Partition temperatures of asymmetric fireballs in π^+p and K^+p collisions

T. F. Hoang
1749 Oxford Street, Berkeley, California 94709

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The pseudorapidity η distributions of π^- from pp, π^+p, and K^+p collisions of a CERN SPS experiment at $P_{\text{lab}}=250$ GeV/c are analyzed in terms of a covariant partition-temperature T_p model, the asymmetry of the distribution being accounted for by a shift parameter η^*. Remarks are made on the properties of T_p and η^*.

Recently, a geometrical model of multiparticle production has been formulated by Chou, Yang, and Yen using the concept of partition temperature T_p, which controls the energy E of particles in the center-of-mass system (c.m.s.) of the collision.[1] They describe the single-particle distribution in the c.m.s. of secondaries from nondiffractive processes as

$$dn \sim g(P_\perp)e^{-E/T_p}\frac{d^3P}{E}\ , \tag{1}$$

$g(P_\perp)$ being a cutoff factor for the transverse momentum: $g(P_\perp)\sim e^{-\alpha P_\perp}$ with $\alpha=2/\langle P_\perp\rangle$. The pseudorapidity $\eta=\ln(\cot\theta_{\text{c.m.}}/2)$ distribution according to their model is

$$\frac{dn}{d\eta}=\frac{N}{\left[\alpha+\dfrac{1}{T_p}\cosh y\ \eta\right]^2}\ , \tag{2}$$

which holds for $\bar{p}p$ and pp collisions as reported previously.[4]

For secondaries of pp or $\bar{p}p$ collisions, the multiplicity is the same on both sides, forward (FD) and backward (BD), in the c.m.s. This property, due to the symmetry of the initial state, holds also for T_p. But the situation is different in the case of π^+p, K^+p, and lepton-p collisions, the fireballs (FB) no longer being symmetric, as is shown by the FD and BD multiplicities $\langle n_-\rangle$ in Fig. 1 for π^+p, as discussed below.

Consider, for instance, the inclusive reaction

$$\pi^+ + p \to \pi^- + \cdots\ .$$

It is well known that $\langle n_-\rangle_{\text{FD}}$ is significantly greater than $\langle n_-\rangle_{\text{BD}}$ as is seen in Fig. 1 for $P_{\text{lab}}=5$ to 250 GeV/c.[2] We will see that the T_p of the π^-'s of the FD side in π^+p c.m.s. is greater than that of the BD side. This difference, nonetheless, is merely a kinematic effect, due to the asymmetry of the fireballs, as will be discussed later.

For this purpose, consider the case of the π^+p collision at $P_{\text{lab}}=250$ GeV/c of the NA22 Collaboration.[3] This experiment is distinguished by simultaneous measurements of $P_\perp$, x, and η distributions of π^- from π^+p, K^+p, and pp collisions, required for our analysis of the present work. The properties of $\langle n_-\rangle_{\text{FD}}$ of this experiment have been reported previously.[4] Their data of the pseudorapidity $\eta=\ln(\cot\theta_{\text{c.m.}}/2)$ distribution in the c.m.s. are reproduced in Fig. 2, together with their $pp\to\pi^-$ at the same energy for comparison. We will use these η distributions to estimate the partition temperature by means of the following formula derived from (1) by Chou, Yang, and Yen (CYY) for zero-mass particles.[5] Their formula (2) may be generalized to describe the asymmetric η distributions, to be discussed below, where N is the normalization coefficient and α the parameter of $P_\perp$ cutoff given by $\alpha=2/\langle P_\perp\rangle$.

We now use (2) to fit separately the FD and the BD η distributions of the NA22 data, assuming $\alpha=2/\langle P_\perp\rangle$. As no account has been taken of the seagull effect on $P_\perp$ in the central region, therefore, as reported elsewhere, we limit (2) to $\eta>1$. The fits thus obtained are shown by the solid curves in Fig. 2, the dashed lines being extrapolations to $\eta=0$. The estimates of T_p (in GeV) are listed in Table I. Note that the fit determines only αT_p. A comparison with the data indicates that the fits are very satisfactory indeed. We find different T_p for forward and

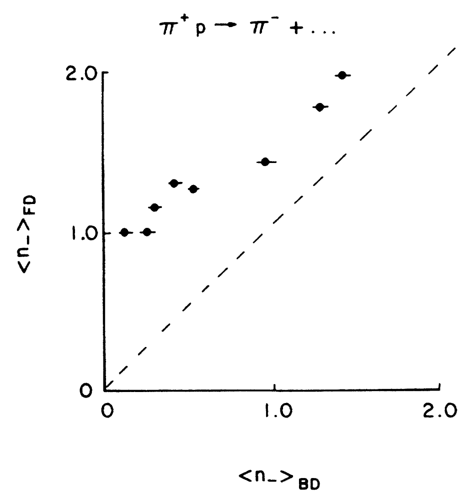

FIG. 1. Plots of forward and backward multiplicities of inclusive $\pi^+p\to\pi^-$ at $P_{\text{lab}}=5$ to 250 GeV/c. Data compiled by the Bologna Group, Ref. 2. The dotted line represents the case of symmetric fireballs of $pp\to\pi^-$, see text.

