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Crystalline-instanton-liquid model of QCD vacuum and its implication on chiral and deconfinement transitions

Zhou Xiaoan
Department of Physics, Simon Fraser University, Burnaby, British Columbia, Canada V5A 1S6
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A crystalline-instanton-liquid model of a QCD vacuum is proposed and its implications on the finite-temperature phase transitions in QCD are explored. It is shown from this model that both the chiral and deconfinement transitions may exist in QCD with $T_{\rm ch} > T_{\rm dec}$ and both may be first order. The critical temperatures are also determined.

Finite-temperature phase transitions in QCD are of particular interest at this time, because of their possible relevance to the physics of heavy-ion collisions and the study of the early Universe. Many theoretical analyses based on the effective-spin models[1] and numerical simulations of lattice QCD (Ref. 2) indicate a first-order deconfinement transition for pure SU(3) gauge theory and a first-order chiral-symmetry-restoration (CSR) transition for gluonic theory with light quarks.[3] The questions to settle are whether both transitions exist in QCD and if they are connected.

One way to understand the mechanism behind the chiral-symmetry breaking in QCD and related physics is by means of the dilute-instanton-gas model.[4] Recent (infrared-finite) calculations using the instanton-liquid model of a QCD vacuum[5] yield very reasonable bulk parameters, and call upon further investigations. However, no explanation for color confinement is made in the instanton-based models so far. On the other hand, our understanding on the confinement and the hadronic structure is by means of some quite different models involving the MIT bag,[6] the phenomenological string,[7] and the recently proposed Polyakov-Kleinert rigid string.[8] It seems that these models have nothing to do with instantons.

An intriguing question is whether these two kinds of models are related. Our answer here is positive. In fact, the crystalline-instanton-liquid model (CILM) proposed in this Brief Report is motivated by the instanton-liquid model[5] and the Polyakov-Kleinert string[8] mentioned above. Our main purpose is to find the connection between these two quite different pictures of a QCD vacuum or the hadronic structure and to describe the CSR and deconfinement transitions within one QCD-motivated model. Formally, the CILM of a QCD vacuum or hadronic structure is the Polyakov-Kleinert string with liquid crystalline order which can be viewed as the continuous theory of instanton liquid crystals. We hope that it can provide an effective description of QCD at large distances and finite temperature. One may ask how come one expects this model to describe CSR since the Polyakov-Kleinert string does not contain fermionic degrees of freedom. Our answer is the rigidity term [the second term in (1)] has its fermionic origin;[9] that is, it appears as a result of integrating out fermions in functional integrals.

Our starting point is the action of the Polyakov-Kleinert string in the first-order form:[8]

$$S_0(g_{ab},\lambda_{ab})=\sigma_0\int g^{1/2}d^2\xi+\frac{1}{2\alpha_0}\int g^{1/2}d^2\xi[(\Delta x^\mu)^2+\lambda^{ab}(g_{ab}-\partial_a x^\mu\partial_b x_\mu)]+\kappa_G\int g^{1/2}d^2\xi R \ . \tag{1}$$

From the Gauss-Bonnet theorem we have

$$\kappa_G\int_{\Gamma_h} d^2\xi\sqrt{g}R=8\pi\kappa_G(1-h) \tag{2}$$

with h being the genus number of the surface Γ_h.

We then restrict our attention to the system with a liquid-crystalline structure, that is, the trace of the strain tensor defined by[10] $u_{ab}=(\partial_a x^\mu\partial_b x^\mu-g_{ab})/2$ vanishes. This corresponds to imposing an *Ansatz* for the Lagrange multiplier,

$$\lambda^{ab}=\lambda g^{ab} \ , \tag{3}$$

and to assume the world-sheet metric to be flat:

$$g_{ab}=\hat{g}_{ab} \ , \tag{4}$$

where $\hat{g}_{ab}$ depend only on the Teichmuller parameters. For a cylinder $\hat{g}_{ab}$ in (4) take the form

$$\hat{g}_{ab}=\begin{bmatrix}1 & 0\\ 0 & \tau_2^2\end{bmatrix}, \quad 0\le\tau_2<\infty \ . \tag{5}$$

We now evaluate the path integral of the model on a cylinder. Since the elastic modulus κ_G increases with distance at low temperature[8,11] and we are interested only in the large-distance behavior of the system, we expect κ_G to be large in the region under consideration. Therefore, contributions from changes of topologies to the path integral will be suppressed for sufficient low temperature.

We use a mean-field approximation to the Lagrange multiplier:

$$\lambda=\langle\lambda\rangle=\lambda_0 \ . \tag{6}$$

Ansätze (3) and (4) impose a liquid-crystalline order in our model while *Ansatz* (6) omits the quantum fluctuations of the λ field. (The quantum fluctuations of the λ field have been considered in Ref. 12. It turns out that these fluctuations only play a role in determining the subleading behavior of the path integral. They would not alter the results obtained in this Brief Report.)

