Leptonic SU(3), grand unification, and higher-dimensionality gravidynamics

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Two considerations pertaining to the electroweak symmetry of leptons, and to higher-dimensionality gravidynamic spacetime-internal unification, lead us to suggest the gauging of SU(15), for each generation of leptons and quarks. On one hand, the electroweak leptonic sector is governed by SU(3), while the quark sector is standard. On the other hand, the Lorentz symmetry of Weyl fermions is generalized to spin-containing SU(2n,C). Sketching the basic elements of the corresponding higher-dimensionality gravidynamics, we point out an associated quark-lepton unification scheme which does not require $V+A$ generations.

I. INTRODUCTION

The theoretical unification of elementary-particle symmetries, and their gauge interactions, is a fundamental task yet to be accomplished. In this paper we treat two diverse aspects of unification which converge on the same result regarding the nature of the grand unification symmetry. One aspect concerns the electroweak gauge symmetry, before quark-lepton grand unification. The other derives from the desire to unify spacetime and internal symmetries in a higher-dimensional framework, and without introducing unobserved mirror fermions.

The currently popular model describing the interactions of quarks and leptons is based on a spontaneously broken SU(2)$\times$U(1) electroweak gauge symmetry,[1–4] and on an exact SU(3)$_c$ color gauge symmetry[5–7] associated with the strong interactions of quarks. In this so-called standard model, quarks carry fractional electric charges and are presumed to be permanently confined inside their hadronic composites. With the purpose of consolidating the SU(2)$\times$U(1) electroweak (EW) gauge model, and fixing the arbitrary mixing angle, we have proposed[8] many years ago an SU(3)$_{\rm EW}$ symmetry. The latter was specifically applied to the (left-handed) leptonic triplet of each generation, consisting of the charged lepton, its antiparticle, and the associated neutrino. At that time we noted the difficulty of applying our SU(3)$_{\rm EW}$ symmetry to quarks that are conventionally characterized by fractional charges, and the model was not pursued much further.[9,10] Part of our work in this paper is to incorporate the standard electroweak interactions of quarks via a separate SU(2)$^q\times$U(1)^q symmetry. We shall describe the breaking of the latter together with our leptonic SU(3)^l down to their common standard factor.[11,12]

Each generation of quarks and leptons consists of 15 Weyl fermions (16 if the neutrino has a Dirac mass). These can be accommodated in a reducible SU(5) multiplet (**15**=**5**+**10***), in the (chiral) spinorial multiplet **16** of O(10), or in the fundamental **15** [**16**] of SU(15) [SU(16)]. The corresponding gauge theories[13–17] have long been recognized as possible frameworks for the unification of electronuclear interactions. The unification of any such internal-symmetry algebra with the Lorentz algebra is an essential prerequisite for the unification of gravity with electronuclear interactions.

Early attempts at a nontrivial unification of the spacetime and the internal elementary-particle (hadronic) symmetries had sought relativistic extensions,[18–22] in four-dimensional spacetime, of the rather successful spin-containing SU(6) symmetry.[23–25] However, it had soon been realized[26] that for such attempts to be successful, one must either extend the Lie-algebraic framework, or must introduce extra spacetime dimensions. Extended superalgebraic theories,[27–31] which unify the Lorentz-Poincaré algebra with an internal-symmetry counterpart (notably among which is the $N=8$ supergravity[30]), have so far been unable to present a realistic quark-lepton model. On the other hand, higher-dimensionality theories,[32–34] which generalize the Einstein-Dirac framework to a spacetime with dimensionality $D>4$, have met the rather enigmatic situation of predicting a right-handed (mirror) counterpart to each quark-lepton generation.[35]

Our second task in this paper is to describe a novel scheme for unifying the internal and the spacetime symmetries of each generation (and possibly several generations) of quarks and leptons. Our scheme is based on a higher-dimensionality generalization of local Lorentz invariance associated with Weyl fermions, and the corresponding gravitational dynamics. Whereas the conventional $D>4$ schemes are compatible with internal O(n) symmetries, our scheme is compatible with spin-containing SU(2n) symmetries, and does not necessarily require mirror quark-lepton generations.

