Universe decay and changing the cosmological constant

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If superspace time is taken to be $\sqrt{\det(\gamma)}$, where γ_{ij} is the three-metric, then the cosmological constant plays the role of a (mass)^2 term in the Wheeler-DeWitt equation. Second quantization allows particles of different mass to decay into one another if they are interacting. Likewise, adding interactions to the third-quantized Lagrangian allows universes with different cosmological constants to decay into one another. We consider a simple model with two types of universes, one of cosmological constant zero and the other nonzero. We compute the probability of decay and production for each type of universe.

I. INTRODUCTION

Following the pioneering work of DeWitt, Misner, Kuchar, Ryan, and others[1–9] third quantization was introduced as a method for dealing with topology-changing processes in quantum gravity. Recently there has been a revival of interest in third quantization.[10–19] This has been motivated in part by the fact that third quantization is deemed necessary in the two-dimensional gravity used in string theory as well as giving a possible solution to certain negative-probability problems which plagued the "Schrödinger-Klein-Gordon" approach to quantum cosmology. Also, universe creation can be shown to occur in third quantization,[15–19] a process analogous to particle creation in second quantization. Here one is led to introduce an object which cannot even be conceived of using second quantization, that of a no-universe state or third-quantized vacuum. In this paper we study a process complementary to universe creation, that of universe decay. A direct consequence of universe decay is that the Universe propagating in an initial state need not be the same type of universe that is propagating in the final-state configuration. In particular, the cosmological constants associated with the initial- and final-state universes need not be the same. This provides us with a mechanism for changing the value of the cosmological constant while keeping its value fixed during any single universe propagation (so that the cosmological constant really is a constant). We will show that universe decay in third quantization is analogous to particle decay in second quantization with the cosmological constant playing the same role as the (mass)^2 of an elementary particle. The analogy seems to suggest that if universe decay is possible then universes with $\Lambda\neq0$ will eventually decay into universes with $\Lambda=0$. After all, massless particles eventually dominate the final states of particle decay in second quantization. We shall show, however, that one must also allow for the inverse process in which a universe with $\Lambda=0$ decays back into universes with $\Lambda\neq0$. The background superspace metric which drives universe creation also makes it possible for universes with $\Lambda=0$ to decay (massless particles do not decay in flat space). One should keep in mind that the proper inclusion of inhomogeneity could substantially alter this result.

This paper is organized as follows. In Sec. II we discuss the role of the cosmological-constant term in the world-volume action. It is shown to play the same role as a (mass)^2 term in the world-line action of a relativistic particle. Upon quantization the correspondence is maintained as the Wheeler-DeWitt equation becomes identified with a Klein-Gordon equation in a background field with (mass)^2 term Λ. In Sec. III we construct an effective three-universe interaction by a second-quantized path integral. This has the effect of adding an interaction term to a third-quantized Lagrangian. The interacting term gives rise to the phenomena of universe decay in which one type of universe can decay into another. We investigate universe decay for the simple case of two interacting universes, one with $\Lambda=0$ and the other with $\Lambda\neq0$. Because of the background superspace metric, universe production is also possible. Owing to the asymmetry of this superspace metric under inversion of conformal superspace time, the decay probabilities are different for universe decay and production and we compute the difference. The distinction between (in) and (out) third-quantized vacuums complicates the computation of the decay probability but can be handled if a suitable cutoff in superspace volume is present. In Appendix A we discuss the fact that for the special case when two of the universes in a three-universe overlap are identical the universal interaction can be implemented by the insertion of a four-dimensional vertex operator on the world volume. In Appendix B we derive the phase space available in universe decay. In Appendix C we consider a model in which a single universe with $\Lambda=0$ decays into two identical universes with $\Lambda\neq0$.

II. THE COSMOLOGICAL CONSTANT AS A (MASS)^2 TERM IN THE WHEELER-DEWITT EQUATION

In Refs. 13 and 18 it was noted that if the Wheeler-DeWitt equation is taken to be analogous to the Klein-Gordon equation then the cosmological constant plays a role identical to a (mass)^2 term for the Klein-Gordon wave function. In this section we wish to explore this analogy a bit further. First, consider the Einstein-Hilbert action with a particular ansatz for the metric corresponding to the given cosmological problem. We shall take this ansatz to be

$$ds^2=-c^2(t)dt^2+a^2(t)[dx_1^2(e^{4\beta'_+ +2\beta'_-}) +dx_2^2(e^{-2\beta'_+ +2\beta'_-}) +dx_3^2(e^{-2\beta'_+ -4\beta'_-})] . \tag{2.1}$$

In the above, $a^3(t)$ denotes the volume of the universe at time t, $\beta'_+(t)$ and $\beta'_-(t)$ are anisotropy parameters which describe its shape. If, for example, we consider a toroidal universe as described by a box with opposite sides identified of length $R_1(t), R_2(t), R_3(t)$ then $a^3 = R_1R_2R_3$, $e^{3\beta'_+}=R_1/R_2$, and $e^{3\beta'_-}=R_2/R_3$. Also $c(t)$ will be seen to be a Lagrange multiplier field (it has no dynamics). Then the Einstein-Hilbert action

$$S=\frac{1}{2\kappa}\int d^4x\sqrt{-g}\,({}^4R-2\Lambda'-2\kappa g_{\mu\nu}T^{\mu\nu}) \tag{2.2}$$

reduces under the above ansatz to

$$S=\frac{1}{2}\frac{1}{l^2}\int dt\, ca^3\left[-\frac{1}{c^2}\left[\frac{\dot a}{a}\right]^2+\frac{1}{9}\frac{1}{c^2}(\dot\beta_+^2+\dot\beta_-^2) +\frac{1}{9}\frac{1}{c^2}(l^2\dot\phi_i^2)-\left[\frac{l^2\rho}{a^4}+\frac{l^2M}{a^2}+9\Lambda\right]\right] , \tag{2.3}$$

where $l^2=\kappa/6$, $\Lambda=3^{-3}\Lambda'$, $\beta_+=3(\frac{1}{2}\beta'_4+\beta'_-)$, and $\beta_-=(3\sqrt{3}/2)\beta'_+$. We have included $(n-3)$ scalar fields ϕ_i so that we have an n-dimensional minisuperspace. The last three terms in S denote, respectively, the matter, radiation, and vacuum contributions to the energy density. In the following we shall keep only the vacuum contribution. We shall find it convenient to define $x=a^3/l^2$ and $g=(cx)^2$. Then the action reduces to

$$S=\tfrac{1}{2}\int dt\sqrt{g}\,[-g^{-1}\dot x^2+g^{-1}x^2(\dot\beta_+^2+\dot\beta_-^2+l^2\dot\phi_i^2)-\Lambda] . \tag{2.4}$$

We define $K_\pm=(1/l)(1/\sqrt{g})x^2\dot\beta_\pm$, $K_i=(1/\sqrt{g})x^2\dot\phi_i$, and $K=(K_+,K_-,K_i)$. Solving the classical equation for g we find the relation

$$g=\frac{\dot x^2-x^2(\dot\beta^2+l^2\dot\phi^2)}{\Lambda}=\frac{x^2\dot x^2}{K^2l^2+\Lambda x^2} . \tag{2.5}$$

And using the definition of K we obtain

$$\frac{d}{dx}\beta_\pm(x)=\frac{1}{x}\frac{lK_\pm}{x}\frac{1}{\left[\frac{K^2}{x^2}l^2+\Lambda\right]^{1/2}}\le\frac{1}{x} . \tag{2.6}$$

Thus this model has a limit for the velocity of the Universe point through superspace. This is the analog of the condition in particle theory:

$$\frac{dx^i}{dx^0}=p^i\frac{1}{\sqrt{p^2+m^2}}\le 1 . \tag{2.7}$$

From (2.4) we see that the action is equivalent to that of a relativistic particle of $(\text{mass})^2=\Lambda$ in a background with metric G_{IJ} derived from the line element

$$\delta s^2=-dx^2+x^2(d\beta_+^2+d\beta_-^2+l^2d\phi_id\phi_i) . \tag{2.8}$$

This is actually the background superspace metric restricted to minisuperspace. Solving (2.6) we find

$$\beta_\pm(x)=\frac{K_+}{|K|}\left[\ln(x/x_0)-\ln\left[\frac{\sqrt{K^2l^2+\Lambda x^2}+|K|l}{\sqrt{K^2l^2+\Lambda x_0^2}+|K|l}\right]\right] . \tag{2.9}$$

For $x \ll |K|l/\sqrt{\Lambda}$ we have

$$\beta_\pm(x)=\frac{K_\pm}{|K|}\ln(x/x_0)+\frac{K_\pm}{|K|}\frac{\Lambda x_0^2}{4K^2l^2}+\cdots \tag{2.10}$$

and for small x the trajectory of the Universe point (β,ϕ) through superspace is identical to that of a universe with a zero cosmological constant.

