Black holes as possible sources of closed and semiclosed worlds

V. P. Frolov
Instituto de Astronomia y Fisica del Espacio, Buenos Aires, Argentina
and P.N. Lebedev Physical Institute, Moscow, Union of Soviet Socialist Republics

M. A. Markov
Institute for Nuclear Research and P.N. Lebedev Physical Institute, Moscow, Union of Soviet Socialist Republics

V. F. Mukhanov
Institute for Nuclear Research, Moscow, Union of Soviet Socialist Republics
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The internal structure of spacetime inside a black hole is investigated on the assumption that some limiting curvature exists. It is shown that the Schwarzschild metric inside the black hole can be attached to the de Sitter one at some spacelike junction surface which may represent a short transition layer. The method of massive thin shells by Israel is used to obtain the characteristics of this layer. It is shown that instead of the singularity the closed world can be formed inside the black hole. It is argued that this property of our model may also be valid in a more general case provided the gravitation theory is asymptotically free and the limiting curvature exists. After passing the deflation stage the closed world in the black-hole interior may begin to inflate and give rise to a new macroscopic universe. The described model may be considered as an example of the creation of a closed or semiclosed world "in the laboratory." The possible fate of the evaporating black hole is also briefly discussed.

I. INTRODUCTION

One of the fundamental problems in classical general relativity is the problem of singularities which inevitably arise in the theoretical description of a massive body (or total Universe) collapse (see, e.g., Refs. 1 and 2). It is generally believed that the appearance of these singularities is usually accompanied by an unlimited increase of spacetime curvature. The singularities in general relativity are usually considered as "a disease" of this classical theory which may be "cured," e.g., by its quantization.[3] There are some reasons for this point of view. It is evident that the classical Einstein equations are not applicable at high curvatures because the quantum corrections to these equations at the Planck curvature become of the same order as the main terms in the equations for the gravitational field. The true effective equations including these "corrections" are still unknown and one may only guess the possible general properties of these equations and their possible consequences. If we believe that the singularities are an artifact of Einstein's theory and they will not be present in the future complete theory it is also reasonable to assume that the curvature for any solution for this theory must be restricted by some maximum limiting value.

The following more general argument is also in favor of this assumption. The quantum fluctuations of a metric at Planck scales $l_{\rm Pl}=(G\hbar/c^3)^{1/2}\sim 10^{-33}$ cm may lead to a radical change of the usual concept of spacetime at high curvatures. It may happen that the notions of time, length, curvature, and so on, lose their sense under these conditions.[3,4] In this case the time intervals less than the Planckian one $t_{\rm Pl}\sim l_{\rm Pl}/c\sim 10^{-43}$ s, the corresponding lengths and extremely high curvatures (higher than the Planckian curvature $\sim 1/l_{\rm Pl}^2$) have no physical meaning. In order to be able to use the habitual notions of space and time one must operate with the effective metric which arises as a result of averaging over scales larger than $l_{\rm Pl}$. It is natural to expect that the curvature for this effective metric will never exceed the Planckian curvature $\sim 1/l_{\rm Pl}^2$. Of course we do not know yet the exact modified equations for this effective metric and cannot verify this assumption. But we can accept this assumption as a hypothesis and investigate its possible consequences.[5]

If we accept this hypothesis the following question inevitably arises. If spacetime curvature is restricted and curvature singularities do not arise, what is a black-hole interior structure? The aim of this paper is to analyze the possible spacetime structure inside a black hole in the framework of the hypothesis on the existence of limiting curvature. We do not suggest any concrete choice of field equations but analyze the general properties of spherically symmetric metrics which may describe the black-hole interior under the assumption that this hypothesis is valid. Our main result is the demonstration that a closed (or semiclosed) world may arise instead of the singularity inside the black hole. This world may give birth to a new expanding macroscopic universe.[6] We now briefly describe the main features of such models.

We consider a black hole which arises as a result of a spherically symmetric gravitational collapse. In the framework of standard 3+1 splitting, the spacetime inside the black hole out of the collapsing matter can be described as an evolution of anisotropic homogeneous three-dimensional space. According to the vacuum Einstein equations the contraction of this space in two directions is accompanied by expansion in the third direction so that spacetime has Kasner-type asymptotic behavior near the singularity. Such behavior is a consequence of the classical vacuum Einstein equations which are valid until the spacetime curvature becomes comparable to the Planckian one. Particle creation and vacuum polarization may change this regime. Quite general arguments allow one to assume that the quantum effects may result in a decrease of the spacetime anisotropy.[7] Poisson and Israel[8] have shown that the anisotropy may be damped during this phase of contraction while the curvature tensor remains of the order of the Planckian value. Unfortunately one cannot hope to prove this result rigorously without knowing the physics at the Planck curvatures. In our model we just assume that the isotropization really takes place and the time required for this process is short and is comparable to the Planck time $\tau_{\rm Pl} \sim l_{\rm Pl}/c$. In order to make our consideration more concrete we assume also at first that the effective equation of state in such a resulting isotropic space of the Planck curvature coincides with the vacuumlike equation of state: $T_\mu^\nu \sim \Lambda\delta_\mu^\nu$, $\Lambda \sim l^{-2}$. This equation of state is generally covariant. It is the simplest possible one violating the energy-dominance condition and hence makes it possible to escape singularities. It is interesting to note, for example, that this de Sitter-type behavior corresponding to the vacuumlike equation of state arises naturally in the consideration of Poisson and Israel.[8]

Under the described assumptions we show that the de Sitter-type world at the stage of its deflation arises inside the black hole. It should be stressed that the described possibility of the junction of the de Sitter-type interior through the transition layer to the Schwarzschild-Kruskal geometry looks rather remarkable and was not considered earlier. The described model may be considered as an example of the "new Universe creation in the laboratory." This example does not contradict the results by Farhi and Guth[9] because their main assumptions are violated in our model. Our consideration shows that a formation of a black hole can be accompanied by the creation of baby universes without violation of any known fundamental physical laws.

For this particular simple model with the de Sitter-type interior one can trace the details of the future evolution of the closed (or semiclosed) world arising inside the black hole which after the stage of deflation may begin inflating. It is important to stress that the main features of the model (e.g., the existence of a closed or semiclosed world in the black-hole interior) are not directly connected with the assumption about a de Sitter-type equation of state at the limiting curvature and may be valid in more general situations. Some more general models are also discussed in this paper.

The paper is organized as follows. We begin by considering the general properties of spherically symmetric spacetimes and by giving a more accurate formulation of our "limiting curvature" hypothesis in such spaces (Sec. II). In the framework of this hypothesis we discuss the possible structure of the spacetime inside an eternal nonrotating black hole. Although the model of the eternal black hole (i.e., the black hole which was not formed by gravitational collapse but exists forever) is oversimplified, it appears to be useful when considering more realistic (but more complicated) situations. A more realistic case of a black hole which arises as a result of the gravitational collapse is considered in Sec. III. Section IV is devoted to the discussion of possible final states of evaporating black holes. The "future evolution" of the black-hole interior and the possibility of creation of a new macroscopic Universe inside a black hole are discussed in Sec. V. It also contains some general remarks concerning the main features of our model. The formulas of the thin-massive-shells approach used in the paper and the details of calculations are collected in Appendixes.

In this paper we use the sign conventions of Misner, Thorne, and Wheeler[3] and Planck's units.

II. ETERNAL BLACK-HOLE INTERIOR

A. Limiting-curvature hypothesis in a spherically symmetric spacetime

We restrict ourselves by considering spherically symmetric black holes. It is instructive at first to discuss the case of an eternal black hole and later to consider a more realistic situation when the black hole arises as a result of a gravitational collapse. The conformal Penrose diagram for the spacetime of such an eternal black hole is shown in Fig. 1. We assume that the mass m of the black hole is large ($m \gg m_{\rm Pl} \sim 10^{-5}$ g) and is invariant in time. In order to suppress the change of the mass due to Hawking radiation one may assume that the black hole is surrounded by a thermal bath, the temperature of which coincides with the black-hole temperature.