In the following section we present a concise account of the basic electroweak gauge model, and review our SU(3)$_{\rm EW}$ gauge model. In Sec. III we discuss a scheme for separate SU(2)$\times$U(1) electroweak symmetries for leptons and quarks. We then promote the leptonic sector to SU(3), and discuss a possible consolidating scheme for quarks and leptons. Section IV treats our spacetime-internal unification scheme in the framework of higher-dimensionality gravidynamics, and the associated grand unification sector. Section V presents a concluding discussion, with some remarks concerning the question of chiral anomalies.

II. STANDARD ELECTROWEAK AND LEPTONIC SU(3)

The standard electroweak model introduces a triplet of SU(2) vector gauge fields $\mathbf{W}$ and a U(1) gauge field $\mathcal{B}$. We shall represent $\mathbf{W}$ by the (traceless and Hermitian) matrix

$$\mathbf{W}=\begin{bmatrix} \frac{1}{2}\mathcal{W}_3 & \frac{1}{\sqrt{2}}\mathcal{W}^+ \\ \frac{1}{\sqrt{2}}\mathcal{W}^- & -\frac{1}{2}\mathcal{W}_3 \end{bmatrix}, \quad (\mathcal{W}^+)^* = \mathcal{W}^- \ , \tag{1}$$

with the normalization

$$\mathrm{Tr}|\mathbf{W}|^2=\tfrac{1}{2}\mathcal{W}_3^2+\mathcal{W}^+\mathcal{W}^- \ . \tag{2}$$

The (electromagnetic) photon field $\mathcal{A}$ and a (massive) neutral counterpart $\mathcal{Z}$ correspond to $\mathcal{B}$ and $\mathcal{W}_3$ via the mixing

$$\mathcal{B}=\mathcal{A}\cos\theta-\mathcal{Z}\sin\theta \ , \tag{3}$$

$$\mathcal{W}_3=\mathcal{A}\sin\theta+\mathcal{Z}\cos\theta \ . \tag{4}$$

This mixing results from the breaking of SU(2)$\times$U(1) down to the electromagnetic $\mathrm{U}(1)_{\mathrm{em}}$, and is triggered by the neutral component of a complex Higgs doublet (ϕ_+,ϕ_0), in which process the photon $\mathcal{A}$ remains massless, while $\mathcal{W}^{\pm}$ and $\mathcal{Z}$ acquire masses according to the relation

$$M_W^2=(M_Z\cos\theta)^2 \ . \tag{5}$$

Leptons and (tricolored) quarks of each generation are introduced as doublets and singlets:

$$\lambda=\begin{bmatrix} \nu \\ l \end{bmatrix}, \quad l^c \ , \tag{6}$$

$$\chi_i=\begin{bmatrix} u_i \\ d_i \end{bmatrix}, \quad u^{ci}, \quad d^{ci}, \quad (i=1,2,3) \ . \tag{7}$$

Here, all fermionic components are left-handed Weyl fields. These acquire the SU(2)$\times$U(1) gauge couplings

$$\begin{aligned}\mathcal{L}=&\, g_2(\bar{\lambda}\mathbf{W}\lambda+\bar{\chi}^i\mathbf{W}\chi_i)\\ &+g_1(a\bar{\lambda}\mathcal{B}\lambda+b\bar{l}^c\mathcal{B}l^c\\ &\quad+x\bar{\chi}^i\mathcal{B}\chi_i+y\bar{u}_i^c\mathcal{B}u^{ci}+z\bar{d}_i^c\mathcal{B}d^{ci}) \ .\end{aligned} \tag{8}$$