The quantization of this model is implemented by replacing the classical constraint $-p_x^2+K^2l^2/x^2+\Lambda=0$ by the Wheeler-DeWitt (WDW) equation. If we define $K=(K_+,K_-,K_{\phi i})$ where $K_\pm$ and $K_{\phi i}$ are eigenvalues of $-i(\partial/l\partial\beta_\pm)$ and $-i(1/l^2)(\partial/\partial\phi_i)$ the WDW equation reduces to

$$\frac{1}{(x/l)^{n-1}}\frac{d}{dx}\left[(x/l)^{n-1}\frac{d}{dx}\psi\right] +\left[\Lambda+\frac{1}{(x/l)^2}(K^2)+\xi\frac{(n-1)(n-2)}{x^2}\right]\psi=0 . \tag{2.11}$$

A modification of the WDW equation by a term proportional to the superspace curvature $(n-1)(n-2)/x^2$ has been included in (2.11). We shall normalize the solutions to (2.11) as in the "Schrödinger-Klein-Gordon" approach to quantum cosmology with the inner product[1,20]

$$(\psi_1,\psi_2)=-i\int d^2\beta\, d^{n-3}\phi\sqrt{-G}\,(\psi_1^*\overleftrightarrow{\partial_x}\psi_2) . \tag{2.12}$$

The WKB solution to (2.11) is given by

$$\psi_K^{\rm WKB}=V^{-1/2}\exp[i(k_+\beta_++k_-\beta_-+k_{\phi j}\phi_j)] \times(x/l)^{(2-n)/2}\chi_K^{\rm WKB} , \tag{2.13}$$

where V is a suitable superspace volume cutoff and

$$\chi_K^{\rm WKB}(x)=\left[2\frac{x}{l}\left\{\frac{l^2}{x^2}\left[K^2+l^{-2}\left[\xi(n-1)(n-2)-\frac{(n-2)^2}{4}\right]\right]+\Lambda\right\}^{1/2}\right]^{-1/2} \times\exp\left[-i\int_{x_0}^{x}dx'\left\{\frac{l^2}{x'^2}\left[K^2+l^{-2}\left[\xi(n-1)(n-2)-\frac{(n-2)^2}{4}\right]\right]+\Lambda\right\}^{1/2}\right] . \tag{2.14}$$

Defining the conformal superspace time η through $x=x_0e^{\eta/l}$ we have

$$\chi_K^{\mathrm{WKB}}(\eta)=\left[2\left[K^2+l^{-2}\left[\xi(n-1)(n-2)-\frac{(n-2)^2}{4}\right]+\Lambda x_0^2l^{-2}e^{2\eta/l}\right]^{1/2}\right]^{-1/2}$$
$$\times\exp\left[-i\int_0^\eta d\eta'\left[K^2+l^{-2}\left[\xi(n-1)(n-2)-\frac{(n-2)^2}{4}\right]+\Lambda x_0^2l^{-2}e^{2\eta'/l}\right]^{1/2}\right]. \quad (2.15)$$

The parameter ξ arises from different choices of measure in the definition of the path integral. The ambiguity is the same as that present in the path integral of a particle in a background field.[21,22] The usual choice $\xi=0$ gives rise to possible instabilities for small superspace momentum K. For $\xi\neq0$ there will be superspace curvature terms present in the WDW equation. The choice $\xi=\frac{1}{4}(n-2)/(n-1)$ is the simplest and amounts to conformal coupling of the universal field to the superspace curvature. In the following we shall choose conformal coupling but other choices can be handled by replacing $|K|$ by

$$\left[K^2+l^{-2}\left[\xi(n-1)(n-2)-\frac{(n-2)^2}{4}\right]\right]^{1/2}$$

in the definition of $\chi(x)$. The choice of (out) wave functions used is determined by requiring that they match the WKB solution for $x\gg\Lambda^{-1/2}$. The choice of (in) wave functions is determined by requiring that for $x\ll\Lambda^{-1/2}$ they match the solution to (2.11) with $\Lambda=0$. This is in accordance with our classical result (2.10). For conformal coupling we obtain

$$\psi_K^{(\mathrm{out})}=\psi_K=V^{-1/2}\exp[i(k_+\beta_++k_-\beta_-+k_{\phi j}\phi_j)]\times(x/l)^{(2-n)/2}\chi_K\,,$$
$$\psi_K^{(\mathrm{in})}=\bar\psi_K=V^{-1/2}\exp[i(k_+\beta_++k_-\beta_-+k_{\phi j}\phi_j)]\times(x/l)^{(2-n)/2}\bar\chi_K\,, \quad (2.16)$$

where

$$\chi_K=(\pi/4)^{1/2}e^{\pi|K|l/2}H^{(2)}_{i|K|l}(x\sqrt{\Lambda})l^{1/2}\,,$$
$$\bar\chi_K=(\pi/2)^{1/2}(\sinh\pi|K|l)^{-1/2}J_{-i|K|l}(x\sqrt{\Lambda})l^{1/2}\,, \quad (2.17)$$
$$\chi_K^0(x)=\frac{1}{\sqrt{2|K|}}\left[\frac{x}{x_0}\right]^{-i|K|l}=\frac{1}{\sqrt{2|K|}}e^{-i|K|\eta}\,.$$

Here $H_\nu^{(2)}(z)$ and $J_\nu(z)$ are Hankel and Bessel functions. The wave function ψ_K^0 is used if the cosmological-constant term is zero. When $\Lambda\neq0$ the (in) and (out) third-quantized vacuums $|^3\bar0\rangle$ and $|^30\rangle$ are not equivalent and we define $\alpha_K=(\bar\psi_K,\psi_K)$ and $\beta_K=-(\bar\psi_K,\psi_K^*)$ (Ref. 23). For our model we have

$$\alpha_K=\left[\frac{e^{\pi|K|l}}{2\sinh|K|\pi l}\right]^{1/2}$$

and (2.18)

$$\beta_K=\left[\frac{e^{-\pi|K|l}}{2\sinh|K|\pi l}\right]^{1/2}.$$

III. UNIVERSE DECAY IN THIRD QUANTIZATION

In this section we extend the discussion of Refs. 10 and 13 on universe decay by using the Hankel-function representation of the one-universe wave functions as opposed to the more general WKB wave function. We shall also see that the distinction between (in) and (out) third-quantized vacua introduces new terms in the decay probability which must be included. A disturbance in superspace is in general due to a combination of the background superspace metric and an explicit third-quantized interaction. It is these disturbances which change the universe number in third quantization. The usual gravitational path integral leads upon second quantization to the WDW equation. Forming a universal Lagrangian which reproduces this equation upon variation and performing a quantization using this Lagrangian one achieves a third-quantized theory of noninteracting universes. We shall consider the possibility of achieving a third-quantized theory of interacting universes by considering the second-quantized path integral. In this language the general expression for the universal propagator may be written

$$K(x,\beta_+,\beta_-,\phi_i;x',\beta'_+,\beta'_-,\phi'_i)=\int_0^\infty dW\int_{\{x(0)=x,\beta_\pm(0)=\beta_\pm,\phi_i(0)=\phi_i\}}^{\{x(W)=x',\beta_\pm(W)=\beta'_\pm,\phi_i(W)=\phi'_i\}}Dg\,Dx\,D\beta_+D\beta_-D\phi_i\times\prod_t\delta(g(t)-1)\times\exp\left[\frac{i}{2}\int_0^W dt\sqrt{g}\,\{g^{-1}[-\dot x^2+x^2(\dot\beta_+^2+\dot\beta_-^2+l^2\dot\phi_i^2)]-\Lambda\}\right]. \quad (3.1)$$