The metric of a static (with the Killing vector $\xi = \xi^\mu \partial_\mu = \partial_t$) spherically symmetric spacetime can be written as

$$ds^2 = -g(r)^{-1}dr^2 + f(r)dt^2 + r^2 d\omega^2$$
$$= \epsilon[-d\tau^2 + F(\tau)dt^2] + r^2(\tau)d\omega^2 , \quad (2.1)$$

where $d\omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$ is a line element on a unit sphere and

$$g(r) \equiv \epsilon|g(r)| = -\nabla r \cdot \nabla r, \quad f(r) = \xi^2 ,$$
$$d\tau = -|g(r)|^{-1/2}dr, \quad \epsilon F(\tau) = f(r(\tau)) . \quad (2.2)$$

One can verify that the Ricci tensor $R_\mu^{\ \nu}$ for this metric is diagonal $R_\mu^{\ \nu} = {\rm diag}(R_0^{\ 0}, R_1^{\ 1}, R_2^{\ 2}, R_2^{\ 2})$, and obeys the inequality

$$4R_\mu^{\ \nu}R_\nu^{\ \mu} - R^2 \equiv (2R_2^{\ 2} - R_0^{\ 0} - R_1^{\ 1})^2 + 2(R_0^{\ 0} - R_1^{\ 1})^2 \geq 0 . \quad (2.3)$$

The left-hand side of Eq. (2.3) vanishes if and only if

$$R_\mu^{\ \nu} = \Lambda\delta_\mu^\nu . \quad (2.4)$$

For a particular case of a spherically symmetric spacetime our main hypothesis about the existence of the limiting curvature may be presented in the form

$$\mathcal{R}^2 \equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \le \alpha l^{-4} , \quad (2.5)$$

where l is the characteristic (Planckian) length and α is a dimensionless parameter comparable to one. One can verify that Eq. (2.5) implies that the other possible quadratic-in-curvature invariants $C^2 \equiv C_{\alpha\beta\gamma\delta}C^{\alpha\beta\gamma\delta}$, $R^{\nu}_{\mu}R^{\mu}_{\nu}$, and R^2 are also positive and limited.

In order to make our consideration more concrete we assume at first (and this is our second hypothesis) that when the curvature reaches its maximum value the equation of state becomes of the vacuumlike type (2.4) or equivalently $R^2 = 4R^{\mu}_{\nu}R^{\nu}_{\mu}$. (A more general model for which this assumption is violated will be considered later in Sec. II D.)

Under these two hypothesis the metric (2.1) describing the spacetime of an eternal black hole allows the following specification. The Schwarzschild metric [i.e., Eq. (2.1) with $g(r)=f(r)=2mr^{-1}-1$] can be used to approximate the geometry for $r \gtrsim r_0$ where

$$r_0 = (12/\alpha)^{1/6}(2m/l)^{1/3}l \quad (2.6)$$

is the value of the radius r at which the invariant $\mathcal{R}^2 = 48m^2r^{-6}$ for the Schwarzschild metric reaches its limiting value αl^{-4}. For $2m \gg l$ one has $l \ll r_0 \ll 2m$. The inequality $r_0 \gg l$ indicates that when describing the geometry of spacetime at $r \lesssim r_0$ (up to $r \sim l$) one may use the classical metric $g_{\mu\nu}$ and neglect its quantum fluctuations. The inequality $r_0 \ll 2m$ means that the surface Σ_0 (where $r=r_0$) is spacelike. It lies inside the event horizon (see Fig. 1) and has a topology $S^2 \times R^1$; i.e., it is an infinite (in direction t) "tube" of a radius r_0. Strictly speaking in order to describe the geometry of the spacetime out and inside the event horizon (including the region near Σ_0) one must use the Kruskal-type analytical continuation of the Schwarzschild metric. But if we are interested in a description of the metric only in the vicinity of Σ_0 it is also possible (and much more convenient for our purpose) to use the Schwarzschild-type element (2.1)

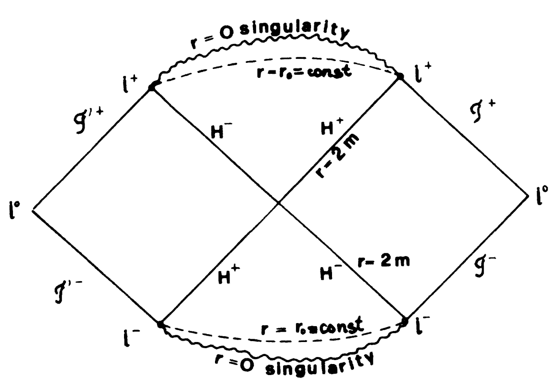

FIG. 1. Conformal diagram for the spacetime of an eternal black hole.

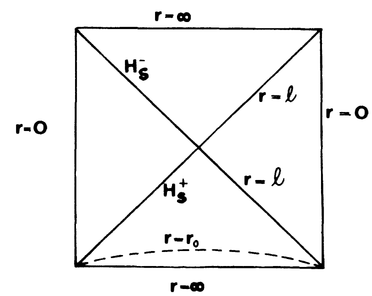

FIG. 2. Conformal diagram for de Sitter spacetime.

with $g=f>0$ so that r is a timelike coordinate. It is worthwhile noting that for the case of a black hole inside the thermal bath the relation $2m \gg l$ implies also that the change of the geometry due to the presence of the thermal radiation at $(2m/l)^2 l \gg r \ge r_0$ can be neglected. As for the future evolution of geometry for $\tau > \tau_0$ ($r < r_0$) we cannot specify two unknown functions in Eq. (2.1) until we know the exact field equations. Nevertheless our second hypothesis guarantees that beginning with some time moment $\tau_1 > \tau_0$ ($r_1 < r_0$) we can approximate these field equations by Eq. (2.4). In the case of a spherically symmetric spacetime it means that the geometry is described by the de Sitter metric which can be written in the form (2.1), with $g(r)=f(r)=(r/l)^2-1$, where $l=(\Lambda/3)^{-1/2}$. Thus the geometry of this region coincides with a part of the complete de Sitter space. The standard conformal Penrose diagram for the complete de Sitter spacetime is shown in Fig. 2. If we assume that the length parameter l in the de Sitter solution coincides with the parameter l in (2.5) then we would have $\alpha = 24$.

B. Transition layer and "massive thin shell" approximation

In the general case the global structure of the spacetime under consideration may depend on details of the evolution in the transition layer $\tau_0 < \tau < \tau_1$. In the particular case when the duration $\Delta\tau = \tau_1 - \tau_0$ of this transition regime is short ($\Delta\tau \sim l$) only some of its integral characteristics become important. In the latter case one may consider this layer as a "thin massive shell" and to sew the Schwarzschild metric ($\tau < \tau_0$) with the de Sitter one ($\tau > \tau_1$) using an approach developed by Israel.[10] (This method is discussed in detail in Ref. 3; see also Appendix A of this paper where the necessary formulas are collected.) It should be stressed that one of the main features of our problem is using a spacelike "shell" to describe the geometry evolution during the transition regime; meanwhile, timelike and null shells representing the motion of some real matter are usually considered (see, e.g., Ref. 11).

According to the "thin shell" approach we assume that $r_1 = r_0$ and consider Σ_0 ($r=r_0$) as a junction surface which separates the Schwarzschild and de Sitter geometries. The junction conditions at this surface require that (i) the three-geometries induced on Σ_0 by both (Schwarzschild and de Sitter) four-geometries are identical and (ii) the jumps $[K_n^m]$ of the external curvature K_n^m $(m,n=1,2,3)$

$$[K_n^m] \equiv (K_n^m)_{\text{de Sitter}} - (K_n^m)_{\text{Schwarzschild}} \tag{2.7}$$

obey the relation

$$[K_n^m] - \delta_n^m [K_k^k] = -8\pi S_n^m , \tag{2.8}$$

where

$$S_n^m = \int_{\tau_0}^{\tau_1} d\tau\, T_n^m . \tag{2.9}$$

The tensor T_μ^ν is the effective energy-momentum tensor which is defined in the transition layer as the right-hand side of the modified gravitational field equations written in the Einstein-type form

$$G_\mu^\nu \equiv R_\mu^\nu - \tfrac{1}{2}\delta_\mu^\nu R = 8\pi T_\mu^\nu . \tag{2.10}$$