Here, g_2 and g_1 are the respective SU(2) and U(1) coupling constants, and (a,b,x,y,z) are the relative U(1) hypercharge eigenvalues of the corresponding particles (we put $b=1$). These can be determined by fixing the electric charges. Hence, noting the couplings of the photon $\mathcal{A}$ in Eq. (8), and prescribing the electric charges $(0,-1,+1)$ to (ν,l,l^c), we obtain

$$a=-\tfrac{1}{2}, \quad g_2=\frac{1}{\sin\theta}, \quad g_1=\frac{1}{\cos\theta} \ . \tag{9}$$

Hence, g_2 and g_1 are traded for the mixing angle θ and the fundamental electromagnetic coupling which we take as a unit $(e=1)$. Likewise, prescribing for the quarks (u_i,d_i,u^{ci},d^{ci}) the respective electric charges $(\frac{2}{3},-\frac{1}{3},-\frac{2}{3},\frac{1}{3})$, obtain

$$x=\tfrac{1}{6}, \quad y=-\tfrac{2}{3}, \quad z=\tfrac{1}{3} \ . \tag{10}$$

Notice that the relation

$$Q=I_3+Y \ , \tag{11}$$

where I_3 is the third generator of SU(2), and Y is the U(1) eigenvalue, is always satisfied.

For later reference we expand Eq. (8) to the form

$$\begin{aligned}\mathcal{L}_I=&-\bar{l}\mathcal{A}l+\bar{l}^c\mathcal{A}l^c+\tfrac{2}{3}(\bar{u}^i\mathcal{A}u_i-\bar{u}_i^c\mathcal{A}u^{ci})-\tfrac{1}{3}(\bar{d}^i\mathcal{A}d_i-\bar{d}_i^c\mathcal{A}d^{ci})\\ &+\frac{1}{2\sin\theta\cos\theta}[\bar{\nu}\mathcal{Z}\nu+(-1+2\sin^2\theta)\bar{l}\mathcal{Z}l+(1-\tfrac{4}{3}\sin^2\theta)\bar{u}^i\mathcal{Z}u_i-(1-\tfrac{2}{3}\sin^2\theta)\bar{d}^i\mathcal{Z}d_i]\\ &+\tan\theta(-\bar{l}^c\mathcal{Z}l^c+\tfrac{2}{3}\bar{u}_i^c\mathcal{Z}u^{ci}-\tfrac{1}{3}\bar{d}_i^c\mathcal{Z}d^{ci})+\frac{1}{\sqrt{2}\sin\theta}(\bar{\nu}\mathcal{W}^+l+\bar{u}^i\mathcal{W}^+d_i+\mathrm{H.c.}) \ .\end{aligned} \tag{12}$$

In our SU(3) electroweak model,[8] the three Weyl fermions of each leptonic generation are represented by the triplet

$$\lambda=\begin{bmatrix} l^c \\ \nu \\ l \end{bmatrix} \ . \tag{13}$$

The $\mathrm{SU}(3)_{\mathrm{EW}}$ gauge fields are represented by the matrix

$$\mathbf{W}=\begin{bmatrix} \left[\mathcal{A}+\frac{1}{\sqrt{3}}\mathcal{Z}\right] & \sqrt{2}\mathcal{U}^+ & \sqrt{2}\mathcal{V}^{++} \\ \sqrt{2}\mathcal{U}^- & -\frac{2}{\sqrt{3}}\mathcal{Z} & \sqrt{2}\mathcal{W}^+ \\ \sqrt{2}\mathcal{V}^{--} & \sqrt{2}\mathcal{W}^- & \left[-\mathcal{A}+\frac{1}{\sqrt{3}}\mathcal{Z}\right] \end{bmatrix}, \tag{14}$$

$$\frac{1}{4}\mathrm{Tr}|\mathbf{W}|^2=\frac{1}{2}\mathcal{A}^2+\frac{1}{2}\mathcal{Z}^2+\mathcal{W}^+\mathcal{W}^-+\mathcal{U}^+\mathcal{U}^-+\mathcal{V}^{++}\mathcal{V}^{--} . \tag{15}$$