We shall find it convenient to define

$$\langle x',\beta'_\pm,\phi'_i,\tau'|x,\beta_\pm,\phi_i,\tau\rangle_\Lambda=\int_{\{x(\tau)=x,\beta_\pm(\tau)=\beta_\pm,\phi_i(\tau)=\phi_i\}}^{\{x(\tau')=x',\beta'_\pm(\tau')=\beta'_\pm,\phi_i(\tau')=\phi'_i\}} Dg\int Dx\int D\beta_+ D\beta_- D\phi_i \prod_t \delta(g(t)-1)$$
$$\times\exp\left[\frac{i}{2}\int_\tau^{\tau'} dt\sqrt{g}\,\{g^{-1}[-\dot{x}^2+x^2(\dot\beta_+^2+\dot\beta_-^2+l^2\dot\phi_i^2)]-\Lambda\}\right] . \quad (3.2)$$

Then we can obtain an effective overlap integral between three universes by forming the following expression:

$$\Gamma(x_3,\beta_3,\phi_3;x_2,\beta_2,\phi_2;x_1,\beta_1,\phi_1)$$
$$=\lambda\int_0^\infty d\tau_3\int_{-\infty}^0 d\tau_2\int_{-\infty}^0 d\tau_1\int dx\, d\beta_+ d\beta_- d^{n-3}\phi$$
$$\times\langle x_3,\beta_3,\phi_3,\tau_3|x,\beta,\phi,0\rangle_{\Lambda_3}\langle x,\beta,\phi,0|x_1,\beta_1,\phi_1,\tau_1\rangle_{\Lambda_1}\langle x,\beta,\phi,0|x_2,\beta_2,\phi_2,\tau_2\rangle_{\Lambda_2} . \quad (3.3)$$

The novel feature of the above formula is that it involves three separate gravitational path integrals each with a different value of the cosmological constant obtained presumably from different particle spectra in each universe. The effective universal interaction can be constructed by composing Γ with three inverse propagators and we obtain the familiar result

$$V(x_3,\beta_3,\phi_3;x_1,\beta_1,\phi_1;x_2,\beta_2,\phi_2)=\lambda\delta(x_3-x_2)\delta^2(\beta_3-\beta_2)\delta^{n-3}(\phi_3-\phi_2)\delta(x_2-x_1)\delta^2(\beta_2-\beta_1)\delta^{n-3}(\phi_2-\phi_1) . \quad (3.4)$$

Notice that this type of universe overlap yields an interaction local in superspace. We believe this is simply an artifact of restricting ourselves to the reduced minisuperspace and do not expect this feature to continue when inhomogeneities are taken into account. This seems to also be the case for an interacting theory of two-dimensional universes used in string theory.[24] The field-theory limit of a string theory, which amounts to neglecting inhomogeneities in the two-dimensional gravity, defines a perfectly local theory in the minisuperspace perceived as space-time. However, in our case there is something artificial about local interactions in superspace because it tells us that only universes of the same size and shape interact (there is a generation gap here as baby universes could not interact with their parents). Other forms of interactions can be formed by propagating the three universes not from the same point in superspace. Say universe 1 forward to $(x+\epsilon,\beta+\epsilon,\phi+\epsilon)$, universe 2 forward to (x,β,ϕ) and universe 3 back to $(x-\epsilon,\beta-\epsilon,\phi-\epsilon)$. This would have the effect of making the interactions in superspace nonlocal. Also the effect of integrating out a universe say with very large Λ could have the effect of making the universal field theory nonlocal in superspace. In this case reinstating this universe would make the interaction local again. In any event universal interactions will still be determined by the overall integral of three solutions to the WDW equation except in the nonlocal case they will not be evaluated at the same point in superspace. Finally if the simplest form of inhomogeneity is introduced: namely, the introduction of universes which contain particles there is some indication that universe decay is frustrated by an accompanying infinite particle production.[7,8]

We shall consider the third-quantized interactions generated by the path integral (3.3) and summarized in the Lagrangian

$$\int dx\, d\beta_+ d\beta_- d^{n-3}\phi\sqrt{-G}\,[\tfrac{1}{2}(G^{IJ}\partial_I\Phi_1\partial_J\Phi_1-\xi R\,\Phi_1\Phi_1-\Lambda_1\Phi_1\Phi_1)+\tfrac{1}{2}(G^{IJ}\partial_I\Phi_2\partial_J\Phi_2-\xi R\,\Phi_2\Phi_2-\Lambda_2\Phi_2\Phi_2)$$
$$+\tfrac{1}{2}(G^{IJ}\partial_I\Phi\partial_J\Phi-\xi R\,\Phi\Phi-\Lambda\Phi\Phi)+\lambda\Phi_1(x,\beta,\phi)\Phi_2(x,\beta,\phi)\Phi(x,\beta,\phi)] . \quad (3.5)$$

In the above Φ,Φ_1,Φ_2 are third-quantized fields used to describe three different universes with different cosmological constants. The different values of the cosmological constant Λ could be due to different values of the particle spectra in each universe. For example, in a linearized theory the cosmological constant is determined to be

$$\Lambda=l^2\tfrac{1}{2}\sum_i(-)^{F_i}\int\frac{d^3k}{(2\pi)^3}(k^2+m_i^2)^{1/2} . \quad (3.6)$$

The parameters of the third-quantized Lagrangian are determined via (3.6) from the second-quantized theory. It is also true that parameters of the second-quantized theory such as mass spectrum m_i are determined by a first-quantized theory say by looking at the eigenvalues of $N+\tilde{N}-2$ in first-quantized string theory. Such an approach suggests a statistical bootstrap formulation of the theory.[17]

The presence of an interaction term in the above Lagrangian gives rise to the phenomena of universe decay with the probability amplitude

$$A(\Phi\to\Phi_1\Phi_2)=\langle {}^3 1^{\Phi_1}1^{\Phi_2}| -i\int dx\, d\beta_+ d\beta_- d^{n-3}\phi\sqrt{-G}\,\lambda \times\Phi_1(x,\beta,\phi)\Phi_2(x,\beta,\phi) \times\Phi(x,\beta,\phi)|{}^3\bar{1}^{\Phi}\rangle \ . \quad (3.7)$$

The above amplitude can be worked out using the operator expansions

$$\Phi(x,\beta,\phi)=\sum_K [\psi_K(x,\beta,\phi)A_K+\psi_K^*(x,\beta,\phi)A_K^\dagger] =\sum_K [\bar\psi_K(x,\beta,\phi)\bar A_K+\bar\psi_K^*(x,\beta,\phi)\bar A_K^\dagger] \quad (3.8)$$

with similar expansions for $\Phi_1(x,\beta,\phi)$ and $\Phi_2(x,\beta,\phi)$.

The following formulas indicate the modification of the presence of the background superspace metric to the decay process:

$$\langle {}^3 0|\Phi(x,\beta,\phi)\bar A_K^\dagger|{}^3\bar 0\rangle=[-\beta_K\alpha_K^{-1}\bar\psi^*_{-K}(x,\beta,\phi)+\bar\psi_K(x,\beta,\phi)]\langle {}^3 0|{}^3\bar 0\rangle \ ,$$
$$\langle {}^3 0|A_{K_1}\Phi_1(x,\beta,\phi)|{}^3\bar 0\rangle=[\beta_{K_1}\alpha_{K_1}^{-1}\psi_{-K_1}(x,\beta,\phi)+\psi_{K_1}^*(x,\beta,\phi)]\langle {}^3 0|{}^3\bar 0\rangle \ , \quad (3.9)$$
$$\langle {}^3 0|A_{K_2}\Phi_2(x,\beta,\phi)|{}^3\bar 0\rangle=[\beta_{K_2}\alpha_{K_2}^{-1}\psi^*_{-K_2}(x,\beta,\phi)+\psi_{K_2}^*(x,\beta,\phi)]\langle {}^3 0|{}^3\bar 0\rangle \ .$$

Putting together (3.7) and (3.9) we have the expression

$$A(\Phi\to\Phi_1\Phi_2)=-i\lambda\int dx\, d\beta_+ d\beta_- d^{n-3}\phi\sqrt{-G}\,[\beta_{K_1}\alpha_{K_1}^{-1}\psi_{-K_1}(x,\beta,\phi)+\psi_{K_1}^*(x,\beta,\phi)] \times[\beta_{K_2}\alpha_{K_2}^{-1}\psi_{-K_2}(x,\beta,\phi)+\psi_{K_2}^*(x,\beta,\phi)][-\beta_K\alpha_K^{-1}\bar\psi^*_{-K}(x,\beta,\phi)+\bar\psi_K(x,\beta,\phi)]\langle {}^3 0|{}^3\bar 0\rangle \ . \quad (3.10)$$