The junction condition (i) can be easily met because the forms of the Schwarzschild and de Sitter metrics in the (t,r,θ,ϕ) coordinates are identical. The junction condition (ii) gives (see Appendix B)

$$S_t^t = \lambda/4\pi ,$$
$$S_\theta^\theta = S_\phi^\phi = (\kappa+\lambda)/8\pi , \tag{2.11}$$
$$S_r^r = 0 ;$$

$$\kappa \equiv [K_t^t] = \frac{r_0}{l^2}\left[\left(\frac{r_0}{l}\right)^2 - 1\right]^{-1/2} + \frac{m}{r_0^2}\left[\frac{2m}{r_0} - 1\right]^{-1/2} \tag{2.12}$$

$$\lambda \equiv [K_\theta^\theta] = \frac{1}{r_0}\left\{\left[\left(\frac{r_0}{l}\right)^2 - 1\right]^{1/2} - \left[\frac{2m}{r_0} - 1\right]^{1/2}\right\}.$$

For $2m \gg l$ these relations read

$$\kappa \simeq l^{-1}(1+\beta/2) ,$$
$$\lambda \simeq l^{-1}(1-\beta) , \tag{2.13}$$
$$\beta \equiv (\alpha/12)^{1/4} .$$

It is worthwhile noting that the "large parameter" $2ml^{-1}$ does not enter these relations so that $S_m^n \sim l^{-1}$ and hence there is no contradiction with our assumption that the proper time interval $\Delta\tau$ of the transition is short. Undeed if we suppose that T_ν^μ in the transition layer reaches the Planckian value $(T_\nu^\mu \sim l^{-2})$ then the proper time duration of this layer can be estimated as $\Delta\tau \sim S_n^m / T_n^m$ and it is comparable with the Planckian time l.

It is interesting to note that the expressions (2.12) are greatly simplified for a particular choice $\alpha = 12$. In this case,

$$\lambda = 0, \quad \kappa = \frac{3}{2l}\frac{\gamma}{\sqrt{\gamma^2 - 1}}, \quad r_0 = \gamma l , \tag{2.14}$$

where $\gamma = (2m/l)^{1/3}$. Although this choice is rather natural we cannot exclude other values for α. That is why in what follows we shall not specify the values of α.

C. Properties of the model. Deflation

The conformal Penrose diagram for the spacetime under consideration is shown in Fig. 3. It can be obtained from diagrams shown in Figs. 1 and 2 by removing the part $r > r_0$ (for the de Sitter space) and the part $r < r_0$ (for the eternal black hole) and by subsequent gluing diagrams together along the hypersurface Σ_0 $(r = r_0)$. For convenience the freedom of the choice of the Penrose conformal coordinates for the spacetime of the eternal black hole (see Fig. 1) is used in order to guarantee the same "coordinate form" of Σ_0 in this diagram as it has in the diagram for the de Sitter space (Fig. 2).

Now let us consider the properties of the spacetime in our model in more detail. First of all it should be noted that the spacelike surface Σ_0 is located in the T_- region: i.e., in the region where $\nabla r \cdot \nabla r < 0$ (it is also known as the domain of trapped surfaces[12]). In this region the radial coordinate r is decreasing along any future-directed causal curve. For $r \ll 2m$ the Schwarzschild metric can be approximated as

$$ds^2 \simeq -d\tau^2 + (-3\tau/4m)^{-2/3} dt^2 + (-3\tau/4m)^{4/3}(2m)^2 d\omega^2 . \tag{2.15}$$

It describes the Kasner-type contraction of spacetime inside the black hole. As the proper time $\tau(<0)$ grows the radius $r \sim (-\tau)^{2/3}$ decreases, while the scale along the t direction increases as $(-\tau)^{-1/3}$. After passing the surface Σ_0 (representing the transition layer) this anisotropic contraction turns into the isotropic de Sitter contraction. Using the relation $r = l\cosh(-\tau/l)$ one can rewrite the de Sitter metric in the form

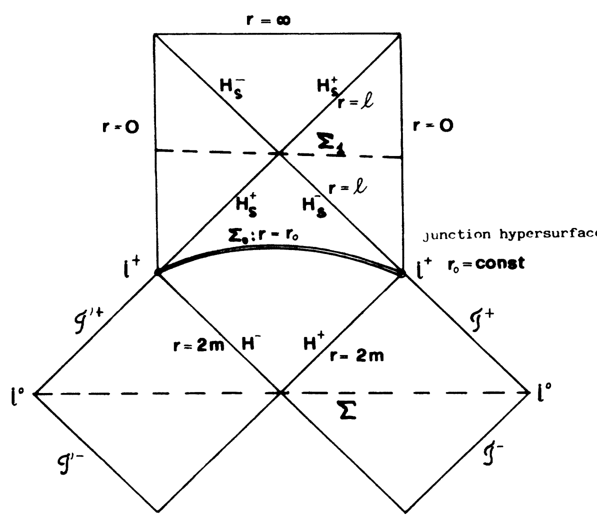

FIG. 3. Conformal diagram for an eternal black hole with the de Sitter-type world in its interior. The surface Σ_0 where $r = r_0 = \text{const}$ is a junction surface which represents a short transition layer. After passing this surface the anisotropic Kasner-type contraction of space inside the black hole is changed into the isotropic de Sitter contraction (deflation). The surface Σ_1 corresponds to the moment of "minimal size" of the de Sitter world. After passing this surface the de Sitter world begins its inflationary expansion.

$$ds^2=-d\tau^2+\sinh^2(-\tau/l)dt^2+l^2\cosh^2(-\tau/l)d\omega^2 . \tag{2.16}$$

At this stage the rate of contraction decreases while the scale parameter characterizing the isotropic contraction is changing as $\sim\exp(-\tau/l)$. This type of evolution is just the opposite to that known in cosmology as inflation. That is why we call it deflation. At the deflation stage the radius r decreases from r_0 to l. The proper time $\tau_0\sim l\ln(2m/l)$ of this evolution is greater than the analogous time in the Schwarzschild spacetime where $\tau_0\sim l$.

The coordinates (τ,t,θ,ϕ) do not cover the complete de Sitter space. The metric (2.16) in these coordinates has a coordinate singularity at $\tau=0$ which corresponds to the null surfaces $H_S^{\pm}$ where $r=l$ and $\nabla r\cdot\nabla r=0$. The spacetime in the future of the surface Σ (see Fig. 3) is regular. It should be stressed that Σ is not a global Cauchy surface. Since $H_S^{\pm}$ are the Cauchy horizons such a global Cauchy surface does not exist at all.

The conformal diagram in Fig. 3 resembles to some extent the maximal analytical continuation of either Reissner-Nordström or Kerr metric in its structure. One of the main differences is that in our case one may expect the stability of the Cauchy horizons $H_S^{\pm}$ while the Cauchy horizons in Reissner-Nordström or Kerr spacetime are shown to be unstable[13] (see also Ref. 14). This instability is related to an infinite blueshift of signals sent into the black hole from external space and registered by an observer crossing the Cauchy horizon. This effect combined with the classical or quantum radiation falling down into a black hole will result in the divergence of T_μ^ν near the Cauchy horizon and its instability. In our case if only the hypothesis about the limiting curvature existence is valid the back reaction of the matter does not allow $\mathcal{R}^2$ (and hence T_μ^ν) to grow without limit and after the curvature reaches its limits we would have the de Sitter-type space. Hence we may expect that in our case there is no such instability.

The T_- region is bounded by the event horizons $H^{\pm}$ and by the Cauchy horizons $H_S^{\pm}$. Beyond the T_- region (at a surface Σ_1) the deflation changes into the de Sitter inflationary expansion. The surface Σ_1 has topology S^3 and in this sense the diagram presented in Fig. 3 describes the closed-world formation inside the black hole. A possible future fate of this world is discussed in Sec. V.

D. Closed-world formation inside the black hole and asymptotic freedom

Before considering the interior of a black hole which arises as a result of a gravitational collapse we make a few general remarks. The model described above is based on the assumption that the equation of state at the limiting curvature is de Sitter type and the transition layer is short. Now we show that our main conclusion about the possibility of the closed-world formation inside the black hole is not directly connected with both these assumptions and may be valid in a more general situation.

Let us write down the function $g(r)\equiv-\nabla r\cdot\nabla r$ which enters the line element (2.1) in the form

$$g(r)=\frac{2m(r)}{r}-1 ,$$

then the field equations (2.10) give

$$\frac{dm}{dr}=-4\pi r^2T_t^t , \tag{2.17a}$$

$$\frac{d}{dr}\ln\left[\frac{f}{g}\right]=\frac{8\pi r}{g}(T_t^t-T_r^r) . \tag{2.17b}$$

The mass function $m(r)$ corresponding to the model with the short transition layer described above can be written as

$$m(r)=m\theta(r-r_0)+\frac{r^3}{2l^2}\theta(r_0-r) . \tag{2.18}$$

It is constant beyond $r>r_0$ and may have jump at $r=r_0$. This jump describes the contribution of the matter in the transition layer to the total mass of the black hole. For the particular choice $\alpha=12$ [see Eq. (2.14)] the mass function is continuous at $r=r_0$. The value $m(r)$ at $r=0$ vanishes. Equation (2.17b) shows that $f=g$ everywhere.

