Moving mirrors, black holes, and cosmic censorship

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We examine negative-energy fluxes produced by mirrors moving in two-dimensional charged-black-hole backgrounds. If there exist no constraints on such fluxes, then one might be able to manipulate them to achieve a violation of cosmic censorship by shooting a negative-energy flux into an extreme $Q=M$ or near-extreme Reissner-Nordström black hole. However, if the magnitude of the change in the mass of the hole $|\Delta M|$, resulting from the absorption of this flux, is small compared to the normal quantum uncertainty in the mass expected from the uncertainty principle $\Delta E \Delta T \geq 1$, then such changes should not be macroscopically observable. We argue that, given certain (physically reasonable) restrictions on the trajectory of the mirror, this indeed seems to be the case. More specifically, we show that $|\Delta M|$ and ΔT, the "effective lifetime" of any naked singularity thus produced, are limited by an inequality of the form $|\Delta M| \Delta T < 1$. We then conclude that the negative-energy fluxes produced by two-dimensional moving mirrors do not lead to a classically observable violation of cosmic censorship.

I. INTRODUCTION

In classical gravity theory, one usually assumes that some energy condition such as the weak-energy condition (positive energy density) is obeyed by matter.[1] These conditions, which are obeyed by known forms of classical matter, are necessary in order to prove singularity theorems. Some restriction on the matter stress tensor must also hold if the cosmic-censorship hypothesis is correct.[2] However, it is well known that quantized matter fields can violate classical energy conditions by having a locally negative energy density.[3] Even free fields in flat spacetime have quantum states in which there is a negative energy density or a flux of negative energy. The fluxes generated by moving mirrors in two-dimensional spacetimes illustrate this effect.[4] If there is no constraint at all on the matter stress tensor, then it seems to be possible to violate not only cosmic censorship but also the second law of thermodynamics[5–8] and causality.[9]

Fortunately, quantum field theory does seem to impose some restrictions on negative energy densities and fluxes. For example, in flat two-dimensional spacetime, negative-energy fluxes appear to satisfy an inequality of the form $|F|\tau^2 \leq 1$, where $|F|$ is the magnitude of a negative-energy flux and τ is the characteristic time during which the flux is negative.[5] Moving mirrors in this spacetime produce negative-energy fluxes which obey the inequality as seen by inertial observers. Such an inequality prevents gross macroscopic violations of the second law. Davies[6] has shown that for a mirror in flat spacetime moving with an acceleration bounded away from zero for an infinite time, the total integrated flux is negative. However, such trajectories have null asymptotes and therefore unphysical characteristics. If the null asymptote is in the past, then the mirror was already accelerating at $t=-\infty$. If it is in the future, then (in two dimensions) the mirror must necessarily collide with any inertial observer trying to measure the flux.[10]

In this paper, we wish to investigate the possibility of violating cosmic censorship using the negative-energy fluxes produced by moving mirrors on a Reissner-Nordström background. If one can shine sufficient negative energy at a charged black hole, then it seems to be possible to decrease its mass enough to convert it into a Reissner-Nordström naked singularity. We will examine this possibility in the context of a two-dimensional model where, unlike in four dimensions, the radiation produced by the moving mirror[4,11] can be exactly computed.

The outline of the paper is as follows. In Sec. II we review the necessary formalism of moving mirrors in two-dimensional curved spacetime. This formalism is used in Sec. III to prove the positivity of the integrated flux from a moving mirror, given certain physically reasonable restrictions on its trajectory. This positivity forces any violation of cosmic censorship to be limited in time. In Sec. IV we analyze the extent to which cosmic censorship can be violated and postulate a criterion for when this violation is classically observable. Our results are summarized and discussed in Sec. V.

II. MOVING MIRRORS IN CURVED SPACE (REVIEW)

Here we briefly review the aspects of the formalism developed by Ottewill and Takagi[12] (OT), Walker,[13] and Davies,[6] which will be useful for our purposes. The method of evaluating the renormalized vacuum expectation value of the stress-energy tensor $T_{\mu\nu}$ due to a conformally invariant field in an arbitrary two-dimensional spacetime has been given by Davies, Fulling, and Unruh.[14]

We examine a mirror of negligible mass moving in the region outside the horizon of a two-dimensional black-hole spacetime (either Schwarzschild or Reissner-Nordström). The regions to the left and right of the mirror are depicted in Fig. 1. The line element is given by

$$ds^2 = -C(u,v)du\,dv \ . \tag{2.1}$$

The mirror's world line is defined by

$$v = p(u) \ , \tag{2.2}$$

where u and v are retarded and advanced null coordinates, respectively, given by

$$\begin{aligned} u &= t - r^* \ , \\ v &= t + r^* \end{aligned} \tag{2.3}$$

with

$$r^* = \int^r C^{-1} dr \ . \tag{2.4}$$

We consider a conformally coupled scalar field ϕ which is assumed to obey either Dirichlet or Neumann boundary conditions (i.e., $\phi=0$ or $n^\mu \nabla_\mu \phi = 0$) on the mirror.

The quantum state of the field in which we are interested may be defined as follows: Suppose that the black hole was formed by collapse in the distant past while the mirror was static. Our state is the no-particle state before the collapse and before the mirror begins to move. This means that it is a vacuum state with respect to the u modes (those $\propto e^{-i\omega u}$) which are to the right of the mirror. It is also a vacuum state with respect to the v modes to the left of the mirror which enter the collapsing body and pass through the center before the horizon forms. Thus in the absence of the mirror, it would be the Unruh vacuum for the black hole; in the absence of the black hole, it would be the in-vacuum state for a mirror trajectory. In fact, the effects of the Hawking radiation from the black hole and of the intrinsic radiation by the mirror add incoherently (a consequence of the thermal nature of the Hawking radiation), as was shown by Walker.[13] Thus the total stress tensor of a quantum field may be expressed as the sum of a Hawking radiation term (including reflection from the mirror) and an intrinsic mirror radiation term. [See Eqs. (2.28) and (2.29) below.]