The inverse process where two universes Φ_1 and Φ_2 merge into a single universe Φ is also possible owing to the background superspace metric. The amplitude for this process is given by

$$A(\Phi_1\Phi_2\to\Phi)=-i\lambda\int dx\, d\beta_+ d\beta_- d^{n-3}\phi\sqrt{-G}\,[\beta_K\alpha_K^{-1}\psi_{-K}(x,\beta,\phi)+\psi_K^*(x,\beta,\phi)] \times[-\beta_{K_1}\alpha_{K_1}^{-1}\bar\psi^*_{-K_1}(x,\beta,\phi)+\bar\psi_{K_1}(x,\beta,\phi)][-\beta_{K_2}\alpha_{K_2}^{-1}\bar\psi^*_{-K_2}(x,\beta,\phi)+\bar\psi_{K_2}(x,\beta,\phi)]\langle {}^3 0|{}^3\bar 0\rangle \ . \quad (3.11)$$

In the following we shall consider in detail the special case where the universes described by Φ_1 and Φ_2 are identical. Interactions can be generated for this model by either a three-universe overlap or the insertion of a four-dimensional vertex operator on the world volume (see Appendix A). This gives us an effective third-quantized action given by

$$\int dx\, d\beta_+ d\beta_- d^{n-3}\phi\sqrt{-G} \times\left[\tfrac{1}{2}(G^{IJ}\partial_I\Psi\partial_J\Psi-\xi R\Psi\Psi)+\tfrac{1}{2}(G^{IJ}\partial_I\Phi\partial_J\Phi-\xi R\Phi\Phi-\Lambda\Phi\Phi)+\frac{\lambda}{2}\Psi(x,\beta,\phi)\Psi(x,\beta,\phi)\Phi(x,\beta,\phi)\right] \ . \quad (3.12)$$

In the above Ψ denotes a third-quantized field describing a universe with zero cosmological constant and Φ denotes a third-quantized field for a universe with cosmological constant Λ. There are modifications to (3.10) when Φ_1 and Φ_2 are identical but these are absent when at least one of the three universes has $\Lambda=0$. So aside from an overall factor of 2 which is compensated by the $\lambda/2$ in (3.12) the amplitude for universe decay [Fig. 1(a)] is given by

$$A(\Phi\to\psi^0\psi^0)=\left\langle {}^3 2^{\psi^0}\left| -i\int dx\, d\beta_+ d\beta_- d^{n-3}\phi\sqrt{-G}\,\frac{\lambda}{2}\Psi(x,\beta,\phi)\Psi(x,\beta,\phi)\Phi(x,\beta,\phi)\right| {}^3\bar 1^{\Phi}\right\rangle$$
$$=-i\lambda\int dx\, d\beta_+ d\beta_- d^{n-3}\phi\sqrt{-G}\,[\psi_{K_1}^{0*}(x,\beta,\phi)\psi_{K_2}^{0*}(x,\beta,\phi)] \times[-\beta_K\alpha_K^{-1}\bar\psi^*_{-K}(x,\beta,\phi)+\bar\psi_K(x,\beta,\phi)]\langle {}^3 0|{}^3\bar 0\rangle \ . \quad (3.13)$$

The inverse process is also possible [Fig. 1(b)] with the amplitude

$$A(\psi^0\psi^0\to\Phi)=-i\lambda\int dx\, d\beta_+ d\beta_- d^{n-3}\phi\sqrt{-G}\,[\beta_K\alpha_K^{-1}\psi_{-K}(x,\beta,\phi)+\psi_K^*(x,\beta,\phi)] \times[\psi_{K_1}^0(x,\beta,\phi)\psi_{K_2}^0(x,\beta,\phi)]\langle {}^3 0|{}^3\bar 0\rangle \ . \quad (3.14)$$

Using our expressions for the one-universe wave functions (2.16) we have

$$A(\Phi\to\psi^0\psi^0)=(2\pi)^{n-1}\delta^{n-1}(K-K_1-K_2)V^{-3/2}(-i\lambda)\langle {}^3 0|{}^3\bar 0\rangle \times\int_0^\infty dx\left[\frac{x}{l}\right]^{(4-n)/2}\frac{1}{\sqrt{2|K_1|}}\left[\frac{x}{x_0}\right]^{i|K_1|l}\frac{1}{\sqrt{2|K_2|}}\left[\frac{x}{x_0}\right]^{i|K_2|l} \times l^{1/2}\left[\frac{\pi}{2}\right]^{1/2}(\sinh\pi|K|l)^{-1/2}[J_{-i|K|l}(\sqrt{\Lambda}x)-\beta_K\alpha_K^{-1}J_{i|K|l}(\sqrt{\Lambda}x)] \ . \quad (3.15)$$

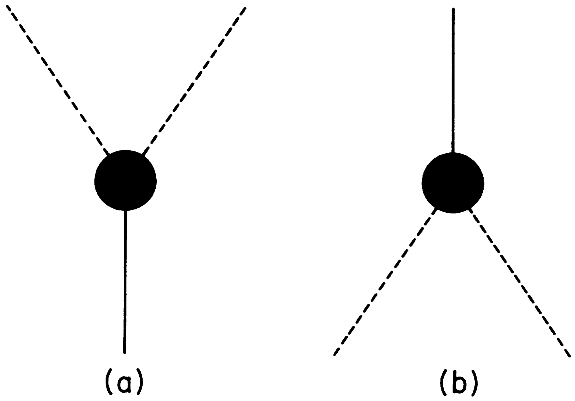

FIG. 1 (a) A universe with nonzero cosmological constant decays into two universes with zero cosmological constant. The solid line denotes the path through superspace of a universe with nonzero cosmological constant. The dashed line denotes the path through superspace of a universe with zero cosmological constant. The shaded circle indicates a disturbance in superspace due to a combination of the background superspace metric and explicit third-quantized interactions. (b) Two universes with zero cosmological constant annihilate and produce a universe with nonzero cosmological constant.

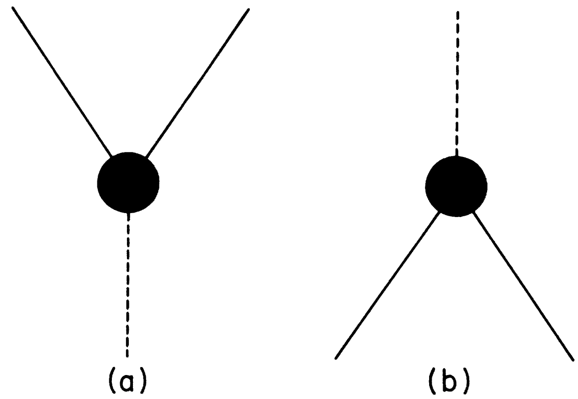

FIG. 2 (a) A universe with zero cosmological constant decays into two universes with nonzero cosmological constant. (b) Two universes with nonzero cosmological constant annihilate and produce a universe with zero cosmological constant.

The integrals can be evaluated by using[25]

$$\int_0^\infty dx\, J_\nu(x) x^{\mu-1} = 2^{\mu-1}\Gamma[\tfrac{1}{2}(\mu+\nu)]\Gamma[\tfrac{1}{2}(\mu-\nu)]\frac{1}{\pi}\sin\frac{\pi}{2}(\mu-\nu) \tag{3.16}$$

and we find

$$A(\Phi\to\psi^0\psi^0) = I(K,K_1,K_2)\left[\sin\pi\left[\frac{1}{2}il(|K_1|+|K_2|+|k|)+\frac{6-n}{4}\right]\right.$$
$$\left. -\beta_K\alpha_K^{-1}\sin\pi\left[\frac{1}{2}il(|K_1|+|K_2|-|K|)+\frac{6-n}{4}\right]\right], \tag{3.17}$$

where we have defined

$$I(K,K_1,K_2) = (2\pi)^{n-1}\delta^{n-1}(K-K_1-K_2)V^{-3/2}(-i\lambda)(l)^{(n-4)/2}\frac{1}{\sqrt{2|K_1|}}\left[\frac{1}{x_0}\right]^{i|K_1|l}\frac{1}{\sqrt{2|K_2|}}\left[\frac{1}{x_0}\right]^{I|K_2|l}$$
$$\times\langle {}^3 0|{}^3\bar{0}\rangle l^{1/2}\left[\frac{\pi}{2}\right]^{1/2}(\sinh\pi|K|l)^{-1/2}\left[\frac{2}{\sqrt{\Lambda}}\right]^{[(6-n)/2+i|K_1|l+i|K_2|l]-1}$$
$$\times\Gamma\left[\frac{1}{2}il(|K_1|+|K_2|+|K|)+\frac{6-n}{4}\right]\Gamma\left[\frac{1}{2}il(K_1|+|K_2|-|K|)+\frac{6-n}{4}\right]\frac{1}{\pi}. \tag{3.18}$$