One can easily generalize this model by assuming that $m(r)$ is an increasing smooth function of r with the asymptotic behavior

$$m(r)\sim\begin{cases} r^3/2l^2, & r\ll l ,\\ m, & r\gg r_0 .\end{cases} \tag{2.19}$$

The corresponding function $g(r)$ is schematically shown in Fig. 4. This function possesses two zeros: $r_+\simeq 2m$ and $r_-\simeq l$ and its maximum is at $r=r_1$. If in addition we choose $f(r)=g(r)$ (and hence $T_t^t=T_r^r$) then the geometry of spacetime is uniquely determined. The Penrose conformal diagram for such spacetime is schematically shown in Fig. 5. This diagram in many respects resembles the one given in Fig. 3. The spacetime contains a closed world inside the black hole. After the deflation stage the size of this world may become comparable with l. In the general case this closed world is nonhomogeneous and anisotropic. It may be completely homogeneous and isotropic only if $m(r)=r^3/2l^2$ beginning with some value $\tilde{r}_0(>l)$. If this value $\tilde{r}_0$ is close

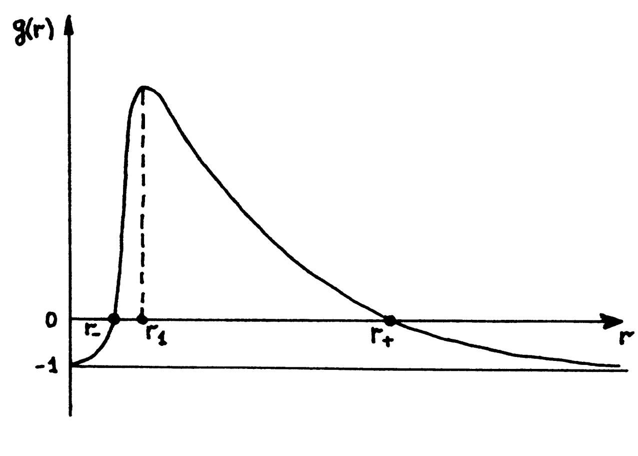

FIG. 4. The function $g(r)=-\nabla r\cdot\nabla r$.

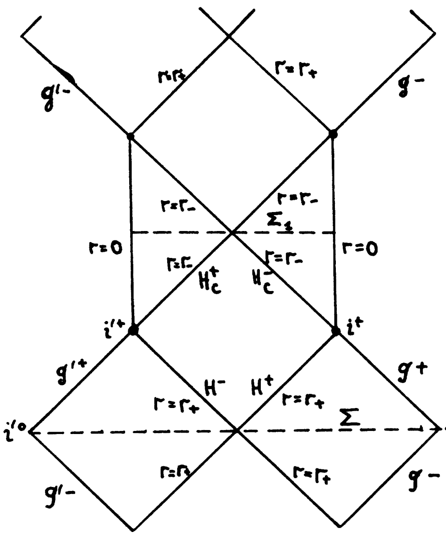

FIG. 5. Conformal diagram for the complete spherically symmetric spacetime which corresponds to the metric (2.1) with $f=g$ for the function $g(r)$ shown in Fig. 4.

enough to r_0 then the solution reproduces all the main features of our model with the short transition layer. It is interesting to note that the mass function obeying conditions (2.19) naturally arises in a simple model considered by Poisson and Israel[8] in which the vacuum-polarization effects are partly taken into account.

The asymptotic behavior $m(r) \sim r^3/2l$ at small distances guarantees that spacetime is locally Euclidean at $r=0$. It means that a regular closed world in the interior of the black hole may arise instead of the singularity only in the case when the mass $m(r)$ at small distances vanishes. The quantity $m(r)$ plays the role of the gravitational charge. Vanishing of the gravitational interaction at small distances is a characteristic property of the so-called asymptotically free theories. The possibility that the gravitation is asymptotically free and that there is no singularity inside the black hole was considered in a number of papers.[15] To summarize our discussion we can conclude that one may expect the formation of a closed world inside the black hole not only in our simple model with a thin transition layer but in a general situation provided the curvature is limited and the gravitation theory is asymptotically free.

After these general remarks we return to our simple model and discuss the problem of a spherically symmetric gravitational collapse.

III. GRAVITATIONAL COLLAPSE AND BLACK-HOLE INTERIOR

In a realistic situation when the black hole is formed by collapsing matter, the spacetime structure differs from that shown in Fig. 3. The difference is due to the matter presence. For the sake of simplicity we consider the spherically symmetric gravitational collapse and assume that the collapsing matter is a homogeneous spherically symmetric dust cloud. According to our assumptions one may use the Einstein equations to describe the evolution of this cloud up to Planck curvatures. The metric inside the matter during this evolution coincides with the metric of a part of the closed Friedmann universe and reads

$$ds^2=a_-^2(\eta_-)(-d\eta_-^2+d\chi^2+\sin^2\chi\, d\omega^2)\,, \tag{3.1}$$

where

$$a_-(\eta_-)=a_0(1-\cos\eta_-) \tag{3.2}$$

and $0\le\chi\le\chi_0<\pi/2$. The parameter $\eta_-=\pi$ corresponds to the maximum expansion of the dust cloud when $a(\pi)=2a_0$. The internal mass of the cloud M is constant during the evolution and is equal to

$$M=\frac{3a_0}{2}(\chi_0-\sin\chi_0\cos\chi_0)\;. \tag{3.3}$$

The external (Schwarzschild) mass m of the cloud is less than M due to the gravitational self-interaction and reads

$$m=a_0\sin^3\chi_0\;. \tag{3.4}$$

The radius of the cloud's boundary evolves according to the law

$$r=a_-(\eta_-)\sin\chi_0\;. \tag{3.5}$$

At the moment of maximal expansion this radius is $r=2a_0\sin\chi_0$. Then the boundary of the cloud begins contracting and after passing the surface $r=2m$ it enters the black hole. The curvature inside the matter grows as

$$\mathcal{R}^2=60\frac{a_0^2}{a_-^6(\eta_-)} \tag{3.6}$$

and at the moment η_-^0 when

$$a_-(\eta_-^0)=\tilde{a}\equiv\left[\frac{60}{\alpha}\right]^{1/6}\left[\frac{a_0}{l}\right]^{1/3} l \tag{3.7}$$

the curvature reaches the limiting value ($\mathcal{R}^2=\alpha l^{-4}$). At this moment the radius of the boundary surface is

$$r=r_0\equiv\left[\frac{15}{\alpha}\right]^{1/6}\left[\frac{2m}{l}\right]^{1/3} l\;. \tag{3.8}$$

It is approximately the same value as r_0 given by Eq. (2.6).

In accordance to our hypothesis we assume that after some short transition layer the geometry in the region occupied by matter would also become de Sitter type. It is convenient to write the de Sitter line element as

$$ds^2=a_+^2(\eta_+)(-d\eta_+^2+d\chi^2+\sin^2\chi\, d\omega^2)\,, \tag{3.9a}$$

where

$$a_+(\eta_+)=l/\sin\eta_+\;. \tag{3.9b}$$

The jump conditions at the junction surface Σ_0 which separates (3.1) and (3.9) can be written in the form (see Appendix C)

$$a_+(\eta_+^0)=a_-(\eta_-^0),\quad S_n^m=-\frac{\lambda}{4\pi}\delta_n^m\ , \tag{3.10}$$

where

$$\lambda=\left[a_+^{-2}\frac{da_+}{d\eta_+}\right]_{\eta_+^0}-\left[a_-^{-2}\frac{da_-}{d\eta_-}\right]_{\eta_-^0}$$
$$=\frac{1}{\tilde a}\left\{\left[2\frac{a_0}{\tilde a}-1\right]^{1/2}-\left[\left[\frac{\tilde a}{l}\right]^2-1\right]^{1/2}\right\}. \tag{3.11}$$

For a particular choice $\alpha=15$ one has $\tilde a=l(2a_0/l)^{1/3}$ and $\lambda=0$. In this case the solutions (3.1) and (3.9) are smoothly sewed together without any "shell." The energy density changes continuously while the curvature invariant $\mathcal{R}^2$ jumps at Σ_0 from $15l^{-4}$ to $24l^{-4}$. For the other values of α the transition layer is needed. For $a_0 \gg l$ we have

$$\lambda \simeq l^{-1}[(\alpha/15)^{1/4}-1]\ . \tag{3.12}$$

As earlier this relation shows that the assumption about the short duration of the transition regime is noncontradictory.