Define u^μ and n^μ to be the unit tangent and (outward)

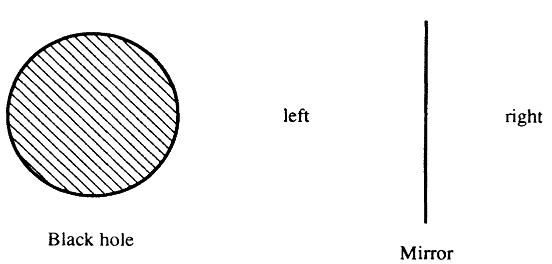

FIG. 1. "Left" and "right" in relation to the mirror and black hole.

normal vectors of the world line, Eq. (2.2), respectively. If we transform to a coordinate system in which the mirror is stationary, given by the coordinates

$$\bar{u} \equiv p(u), \quad \bar{v} \equiv v \ , \tag{2.5}$$

then our line element, Eq. (2.1), becomes

$$\begin{aligned} ds^2 &= -\bar{C}(u,v) d\bar{u}\, d\bar{v} \ , \\ \bar{C} &\equiv C/p', \quad p' \equiv dp(u)/du \ . \end{aligned} \tag{2.6}$$

Equations (2.1), (2.2), (2.5), and (2.6) describe the region to the right of the mirror. The region to the left of the mirror is described by the same equations with u and v reversed. Note, however, that the definitions of u and v given by Eq. (2.3) are the same on both sides of the mirror, as is the direction of increasing r^*. In the coordinates of Eq. (2.5), the vectors u^μ, n^μ take the simple forms

$$u^\mu = (u^{\bar{u}}, u^{\bar{v}}) = \bar{C}^{-1/2}(1,1) \ , \tag{2.7}$$

$$n^\nu = (n^{\bar{u}}, n^{\bar{v}}) = \bar{C}^{-1/2}(1,-1) \ . \tag{2.8}$$

The energy flux seen by an observer comoving with the mirror is given by

$$F_c \equiv -T_{\mu\nu} u^\mu n^\nu = (T_{uu} - T_{vv}) \bar{C}^{-1} \ . \tag{2.9}$$

Unruh and Wald[15] have shown that the comoving flux (to the right of the mirror) is given by

$$F_c = \frac{-1}{12\pi} \frac{da}{d\tau} \ , \tag{2.10}$$

where $a \equiv a^\mu n_\mu$, a^μ is the two-acceleration of the mirror's world line, and τ is the proper time of the mirror. In our (i.e., OT's) formalism, the spatial component of n^μ changes sign to the left of the mirror, while the sign of the spatial component of a^μ is left unchanged. As a result, the comoving flux to the left of the mirror is given by

$$F_{c,L} = + \frac{1}{12\pi} \frac{da}{d\tau} \ . \tag{2.11}$$

The energy flux F_s, seen by a static observer at rest at radial position r, is given by

$$F_s \equiv -T_{\mu\nu} u_s^\mu n_s^\nu = (T_{uu} - T_{vv}) C_s^{-1} \ , \tag{2.12}$$

where u_s^μ and n_s^ν are the tangent and normal vectors, respectively, to this observer's world line. They are given by Eqs. (2.7) and (2.8) with the barred quantities replaced by unbarred quantities on the right-hand side. The quantity C_s represents the value of the conformal factor at the static observer's location. Note that the sign of the spatial component of n_s^ν is always the same irrespective of the direction of the flux, i.e., the spatial component of n_s^ν is always assumed to point in the direction of increasing r (to the right). We assume that the static observer always remains on the same side of the mirror, i.e., that the mirror does not cross his position.

From Eqs. (2.2), (2.3), and (2.6) it follows that

$$p'(u) = \frac{1+V}{1-V} \ , \tag{2.13}$$

where

$$V \equiv dr^*/dt \; . \tag{2.14}$$

The quantity $p'(u)$ represents a Doppler shift which is a "redshift" $[p'(u)<1]$ when $V<0$ and a "blueshift" $[p'(u)>1]$ when $V>0$. The stress tensor $t_{\mu\nu}$ defined by

$$t_{uu} = D_u[p'(u)], \quad t_{uv} = t_{vv} = 0 \; , \tag{2.15}$$

$$D_x[y] \equiv \frac{1}{12\pi} y^{1/2} \frac{\partial^2 (y^{-1/2})}{\partial x^2} \tag{2.16}$$

(to the right of the mirror) is exactly the same as the stress tensor due to a mirror moving along the world line of Eq. (2.2) in flat spacetime with Minkowskian null coordinates (u,v). This will be demonstrated below. We will henceforth refer to $t_{\mu\nu}$ as the "intrinsic mirror radiation." The intrinsic radiation to the left of the mirror is described by Eq. (2.15) with u and v reversed, where

$$p'(v) \equiv \frac{dp(v)}{dv} = \frac{1-V}{1+V} \; , \tag{2.17}$$

to the left of the mirror.

Lastly, we define $T^{(B)}_{\mu\nu}$ to be the stress tensor in the Boulware vacuum, whose components are given by

$$T^{(B)}_{uu} = T^{(B)}_{vv} = \frac{1}{48\pi} C^{3/2} (C^{1/2})'' \; , \tag{2.18}$$

with

$$(C^{1/2})'' = \tfrac{1}{4} C^{-3/2} [2CC'' - (C')^2] \tag{2.19}$$

and

$$C' \equiv dC/dr \; . \tag{2.20}$$

(We will not need the component $T^{(B)}_{uv}$.) Equations (2.18)–(2.20) hold on both sides of the mirror.