The inverse decay can similarly be evaluated and shown to yield

$$A^*(\psi^0\psi^0\to\Phi) = I(K,K_1,K_2)\alpha_K^{-1}\sin\pi\left[\frac{1}{2}il(|K_1|+|K_2|+|K|)+\frac{6-n}{4}\right]. \tag{3.19}$$

The asymmetry $a=|A(\Phi\to\psi^0\psi^0)|^2-|A(\psi^0\psi^0\to\Phi)|^2$ tells us whether it is more probable for a universe with $\Lambda=0$ to be created from or absorbed into the third-quantized vacuum. This is the analog of asymmetries between particle decay and inverse particle decay in a background gravitational field.[26] After some algebra we obtain

$$a=|A(\Phi\to\psi^0\psi^0)|^2-|A(\psi^0\psi^0\to\Phi)|^2$$
$$=|A(\psi^0\psi^0\to\Phi)|^2\left[\left[\frac{1-\exp(-2\pi l|K|)}{1-\exp[-\pi l(|K_1|+|K_2|+|K|)]}\right]^2\left[1+\frac{\sin^2[\pi(6-n)/4]}{\sinh^2[\pi(|K_1|+|K_2|+|K|)/2]}\right]^{-1}-1\right]. \tag{3.20}$$

In this form it is clear that $a<0$ and it is more probable to absorb a universe with $\Lambda=0$ than it is to create one. This does not mean that the probability of creation is negligible. Indeed for large $|K_1|$, $|K_2|$, and $|K|$ production and decay are comparable processes.

In general the asymmetry need not be negative. For example, in Appendix C we discuss a model for which a universe with $\Lambda=0$ can decay into two universes with $\Lambda\neq 0$ [Fig. 2(a)]. Conversely two universes with $\Lambda\neq 0$ can annihilate and create a universe with $\Lambda=0$ [Fig. 2(b)]. The asymmetry in this case is obtained from

$$a'=|A(\Phi\Phi\to\psi^0)|^2-|A(\psi^0\to\Phi\Phi)|^2\ . \tag{3.21}$$

For $n=4k+6$, where k a non-negative integer we find

$$a'=|A(\psi^0\to\Phi\Phi)|^2\left[\left|\alpha_{K_1}\alpha_{K_2}-\beta_{K_2}\alpha_{K_1}\frac{\sinh[\pi l(|K_1|+|K_2|+|K|)/2]}{\sinh[\pi l(|K_1|-|K_2|+|K|)/2]}\right.\right.$$
$$\left.\left.-\beta_{K_1}\alpha_{K_2}\frac{\sinh[\pi l(|K_1|+|K_2|+K|)/2]}{\sinh[\pi l(|K_2|-|K_1|+|K|)/2]}+\beta_{K_1}\beta_{K_2}\frac{\sinh[\pi l(|K_1|+|K_2|+|K|)/2]}{\sinh[\pi l(|K|-|K_1|-|K_2|)/2]}\right|^2-1\right]\ . \tag{3.22}$$

For $|K|\ll l$ and $|K_1|=|K_2|$ this reduces to

$$a'\approx\left[\frac{16}{\pi^2 l^2|K|^2}-1\right]|A(\psi^0\to\Phi\Phi)|^2\ . \tag{3.23}$$

So that for some models and in some regions of phase-space emission into universes with $\Lambda=0$ is a dominant process, which is one way of addressing the cosmological-constant problem.

We shall now discuss the integrated probability. Defining $M(\Phi\to\psi^0\psi^0)$ by

$$A(\Phi\to\psi^0\psi^0)=(2\pi)^{n-1}\delta^{n-1}(K-K_1-K_2)V^{-3/2}M(\Phi\to\psi^0\psi^0)$$

we find the total decay probability by integrating

$$\text{Prob}(\Phi\to\psi^0\psi^0)=V\int\frac{d^{n-1}K_1}{(2\pi)^{n-1}}V\int\frac{d^{n-1}K_2}{(2\pi)^{n-1}}|A(\Phi\to\psi^0\psi^0)|^2$$
$$=V\int\frac{d^{n-1}K_1}{(2\pi)^{n-1}}V\int\frac{d^{n-1}K_2}{(2\pi)^{n-1}}V(2\pi)^{n-1}\delta^{n-1}(K-K_1-K_2)V^{-3}|M(\Phi\to\psi^0\psi^0)|^2\ . \tag{3.24}$$

The phase-space integrals are derived in Appendix B and we have finally

$$\text{Prob}(\Phi\to\psi^0\psi^0)=2\frac{2\pi^{(n-2)/2}}{\Gamma((n-2)/2)}(2\pi)^{1-n}\int_0^\infty|K_1|d|K_1|\int_{||K_1|-|K||}^{|K_1|+|K|}|K_2|d|K_2|\frac{1}{2|K|}$$
$$\times\left[K_1^2-\left[\frac{K^2+K_1^2-K_2^2}{2K}\right]^2\right]^{n-4/2}|M|^2\ . \tag{3.25}$$

The quantity $|M(\Phi\to\psi^0\psi^0)|^2$ takes on a rather simple form for certain values of the minisuperspace dimension n. For $n=4+4k$ where k is a non-negative integer we find

$$|M(\Phi\to\psi^0\psi^0)|^2=\lambda^2 l^{n-4}\frac{1}{2|K_1|}\frac{1}{2|K_2|}\frac{\pi}{2}(\sinh\pi|K|l)^{-1}l\Lambda^{(n-6)/2}2^{4-n}$$
$$\times|\langle{}^3 0|{}^3\bar{0}\rangle|^2\prod_{s=1}^{(n-4)/4}\frac{1}{(s-\frac{1}{2})^2+\frac{1}{4}l^2(|K_1|+|K_2|+|K|)^2}\frac{1}{(s-\frac{1}{2})^2+\frac{1}{4}l^2(|K_1|+|K_2|-|K|)^2}$$
$$\times\left[\frac{\cosh\pi\frac{1}{2}l(|K_1|+|K_2|+|K|)}{\cosh\pi\frac{1}{2}l(|K_1|+|K_2|-|K|)}\right.$$
$$\left.+e^{-2\pi|K|l}\frac{\cosh\pi\frac{1}{2}l(|K_1|+|K_2|-|K|)}{\cosh\pi\frac{1}{2}l(|K_1|+|K_2|+|K|)}-2e^{-\pi|K|l}\right]\ . \tag{3.26}$$

While for $n=6+4k$ where k is a non-negative integer we find

$$|M(\Phi\rightarrow\psi^0\psi^0)|^2=\lambda^2 l^{n-4}\frac{1}{2|K_1|}\frac{1}{2|K_2|}\frac{\pi}{2}(\sinh\pi|K|l)^{-1}l\Lambda^{(n-6)/2}2^{4-n}\frac{2l^{-1}}{|K_1|+|K_2|+|K|}\frac{2l^{-1}}{|K_1|+|K_2|-|K|}$$
$$\times|\langle{}^3 0|{}^3\bar{0}\rangle|^2\prod_{s=1}^{(n-6)/4}\frac{1}{s^2+\frac{1}{4}l^2(|K_1|+|K_2|+|K|)^2}\frac{1}{s^2+\frac{1}{4}l^2(|K_1|+|K_2|-|K|)^2}$$
$$\times\left[\frac{\sinh\pi\frac{1}{2}l(|K_1|+|K_2|+|K|)}{\sinh\pi\frac{1}{2}l(|K_1|+|K_2|-|K|)}+e^{-2\pi|K|l}\frac{\sinh\pi\frac{1}{2}l(|K_1|+|K_2|-|K|)}{\sinh\pi\frac{1}{2}l(|K_1|+|K_2|+|K|)}-2e^{-\pi|K|l}\right]. \tag{3.27}$$