The Penrose conformal diagram for the spacetime of a black hole which arises as a result of the gravitational collapse is shown in Fig. 6. It contains three distinct domains denoted as K, S, and F in which the metric coincides with Schwarzschild-Kruskal, de Sitter, and Friedmann metrics, correspondingly. The jump conditions at the junction surface FS separating the Friedmann and de Sitter domains are given by Eqs. (3.10) and (3.11). The jump conditions at the surface KS outside the collapsing matter were considered in the previous section [Eqs. (2.11) and (2.12)]. The boundary FK between the dust and vacuum is regular and does not require any "shells." The corresponding jump conditions at FK lead to the relations (3.4) and (3.5). The boundary of the T_- region outside the matter coincides with the part of the event horizon H^+ and with the Cauchy horizons $H_S^\pm$. It is nonstatic inside the matter. This boundary is shown in Fig. 6 by the solid line.

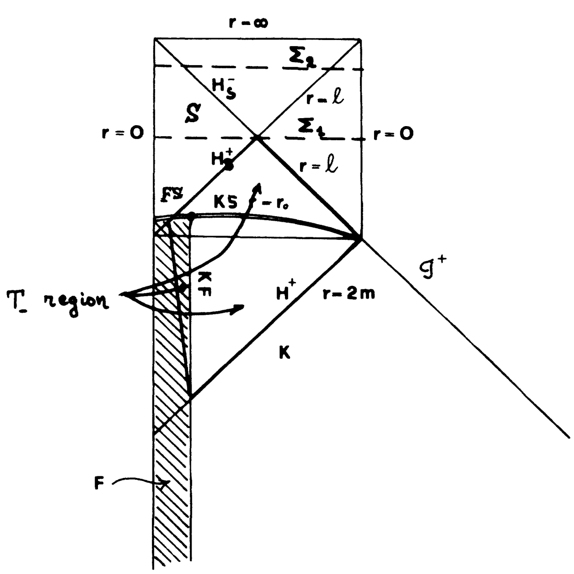

FIG. 6. Conformal diagram for the spacetime of a black hole formed by a collapsing spherically symmetric dust cloud. The domains corresponding to the Schwarzschild-Kruskal, Friedmann, and de Sitter solutions are denoted by K, F, and S, respectively. The boundaries FK, FS, and KS which separate these domains are also shown.

IV. FINAL STATE OF EVAPORATING BLACK HOLE

Let us discuss now what happens when the mass of a black hole decreases due to the process of its quantum evaporation. The radiation of energy to infinity in this process is accompanied by the negative-energy flux through the horizon into the black hole. As a result the mass of the black hole decreases. The exact solution of the gravitation equations describing this process is not known. In order to describe the black hole with variable mass we shall use the Vaidya metric[16] which we write in the form

$$ds^2=f\,dv^2+2\,dv\,dr+r^2d\omega^2\ , \tag{4.1}$$

$$f\equiv-\nabla r\cdot\nabla r=2m(v)r^{-1}-1\ . \tag{4.2}$$

Here v is the advanced time coordinate (at infinity $f=-1$ and $v=t+r$). This metric is a solution of Einstein's equations for the energy-momentum tensor:

$$T_{\mu\nu}=\frac{1}{4\pi r^2}\frac{dm}{dv}v_{,\mu}v_{,\nu}\ . \tag{4.3}$$

Such an energy-momentum tensor describes a spherically symmetric flow of radiation into the black hole. When $dm/dv<0$ the energy density of this flow is negative. Of course the metric (4.1), (4.2) is not the solution for a spacetime of an evaporating black hole. Nevertheless, it is often used as a model metric to describe the decrease of the mass during the evaporation process (see, e.g., Ref. 17).

When the black-hole mass is variable the spacetime structure may differ from that described in the previous section. According to our model this spacetime can be obtained by attaching the Vaidya metric (4.1), (4.2) to the de Sitter one. As earlier the junction surface Σ_0 is defined by the condition $\mathcal{R}^2=\alpha l^{-4}$. The invariant $\mathcal{R}$ for the Vaidya metric takes the simple form

$$\mathcal{R}^2=\frac{48m^2(v)}{r^6} \tag{4.4}$$

and the equation of Σ_0 is

$$r=r_0(v)\equiv\left[\frac{12}{\alpha}\right]^{1/6}\left[\frac{2m(v)}{l}\right]^{1/3}l\ . \tag{4.5}$$

The jump conditions and the parameters of the "thin massive shell" at Σ_0 are given in Appendix B.

The junction surface Σ_0 lies inside the apparent horizon ($f=0$) until the advanced time reaches the value v_1 defined by the condition $r_0(v_1)=\beta^{-1}l$, where $\beta=(\alpha/12)^{1/4}$. If the evaporation ends before this time the resulting possible structure of spacetime is qualitatively the same as presented in Fig. 6 or as given in Fig. 7.

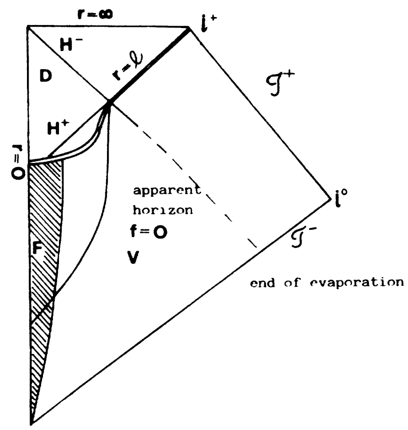

FIG. 7. Conformal diagram for a possible spacetime structure in the case when after the end of a black-hole evaporation there is a stable remnant with the mass $m_{\min} = l/2$.

In this case the black hole of a minimum possible mass $m_{\min} \geq l/2$ remains ("maximon"[18] or "elementary black hole"[19]). If the final mass $m_{\min}$ is smaller than $l/2$ then there arises a version of a "semiclosed" world. It is necessary to stress that the "massive thin shell" approach in such a situation becomes questionable and one must treat the results obtained in the framework of this approach with cautions. If stable "maximons" do not exist then one may expect that the remnant of a black hole may just disappear at the final stage of evaporation. This pure quantum effect would change the topology of space and hence it does not allow the regular classical description.

V. FATE OF BLACK-HOLE INTERIOR

Now we briefly discuss the possible fate of the de Sitter world which according to our model may be present in the interior of a black hole. First of all it should be noted that de Sitter space is usually unstable.[20] If the hypothesis about the limiting curvature existence is valid then such an instability at the stage of deflation might be suppressed. There is a possibility that at the end of deflation when the closed world has Planckian dimensions it can just disappear in the process of quantum annihilation. If it does not happen then the decay of this world which begins its inflationary expansion may create a new macroscopic Universe in the same manner as it happens in the usual inflation models.[20] The result of this decay depends on the effective hypersurface at which it occurs and hence on the nature of the Λ term. In particular one may expect that a new macroscopic closed Friedmann universe will arise as a result of this process. In this case de Sitter space decays at some hypersurface Σ_2 (see Fig. 6). The spacetime in the future with respect to Σ_2 will coincide with the spacetime of an expanding closed Friedmann universe. Another possibility is the creation of a white hole in a new asymptotically flat universe which lies in the absolute future with respect to the original asymptotically flat space. Such a process of white-hole creation may occur only if the de Sitter phase decays on the surface $r=$const.

The considered model may be interpreted as "the creation of the universe in a laboratory" via a black hole which may be formed by contraction of matter up to high density. This conclusion does not contradict the theorem of Ref. 9. The reason is that in our case the assumptions of this theorem (in particular the existence of a global Cauchy surface as well as the condition of energy dominance) may be violated.