Using an argument of OT, it can be shown that, to the left of the mirror,

$$t_{vv} = (p'(v) C F_{c,L} + \{[p'(v)]^2 - 1\} T^{(B)}_{vv})|_M \; , \tag{2.21}$$

where the left-hand side is evaluated at $r = r_s$, the location of a static observer near the horizon, and the right-hand side is evaluated at the location of the mirror, both along the *same* v=const line. The result, Eq. (2.21), can be derived from the fact that $t_{\mu\nu}$ is a conserved traceless tensor, which implies that the value of t_{vv} along any fixed v=const line remains constant. Using Eqs. (2.11) and (2.18), we can rewrite Eq. (2.21) as

$$t_{vv} = \frac{1}{48\pi} \left[4Cp'(v) \left(\frac{da}{d\tau} \right) + \{[p'(v)]^2 - 1\} C^{3/2} (C^{1/2})'' \right] \Bigg|_M \; . \tag{2.22}$$

It will be useful to have Eq. (2.22) expressed in terms of V and C. A very tedious calculation then reveals that

$$t_{vv} = \frac{-(V^2 \dddot{V} - \dddot{V} - 3V\ddot{V}^2)}{12\pi (1-V)^2 (1+V)^4} \; , \tag{2.23}$$

which can be written as

$$t_{vv} = \frac{(1-V^2)^{1/2}}{12\pi (1+V)^2} \frac{d}{dt} \left[\frac{\dot{V}}{(1-V^2)^{3/2}} \right] . \tag{2.24}$$

Remarkably, C drops out. The analogous formula for the region to the right of the mirror can be obtained by interchanging u and v on the left-hand side and letting $V \to -V$ on the right-hand side. The physical interpretation of Eq. (2.24) is that the intrinsic flux radiated by a mirror in any curved spacetime (i.e., not counting any reflected Hawking or thermal radiation), when expressed in terms of $V \equiv dr^*/dt$, is the same as in flat spacetime. This result and its physical implications were first obtained by Walker[13] for the region to the right of the mirror, using a slightly different method.

Davies[6] has shown that for a mirror moving in flat spacetime, the total (intrinsic) energy flux radiated to either side of the mirror is always positive for trajectories where the mirror is either asymptotically static or asymptotically moving with constant velocity. Since the intrinsic energy flux in curved spacetime depends on the trajectory in the same way as in flat spacetime as shown by Eq. (2.24), Davies' discussion is transferable to curved spacetime as well. His argument (here given for the left side of the mirror) is as follows.

In the region to the left of the mirror,

$$t_{vv} = \frac{1}{12\pi} \left[\frac{3}{4} \left(\frac{p''}{p'} \right)^2 - \frac{1}{2} \frac{p'''}{p'} \right] , \qquad t_{uu} = t_{uv} = 0 \; , \tag{2.25}$$

where we have used Eqs. (2.15)–(2.17). The integrated (left-moving) intrinsic flux, I, seen by a static observer to the left of the mirror is given by

$$I = -\int_{t_1}^{t_2} dt\, C_s^{1/2} F_s = +C_s^{-1/2} \int_{v_1}^{v_2} dv\, t_{vv} \; , \tag{2.26}$$

from Eqs. (2.12) and (2.25). The minus sign on the left-hand side arises because we are interested in the sign of the left-moving flux. Using Eq. (2.25) and integrating by parts, we can rewrite Eq. (2.26) as

$$I = C_s^{-1/2} \int_{v_1}^{v_2} dv\, t_{vv} = \left[\frac{1}{24\pi} \left(\frac{p''}{p'} \right) \Bigg|_{v_1}^{v_2} + \frac{1}{48\pi} \int_{v_1}^{v_2} \left(\frac{p''}{p'} \right)^2 dv \right] C_s^{-1/2} \; . \tag{2.27}$$

For trajectories where the mirror is either asymptotically static or asymptotically moving with constant velocity (i.e., $V \equiv dr^*/dt$ equals zero or a constant at both v_1 and v_2), the first (boundary) term in Eq. (2.27) will vanish since, from Eq. (2.17), $p'(v)$ is constant if V is constant. Consequently, these trajectories radiate net positive intrinsic energy.

For the case of a mirror moving in an arbitrary two-dimensional black-hole spacetime, there will in general be Hawking radiation to the left of the mirror. Such radiation will be reflected, and Doppler shifted depending on the mirror's velocity, as discussed by Walker.[13] His result, adapted here for the region to the left of the mirror, is

$$T_{vv}=[p'(v)]^2\frac{\pi T_{\rm BH}^2}{12}+T_{vv}^{(B)}+t_{vv}\ , \tag{2.28}$$

$$T_{uu}=T_{uu}^{(B)}+\frac{\pi T_{\rm BH}^2}{12}\ , \tag{2.29}$$

where $T_{\rm BH}$ represents the Hawking temperature of the black hole. The first term in Eq. (2.28) represents Hawking radiation which has been reflected from the left side of the mirror back into the black hole after being Doppler shifted. The second term represents the left-moving component of the static vacuum polarization. The last term represents the intrinsic radiation emitted to the left of the mirror. The sum of these three contributions make up the radiation flowing away from the mirror to the left. Note that in this two-dimensional treatment the mirror prevents the Hawking radiation from reaching infinity and also blocks off any radiation which might be present on the right side of the mirror from reaching the left side.

From Eqs. (2.12), (2.28), and (2.29), the net left-moving flux seen by a static observer stationed near the horizon at $r=r_s$ is

$$F_{s,L}\equiv -F_s$$
$$=\left[\{[p'(v)^2]^2-1\}\frac{\pi T_{\rm BH}^2}{12}+t_{vv}\right]C_s^{-1}\ . \tag{2.30}$$

We note in passing that since $T_{uu}^{(B)}=T_{vv}^{(B)}$ [from Eq. (2.18)], the static vacuum polarization does not contribute to the net flux, Eq. (2.30), seen by a static observer.[16]

III. A CONSTRAINT ON INTEGRATED FLUXES FROM MOVING MIRRORS

Our main purpose in this paper is to determine whether the negative-energy fluxes from moving mirrors can be manipulated in such a way so as to achieve a violation of cosmic censorship. Therefore, let us now consider a nonextreme ($Q<M$) Reissner-Nordström black hole in the presence of a mirror. Since the hole is nonextreme, there will be Hawking radiation in the region to the left of the mirror which will be reflected from the mirror and back into the black hole. This radiation will be Doppler redshifted (blueshifted) upon reflection if the mirror is moving away from (toward) the black hole.