These matrix elements do not fall off fast enough in order to define a total decay probability from (3.25). The best we can do is to integrate over $|K_2|$ and obtain a probability distribution for one of the final-state universes with $\Lambda=0$ to have superspace momentum $|K_1|$. For the simple case of $n=4$, i.e., one scalar field, we find

$$\frac{d\,\mathrm{Prob}\,(\Phi\rightarrow\psi^0\psi^0)}{d|K_1|}=(2\pi)^{-2}\lambda^2\frac{\pi}{2}(\sinh\pi|K|l)^{-1}l\Lambda^{-1}\frac{1}{2}\frac{1}{2|K|}$$
$$\times\Bigg[\theta(|K_1|-|K|)\Bigg\{(e^{\pi|K|l}+e^{-3\pi|K|l}-2e^{-\pi|K|l})2|K|-\frac{2}{l\pi}\sinh\pi|K|l\left[\ln\left[\frac{1+\exp[-\pi 2l(|K_1|-|K|)]}{1+\exp(-\pi 2l|K_1|)}\right]+e^{-2\pi|K|l}\ln\left[\frac{1+\exp[-\pi 2l(|K_1|+|K|)]}{1+\exp(-\pi 2l|K_1|)}\right]\right]\Bigg\}$$
$$+\theta(|K|-|K_1|)\Bigg\{(e^{\pi|K|l}+e^{-3\pi|K|l}-2e^{-\pi|K|l})2|K_1|-\frac{2}{l\pi}\sinh\pi|K|l\left[\ln\left[\frac{2}{1+\exp(-\pi 2l|K_1|)}\right]+e^{-2\pi|K|l}\ln\left[\frac{1+\exp[-\pi 2l(|K_1|+|K|)]}{1+\exp(-\pi 2l|K|)}\right]\right]\Bigg\}\Bigg]. \tag{3.28}$$

Although it appears that the superspace volume cutoffs have disappeared from the above expression for the integrated probability this is actually not the case. Decay probabilities as well as the probabilities for universe creation depend sensitively on the superspace volume cutoff V. For example the probability that no universes are created out of the third-quantized vacuum $|{}^3\bar{0}\rangle$ is given by[18]

$$P(\{0\})=|\langle{}^3 0|{}^3\bar{0}\rangle|^2=\prod_K\frac{1}{\alpha_K^2}=\prod_K\left[\frac{2\sinh\pi|K|l}{\exp\pi|K|l}\right]=\prod_K(1-e^{-2\pi|K|l}). \tag{3.29}$$

If K is quantized say like $K=2\pi m/L$ so that the superspace volume is $V=L^{n-1}$ then we can make sense of the above product and

$$P[(0)]=|\langle{}^3 0|{}^3\bar{0}\rangle|^2=\prod_m(1-e^{-2\pi|m|l/L})=\exp\left[\sum_m\ln(1-e^{-2\pi|m|l/L})\right]. \tag{3.30}$$

Clearly as $L\rightarrow\infty$ then $|\langle{}^3 0|{}^3\bar{0}\rangle|^2\rightarrow 0$. Particle decay in curved space is afflicted with the same problem.[27] It may happen, however, that k really is quantized and that the superspace volume is physically cut off. This is the case in the mixmaster universe considered by Misner.[2] In that case

$$K=(K_+,K_-)\sim\frac{1}{l}\frac{1}{\ln(x/l)}(n_+,n_-). \tag{3.31}$$

The superspace volume cutoff is then the area of the equilateral triangle to which the universes are confined $V=\sqrt{3}3l^2[\ln(x/l)]^2$ so that $\langle{}^3 0|{}^3\bar{0}\rangle$ together with other universe creation amplitudes can be defined in this model.[18]

Another way to avoid the superspace cutoff problems in universe decay is to work with what are called added-up probabilities.[28,29] This method consists of identifying the (in) and (out) states for universes with $\Lambda=0$ (possible if they are conformally coupled) and summing over arbitrary numbers of universes with $\Lambda\neq 0$ in the final state. The method makes use of the identity

$$\sum_{(N)} |\langle N^{\Phi} 2^{\psi^0}| - i\lambda \int dx\, d\beta_+ d\beta_- d^{n-3}\phi \sqrt{-G}\, \Psi\Psi\Phi | \bar{1}^{\Phi}\rangle|^2 = \sum_{(N)} |\langle \bar{N}^{\Phi} 2^{\psi^0}| - i\lambda \int dx\, d\beta_+ d\beta_- d^{n-3}\phi \sqrt{-G}\, \Psi\Psi\Phi | \bar{1}^{\Phi}\rangle|^2 . \tag{3.32}$$

Although the added-up probability method is of some use in universe decay as it allows one to use an (in-in) or (out-out) prescription in its computation it is not expected to help in discussions of universe creation. This is because the corresponding equality

$$\sum_{(N)} |\langle N^{\Phi} | {}^3\bar{0}\rangle|^2 = \sum_{(N)} |\langle \bar{N}^{\Phi} | {}^3\bar{0}\rangle|^2 = 1$$

amounts to a trivial identity.

IV. CONCLUSIONS

We have shown that if universes with zero cosmological constant are not created directly out of the third-quantized vacuum they can still emerge from the decay of a universe with nonzero cosmological constant. Owing to the background superspace metric we also find that it is possible for universes with $\Lambda=0$ to decay again into universes with nonzero cosmological constant but the inclusion of inhomogeneities could substantially alter this result. Also because the background superspace metric is not invariant with respect to inversion in conformal superspace time the probability of creation of universes with $\Lambda=0$ is not necessarily equal to their probability of absorption. We studied this effect in detail for a simple model of two universes one with zero cosmological constant and the other nonzero. We found that the decay and absorption probabilities do indeed differ and give an explicit expression for the decay and absorption amplitudes. We discussed the general expression for the integrated decay probabilities and derived the form of the phase space governing universe decay. Finally we discussed the dependence of the decay probabilities on an arbitrary cutoff in the volume of superspace. We argued that the presence of a physical cutoff in superspace as in the mixmaster universe could possibly remove this ambiguity.

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APPENDIX A: INSERTION OF FOUR-DIMENSION VERTEX OPERATORS

For the case when two of the three universes in the overlap integral of (3.3) are identical we can write

$$\begin{aligned}\Gamma(x_3,\beta_3,\phi_3;x_2,\beta_2,\phi_2;x_1,\beta_1,\phi_1) = {} & \int_0^\infty d\tau_3 \int_{-\infty}^0 d\tau_2 \int_{-\infty}^0 d\tau_1 \int_{\{x(\tau_1)=x_1,\beta_\pm(\tau_1)=\beta_{\pm 1},\phi_i(\tau_1)=\phi_{i1}\}}^{\{x(\tau_3)=x_3,\beta_\pm(\tau_3)=\beta_{\pm 3},\phi_i(\tau_3)=\phi_{i3}\}} Dg\, Dx\, D\beta_+ D\beta_- D\phi_i \\ & \times \prod_t \delta(g(t)-1) \exp\left[\frac{i}{2}\int_{\tau_1}^{\tau_3} dt\sqrt{g}\,\{g^{-1}[-\dot{x}^2 + x^2(\dot\beta_+^2 + \dot\beta_-^2 + l^2\dot\phi_i^2)] - \Lambda\}\right] \\ & \times \left[\int_{\{x'(\tau_2)=x_2,\beta'_\pm(\tau_2)=\beta_{\pm 2},\phi_i(\tau_2)=\phi_{i2}\}}^{\{x'(0)=x(0),\beta'_\pm(0)=\beta'_\pm(0),\phi'_i(0)=\phi_{i0})\}} Dg\, Dx'\, D\beta'_+ D\beta'_- D\phi'_i \right. \\ & \left. \times \prod_t \delta(g(t)-1) \exp\left[\frac{i}{2}\int_{\tau_2}^{0} dt\sqrt{g}\,\{g^{-1}[-\dot{x}'^2 + x'^2(\dot\beta'^2_+ + \dot\beta'^2_- + l^2\dot\phi'^2_i)] - \Lambda'\}\right]\right],\end{aligned} \tag{A1}$$