In conclusion it should be stressed once again that the consideration in this paper is based on rather restrictive assumptions about the properties of the effective gravitational equations at high curvatures. For example at small distances it may become important that real spacetime dimensionality is higher than 4. We cannot exclude also the formation of a few or many closed baby universes inside a black hole. Nevertheless we hope that the described model with a closed world in the interior of a black hole may be useful and this picture or its main features will survive in a future theory. If it happens then the possibility (which was discussed earlier in connection with the Reissner-Nordström or Kerr spacetime) "to travel" from our Universe into a new one which is in the absolute future with respect to us may still become open.

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APPENDIX A

In this appendix we collect the main formulas of the massive thin shell approach. Let Σ be a surface which separates two spacetime domains $U^{(+)}$ and $U^{(-)}$. We shall use δ to distinguish the quantities which are defined in these domains by assuming that $\delta=+$ for $U^{(+)}$ and $\delta=-$ for $U^{(-)}$. Thus the line element $ds^{(\delta)2}$ in $U^{(\delta)}$ reads

$$ds^{(\delta)2}=g^{(\delta)}_{\mu\nu}dx^{(\delta)\mu}dx^{(\delta)\nu} . \tag{A1}$$

We assume that the surface Σ is either spacelike or timelike.[21] The Gaussian normal coordinates $(q,y^{(\delta)i})$ can be introduced in $U^{(\delta)}$ in which

$$ds^{(\delta)2}=-\nu\, dq^2+h^{(\delta)}_{ij}(q,y^{(\delta)k})dy^{(\delta)i}dy^{(\delta)j} , \tag{A2}$$

$q=0$ is the equation of Σ, $\nu=+1$ for the spacelike surface, and $\nu=-1$ for the timelike surface. Denote by n^{μ} the unit vector which is normal to Σ and is directed from $U^{(-)}$ to $U^{(+)}$:

$$n_{\mu}n^{\mu}=-\nu . \tag{A3}$$

In the Gaussian normal coordinates $n^{\mu}\partial_{\mu}=\partial_q$.

The three-geometries $dh^{(\delta)2}$ induced on Σ by the four-dimensional metrics $g^{(\delta)}_{\mu\nu}$ can be written as

$$dh^{(\delta)2}=h^{(\delta)}_{ij}(0,y^{(\delta)k})dy^{(\delta)i}dy^{(\delta)j} . \tag{A4}$$

In general relativity these metrics must be isometric while the external curvatures may have jumps at Σ. One can use the freedom in the choice of the coordinates $y^{(\delta)i}$ in order to imply that $h_{ij}^{(+)}=h_{ij}^{(-)}$. We write the three-dimensional metric dh^2 on Σ in this coordinate as

$$dh^2=h_{ij}(x^k)dy^i dy^j \ . \tag{A5}$$

The external curvature $K_{\mu\nu}^{(\delta)}$ of the surface Σ is defined as

$$K_{\mu\nu}^{(\delta)}=-P^{(\delta)\alpha}{}_{\mu}P^{(\delta)\beta}{}_{\nu}\nabla_\alpha^{(\delta)}n_\beta \ , \tag{A6}$$

where

$$P^{(\delta)\nu}{}_{\mu}=\delta_\mu^\nu-\nu n_\mu n^\nu \ . \tag{A7}$$

In the Gaussian normal coordinates (A2) one has

$$K_{ij}^{(\delta)}=-\frac{1}{2}\frac{\partial h_{ij}^{(\delta)}}{\partial q} \ . \tag{A8}$$

To obtain the jumps of the external curvature at Σ one can use Einstein's equations

$$G_{\mu\nu}=R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R=8\pi T_{\mu\nu} \tag{A9}$$

which in $(3+1)$ form read

$$G_j^i={}^{(3)}G_j^i-\nu\partial_q(K_j^i-\delta_j^iK)-\nu[KK_j^i-\tfrac{1}{2}\delta_j^i(K_k^mK_m^k+K^2)]$$
$$=8\pi T_j^i \ , \tag{A10}$$

$$G_j^n=\nu(K_{j|i}^i-K_{|j})=8\pi T_j^n \ , \tag{A11}$$

$$G_n^n=-\tfrac{1}{2}{}^{(3)}R-\tfrac{1}{2}\nu(K^2-K_j^iK_i^j)=8\pi T_n^n \ , \tag{A12}$$

where ${}^{(3)}G_j^i={}^{(3)}R_j^i-\frac{1}{2}\delta_j^i\,{}^{(3)}R$, ${}^{(3)}R_j^i$ is the Ricci tensor for h_{ij}, $(\ )_{|i}$ denotes the covariant derivative with respect to the metric h_{ij} and $A^{n\cdots}_{\cdots}\equiv n_\mu A^{\mu\cdots}_{\cdots}$. If we denote

$$[K_{ij}]=K_{ij}^{(+)}-K_{ij}^{(-)} \ , \tag{A13}$$

then by integrating (A10) over q in the vicinity of Σ we get

$$-\nu([K_j^i]-\delta_j^i[K])=8\pi S_j^i \ , \tag{A14}$$

where

$$S_j^i=\lim_{\epsilon\to 0}\int_{-\epsilon}^{\epsilon}T_j^i dq \ . \tag{A15}$$

By calculating the jump of Eq. (A11) at Σ and using Eq. (A14) we get

$$S_{j|i}^i=-[T_j^n] \ , \tag{A16}$$

where $[T_j^n]=T^{(+)}{}_j^n-T^{(-)}{}_j^n$.

Equation (A14) gives the relation between the jumps of the external curvature at Σ and the integral characteristics of the "shell" while Eq. (A16) provides the "equation of motion" of this "shell." If the field equations (A10)–(A12) are satisfied in both domains $U^{(+)}$ and $U^{(-)}$ then other relations which can be obtained from (A10)–(A12) are just identities.

APPENDIX B

This appendix contains the details of the calculations of the external curvatures and jump conditions at the junction surfaces in spherically symmetric spacetimes for the cases which are considered in the paper. For our purpose it is convenient to write the metric in the form (see, also, Ref. 22)

$$ds^2=f\,dv^2+2\,dv\,dr+r^2d\omega^2 \ , \tag{B1}$$

where $d\omega^2=d\theta^2+\sin^2\theta\,d\phi^2$, $f\equiv f(r,v)=-\nabla r\cdot\nabla r$, and v is an advanced time coordinate which is normalized at infinity (where $f=-1$) by the condition $\nabla r\cdot\nabla v=1$.

Let Σ be a surface in this space defined by the equation $\Phi=0$ where

$$\Phi\equiv R(v)-r \ . \tag{B2}$$

We denote by $U^{(+)}(U^{(-)})$ the domain of the spacetime where $\Phi>0$ ($\Phi<0$). The internal geometry on Σ which is induced by the metric (B1) is

$$dh^2=\nu\epsilon^2(v)dv^2+R^2(v)d\omega^2$$
$$=\nu\,dq^2+\rho^2(q)d\omega^2 \ . \tag{B3}$$

where

$$\nu\epsilon^2(v)=f(R(v),v)+2\frac{dR}{dv}\equiv F(v)+2\frac{dR}{dv} \ ,$$
$$\nu\equiv\mathrm{sgn}(\nu\epsilon^2) \ , \tag{B4}$$
$$dq=\epsilon(v)dv, \quad \rho(q)=R(v(q)) \ .$$

The surface Σ is a spacelike one in the case $\nu=+1$ and it is a timelike surface for $\nu=-1$.