The conformal factor for this spacetime is

$$C=\left[1-\frac{2M}{r}+\frac{Q^2}{r^2}\right]\quad\text{with } Q<M\ , \tag{3.1}$$

where M,Q are the mass and charge of the black hole, respectively. We treat the charge Q of the hole as constant.[17] The Hawking temperature of such a black hole is given by

$$T_{\rm BH}=\frac{(M^2-Q^2)^{1/2}}{2\pi r_+^2}\ , \tag{3.2}$$

where

$$r_+=M+(M^2-Q^2)^{1/2}\ . \tag{3.3}$$

It follows from Eqs. (2.30) and (3.2) that the integrated left-moving flux during some time interval t_2-t_1, seen by a stationary observer near the horizon, is

$$E_{s,L}=-\int_{t_1}^{t_2}dt\, C_s^{1/2}F_s$$
$$=C_s^{-1/2}\left[\int_{v_1}^{v_2}\{[p'(v)]^2-1\}\left[\frac{M^2-Q^2}{48\pi r_+^4}\right]dv+\int_{v_1}^{v_2}t_{vv}dv\right]\ . \tag{3.4}$$

From here on, we will always assume that any change ΔM in the mass of the black hole is small compared with the original mass M, i.e.,

$$\Delta M \ll M\ . \tag{3.5}$$

The condition, Eq. (3.5), is assumed in order that we may effectively ignore the back reaction of the change in the black-hole mass on the spacetime geometry.

We now would like to compare Eq. (3.4) with the integrated negative-energy flux, seen by the same observer, due to the Hawking radiation process when the black hole is simply radiating into empty space with no mirror present. In that case (see, for example, Hiscock[18]),

$$(E_s)_{\rm rad}=-\int_{t_1}^{t_2}dt\, C_s^{1/2}(F_s)_{\rm rad}$$
$$=-C_s^{-1/2}\int_{v_1}^{v_2}dv\left[\frac{M^2-Q^2}{48\pi r_+^4}\right]\ . \tag{3.6}$$

The time interval t_2-t_1 is always assumed to be in the range where the semiclassical description of black-hole evaporation is valid.

Examining the first term in Eq. (3.4), which represents the contribution from the reflected Hawking radiation, we see that the maximum negative value of the integrand occurs when $p'(v)=0$, which corresponds to $V\rightarrow 1$ (recall that $|V|<1$ for timelike trajectories). This would correspond to the limit of a mirror moving away from the black hole at the speed of light. Since the maximum possible negative value of $\{[p'(v)]^2-1\}$ is -1, then the first term in Eq. (3.4) is always $\geq (E_s)_{\rm rad}$. This implies that the "redshift term" alone can never allow us to decrease the mass of the black hole at a greater rate than the normal Hawking evaporation rate (at least in two dimensions).

The second term in Eq. (3.4) represents the contribution due to the "intrinsic" mirror radiation. Let us assume that the trajectory of the mirror is such that the mirror is either asymptotically static or asymptotically moving with constant velocity ($V=dr^*/dt=$const) in the past and future. This is a physically reasonable restriction since otherwise the mirror will either be accelerating in the arbitrarily far past/future or the mirror will eventually fall into the black hole. The latter possibility involves two drawbacks: (a) the mirror is unrecoverable and (b) the finite (although small) rest mass of the mirror can no longer be ignored, as we have been implicitly doing up to now.

If we confine our attention to these types of trajectories, then from our argument following Eq. (2.27) we know that the second term in Eq. (3.4) is positive:

$$C_s^{-1/2}\int_{v_1}^{v_2} t_{vv}\,dv > 0 \ . \tag{3.7}$$

A consequence of Eq. (3.7), which will prove to be of crucial use the next section, is the fact that for these types of trajectories, any initial intrinsic negative-energy flux must always be more than compensated for by a subsequent intrinsic positive-energy flux.

In summary, for mirror trajectories in which the mirror is asymptotically either static or moving with constant velocity in the past and future,

$$E_{s,L} \geq (E_s)_{\rm rad} \ . \tag{3.8}$$

In physical terms this means that, for the trajectories specified, the integrated energy flux due to a moving mirror (including reflected Hawking radiation), as seen by a stationary observer near the horizon, will never be more negative than the ingoing negative-energy flux that would occur due to ordinary Hawking evaporation of the black hole into empty space. The latter is a process which does not produce an extreme ($Q=M$) Reissner-Nordström black hole in a finite amount of time (see Davies[19]). Note that Eq. (3.8) is also true for a Schwarzschild black hole, since this is just the special case when $Q=0$.

Since our result, Eq. (3.8), is a restriction only on integrated negative-energy fluxes, it does not by itself necessarily preclude the formation of a naked singularity in this process. However, it does imply that any naked singularity so formed would be exposed only for a limited time and would eventually be "reclothed" by an event horizon. In keeping with the original spirit of Penrose's terminology, we refer to this possibility as "cosmic flashing."

IV. TRANSIENT COSMIC-CENSORSHIP VIOLATION: COSMIC FLASHING

We wish to investigate the possibility of a violation of cosmic censorship in a Reissner-Nordström spacetime. If a charged black hole were to be formed by gravitational collapse, the spacetime is of the form of those illustrated in Figs. 2 and 3. Figure 2 shows a black-hole spacetime in which the $r=0$ singularity forms on the right; the left-hand singularity does not exist. Hiscock[20] has argued that this is likely to be the generic case. In this spacetime, no violation of cosmic censorship is possible, and injecting negative energy into the black hole cannot produce a violation. That is, no amount of negative energy can cause a null ray leaving the singularity to reach $\mathscr{J}^+$. However, it is not clear that Fig. 2 represents the only possible collapse-to-charged-black-hole spacetime. De Felice and Maeda[21] have argued that with certain initial conditions the spacetime will be of the form of Fig. 3, with the singularity forming on the left. In this case, negative energy injected after the collapse could produce a violation of cosmic censorship if it allows any null ray to connect the singularity to $\mathscr{J}^+$. The case in which a violation of cosmic censorship would be easiest to produce with negative energy is the eternal extreme ($Q=M$) Reissner-Nordström spacetime, the Penrose diagram for which is shown in Fig. 4. Here the future horizon H^+ is,

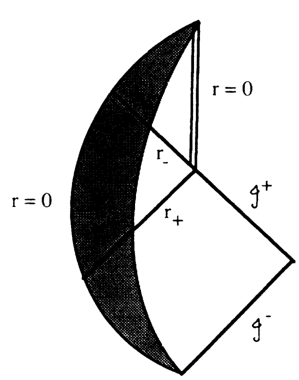

FIG. 2. A spacetime in which the singularity forms on the right as a result of gravitational collapse. The shaded region is the collapsing star.