In addition to the photon $\mathcal{A}$, its massive counterpart $\mathcal{Z}$, and the charged-current mediators $\mathcal{W}^\pm$, new particles $\mathcal{U}^\pm$ and $\mathcal{V}^{\pm\pm}$ are predicted. The gauge couplings of all these particles to the leptons are given (in units of the electromagnetic coupling constant) by

$$\bar{\lambda}\mathbf{W}\lambda=(-\bar{l}\mathcal{A}l+\bar{l}^c\mathcal{A}l^c)+\frac{1}{\sqrt{3}}(\bar{l}\mathcal{Z}l+\bar{l}^c\mathcal{Z}l^c-2\bar{\nu}\mathcal{Z}\nu)+\sqrt{2}(\bar{\nu}\mathcal{W}^+l+\bar{l}^c\mathcal{U}^+\nu+\bar{l}^c\mathcal{V}^{++}l+\mathrm{H.c.}) . \tag{16}$$

Comparing Eq. (16) with Eq. (12) we note that the couplings of $\mathcal{A}$, $\mathcal{Z}$, and $\mathcal{W}^\pm$ correspond to those of the standard SU(2)×U(1) electroweak model with the mixing angle fixed at 30°, ($\sin^2\theta=\frac{1}{4}$).

Equation (16) also shows that the new couplings with $\mathcal{U}^\pm$ and $\mathcal{V}^{\pm\pm}$ would lead, when two generations are involved, to

$$\mu^-\to\mathcal{U}^-+\nu_\mu^c\to(e^-+\nu_e)+\nu_\mu^c , \tag{17}$$

$$e^-+e^-\to\mathcal{V}^{--}\to\mu^-+\mu^- . \tag{18}$$

Evidently, these reactions violate separate muon-type and electron-type lepton numbers, and would have to be suppressed. From the lower bound on the branching ratio of the decay (17) relative to the usual decay ($\mu^-\to e^-+\nu_e^c+\nu_\mu$) mediated by $\mathcal{W}^-$, we could estimate the mass of $\mathcal{U}^\pm$ to be greater than about $\frac{3}{2}M_W$. The mass of $\mathcal{V}^{\pm\pm}$ can also be estimated according to our breaking scheme of $\mathrm{SU(3)_{EW}}$ down to SU(2)×U(1) and further to $\mathrm{U(1)_{em}}$.

In our scheme of breaking $\mathrm{SU(3)_{EW}}$, we have a strong breaking down to SU(2)×U(1), and a weaker breaking directly to $\mathrm{U(1)_{em}}$. For the first breaking we utilize a Higgs field in the adjoint representation, whose vacuum value is represented by the diagonal matrix

$$\langle\Phi\rangle=M\begin{bmatrix}2&0&0\\0&-1&0\\0&0&-1\end{bmatrix} . \tag{19}$$

From the gauge couplings of $\mathbf{W}$, represented by Eq. (14), we obtain the mass terms

$$|[\mathbf{W},\langle\Phi\rangle]|^2\Longrightarrow M^2(\mathcal{U}^+\mathcal{U}^-+\mathcal{V}^{++}\mathcal{V}^{--}) . \tag{20}$$

For the weaker breaking of $\mathrm{SU(3)_{EW}}$ directly down to $\mathrm{U(1)_{em}}$, we utilize a complex Higgs field ϕ in the symmetric 6-plet representation, whose vacuum value is represented by the matrix

$$\langle\phi_{ij}\rangle=\mu\begin{bmatrix}0&0&1\\0&0&0\\1&0&0\end{bmatrix} . \tag{21}$$

In contrast with Φ, this Higgs field couples to fermions, providing the charged leptons with their masses, via the SU(3)-invariant Yukawa coupling

$$(\lambda_i^TC^{-1}\lambda_i\phi^{ij}+\mathrm{H.c.}) . \tag{22}$$

The gauge coupling of $\mathbf{W}$ to ϕ gives such mass terms

$$|(\mathbf{W})_i^m\phi_{mj}+(i\leftrightarrow j)|^2\Longrightarrow\mu^2(4\mathcal{V}^{++}\mathcal{V}^{--}+\mathcal{U}^+\mathcal{U}^-+\mathcal{W}^+\mathcal{W}^-+\tfrac{2}{3}\mathcal{Z}^2) . \tag{23}$$