Using our expression for the universal propagator (3.2) we have

$$\begin{aligned}&\Gamma(x_3,\beta_3,\phi_3;x_2,\beta_2,\phi_2;x_1,\beta_1,\phi_1) \\ &\quad = \int_0^\infty d\tau_3 \int_{-\infty}^0 d\tau_2 \int_{-\infty}^0 d\tau_1 \int_{\{x(\tau_1)=x_1,\beta_\pm(\tau_1)=\beta_{\pm 1},\phi_i(\tau_1)=\phi_{i1}\}}^{\{x(\tau_3)=x_3,\beta_\pm(\tau_3)=\beta_{\pm 3},\phi_i(\tau_3)=\phi_{i3}\}} Dg\, Dx\, D\beta_+ D\beta_- D\phi_i \\ &\qquad \times \prod_t \delta(g(t)-1) \exp\left[\frac{i}{2}\int_{\tau_1}^{\tau_3} dt\sqrt{g}\,\{g^{-1}[-\dot{x}^2 + x^2(\dot\beta_+^2 + \dot\beta_-^2 + l^2\dot\phi_i^2)] - \Lambda\}\right] \langle x(0),\beta(0),\phi(0);0|x_2,\beta_2,\phi_2;\tau_2\rangle_{\lambda'} .\end{aligned} \tag{A2}$$

The insertion of the propagator inside the path integral acts like a potential term[30] so that

$$\Gamma(x_3,\beta_3,\phi_3;x_2,\beta_2,\phi_2;x_1,\beta_1,\phi_1) = \int_0^\infty d\tau_3 \int_{-\infty}^0 d\tau_2 \int_{-\infty}^0 d\tau_1 \langle x_3,\beta_3,\phi_3;\tau_3 | \langle \hat{x}(0),\hat{\beta}(0),\hat{\phi}(0);0|x_2,\beta_2,\phi_2;\tau_2\rangle_{\Lambda'} |x_1,\beta_1,\phi_1,\tau_1\rangle \ . \tag{A3}$$

Here $\hat{x}(0),\hat{\beta}(0),\hat{\phi}(0)$ are taken to be operators and this turns the intermediate propagator into an operator. Now defining the states $|K,E\rangle$ by $\langle x,\beta,\phi;\tau|K,E\rangle=\psi_{K,E}(x,\beta,\phi;\tau)$ where

$$2i\frac{\partial}{\partial\tau}\psi+\frac{1}{(x/l)^{n-1}}\frac{d}{dx}\left[(x/l)^{n-1}\frac{d}{dx}\psi\right]+\left[\frac{1}{(x/l)^2}(K^2)+\xi\frac{(n-1)(n-2)}{x^2}\right]\psi=0 \ . \tag{A4}$$

We can reexpress (A3) as

$$\begin{aligned}
&\int_0^\infty d\tau_3\int_{-\infty}^0 d\tau_2\int_{-\infty}^0 d\tau_1 e^{i(-\tau_3\Lambda+\tau_2\Lambda'+\tau_1\Lambda)/2}\int\frac{d^{n-1}K_3}{(2\pi)^{n-1}}\int\frac{d^{n-1}K_2}{(2\pi)^{n-1}}\int\frac{d^{n-1}K_1}{(2\pi)^{n-1}}\int_0^\infty dE_3\int_0^\infty dE_2\int_0^\infty dE_1\\
&\qquad\times\langle x_3,\beta_3,\phi_3;\tau_3|K_3E_3\rangle\langle K_3E_3|\\
&\qquad\times\langle\hat{x}(0),\hat{\beta}(0),\hat{\phi}(0);0|K_2E_2\rangle\langle K_2E_2|x_2,\beta_2,\phi_2;\tau_2\rangle_\Lambda|K_1E_1\rangle\langle K_1E_1|x_1,\beta_1,\phi_1,\tau_1\rangle\\
&=\int_0^\infty d\tau_3\int_{-\infty}^0 d\tau_2\int_{-\infty}^0 d\tau_1 e^{i(\tau_2\Lambda'-\tau_3\Lambda+\tau_1\Lambda)/2}\int\frac{d^{n-1}K_3}{(2\pi)^{n-1}}\int\frac{d^{n-1}K_2}{(2\pi)^{n-1}}\int\frac{d^{n-1}K_1}{(2\pi)^{n-1}}\int_0^\infty dE_3\int_0^\infty dE_2\int_0^\infty dE_1\\
&\qquad\times\psi_{K_3E_3}(x_3,\beta_3,\phi_3;\tau_3)\psi^*_{K_2E_2}(x_2,\beta_2,\phi_2;\tau_2)\psi^*_{K_1E_1}(x_1,\beta_1,\phi_1,\tau_1)\\
&\qquad\times\langle K_3E_3|\langle\hat{x}(0),\hat{\beta}(0),\hat{\phi}(0);0|K_2E_2\rangle|K_1E_1\rangle \ .
\end{aligned}\tag{A5}$$

The final factor can be written as

$$\langle K_3E_3|V^{-1/2}\exp[i(k_{+2}\hat{\beta}_+ + k_{-2}\hat{\beta}_- + k_{\phi j 2}\hat{\phi}_j)](x/l)^{(2n-n)/2}\chi_{K_2E_2}(\hat{x})|K_1E_1\rangle \ , \tag{A6}$$

where the operator $\chi_{KE}(\hat{x})$ is given by

$$\chi_{K_2E_2}(\hat{x})=(\pi/4)^{1/2}e^{\pi|K_2|l/2}H^{(2)}_{i|K_2|l}(\hat{x}\sqrt{2E_2})l^{1/2} \ . \tag{A7}$$

In this form we see that the modification of the path integral to include universal interactions can be implemented by the insertion of a four-dimensional vertex operator.

APPENDIX B: PHASE SPACE FOR UNIVERSE DECAY

In this appendix we derive the phase space available in universe decay. Inserting $1=\int dp_1^2\delta(p_1^2-K_1^2)$ and $1=\int dp_2^2\delta(p_2^2-K_2^2)$ into (3.24) we have

$$\begin{aligned}
\text{Prob}\,(\Phi\to\psi^0\psi^0)&=\int\frac{d^{n-1}K_1}{(2\pi)^{n-1}}\int\frac{d^{n-1}K_2}{(2\pi)^{n-1}}(2\pi)^{n-1}\delta^{n-1}(K-K_1-K_2)|M(\Phi\to\psi^0\psi^0)|^2\\
&=\int dp_1^2\int dp_2^2\int\frac{d^{n-1}K_1}{(2\pi)^{n-1}}\int\frac{d^{n-1}K_2}{(2\pi)^{n-1}}\delta(p_1^2-K_1^2)\delta(p_2^2-K_2^2)\\
&\qquad\times(2\pi)^{n-1}\delta^{n-1}(K-K_1-K_2)|M(\Phi\to\psi^0\psi^0)|^2 \ .
\end{aligned}\tag{B1}$$

We choose K to be of the form $(K_+,0)$. The $O(n-1)$ invariance of the above expression means that we will be able to obtain the general form by the replacement of K_+ by $|K|$. Integration over K_2 using the delta function yields

$$\begin{aligned}
\text{Prob}\,(\Phi\to\psi^0\psi^0)&=\int dp_1^2\int dp_2^2\int\frac{d^{n-1}K_1}{(2\pi)^{n-1}}\delta(K_1^2-p_1^2)\delta(K_+^2+p_1^2-2K_+K_{1_+}-p_2^2)|M(\Phi\to\psi^0\psi^0)|^2\\
&=\int dp_1^2\int dp_2^2\int dK_{1_+}\tfrac{1}{2}\int d(\bar{K}_1^2)|\bar{K}_1|^{n-4}\Omega_{d-3}\frac{1}{(2\pi)^{n-1}}\\
&\qquad\times\delta(K_1^2-p_1^2)\delta(K_+^2+p_1^2-2K_+K_{1_+}-p_2^2)|M(\Phi\to\psi^0\psi^0)|^2 \ ,
\end{aligned}\tag{B2}$$

where we have split $K_1=(K_{1_+},\bar{K})$ and

$$\Omega_{n-3}=\frac{2\pi^{(n-2)/2}}{\Gamma((n-2)/2)}$$

is the volume of an $(n-3)$ dimensional sphere. Integration over K_{1_+} yields

$$\text{Prob}\,(\Phi\to\psi^0\psi^0)=\int dp_1^2\int dp_2^2\tfrac{1}{2}\int d(\bar{K}_1^2)|\bar{K}_1|^{n-4}\Omega_{d-3}\frac{1}{(2\pi)^{n-1}}\times\delta\left[\bar{K}_1^2+\left[\frac{K_+^2+p_1^2-p_2^2}{2K_+}\right]^2-p_1^2\right]\frac{1}{2K_+}|M(\Phi\to\psi^0\psi^0)|^2\ . \tag{B3}$$

And finally integration over $\bar{K}_1^2$ yields

$$\text{Prob}\,(\Phi\to\psi^0\psi^0)=\int dp_1^2\int dp_2^2\tfrac{1}{2}\Omega_{d-3}\frac{1}{(2\pi)^{n-1}}\left[p_1^2-\left[\frac{K_+^2+p_1^2-p_2^2}{2K_+}\right]^2\right]^{(n-4)/2}\frac{1}{2K_+}|M(\Phi\to\psi^0\psi^0)|^2\ . \tag{B4}$$

Replacing K_+ by $|K|$ and renaming p_1,p_2 by K_1,K_2 we obtain (3.25).