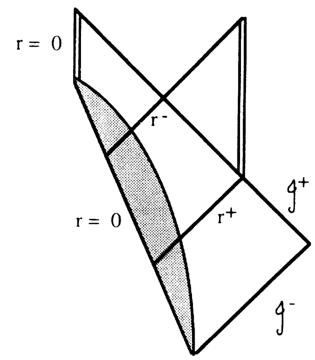

FIG. 3. A spacetime in which the singularity forms on the left as a result of gravitational collapse.

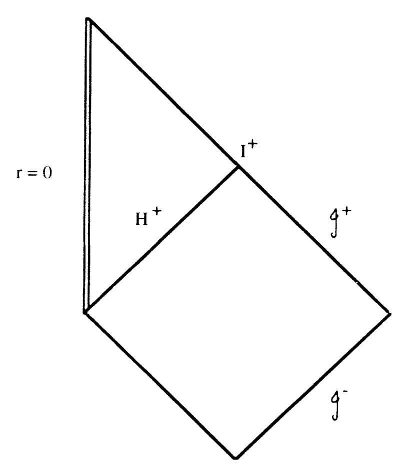

FIG. 4. The spacetime of an eternal, extreme ($Q=M$) Reissner-Nordström black hole. Here H^+ is the future event horizon, and I^+ is future timelike infinity. The 45° line above I^+ is a Cauchy horizon; the spacetime may be extended beyond this line, although this is not shown in the figure.

on one hand, the limit of all future-directed null rays which leave the singularity and is, on the other, the limit of all null rays which reach $\mathscr{I}^+$. Thus an infinitesimal perturbation of the spacetime could allow a null ray to connect the singularity with $\mathscr{I}^+$. We can regard this spacetime as the limit of the more realistic $Q<M$ black-hole spacetimes formed by gravitational collapse—the limit in which the cosmic-censorship hypothesis becomes most vulnerable.

For the $Q=M$ black hole, there is no Hawking radiation, so we need only be concerned with the intrinsic mirror radiation given by Eq. (2.24). As discussed in the preceding section, the net radiation emitted by a mirror for which $dr^*/dt \to \text{const}$ as $t\to\pm\infty$ must be non-negative. Thus any appearance of a naked singularity must be limited in time ("cosmic flashing"). A mirror trajectory which generates negative energy followed at a later time by compensating positive energy will produce cosmic flashing. Thus if the spacetime is treated as being strictly classical, cosmic censorship fails: An observer at $\mathscr{I}^+$ receives signals from the singularity.

However, the serious implications of this conclusion demand that we examine it more closely. In particular, the spacetime is in reality subject to quantum metric fluctuations which may render the violation of cosmic censorship unobservable. A full treatment of the effects of metric fluctuations would require a quantum theory of gravity. Nonetheless, we can postulate a reasonable criterion for the observability of the singularity: Let $|\Delta M|$ be the magnitude of the decrease in mass of the Reissner-Nordström spacetime due to the absorption of negative energy, and let ΔT be the time scale over which this decrease in mass persists. Then we expect

$$|\Delta M|\Delta T \gtrsim 1 \tag{4.1}$$

should be a necessary condition that the cosmic-censorship violation be observable. We will refer to Eq. (4.1) as the "classical observability requirement." The physical content of this requirement is that if an observer at $\mathscr{I}^+$ makes an observation on a time scale ΔT, quantum fluctuations on this time scale could obscure the effect of any change, ΔM, in the mass of the black hole unless $|\Delta M| \gtrsim 1/\Delta T$.

The limit in which this classical observability requirement will be easiest to satisfy will be when the negative- and positive-energy pulses are separated in time as much as possible. The extreme limit is the case of a δ-function pulse of negative energy, followed a time ΔT later by a compensating pulse of positive energy. The spacetime in which this occurs is illustrated in Fig. 5. Here an extreme $Q=M$ black-hole metric is converted into a $Q>M$ naked singularity metric by the negative-energy pulse $v=v_1$ and is subsequently converted back to a $Q=M$ metric by the positive-energy pulses at $v=v_2$.[22] Between retarded times u_1 and u_2, outgoing null rays from the singularity reach $\mathscr{I}^+$; this is the period of cosmic flashing.

What an observer at $\mathscr{I}^+$ would actually see during this period cannot be predicted, as there is no theory that predicts what the singularity will emit. A possible ansatz is to suppose that the singularity behaves as a constant-luminosity source. Then the observer at $\mathscr{I}^+$ will see a pulse of the form of that illustrated in Fig. 6. That is, this observer sees a source which turns on instantaneously at $u=u_1$; the output of this source appears to be a constant for a time $\Delta v = v_2 - v_1$, and then begins to decay, decreasing as $1/u$. The details of the derivation of this result are given in the Appendix. The characteristic time scale over which the source decays is determined by the parameters of the Reissner-Nordström metrics on either side of the boundary and is hence of order M. Thus the time scale ΔT over which cosmic flashing occurs is at least of order M. However, to increase ΔT as much as possible, we should arrange to have $\Delta v \gg M$, in which case $\Delta T \approx \Delta v$.

We now wish to investigate whether the classical observability requirement, Eq. (4.1), can be satisfied if the pulses of energy are generated by a moving mirror. First let

$$\tilde{a} = \frac{\dot{V}}{(1-V^2)^{3/2}} \ . \tag{4.2}$$

Note that in flat spacetime $\tilde{a}=a=a^\mu n_\mu$. As long as the mirror is moving on a trajectory for which $\tilde{a}$ is constant,

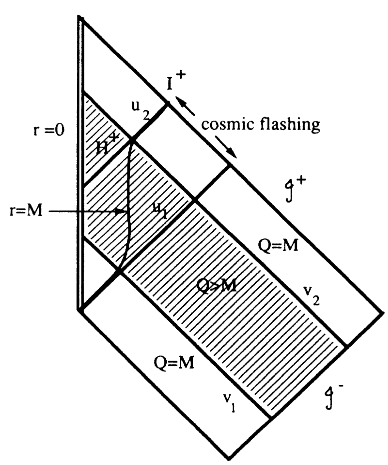

FIG. 5. A spacetime which exhibits cosmic flashing, the transient receipt of null rays at $\mathscr{I}^+$ from the singularity at $r=0$. A negative-energy pulse at advanced time $v=v_1$ converts a $Q=M$ Reissner-Nordström metric into a $Q>M$ metric; a positive-energy pulse at $v=v_2$ reconverts it. Between retarded time $u=u_1$, and $u=u_2$, outgoing null rays from the singularity reach $\mathscr{I}^+$. The future horizon H^+ is the $u=u_2$ line.