From Eqs. (20) and (23) we find that the vector-boson masses have acquired relations of the form

$$M_Z=\frac{2}{\sqrt{3}}M_W , \tag{24}$$

$$M_U=M_W+\Delta,\quad M_V=2M_W+\Delta . \tag{25}$$

Equation (24) gives the usual relation between M_Z and M_W in the standard model, given by Eq. (5), however, with $\theta=30°$. Equations (25) give M_U and M_V in terms of M_W and some arbitrary shift Δ. As noted after Eq. (18) above, Δ can be estimated to be greater than $\frac{1}{2}M_W$. Hence, we have the modest estimates

$$M_U>\tfrac{3}{2}M_W,\quad M_V>\tfrac{5}{2}M_W . \tag{26}$$

Hence, our leptonic $\mathrm{SU(3)_{EW}}$ puts $\mathcal{U}^\pm$ and $\mathcal{V}^{\pm\pm}$ above 120 and 200 GeV, respectively.

As we wish to maintain the standard scheme for quarks, we propose separate leptonic and quark electroweak symmetries at high energies, which would break down to the standard common factor.

III. SEPARATE LEPTON AND QUARK SYMMETRIES

We begin by considering the electroweak gauge group $\mathrm{SU(2)}^l\times\mathrm{U(1)}^l\times\mathrm{SU(2)}^q\times\mathrm{U(1)}^q$, which consists of separate standard symmetries for leptons and quarks. The associated gauge fields will be represented by the matrices

$$\mathbf{W}^l=\begin{bmatrix}\frac{1}{2}\mathcal{W}_3^l+y\mathcal{B}^l & \frac{1}{\sqrt{2}}\mathcal{W}_+^l\\ \frac{1}{\sqrt{2}}\mathcal{W}_-^l & -\frac{1}{2}\mathcal{W}_3^l+y\mathcal{B}^l\end{bmatrix} , \tag{27}$$

$$\mathbf{W}^q=\begin{bmatrix}\frac{1}{2}\mathcal{W}_3^q+z\mathcal{B}^q & \frac{1}{\sqrt{2}}\mathcal{W}_+^q\\ \frac{1}{\sqrt{2}}\mathcal{W}_-^q & -\frac{1}{2}\mathcal{W}_3^q+z\mathcal{B}^q\end{bmatrix} . \tag{28}$$

Here, y and z represent the $\mathrm{U(1)}^l$ and the $\mathrm{U(1)}^q$ eigenvalues, respectively, which are associated with the various matter fields.

The breaking of the above symmetry down to the familiar SU(2)×U(1) is achieved by utilizing a complex Higgs multiplet Φ in the representation (**2**,**2**) of $\mathrm{SU(2)}^l\times\mathrm{SU(2)}^q$. This takes the $\mathrm{U(1)}^l\times\mathrm{U(1)}^q$ eigenvalues ($y=\frac{1}{2}$, $z=\frac{1}{2}$). The vacuum value acquired by Φ takes the form

$$\frac{1}{M}\langle\Phi\rangle=\begin{bmatrix}1\\0\end{bmatrix}_l\otimes\begin{bmatrix}0\\1\end{bmatrix}_q+\begin{bmatrix}0\\1\end{bmatrix}_l\otimes\begin{bmatrix}1\\0\end{bmatrix}_q . \tag{29}$$

Here, M is a mass scale.