backward π^-'s. Their ratio 2.50 ± 0.43 is indeed very large. In this respect, we note that the BD T_p is consistent with those of pp and K^+p as listed in Table I: The average being 0.463 ± 0.036 GeV. This is expected from the well-known property of limiting fragmentation of the target proton of a previous investigation.[5] Note that the errors on T_p are rather large, as is seen from the pp case.

As mentioned before, the difference in T_p thus observed is rather due to the asymmetry of the two fireballs associated with the π^+ projectile and the p target. As $\langle n_-\rangle_{FD} \neq \langle n_-\rangle_{BD}$; their c.m.s. is not the same as the initial c.m.s. of the colliding π^+ and p. Therefore, we have to use an appropriate system to estimate T_p as far as the partition energy of produced particles is concerned.

For this purpose, we consider the covariant Boltzmann factor as in the case of the conventional statistical model discussed before,[6] namely,

$$f \sim e^{(E-\beta_F P_\parallel)/T}, \tag{3}$$

where T is the conventional temperature[8] and β_F is the velocity of the *rest frame* of secondaries (on both sides) with respect to the c.m.s. We recall that β_F is related to the Feynman-Yang scaling and that it may be estimated by the x distribution (see below). If T_p transforms like T,[9] we may write the passage to the rest frame of secondaries, specified by an asterisk, as

$$E^* = \gamma_F(E-\beta_F P_\parallel), \quad T_p^* = \gamma_F T_p \tag{4}$$

so that Eq. (1) becomes

$$dn \sim g(P_\perp) e^{(E-\beta_F P_\parallel)/T_p} \frac{d^3P}{E}, \tag{5}$$

which has been considered by Li and Young[7] in the context of their partition temperature model for p-nucleus reactions.

We may set

$$\beta_F = \tanh\eta^* \tag{6}$$

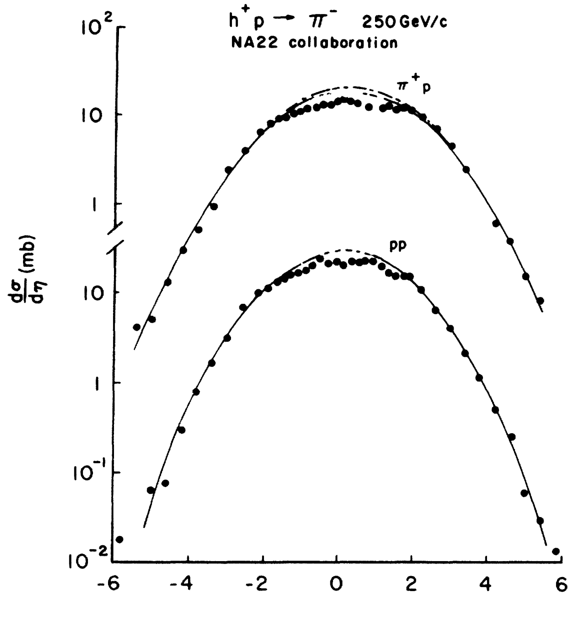

FIG. 2. The c.m.s. pseudorapidity distribution of π^- from π^+p and pp at $p_{\rm lab}=250$ GeV/c (data of NA22 Collaboration, Ref. 3). Typical errors $\sim 2\%$. The solid curves are fits with (5) for $\eta > 1$; the dotted lines represent extrapolations to $\eta=0$. The parameters of fit are listed in Table I. The discontinued curve is the overall fit with (14) to estimate T_p^* in the FB c.m.s. The parameters are listed in Table I.

and rewrite (5) in terms of $P_\perp$ and η to generalize the CYY formula. By a simple integration, we get

$$\frac{dn}{d\eta} = \frac{N}{\left[\alpha + \frac{1}{T_p^*}\cosh(\eta-\eta^*)\right]^2}, \tag{7}$$

TABLE I. Parameters of inclusive $h^+p \to \pi^-$ at $P_{\rm lab}=250$ GeV/c from NA22 Collaboration, Ref. 3.