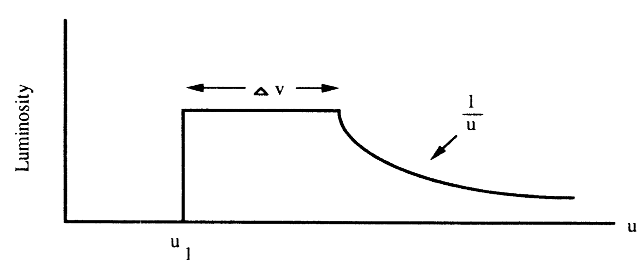

FIG. 6. The luminosity profile of the pulse that reaches $\mathscr{I}^+$, assuming a constant output from the singularity. The pulse turns on at $u=u_1$, is constant for a time $\Delta v = v_2 - v_1$, and then begins to decay inversely with time.

no radiation is emitted, as may be seen from Eq. (2.24).[23] However, if $\tilde{a}$ changes discontinuously, then a δ-function pulse of energy is emitted. If the change corresponds to an increase in $\tilde{a}$ (increasing acceleration *away* from the black hole), then the pulse emitted *toward* the black hole is positive; a decrease in $\tilde{a}$ emits a negative-energy pulse toward the black hole. A trajectory on which $\tilde{a}$ is constant is a hyperbola in the (t,r^*) coordinates; this may be seen by recalling that in flat spacetime, constant acceleration trajectories are hyperbolae. A trajectory which produces a negative δ-function pulse, followed later by a positive pulse, is shown in Fig. 7. Here the mirror is taken to be at rest at $r^*=r_0^*$, far from the horizon, where $C\approx 1$, for $t<0$. At time $t=0$, it starts accelerating toward the black hole with $\tilde{a}$ constant, and continues to do so for a finite time, after which it suddenly stops accelerating at a point outside $r=M$. Just after $t=0$, the mirror is still nearly at rest ($V \ll 1$), although later it may move relativistically. [I.e., suppose that we give the mirror a kick over a finite but short time interval Δt_1. During this interval, while the mirror is far from the black hole, we can let $\ddot{V}=\text{const}=\alpha$, so $\tilde{a}=\alpha\Delta t_1$ and the change in velocity is $\Delta V=\alpha(\Delta t_1)^2/2=\tilde{a}\Delta t_1/2$. For $\Delta t_1 \to 0$ with $\tilde{a}$ constant, $\Delta V \to 0$.] Thus, from Eqs. (2.24) and (4.2), the magnitude of the negative-energy pulse emitted at $t=0$, as measured at infinity, is

$$|\Delta E| = \frac{\tilde{a}}{12\pi} \ . \tag{4.3}$$

Let the advanced time interval between the negative- and positive-energy pulses be Δv. From Fig. 7, we can see that

$$\Delta v < \frac{1}{\tilde{a}} \ . \tag{4.4}$$

If $\Delta T \approx \Delta v$, then

$$|\Delta E| \Delta T < \frac{1}{12\pi} \ . \tag{4.5}$$

However, because $\Delta E = \Delta M$, the change in the black-hole mass,

$$|\Delta M| \Delta T < 1 \ . \tag{4.6}$$

Consequently, the classical observability criterion, Eq. (4.1), is *not* satisfied.

The trajectory in Fig. 7 assumes that the mirror was at rest far from the black hole for $t<0$. If the mirror was initially moving at a constant velocity, our conclusions will be unchanged. This may be seen by noting that the above discussion is formally identical to the case of a mirror in flat spacetime (where $a=\tilde{a}$ and $r=r^*$). The product $\Delta E \Delta t$ in flat spacetime is a Lorentz-invariant quantity which may be expressed as $p^\mu \Delta x_\mu$, where p^μ is the energy-momentum four-vector of the pulse of radiation and Δx_μ is a four-vector which has only a time component Δt in a Lorentz frame in which the mirror is initially at rest. Because this quantity is Lorentz invariant, it will be unchanged for a mirror which was initially moving at constant velocity. This can be physically interpreted as a cancellation between a Doppler-shift factor in ΔE and a time dilation factor in Δt. Finally, the formal equivalence of mirrors in flat spacetime to those in a Reissner-Nordström background, from Eq. (2.24), allows us to conclude that if the mirror in the latter case was initially moving with respect to the (t,r^*) coordinates with $V=dr^*/dt=\text{const}$, then Eq. (4.5) still applies.

We also have an upper bound on $\tilde{a}$: $\tilde{a}/(12\pi) \ll M$, where $M=Q$, This bound comes from the requirement $\Delta M \ll M$, Eq. (3.5), so that we may safely ignore back reaction. Note too that, other than the assumption that the mirror initially started far from the black hole, our conclusion, Eq. (4.6), does not depend on exactly where the mirror started.