From the gauge couplings of $\mathbf{W}^l$ and $\mathbf{W}^q$ to Φ, we obtain the mass terms

$$|(\mathbf{W}^l+\mathbf{W}^q)\langle\Phi\rangle|^2 \Longrightarrow M^2[\tfrac{1}{2}(\mathcal{W}_3^l-\mathcal{W}_3^q)^2+\tfrac{1}{2}(\mathcal{B}^l+\mathcal{B}^q)^2 + (\mathcal{W}_+^l+\mathcal{W}_+^q)(\mathcal{W}_-^l+\mathcal{W}_-^q)] . \tag{30}$$

The four gauge coupling constants, which we have suppressed so far, will be traded for the electromagnetic coupling constant (taken as unit), and three mixing angles (θ^l combining $\mathcal{W}_3^l$ and $\mathcal{B}^l$; θ^q combining $\mathcal{W}_3^q$ and $\mathcal{B}^q$; and φ combining the leptonic gauge fields with the corresponding quark gauge fields). Hence, reinstating the gauge couplings in Eq. (30), we obtain, for the right-hand side (RHS),

$$\left[\frac{M}{\sin\varphi\cos\varphi}\right]^2 [\tfrac{1}{2}(\mathcal{W}_3^H)^2+\tfrac{1}{2}(\mathcal{B}^H)^2+\mathcal{W}_+^H\mathcal{W}_-^H] , \tag{31}$$

where

$$\mathcal{W}_3^H=\mathcal{W}_3^l\cos\varphi-\mathcal{W}_3^q\sin\varphi , \tag{32a}$$

$$\mathcal{W}_\pm^H=\mathcal{W}_\pm^l\cos\varphi+\mathcal{W}_\pm^q\sin\varphi , \tag{32b}$$

$$\mathcal{B}^H=\mathcal{B}^l\cos\varphi+\mathcal{B}^q\sin\varphi . \tag{32c}$$

The combinations defined by Eqs. (32) are the heavy counterparts to the familiar SU(2)×U(1) gauge particles. The latter, which remain massless in the first stage of symmetry breaking, are defined by the orthogonal (light) combinations

$$\mathcal{W}_3^L=\mathcal{W}_3^l\sin\varphi-\mathcal{W}_3^q\cos\varphi , \tag{33a}$$

$$\mathcal{W}_\pm^L=-\mathcal{W}_\pm^l\sin\varphi+\mathcal{W}_\pm^q\cos\varphi , \tag{33b}$$

$$\mathcal{B}^L=-\mathcal{B}^l\sin\varphi+\mathcal{B}^q\cos\varphi . \tag{33c}$$

The second stage of symmetry breaking is standard, and could be triggered by complex Higgs fields in the representations $(\mathbf{2},\mathbf{1})$ and $(\mathbf{1},\mathbf{2})$ of $SU(2)^l\times SU(2)^q$. These are the ones which couple to leptons and quarks giving them their masses. However, it is possible that at the tree level only quarks (or leptons) would acquire mass. In this connection, the model of Ref. 36, where a single (leptonic) Higgs doublet is used, suggests that the heavier vector bosons can be as light as several hundred GeV, and would couple primarily to quarks.

We now turn to the case where the leptonic electroweak symmetry is promoted to the SU(3) of the preceding section. We represent the associated gauge fields by the matrix

$$\mathbf{W}^l=\begin{pmatrix} -\mathcal{B}^l & \frac{1}{\sqrt{2}}\mathcal{U}^+ & \frac{1}{\sqrt{2}}\mathcal{V}^{++} \\ \frac{1}{\sqrt{2}}\mathcal{U}^- & \tfrac{1}{2}(\mathcal{W}_3^l+\mathcal{B}^l) & \frac{1}{\sqrt{2}}\mathcal{W}_+^l \\ \frac{1}{\sqrt{2}}\mathcal{V}^{--} & \frac{1}{\sqrt{2}}\mathcal{W}_-^l & \tfrac{1}{2}(-\mathcal{W}_3^l+\mathcal{B}^l) \end{pmatrix} . \tag{34}$$

The breaking of $SU(3)^l\times SU(2)^q\times U(1)^q$ down to SU(2)×U(1) will be triggered by a complex Higgs field Φ in the representation $(\mathbf{3},\mathbf{2})$, with a $U(1)^q$ eigenvalue of $(z=\tfrac{1}{2})$. This takes the vacuum value

$$\frac{1}{M}\langle\Phi\rangle=\begin{bmatrix}0\\1\\0\end{bmatrix}_l\otimes\begin{bmatrix}0\\1\end{bmatrix}_q+\begin{bmatrix}0\\0\\1\end{bmatrix}_l\otimes\begin{bmatrix}1\\0\end{bmatrix}_q . \tag{35}$$