The unit vector n^μ which is normal to Σ and directed from $U^{(-)}$ to $U^{(+)}$ is

$$n^\mu=\frac{\nu}{\epsilon}\left[-1,F+\frac{dR}{dv},0,0\right] \ . \tag{B5}$$

It is convenient to consider this vector as one of the vectors $(e_{\hat 0}^\mu)$ of the tetrad $e_{\hat a}^\mu$ ($\hat a=0,1,2,3$) the other vectors of which are defined as

$$e_{\hat 1}^\mu=\epsilon^{-1}(1,dR/dv,0,0) \ ,$$
$$e_{\hat 2}^\mu=R^{-1}(0,0,1,0) \ , \tag{B6}$$
$$e_{\hat 3}^\mu=(R\sin\theta)^{-1}(0,0,0,1) \ .$$

These vectors obey the normalization conditions

$$e_{\hat a}^\mu e_{\hat b\mu}=\eta_{\hat a\hat b} \ , \tag{B7}$$

where

$$\eta_{\hat a\hat b}=\mathrm{diag}(-\nu,\nu,1,1) \ . \tag{B8}$$

The external curvature tensor K_{ij} for the hypersurface Σ has the following nonzero tetrad components:

$$K_{\hat 1}^{\hat 1}=\frac{1}{\sigma}\left[\ddot\rho+\frac{\nu}{2}\frac{\partial f}{\partial r}+\frac{1}{2\epsilon^2}\frac{\partial f}{\partial v}\right], \quad K_{\hat 2}^{\hat 2}=K_{\hat 3}^{\hat 3}=\sigma\rho^{-1} \ , \tag{B9}$$

where

$$\sigma = -n^\mu \nabla_\mu r = \frac{\nu}{\epsilon}\left[F + \frac{dR}{dv}\right] \tag{B10}$$

and the overdot denotes the differentiation in the direction of e_1^μ:

$$(\ )^{\cdot} \equiv e_1^\mu \nabla_\mu(\ ) = \epsilon^{-1}\frac{d}{dv}(\ ) \ . \tag{B11}$$

Using Eqs. (B10) and (B11) it is easy to find that

$$\epsilon = \sigma + \nu\dot{\rho} \ . \tag{B12}$$

We discuss at first the case of the short transition layer which separates the Schwarzschild ($\delta=-$) and de Sitter ($\delta=+$) metrics and obtain the parameters of the corresponding "massive thin shell." In this case $R(v)=r_0$, $l<r_0<2m$. The Schwarzschild metric can be rewritten in the form (B1) where

$$f^{(-)} = 2mr^{-1} - 1 \ . \tag{B13}$$

It is easy to verify that

$$\epsilon = \epsilon^{(-)} \equiv (2mr_0^{-1}-1)^{1/2} \ , \quad \sigma = \sigma^{(-)} \equiv (2mr_0^{-1}-1)^{1/2} \ , \tag{B14}$$

and

$$dh^{(-)2} = dq^2 + r_0^2 d\omega^2 \ . \tag{B15}$$

The nonzero components of the extrinsic curvature $K_{ij}^{(-)}$ are equal to

$$K^{(-)}{}_1^1 = -\frac{m}{r_0^2}\left[\frac{2m}{r_0}-1\right]^{-1/2} , \quad K^{(-)}{}_2^2 = K^{(-)}{}_3^3 = \frac{1}{r_0}\left[\frac{2m}{r_0}-1\right]^{1/2} . \tag{B16}$$

For the de Sitter metric we find

$$f^{(+)} = (r/l)^2 - 1 \ , \quad dh^{(+)2} = dq^2 + r_0^2 d\omega^2 \ , \quad \epsilon^{(+)} = \sigma^{(+)} = [(r_0/l)^2-1]^{1/2} \ , \quad K^{(+)}{}_1^1 = \frac{r_0}{l^2}[(r_0/l)^2-1]^{-1/2} \ , \quad K^{(+)}{}_2^2 = K^{(+)}{}_3^3 = \frac{1}{r_0}[(r_0/l)^2-1]^{1/2} \ . \tag{B17}$$

If we introduce the notation

$$\kappa \equiv [K_1^1] = \frac{r_0}{l^2}[(r_0/l)^2-1]^{-1/2} + \frac{m}{r_0^2}[(2m/r_0)-1]^{-1/2} \ , \tag{B18}$$

$$\lambda \equiv [K_2^2] = -\frac{1}{r_0}\{[(2m/r_0)-1]^{1/2} - [(r_0/l)^2-1]^{1/2}\} \ , \tag{B19}$$

then

$$S_1^1 = \lambda/4\pi, \quad S_2^2 = S_3^3 = (\kappa+\lambda)/8\pi \ . \tag{B20}$$

It is easy to verify that $[T_i^n]=0$ and correspondingly Eq. (A16) is satisfied identically.

Now we turn to the case when the surface Σ is the boundary between the Vaidya ($\delta=-$) and de Sitter ($\delta=+$) metrics. For this case,

$$f^{(-)} = 2m(v)r^{-1} - 1 \tag{B21}$$

and we have

$$\nu\epsilon^{(-)2}(v) = 2m(v)/R(v) - 1 + 2\,dR/dv \ , \quad \sigma^{(-)} = \nu(\epsilon^{(-)})^{-1}[2m(v)/R(v) - 1 + dR/dv] \ , \tag{B22}$$

and

$$K^{(-)}{}_1^1 = \frac{1}{\sigma^{(-)}}\left[\ddot{\rho} - \frac{\nu m}{\rho^2} + \frac{\dot{m}}{\epsilon^{(-)}\rho}\right] , \quad K^{(-)}{}_2^2 = K^{(-)}{}_3^3 = \frac{\sigma^{(-)}}{\rho} \ . \tag{B23}$$

It is easy to find the following expressions for the de Sitter metric:

$$f^{(+)} = (r/l)^2 - 1 \ , \quad \nu\epsilon^{(+)2} = [R(v)/l]^2 - 1 + 2\,dR/dv \ , \quad \sigma^{(+)} = \frac{\nu}{\epsilon^{(+)}}\{[R(v)/l]^2 - 1 + dR/dv\} \ , \tag{B24}$$

and

$$K^{(+)}{}_1^1 = \frac{1}{\sigma^{(+)}}\left[\ddot{\rho} + \frac{\nu\rho}{l^2}\right], \quad K^{(+)}{}_2^2 = K^{(+)}{}_3^3 = \frac{\sigma^{(+)}}{\rho} \ . \tag{B25}$$

For the jumps of the extrinsic curvature we obtain

$$\kappa \equiv [K_1^1] = \frac{1}{\sigma^{(+)}}\left[\ddot{\rho} + \frac{\nu\rho}{l^2}\right] - \frac{1}{\sigma^{(-)}}\left[\ddot{\rho} - \frac{\nu m}{\rho^2} + \frac{\dot{m}}{\epsilon^{(-)}\rho}\right] , \quad \lambda \equiv [K_2^2] = \rho^{-1}(\sigma^{(+)} - \sigma^{(-)}) \ , \tag{B26}$$

and

$$S_1^1 = \lambda/4\pi, \quad S_2^2 = S_3^3 = (\kappa+\lambda)/8\pi \ . \tag{B27}$$

The "conservation law" (A6) takes the form

$$\dot{\lambda}\rho^2 + (\lambda - \kappa)\rho\dot{\rho} = -\nu\epsilon^{-1}\dot{m} \ . \tag{B28}$$

In the simplest case, when $\lambda=0$, Eq. (B28) gives

$$\kappa = -\frac{\nu}{\rho\epsilon}\frac{dm}{d\rho} \ . \tag{B29}$$

Taking into account (B26), we find in this case that $\sigma^{(-)}=\sigma^{(+)}$ and correspondingly [see (B12)] $\epsilon^{(-)}=\epsilon^{(+)}$ or

$$\rho = (2m(v)/l)^{1/3} l \ . \tag{B30}$$

If we use this equation, then Eq. (B29) can be written in the form

$$\kappa = \frac{3\nu p}{2\epsilon l^2} \ . \tag{B31}$$

APPENDIX C

In this appendix the problem of matching the closed Friedmann universe metric to the de Sitter one is considered. We write the metric for these spaces in the form

$$ds^{(\delta)2} = a_\delta^2(\eta_\delta)(-d\eta_\delta^2 + d\chi^2 + \sin^2\chi\, d\omega^2) \ . \tag{C1}$$

One has, for the closed Friedmann universe ($\delta = -$) with dustlike matter ($p=0$),

$$a_-(\eta_-) = a_0(1-\cos\eta_-) \ , \tag{C2}$$

while for the de Sitter space ($\delta = +$),

$$a_+(\eta_+) = l/\sin\eta_+ \ . \tag{C3}$$

We suppose that the junction surface is defined by

$$\eta_\delta = \eta_\delta^0 = \text{const} \ . \tag{C4}$$

The three-geometries induced by the metrics (C1) at this surface are identical provided

$$a_-(\eta_-^0) = a_+(\eta_+^0) \ . \tag{C5}$$

The components of the external curvatures $K^{(\delta)}{}_i^j$ can be written as

$$K^{(\delta)}{}_j^i = -\left[\frac{1}{a_\delta^2(\eta_\delta)} \frac{da_\delta}{d\eta_\delta} \right]_{\eta_\delta^0} \delta_j^i \ . \tag{C6}$$

Using this expression one finds

$$S_j^i = -\frac{\lambda}{4\pi}\delta_j^i \ , \tag{C7}$$

$$\lambda = (a_+^{-2} da_+/d\eta_+)_{\eta_+=\eta_+^0} - (a_-^{-2} da_-/d\eta_-)_{\eta_-=\eta_-^0} \ . \tag{C8}$$

The junction condition (C5) and the explicit expressions (C2) and (C3) for $a_\delta(\eta_\delta)$ allows one to rewrite Eq. (C8) in the form (3.11).