The path integral on a cylinder can then be evaluated in a standard fashion.[12,13] Using ζ-function regularization, we find

$$Z_{\rm cylin}=\left[\frac{\lambda_0}{\alpha_0}\right]^{d/2}\prod_\mu L^\mu\int_0^\infty\frac{d\tau_2}{2\tau_2}(2\pi\tau_2)^{-d/2}\left|\prod_{n=1}^\infty(1-e^{-2\pi n\tau_2})\right|^{2-d}\left|\prod_{n=1}^\infty(1-e^{-2\pi\tilde{n}\tau_2})\right|^{2-d}(1-e^{-\tau_2 a})^{2-d}e^{-\sigma_{\rm eff}A}\,, \tag{7}$$

where $A=\tau_2\beta^2/2$, $a^2=\lambda_0\beta^2$, $\tilde{n}=\sqrt{n^2+a^2/4\pi^2}$,

$$\sigma_{\rm eff}=\sigma_0-\frac{\lambda_0}{\alpha_0}+\frac{(d-2)\lambda_0}{8\pi}\left[1+\ln\frac{\Lambda^2}{\lambda_0}\right]-\frac{\pi(d-2)}{6\beta^2}[1-6I(a)]+\frac{\sqrt{\lambda_0}(d-2)}{\beta}\,, \tag{8}$$

and

$$I(a)=4\int_0^\infty\frac{dy}{1-e^{2\pi(y+a/2\pi)}}y^{1/2}(y+a/\pi)^{1/2}=-\frac{a}{\pi^2}\sum_{n=1}^\infty\frac{1}{n}K_1(na) \tag{9}$$

with $K_1(na)$ being the modified Bessel function.

A remark on our result of the path integral [Eqs. (7)–(9)] is now in order. It differs from our former result, Eq. (2.28) of Ref. 13, by the fact that some important terms in (8) arising from the zero-point transverse fluctuation of the stretched string were missing there. Results similar but not as complete as (8) have been obtained by Pisarski and David and Guitter[14] where the important β-dependent terms in (8) have been overlooked by using the Pauli-Villars and dimensional regularizations. It is worth mentioning that (8) agrees with our most recent result for the effective string tension obtained from a single rigid string with fixed density [Eq. (16) in Ref. 15]. The only difference is that the numerical factor d in (21) of Ref. 15 is replaced by $d-2$ in (8) here. This discrepancy is because a large-d expansion was used in Ref. 15.

The free energy of the system is defined by

$$F(\beta)=-\left[\prod_\mu L^\mu\right]^{-1}Z_{\rm cylin}\,, \tag{10}$$

which can be calculated from the path integral (7) and is then compared with the expression for a collection of free particles. We find[12,13]

$$F(\beta,m^2)=\frac{1}{\beta}\int\frac{d^{d-1}k}{(2\pi)^{d-1}}\ln(1-e^{-\beta\omega_k})=-\int_0^\infty\frac{ds}{s}(2\pi s)^{-d/2}\sum_{r=1}^\infty e^{-m^2s/2-r^2\beta^2/2s}\,, \tag{11}$$

where $s=\alpha_0\tau_2/\lambda_0$ and the mass spectrum is given by

$$m^2(\lambda)=2\pi\frac{\lambda_0}{\alpha_0}\left[\sigma_{\rm eff}\frac{\beta^2}{2\pi}+\sum_{i=1}^{d-2}\sum_{n=1}^\infty(nN^{(1)}_{n_i}+\tilde{n}N^{(2)}_{\tilde{n}_i})+\frac{a}{2\pi}\sum_{i=1}^{d-2}N^{(3)}_i\right]. \tag{12}$$

From the mass spectrum (12), it is easy to read off the ground state

$$m_0^2=a^2\sigma_{\rm eff}/\alpha_0\,. \tag{13}$$

We immediately see that the theory is free of tachyons if

$$\sigma_{\rm eff}=0\,, \tag{14}$$

which can be satisfied if the mean-field solutions[15] are achieved:

$$\alpha_0=\frac{8\pi/(d-2)}{\ln\Lambda^2/\lambda_0}\,, \tag{15}$$

$$\lambda_0+\frac{8\pi}{\beta}\sqrt{\lambda_0}=-\frac{8\pi\sigma_0}{d-2}+\frac{4\pi^2}{3\beta^2}[1-6I(a)]\,. \tag{16}$$

From (15), we see that λ_0 has an interpretation of the inverse-squared persistence length. Using the limit value of $I(a)$ in (9), we see from (16) that λ_0 approaches zero as the temperature reaches its critical value from above

$$\lambda_0\underset{T\to T_c^+}{\sim}1-T_c^2/T^2\,, \tag{17}$$

where

$$T_c=\left[\frac{3\sigma_0}{\pi(d-2)}\right]^{1/2}. \tag{18}$$

As explained in Ref. 15, λ_0 remains zero below T_c. Since λ_0 vanishes below T_c and becomes finite above T_c, it can serve as an order parameter to characterize a finite-temperature transition. The critical temperature (18) has been obtained by a number of authors[15,16] although as we see presently that it may be associated with transitions with different natures. In our case, the critical temperature (18) is related to a smooth-rough transition which is analogous to a smectic-nematic transition in liquid crystals.