		π^+p	K^+p	pp
$\langle n_{\rm ch}\rangle$		8.43 ± 0.02	8.38 ± 0.04	7.85 ± 0.08
$\langle P_\perp\rangle$ (MeV/c)		354 ± 4	358 ± 8	362 ± 8
T (MeV)		142 ± 2	144 ± 2	146 ± 2
α [(GeV/c)$^{-1}$]		5.65	5.58	5.56
T_p (GeV)	FD	1.270 ± 0.083	0.995 ± 0.043	0.479 ± 0.032
	BD	0.504 ± 0.080	0.425 ± 0.051	0.409 ± 0.024
a ($x>0.2$)	FD	8.84 ± 0.58	10.86 ± 0.60	13.85 ± 0.60
	BD	11.88 ± 0.52	13.55 ± 0.52	13.55 ± 0.52
β_F	FD	0.882 ± 0.008	0.848 ± 0.006	0.817 ± 0.005
	BD	0.842 ± 0.006	0.823 ± 0.005	0.819 ± 0.004
$\Delta\eta_F$		0.15 ± 0.02	0.11 ± 0.02	0.00 ± 0.02
Overall fit	η^*	0.19 ± 0.03	0.17 ± 0.05	$=0$
FB (c.m.s.) T_p^* (GeV)		0.713 ± 0.018	0.643 ± 0.031	0.521 ± 0.025

where α remains the same.

We have made *overall* fits with (7) using the same criteria $|\eta| > 1$ as before and assuming η^* as a free parameter. The values of T_p^* and η^* are listed in Table I, α being the same as before. As an illustration, the fit for π^+p is shown in Fig. 2 by the discontinued curve that is distinct from the previous fits for $|\eta| < 2$.

It is interesting to note that the estimates of T_p^* and T_p are not independent. Indeed, from the values in Table I, we get approximately

$$(T_p^*)^2 \simeq (T_p)_{\rm FD}(T_p)_{\rm BD} \tag{8}$$

as is required by the passage from the initial frame for T_p^* to the final frame for T_p in the c.m.s., where the FD and the BD π's are observed. This property is in contradistinction with the conventional temperature T which is invariant so that (8) is trivial.[9]

With regard to the parameter η^*, it is to be noted that $dn/d\eta$ is related to dn/dx ($x = 2P_{\parallel}/\sqrt{s}$) of the FD and BD π^- by a change of variable. We have analyzed the data of the NA22 experiment,[3] Fig. 3, using the covariant Boltzmann factor (3) and assuming classical phase space and $m = 0$; this leads to

$$\frac{dn}{dx} \sim x e^{-ax} , \tag{9}$$

where

$$a = (1-\beta_F)\sqrt{s}/2T , \tag{10}$$

T being the conventional temperature. We have fitted the data of NA22 Collaboration[3] with (8), for $x > 0.2$. The results are shown in Fig. 3. The fits are very satisfactory. The estimates of β_F and the corresponding γ_F thus obtained are listed in Table I, together with $\eta_F = \text{arctanh}\beta_F$ and

$$\Delta\eta_F = (\eta_F)_{\rm FD} - (\eta_F)_{\rm BD} . \tag{11}$$

We find

$$\eta^* \simeq \Delta\eta_F \tag{12}$$

as expected; namely, η^* corresponds to the pseudorapidity of the original FB with respect to the c.m.s. before it splits into two parts moving in opposite directions which belong to the projectile and the target. This property (11) justifies *a posteriori* our parametrization of the generalized CYY formula.

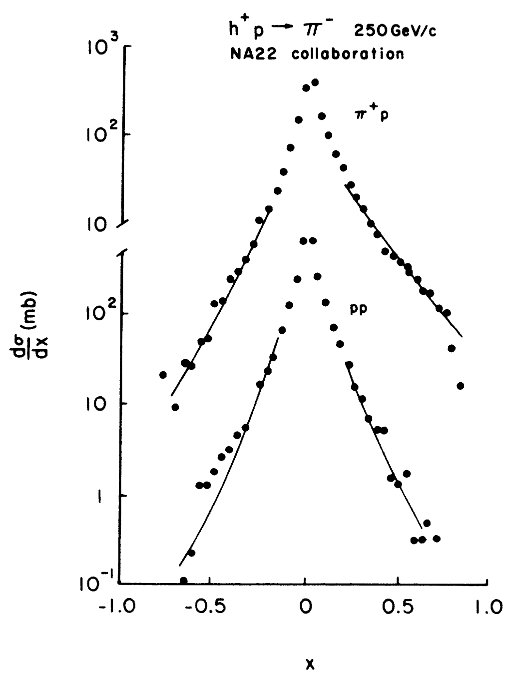

FIG. 3. The x distribution of inclusive π^+p and $pp \to \pi^-$ at $P_{\rm lab} = 250$ GeV/c (data of NA22 Collaboration, Ref. 3). Typical errors $\sim 3\%$. The curves are fits with (9) for $x \geq 0.2$. The parameters are listed in Table I.

Finally, we note that the generalized CYY formula (7) may be used to describe the asymmetry of the η distribution of secondaries of nuclear reactions. We note in passing that the shift parameter η^* for nuclear reactions is found to be energy independent and that it depends only on the relative size of the projectile and the target to be reported elsewhere.[10]

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