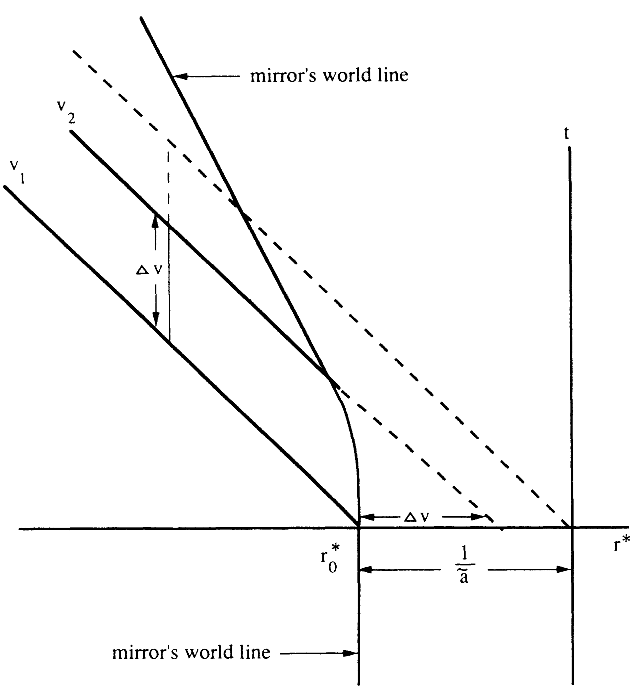

FIG. 7. The world line of a mirror which generates the pulses of negative and positive energy in Fig. 5. The mirror is at rest at $r^*=r_0^*$ for $t<0$. It then accelerates with $\tilde{a}=\text{const}$ for a finite time, and subsequently moves at a constant velocity. Negative- and positive-energy pulses are emitted at $v=v_1$ and $v=v_2$, respectively.

V. SUMMARY AND DISCUSSION

We have seen that negative-energy fluxes, such as those produced by moving mirrors, have the possible ability to violate cosmic censorship. We have focused our attention on the case of an extreme, $Q=M$, eternal Reissner-Nordström black hole as the case in which a naked singularity would be easiest to produce. Negative energy due to quantum coherence effects is not produced in isolation, but tends to be accompanied by compensating positive energy. This was demonstrated explicitly in Sec. III for a wide class of mirror trajectories on the Reissner-Nordström background. Consequently, any violation of cosmic censorship must be limited in time. This interval when signals may travel from the singularity to $\mathscr{I}^+$ we have termed "cosmic flashing." We have postulated a "classical observability requirement," Eq. (4.1), which should be satisfied in order that the "cosmic flash" not be lost in the background noise of quantum fluctuations. It was shown that this requirement is not satisfied.

We therefore conclude that the breakdown of predictability implied by a naked singularity probably does not occur. This conclusion depends upon the classical observability requirement, which is an uncertainty-principle-type inequality which seems to be a reasonable estimate of the scale of quantum metric fluctuations. However, it would be desirable to derive this condition from a more formal theory of quantum gravity. Our conclusions are in the context of a two-dimensional model. It would also be desirable to explore four-dimensional analogs of our models. These are topics for future research.

ACKNOWLEDGMENTS

We would like to thank B. Allen, A. Ottewill, A. Vilenkin, and R. Wald for helpful discussions. This work was supported in part by National Science Foundation Grant No. PHY-8905400 (L.F.) and by a Central Connecticut State University (CCSU) Faculty Research Grant (T.A.R.).

APPENDIX

In this appendix we present the matching of Reissner-Nordström metrics across a null line where there is an incoming δ-function pulse of energy. This matching leads to the derivation of the intensity profile illustrated in Fig. 6. The spacetime of interest is that shown in Fig. 5. We are concerned with the matching of metrics across the line $v=v_2$, where the naked singularity metric of mass $M_1<Q$ ($v_1<v<v_2$) is joined to the extreme black-hole metric of $M_2=Q$ ($v>v_2$). Let the line elements for these two regions be ds_1^2 and ds_2^2, respectively, where

$$ds_1^2=-C_1dt'^2+C_1^{-1}dr^2=C_1(-dt'^2+dr_1^{*2})\ , \tag{A1}$$

and

$$ds_2^2=-C_2dt^2+C_2^{-1}dr^2=C_2(-dt^2+dr_2^{*2})\ . \tag{A2}$$

Here

$$C_1=1-\frac{2M_1}{r}+\frac{Q^2}{r^2}$$

and

$$C_2=\left[1-\frac{M_2}{r}\right]^2\ ,$$

and t' and t are the time coordinates of the two regions. Note that the Schwarzschild-like radial coordinate r is the same in both regions, but the tortoise coordinates $r_1^*=\int C_1^{-1}dr$ and $r_2^*=\int C_2^{-1}dr$ are not.

Let $r=R(t')$ [equivalently $r_1^*=R_1^*(t')$] be the path of the incoming pulse ($v=v_2$). The line elements must agree on this boundary. Equate $ds_1^2=ds_2^2$ at $r=R(t')$ and use the fact that $dR/dt'=C_1dR_1^*/dt'=-C_1$. The result may be expressed, after some algebra, as

$$\frac{dt}{dt'}=\frac{C_1}{C_2}\ , \tag{A3}$$

where C_1 and C_2 are now understood to be evaluated at $r=R(t')$. The solution of Eq. (A3) yields the functional relation of the two time coordinates, $t(t')$. For our purposes, we need only find an approximate solution valid near the point where the $v=v_2$ line crosses $r=M$. Let the time coordinate of this point be $t'=T'$, which is finite in the t' coordinate. Thus there must be a Taylor expansion of the form

$$R(t')=M+a(T'-t')+O((T'-t')^2)\ , \tag{A4}$$

where the constant a is determined by M_1 and M_2. Integrating Eq. (A3) yields

$$t=\int\frac{C_1(R(t'))R^2(t')}{[R(t')-M]^2}dt'\ . \tag{A5}$$

If we now substitute the expansion for $R(t')$, Eq. (A4), into Eq. (A5), we find, to leading order,

$$t\approx\frac{C_1(M)M^2}{a^2(T'-t')}\ . \tag{A6}$$

Thus $t\to\infty$ as $t'\to T'$; this is to be expected, as it takes an infinite amount of t time for the incoming pulse to cross $r=M$.

We now need to find the relationship between r_2^* and r near $r=M$; it is

$$r_2^*=\int C_2^{-1}(r)dr\sim-\frac{M^2}{r-M}\ . \tag{A7}$$

From Eq. (A4),

$$R_2^*(t')\sim-\frac{M^2}{a(T'-t')}\ . \tag{A8}$$

An outgoing null ray has $u=t-r_2^*=\text{const}$ or $u'=t'-r_1^*=\text{const}$. We wish to find the relation between the values of u and of u' for a ray which crosses the $v=v_2$ line near $r=M$, that is, the outgoing rays that escape to $\mathcal{I}^+$ just before the horizon forms. First express u in terms of t':

$$u(t')=t(t')-R_2^*(t)\underset{t'\to T'}{\sim}\frac{M^2[C_1(M)+a]}{a^2(T'-t')}\ , \tag{A9}$$

using Eqs. (A6) and (A8). Similarly, $u'(t')=t'-r_1^*(t')$.