We have defined the T_- region as a domain where $\nabla r \cdot \nabla r < 0$. The boundary ∂T_- of this domain is the surface where $\nabla r \cdot \nabla r = 0$. One can write the equation for ∂T_- in the spacetime (C1) as

$$a_\delta^{-1} da_\delta / d\eta_\delta = \pm\cot\chi \ . \tag{C9}$$

The solutions of these equations can be written in the form

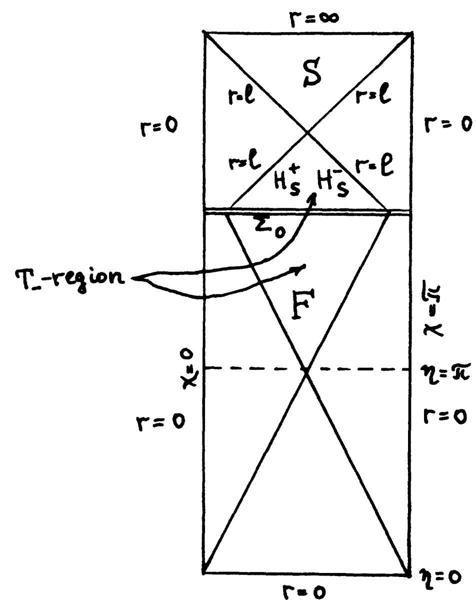

FIG. 8. Conformal diagram for the spacetime of the Friedmann-de Sitter universe. The surface Σ_0 is a junction surface at which the Friedmann metric is matched to the de Sitter one. The T_- region shown in this picture corresponds to the particular choice of the parameter $\alpha = 15$.

$$\chi = \zeta_-^\pm(\eta_-) \equiv \tfrac{1}{2}\pi \pm \tfrac{1}{2}(\eta_- - \pi) \ , \tag{C10}$$

$$\chi = \zeta_+^\pm(\eta_+) \equiv \tfrac{1}{2}\pi \pm \tfrac{1}{2}(\eta_+ - \pi/2) \ . \tag{C11}$$

For the particular choice $\alpha = 15$ ($\lambda = 0$) one has

$$\zeta_-^\pm(\eta_-^0) = \zeta_+^\mp(\eta_+^0) \ . \tag{C12}$$

The conformal diagram for spacetime for this case is shown in Fig. 8. The spacetime structure is qualitatively the same for other choices of the parameter α. The main difference is that in the general case the relation (C12) is violated, so that there is a nontrivial part of the surface ∂T_- which lies on Σ_0.

It is also easy to show that when one considers not the complete closed world but only part of it ($0 \le \chi \le \chi_0 < \pi/2$) then the surface ∂T_- intersects the dust cloud boundary $\chi = \chi_0$ at the point where

$$r = 2m \ . \tag{C13}$$

Here r and m are given by Eqs. (3.5) and (3.4), correspondingly.

[1]R. Penrose, in *Battelle Rencontres*, edited by C. M. DeWitt and J. A. Wheeler (Benjamin, New York, 1968).

[2]S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space-Time* (Cambridge University Press, Cambridge, England, 1973).

[3]C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[4]J. A. Wheeler, *Geometrodynamics* (Academic, New York, 1962); J. B. Hartle, Phys. Rev. D **38**, 2985 (1988).

[5]For such an approach in cosmology, see M. A. Markov, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 214 (1982) [JETP Lett. **36**, 265 (1982)]; M. A. Markov, Ann. Phys. (N.Y.) **155**, 333 (1984); E. G. Aman and M. A. Markov, Teor. Mat. Fiz. **58**, 97 (1984); M. A. Markov and V. F. Mukhanov, Nuovo Cimento **86B**, 163 (1985); M. A. Markov, in *Quantum Gravity*, proceedings of the Third Seminar, Moscow, USSR, 1984, edited by V. A. Berezin, V. P. Frolov, and M. A. Markov (World Scientific, Singapore, 1985).

[6]V. P. Frolov, M. A. Markov, and V. F. Mukhanov, Phys. Lett. B **216**, 272 (1989).

[7]Ya. B. Zeldovich and I. D. Novikov, *The Structure and Evolution of the Universe* (Chicago University Press, Chicago, 1983), Vol. 2; B. L. Hu and L. Parker, Phys. Rev. D **17**, 3292 (1978); J. B. Hartle and B. L. Hu, *ibid.* **20**, 1772 (1979); **21**, 2756 (1980); B. L. Hu, in Lectures given at the 11th Course of the International School on Cosmology and Gravitation, Quantum Mechanics in Curved Spacetime, Erice, Italy, 1989 (unpublished).

[8]E. Poisson and W. Israel, Class. Quantum Grav. **5**, L201 (1988).

[9]E. Farhi and A. H. Guth, Phys. Lett. B **183**, 149 (1987).

[10]W. Israel, Nuovo Cimento **44B**, 1 (1966); **48B**, 463 (1967).

[11]V. A. Berezin, V. A. Kuzmin, and I. I. Tkachev, Phys. Rev. D **23**, 2919 (1987).

[12]T. A. Roman and P. G. Bergmann, Phys. Rev. D **28**, 1265 (1983).

[13]Y. Gursel, V. D. Sandberg, I. D. Novikov, and A. A. Starobinsky, Phys. Rev. D **19**, 413 (1979); **120**, 1260 (1979); S. Chandrasekhar and J. B. Hartle, Proc. R. Soc. London **A384**, 301 (1982); I. D. Novikov and A. A. Starobinsky, Zh. Eksp. Teor. Fiz. **78**, 3 (1980) [Sov. Phys. JETP **51**, 1 (1980)].

[14]I. D. Novikov and V. P. Frolov, *Black Holes Physics* (Moscow, Nauka, 1986); English edition published by Reidel, Holland, 1989.

[15]E. S. Fradkin and G. A. Vilkovisky, Phys. Lett. **78B**, 262 (1978); V. P. Frolov and G. A. Vilkovisky, in *Proceedings of the Second Marcel Grossmann Meeting on the Recent Developments of General Relativity,* Trieste, Italy, 1979, edited by R. Ruffini (North-Holland, Amsterdam, 1982), p. 455; Phys. Lett. **106B**, 307 (1981); E. Tomboulis, *ibid.* **97B**, 77 (1980); B. Hasslacher and E. Mottola, *ibid.* **99B**, 221 (1981); E. S. Fradkin and A. A. Tseytlin, *ibid.* **104B**, 377 (1981).

[16]P. C. Vaidya, Phys. Rev. **83**, 10 (1951); Proc. Ind. Acad. Sci. **A33**, 264 (1951); R. W. Lindquist, R. A. Schwarz, and C. W. Misner, Phys. Rev. **137**, 1364 (1965).

[17]I. V. Volovich, V. A. Zagrebnov, and V. P. Frolov, Teor. Mat. Fiz. **29**, 191 (1976); R. Nityananda and R. Narayan, Phys. Lett. **82A**, 1 (1981); W. A. Hiscock, Phys. Rev. D **23**, 2813 (1981); P. Hajicek, *ibid.* **36**, 1065 (1987).

[18]M. A. Markov, Suppl. Progr. Theor. Phys. 85 (1965).

[19]S. W. Hawking, Mon. Not. R. Astron. Soc. **152**, 75 (1971).

[20]A. D. Linde, Phys. Lett. **108B**, 389 (1982); **129B**, 177 (1983); A. A. Starobinsky, *ibid.* **91B**, 99 (1980); V. F. Mukhanov and G. V. Chibisov, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 549 (1981) [JETP Lett. **33**, 532 (1981)]; Zh. Eksp. Teor. Fiz. **83**, 475 (1982) [Sov. Phys. JETP **56**, 258 (1982)].

[21]For discussion of matching of the solutions of the Einstein equations at null surfaces, see C. J. Clarke and T. Dray, Class. Quantum Grav. **4**, 265 (1987).

[22]V. P. Frolov, Zh. Eksp. Teor. Phys. **66**, 813 (1974) [Sov. Phys. JETP **39**, 393 (1974)].