APPENDIX C: UNIVERSE DECAY FROM A $\Phi\Phi\Psi^0$ INTERACTION

In this appendix we work out the amplitudes for universe decay in which a universe with $\Lambda=0$ is allowed to decay into two identical universes with $\Lambda\neq 0$ [Fig. 2(a)]. Owing to the superspace time dependence of the background superspace metric the inverse process can also occur [Fig. 2(b)]. The third-quantized Lagrangian we use is identical to (3.12) with the interaction term replaced by

$$\int dx\,d\beta_+d\beta_-d^{n-3}\phi\sqrt{-G}\,\frac{\lambda'}{2}\Phi(x,\beta,\phi)\Phi(x,\beta,\phi)\Psi(x,\beta,\phi)\ . \tag{C1}$$

The amplitude for the decay of universes with $\Lambda=0$ is then

$$A(\psi^0\to\Phi\Phi)=\left\langle {}^3 2^{\Phi}\left|-i\int dx\,d\beta_+d\beta_-d^{n-3}\phi\sqrt{-G}\,\frac{\lambda'}{2}\Phi(x,\beta,\phi)\Phi(x,\beta,\phi)\Psi(x,\beta,\phi)\right|\bar{1}^{\psi^0}\right\rangle^3\ . \tag{C2}$$

Inserting the operator expansions (3.8) for Φ and ψ we obtain

$$A(\psi^0\to\Phi\Phi)=-i\lambda'\int dx\,d\beta_+d\beta_-d^{n-3}\phi\sqrt{-G}\,[\beta_{K_1}\alpha_{K_1}^{-1}\psi_{-K_1}(x,\beta,\phi)+\psi_{K_1}^*(x,\beta,\phi)]\times[\beta_{K_2}\alpha_{K_2}^{-1}\psi_{-K_2}(x,\beta,\phi)+\psi_{K_2}^*(x,\beta,\phi)][\psi_K^0(x,\beta,\phi)]\langle{}^3 0|{}^3\bar{0}\rangle\ . \tag{C3}$$

Similarly the inverse decay can be computed

$$A(\Phi\Phi\to\psi^0)=\left\langle {}^3 1^{\psi^0}\left|-i\int dx\,d\beta_+d\beta_-d^{n-3}\phi\sqrt{-G}\,\frac{\lambda'}{2}\Phi(x,\beta,\phi)\Phi(x,\beta,\phi)\Psi(x,\beta,\phi)\right|\bar{2}^{\Phi}\right\rangle^3\ . \tag{C4}$$

Again using their operator expansions we obtain

$$A(\Phi\Phi\to\psi^0)=-i\lambda'\int dx\,d\beta_+d\beta_-d^{n-3}\phi\sqrt{-G}\,[\psi_K^{0*}(x,\beta,\phi)][-\beta_{K_1}\alpha_{K_1}^{-1}\bar{\psi}^*_{-K_1}(x,\beta,\phi)+\bar{\psi}_{K_1}(x,\beta,\phi)]\times[-\beta_{K_2}\alpha_{K_2}^{-1}\bar{\psi}^*_{-K_2}(x,\beta,\phi)+\bar{\psi}_{K_2}(x,\beta,\phi)]\langle{}^3 0|{}^3\bar{0}\rangle\ . \tag{C5}$$

The overlap integral

$$\begin{aligned}J(K,K_1,K_2)&=-i\lambda'\int dx\,d\beta_+d\beta_-d^{n-3}\phi\sqrt{-G}\,\psi_K^{0*}(x,\beta,\phi)\bar{\psi}_{K_1}(x,\beta,\phi)\bar{\psi}_{K_2}(x,\beta,\phi)\langle{}^3 0|{}^3\bar{0}\rangle\\&=(2\pi)^{n-1}\delta^{n-1}(K-K_1-K_2)V^{-3/2}(-i\lambda')\int_0^\infty dx(x/l)^{(4-n)/2}\frac{1}{\sqrt{2|K|}}\left[\frac{x}{x_0}\right]^{i|K|l}\\&\quad\times l\frac{\pi}{2}(\sinh\pi|K_1|l)^{-1/2}(\sinh\pi|K_2|l)^{-1/2}\\&\quad\times J_{-i|K_1|l}(\sqrt{\Lambda}x)J_{-i|K_2|l}(\sqrt{\Lambda}x)\end{aligned} \tag{C6}$$

can be evaluated using[25]

$$\int_0^\infty dx\, J_\mu(ax)J_\nu(ax)x^{-\rho} = \frac{(a/2)^{\rho-1}\Gamma(\rho)\pi}{\sin\frac{\pi}{2}(\nu+\mu+1-\rho)}[2\Gamma(\tfrac{1}{2}(1+\nu-\mu+\rho))\Gamma(\tfrac{1}{2}(1+\nu+\mu+\rho))\Gamma(\tfrac{1}{2}(1-\nu+\mu+\rho))\Gamma(\tfrac{1}{2}(1-\nu-\mu+\rho))]^{-1} \quad \text{(C7)}$$

and we obtain the following expression for $J(K,K_1,K_2)$:

$$\begin{aligned} J(K,K_1,K_2) &= (2\pi)^{n-1}\delta^{n-1}(K-K_1-K_2)V^{-3/2}(-i\lambda')(l)^{(n-4)/2}\frac{1}{\sqrt{2|K|}}\left[\frac{1}{x_0}\right]^{i|K|l} \\ &\times l\frac{\pi}{2}(\sinh\pi|K_1|l)^{-1/2}(\sinh\pi|K_2|l)^{-1/2}\left[\frac{2}{\sqrt{\Lambda}}\right]^{i|K|l+(6-n)/2} \\ &\times\Gamma\left[-il|K|+\frac{n-4}{4}\right]\frac{\pi}{\sin\frac{\pi}{2}\left[-il(|K_1|+|K_2|+|K|)+\frac{6-n}{2}\right]} \\ &\times\left[2\Gamma\left[\frac{1}{2}il(-|K_1|-|K_2|-|K|)+\frac{n-2}{4}\right]\Gamma\left[\frac{1}{2}il(-|K_1|+|K_2|-|K|)+\frac{n-2}{4}\right]\right. \\ &\quad\left.\times\Gamma\left[\frac{1}{2}il(|K_1|-|K_2|-|K|)+\frac{n-2}{4}\right]\Gamma\left[\frac{1}{2}il(|K_1|+|K_2|-|K|)+\frac{n-2}{4}\right]\right]^{-1} . \end{aligned} \quad \text{(C8)}$$

Using this integral the amplitudes for decay and inverse decay are determined to be

$$\begin{aligned} A^*(\psi^0\to\Phi\Phi) &= J(K,K_1,K_2)\alpha_{K_1}^{-1}\alpha_{K_2}^{-1} , \\ A(\Phi\Phi\to\psi^0) &= J(K,K_1,K_2)\left[1-\beta_{K_2}\alpha_{K_2}^{-1}\frac{s(|K_1|,|K_2|)}{s(|K_1|,-|K_2|)}-\beta_{K_1}\alpha_{K_1}^{-1}\frac{s(|K_1|,|K_2|)}{s(-|K_1|,|K_2|)}\right. \\ &\quad\left.+\beta_{K_1}\alpha_{K_1}^{-1}\beta_{K_2}\alpha_{K_2}^{-1}\frac{s(|K_1|,|K_2|)}{s(-|K_1|,-|K_2|)}\right] , \end{aligned} \quad \text{(C9)}$$

where we have defined

$$s(|K_1|,|K_2|)=\sin\frac{\pi}{2}\left[-il(|K_1|+|K_2|+|K|)+\frac{6-n}{2}\right] .$$

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