However, u' approaches a finite constant as $t' \to T'$, which we call u'_0. If we now eliminate t' and express u as a function of u' near $u' = u'_0$, we must have a relation of the form

$$u(u') \sim \frac{A}{u'_0 - u'} , \tag{A10}$$

where A is a constant.

Suppose that we have a source at $r = M$ which is emitting waves of frequency ω'. These waves reach $\mathcal{I}^+$ with a frequency ω. The phase of these waves is $\omega' u' = \omega u$, as the phase does not change across the $v = v_2$ line. Hence,

$$\omega = \left[\frac{u'}{u} \right] \omega' \sim \frac{u'_0}{u} \omega' \quad \text{as } u' \to u'_0, u \to \infty . \tag{A11}$$

Thus the observer at $\mathcal{I}^+$ sees the waves redshifted as u^{-1}, as illustrated in Fig. 6. This inverse power falloff of signals from an extreme Reissner-Nordström black hole was first obtained by Bicak.[24] For nonextreme black holes, the falloff is exponential in time.

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[1]S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Spacetime* (Cambridge University Press, London, 1973), pp. 88–96.

[2]R. M. Wald, *General Relativity* (The University of Chicago Press, Chicago, 1984), p. 303.

[3]H. Epstein, V. Glaser, and A. Jaffe, Nuovo Cimento **36**, 2296 (1965).

[4]S. A. Fulling and P. C. W. Davies, Proc. R. Soc. London **A348**, 393 (1976); P. C. W. Davies and S. A. Fulling, *ibid.* **A356**, 237 (1977).

[5]L. H. Ford, Proc. R. Soc. London **A364**, 227 (1978).

[6]P. C. W. Davies, Phys. Lett. **113B**, 215 (1982).

[7]D. Deutsch, A. C. Ottewill, and D. W. Sciama, Phys. Lett. **119B**, 72 (1982).

[8]P. Grove, Class. Quantum Grav. **5**, 1381 (1988).

[9]M. Morris, K. Thorne, and U. Yurtsever, Phys. Rev. Lett. **61**, 1446 (1988).

[10]To observe fluxes of net negative intrinsic energy in flat two-dimensional spacetime, an observer would have to "run ahead" of the mirror and would therefore be accelerating at least at the same rate as the mirror. It is well known that noninertial observers in flat spacetime see additional effects, e.g., the Unruh acceleration radiation, making it unclear as to whether or not such an observer running ahead of the mirror would actually measure a net negative flux. In any case, these observers would seem to be unphysical since they would require an infinite supply of energy to maintain their acceleration indefinitely.

[11]The four-dimensional analog of a mirror in a two-dimensional black-hole spacetime is a spherical mirror surrounding a black hole. However, the radiation from a moving mirror in four dimensions is not known exactly even in flat spacetime, although it can be calculated in special cases. [L. H. Ford and A. Vilenkin, Phys. Rev. D **25**, 2569 (1982); V. P. Frolov and E. M. Serebriany, J. Phys. A **12**, 2415 (1979).]

[12]A. Ottewill and S. Takagi, Prog. Theor. Phys. **79**, 429 (1988).

[13]W. Walker, Class. Quantum Grav. **2**, L37 (1985).

[14]P. C. W. Davies, S. A. Fulling, and W. G. Unruh, Phys. Rev. D **13**, 2720 (1976).

[15]W. G. Unruh and R. M. Wald, Phys. Rev. D **25**, 942 (1982).

[16]Although, as OT have shown, it does contribute to the net flux seen by an observer comoving with the mirror, since this observer is moving relative to the static negative-energy background vacuum polarization.

[17]In four dimensions, a charged black hole must have $M \gtrsim 10^5 M_\odot$, so that the hole does not discharge itself by electron-positron pair production due to its electric field. See, for example, G. Gibbons, Commun. Math. Phys. **44**, 245 (1975).

[18]W. Hiscock, Phys. Rev. D **15**, 3054 (1977).

[19]P. C. W. Davies, Proc. R. Soc. London **A353**, 499 (1977).

[20]W. Hiscock, J. Math. Phys. **22**, 215 (1981).

[21]F. de Felice and K. Maeda, Prog. Theor. Phys. **68**, 1967 (1982).

[22]In actuality, for pulses generated by a mirror, this represents a limiting case. For the trajectories of the mirror considered in Sec. III, we know that the integrated intrinsic flux is always positive—which implies that the final spacetime will be $Q < M$ rather than $Q = M$.

[23]In flat spacetime, mirror trajectories with constant proper acceleration (a=const) yield zero intrinsic radiation. That this is not the case in curved spacetime may be seen from examination of Eqs. (2.11) and (2.21) with a=const. The radiation emitted in these cases has surprising features which will be discussed in a later publication.

[24]J. Bicak, Czech. J. Phys. B **28**, 1 (1978); Gen. Relativ. Gravit. **3**, 331 (1972).

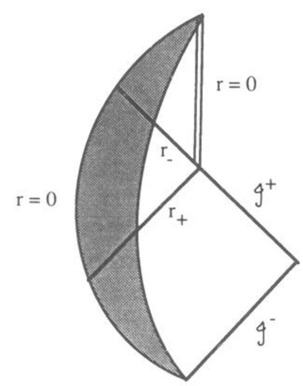

FIG. 2. A spacetime in which the singularity forms on the right as a result of gravitational collapse. The shaded region is the collapsing star.

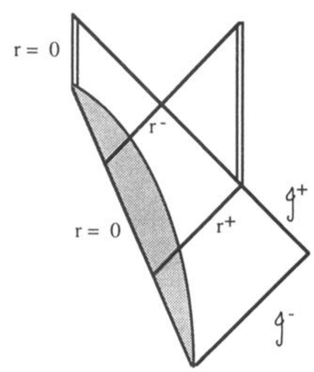

FIG. 3. A spacetime in which the singularity forms on the left as a result of gravitational collapse.