We identify this transition as the deconfinement transition in QCD. The reason is the following. First, the smooth (smectic) phase corresponds to the confined phase since both are characterized by a nonvanishing string tension while the rough (nematic) phase corresponds to the deconfined phase since both are characterized by a vanishing string tension. The resemblance between these two transitions becomes more close if we look at the symmetry breaking associated with these transitions. The model in the smooth phase has a local SU(∞) symmetry which is known as the area-preserving symmetry existing in the Dirac membrane in the light-cone gauge.[17] This is not hard to prove. Under an infinitesimal general motion (both tangential and normal motion) of the string sheet, the dilation is given by[18] $\delta\phi = \nabla_a \delta\epsilon_a + 2nH$ (H is the mean curvature of the sheet). In the smooth phase $\lambda_0 \to 0$, $\alpha_0 \to 0$, and $1/\alpha_0 \to \infty$ which implies $H \to 0$. Therefore, the requirement of fixed density in the smooth phase implies $\delta\phi = \nabla_a \delta\epsilon_a = 0$. All the solutions of $\delta\epsilon_a$ form a group which is the area-preserving group SU(∞) (Ref. 17). We here emphasize that in the case of the CILM the area-preserving symmetry is a true symmetry of the system in the smooth phase which has nothing to do with any gauge chosen, as is the case for the Dirac membrane. The smooth-rough (smectic-nematic) transition associates precisely with the breaking of this symmetry or its center group O(2) or U(1) which is equivalent to $Z(N_c)$ as N_c becomes large,[1] which associates with the deconfinement transition in QCD.

What is the transition order then? A smooth-rough transition can be second order[15] while the deconfinement transition in QCD is possibly first order.[1,2] So far we have only considered the genus-one surface contribution to the free energy. However, as argued by Atick and Witten,[19] the breaking or melting of strings may make holes in Riemann surfaces. This change in topology contributes to the free energy in the deconfined phase. Since we expect κ_G to be large up to the temperature $T_{\rm dec}$, the dominant contribution of changing topology is, as denoted by $\Sigma(T)$,

$$\Sigma(T) = \begin{cases} -8\pi\kappa_G & \text{as } T \ge T_c^+ , \\ 0 & \text{as } T \le T_c^- . \end{cases} \tag{19}$$

Adding such a term to the free energy does not change the critical temperature (18) but makes λ_0 discontinuous at the transition:

$$\lambda_0 = \begin{cases} 8\pi\kappa_G T_c^2 & \text{as } T \to T_c^+ , \\ 0 & \text{as } T \to T_c^- . \end{cases} \tag{20}$$

Since λ_0 is the order parameter, its discontinuous at the transition simply means the deconfinement transition to be first order.

As for any stringlike theory, a Hagedorn temperature can be calculated for our model. Following a procedure of calculations parallel to that in Ref. 20, we find

$$\beta_H = \left[1 + \frac{1}{a+1} \right]^{1/2} \left[\frac{(d-2)\pi\alpha_0}{3\lambda_0} \right]^{1/2} . \tag{21}$$

Unlike the deconfinement temperature (18) which is determined from the lowest-mass state of the free energy, the Hagedorn temperature (21) is obtained from the high-mass-level state. Since both α_0 and λ_0 in (21) are variables, β_H can be determined by its extreme value. Using the renormalization-group flow of (15), we find that β_H has a well-defined minimum. From

$$\frac{d}{d\lambda}\beta_H = 0 \quad \text{and} \quad \frac{d^2}{d\lambda^2}\beta_H < 0 \quad \text{with} \quad \beta_H \neq 0 \tag{22}$$

it is easy to find

$$\lambda_0 = \frac{\Lambda^2}{e}, \quad \alpha_0 = \frac{8\pi}{d-2}, \quad \text{and} \quad \beta_H \approx \left[\frac{8\pi^2 e}{3\Lambda^2} \right]^{1/2} . \tag{23}$$

It should be mentioned that the saddle-point value of α_0 in (23) coincides with the ultraviolet fixed-point value found by David and Guitter[10] and was associated with a second-order crumpling transition there.

