

$$\mathrm{Re}H_f=\frac{T^2}{6}\left[1-\frac{k_0}{2k}\ln\frac{k_+}{k_-}\right]+\text{const} ,$$

$$\mathrm{Re}G_b=\frac{T^2}{3}+\text{const} ,$$

$$\mathrm{Re}G_f=\frac{T^2}{6}+\text{const} .$$

In order to calculate the imaginary part of those expressions one should make an analytical continuation to avoid the cuts at the real axis. In order to obtain the correct expressions from the point of view of causality,[20] one has to continue the frequencies into the upper half plane ($k_0=k_0+i\epsilon$). This is connected with the prescription of avoiding the singularities $1/p^2$, that we have discussed in Sec. III B.

The singular expressions in the above integrals take the form

$$\ln\frac{l-k_-}{l-k_+}=\ln\frac{|l-k_-|}{|l-k_+|}+\pi i .$$

A simple calculation shows that

$$\mathrm{Im}H_b=\int_{k_-}^{k_+}\frac{dl}{8\pi k}[(2l-k_0)^2-2k^2]n_b(l)$$

$$\simeq\frac{T}{4\pi}\left[\frac{k_0}{2k}\ln\frac{k_+}{k_-}-1\right]+\frac{kT}{4\pi}\ln\frac{k_+}{k_-}+\text{const} ,$$

$$\mathrm{Im}G_b=\int_{k_-}^{k_+}dl\frac{5(k_0^2-k^2)T}{8\pi k^2}n_b(l)$$

$$\simeq\frac{5(k_0^2-k^2)T}{k^2}\ln\frac{k_+}{k_-}+\text{const} ,$$

$$\mathrm{Im}H_f\sim 1,\quad \mathrm{Im}G_f\sim 1 .$$

2. Calculation of the integrals in the limit $\mu \gg T$

For fermionic integrals in the limit $\mu \gg T$ one has to use the Sommerfeld expansion. Let us discuss an expression of the form

$$K=\int_0^{\infty}f(l)\frac{dl}{\exp[\beta(l-\mu)]+1} .$$

Let us now change the variables $x=\beta l$,

$$K=T\int_0^{\infty}f(Tx)\frac{dx}{\exp(x-\beta\mu)+1} ,$$

and perform the integration by parts:

$$K=\int_0^{\infty}\frac{F(Tx)\exp(x-\beta\mu)}{[\exp(x-\beta\mu)+1]^2}dx ,$$

where F is the primitive of f. Changing again the variables $y=x-\beta\mu$ one obtains

$$K=\int_{-\beta\mu}^{\infty}\frac{F(T(y+\beta\mu))e^y}{(e^y+1)^2}dy ,$$

i.e.,

$$K=\int_{-\infty}^{\infty}\frac{F(T(y+\beta\mu))e^y}{(e^y+1)^2}dy+O(e^{-\beta\mu}) .$$

If in the above expression we make an expansion of the function $F(T(y+\beta\mu))$ around $y=0$, then we obtain the leading powers of $\beta\mu$ in the expansion of K. Using this procedure for the integrals obtained in Sec. IV D we obtain

$$\mathrm{Re}H_f=\frac{7\mu^2}{12\pi^2}+\text{const},\quad \mathrm{Re}G_f=\frac{\mu^2}{2\pi^2}+\text{const} .$$

In the leading terms of the expansion,

$$\mathrm{Im}H_f=\frac{k_0^2}{16\pi}=O_T(1),\quad \mathrm{Im}G_F=O(e^{-\beta\mu}) .$$

μ does not occur in the imaginary part of those integrals. Hence, one can neglect the imaginary part of the quark term also in this area of the μ-T diagram.

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