As the Polyakov-Kleinert string,[8] our model is characterized by two parameters: the string tension at zero temperature σ_0 and the stiffness α^{-1}. Since α^{-1} is dimensionless and asymptotically free, the so-called dimensional transmutation occurs: the dimensionless $\alpha^{-1}(\beta)$ and the dimensional scale Λ can be traded with each other. This can be seen explicitly from (21) and (23). It is easy to convince ourselves that

$$T_H > T_c \quad (T_c = T_{\rm dec}) \tag{24}$$

since λ_0 is an increasing function with T [see Eq. (16)] and $\lambda_0 = \Lambda^2/e$ is the maximum value at which β_H achieves its minimum. So that, $\lambda_0 = \Lambda^2/e > 8\pi\kappa_G T_c^2$ [see Eq. (2)]. This proves Eq. (24).

A discussion on the physical implication of our result (21)–(24) is now in order. The main result found in this Brief Report is that there exist two phase transitions in the model: the smooth-rough (smectic-nematic) transition and the Hagedorn transition with the critical temperature $T_H > T_c$. This differs from that obtained from the Nambu-Goto string[19,21] and the Polyakov-Kleinert string[16,20,22] where only one transition is present. The reason for this is not hard to find. In the case of the Nambu-Goto string, only one parameter σ_0 is involved while in the case of the Polyakov-Kleinert string where two parameters σ_0 and α_0 are involved, the transitions are related to tachyonic singularities.[16,20,22] Such transitions related with tachyons must coincide. Since if $T_H \neq T_c$, suppose $T_H > T_c$, there exists a region such that $T_H > T > T_c$ within which the theory is ill defined due to the tachyonic singularity. This explains why only one transition is allowed to exist in these string models. In the present model, however, none of the transitions are related with tachyons. Beyond T_c [e.g., Eq. (18)], $\sigma_{\rm eff}$ remains zero rather than becomes negative [e.g., Eqs. (8) and (14)]. This is because the condensate of the Lagrange multiplier which compensates the influences of thermal fluctuations. As is well known, such a rough phase with $\sigma_{\rm eff}=0$ is a desired phase for fluid membranes.[23] In the case of QCD strings, though the string is melted ($\sigma_{\rm eff}=0$) by thermal fluctuations in this phase, the instantons may still exist and play an important role.[4,5] It is the existence of such a phase which is the major motivation for us to call the model as the crystalline-instanton-liquid model. (We remind the reader that, though $\sigma_{\rm eff}=0$ in the rough phase, both the bending rigidity $1/\alpha$ and the dilation modulus λ_0/α take finite values, that is, something stringy survives in this phase. More explicitly, the mass spectrum (12) shows that the system contains a whole set of massive excited states in this phase though the ground-state mass remains zero.

From (22), we see that $d\lambda/d\beta|_{\beta=\beta_H}\rightarrow\infty$, which means that λ_0 jumps at $T_H^\dagger$. Since the theory has a cutoff Λ, we expect that λ_0 jumps from Λ^2/e to Λ^2 at $T_H^\dagger$. This in turn means that $1/\alpha$ jumps from $(d-2)/8\pi$ to zero at $T_H^\dagger$. We thus see that λ_0/α_0 can serve as an order parameter to characterize the Hagedorn transition associated with the O(4)-symmetry breaking and unbreaking. Above T_H, the hadrons enter the isotropic phase or the crumpling phase with the Hausdorff dimension infinite where the entropy dominates. This is the expected phase of quark-gluon plasma. In this phase, the chiral symmetry is restored due to isotropic orientation of the instantons or instanton molecules.[5] [We remind the reader that due to the special property of instantons, the orientation of the instantons in the color space is identical to that in space at least for $SU_c(2)$.] We then identify the Hagedorn transition with the chiral transition in QCD which is also analogous to the nematic-isotropic transition in liquid crystals. Since the order parameter λ_0/α_0 jumps at T_H, the transition is of first order.

It is also interesting to note the following isomorphism:

$$O(4)\sim\frac{SU(2)\times SU(2)}{Z_2}\ . \tag{25}$$

This relation implies that our CILM describes effectively QCD with two light quarks (i.e., u and d quarks).

Finally, we make some numerical estimates of our results. Taking $\sigma_0=(400\ \text{MeV})^2$ and $T_H=M_{\text{constituent quark}}=340$ MeV, we obtain $T_{\text{dec}}\approx 276$ MeV and $\Lambda\approx 2.88\times 10^3$ MeV.

Our conclusion is that both the CSR and deconfinement transitions exist in QCD and both are connected and of first order with $T_{\text{ch}}>T_{\text{dec}}$. We have shown in our CILM that both instantons and strings are very useful concepts in describing QCD vacuum or hadronic structure. In fact, they are nothing but the simple collective coordinates which characterize the extremely complicated (inhomogeneous) QCD vacuum or the hadronic structure in different (temperature) phases or spatial regions. Our results qualitatively agree with most of the phenomenology available for QCD.

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