From the gauge couplings of $\mathbf{W}^l$ and $\mathbf{W}^q$ to Φ, we obtain the mass terms

$$\left[\frac{M}{\sin\varphi\cos\varphi}\right]^2 [\tfrac{1}{2}\mathcal{U}^+\mathcal{U}^-\cos^2\varphi+\tfrac{1}{2}\mathcal{V}^{++}\mathcal{V}^{--}\cos^2\varphi + \mathcal{W}_+^H\mathcal{W}_-^H+\tfrac{1}{2}(\mathcal{W}_3^H)^2+\tfrac{1}{2}(\mathcal{B}^H)^2] , \tag{36}$$

where we made use of Eqs. (32).

Hence, in the first stage of symmetry breaking, the vector bosons ($\mathcal{U}^\pm$, $\mathcal{V}^{\pm\pm}$, $\mathcal{W}_\pm^H$, $\mathcal{W}_3^H$, and $\mathcal{B}^H$) acquire masses, while the light combinations given by Eqs. (33) remain massless. The second stage breaking SU(2)×U(1) down to $U(1)_{em}$ can be triggered by the complex doublet giving masses to quarks, and the 6-plet giving masses to leptons. As we noted before, only one type of Higgs field (leptonic or quarkionic) may be needed to trigger the second stage of symmetry breaking. In such framework, it would be interesting to find out whether radiative corrections could relate the masses of quarks and leptons.

IV. HIGHER-DIMENSIONALITY WEYL FERMION UNIFICATION AND SU(2n,C) CHIRAL GRAVIDYNAMICS

We now turn to the spacetime-internal aspect of unification. The kinetic action of a (left-handed) Weyl fermion ψ, is given by[37]

$$\psi^\dagger i l^a\partial_a\psi , \tag{37}$$

where $l^a=(1,\lambda_r)$, and λ_r are normally the three Pauli matrices. Here, we shall regard λ_r as the $2n\times 2n$ Hermitian and traceless matrices[38] of $SU(2n)$; hence $(r=1,\ldots,4n^2-1)$ and spacetime has $4n^2$ dimensions $(a=0,r)$. Our generalized fermions are $2n$ dimensional, and transforming like

$$\psi\rightarrow e^{-\Omega_r\lambda_r/2}\psi,\quad \psi^\dagger\rightarrow\psi^\dagger e^{-\Omega_r^*\lambda_r/2} , \tag{38}$$

where $\Omega_r=(w_r+iv_r)$ are complex parameters. Notice that whereas v_r correspond to rotations in our extended $SU(2n)$ space of $4n^2-1$ dimensions, w_r would correspond to generalized Lorentz boosts connecting the space coordinates to time. The $SU(2n)$ symmetry would contain spin SU(2), as well as an internal $SU(n)$, in the decomposition

$$SU(2n)\rightarrow SU(2)\times SU(n) . \tag{39}$$

The extension to *local* Lorentz invariance, as well as Einstein general coordinate invariance, is achieved by replacing (37) by the Lagrangian density

$$g^{1/2}\psi^\dagger i\mathcal{E}^\mu\nabla_\mu\psi+\text{H.c.},\quad \nabla_\mu\psi=(\partial_\mu-i\mathcal{C}_\mu)\psi . \tag{40}$$

Here, $\mathcal{E}^\mu$ and $\mathcal{C}_\mu$ are the vielbein and the generalized *Lorentz connection*:

$$\mathcal{E}^{\mu}=\mathcal{E}^{\mu}_{a}l^{a}, \quad \mathcal{C}_{\mu}=\tfrac{1}{2}\lambda_r\mathcal{C}^{r}_{\mu} . \tag{41}$$

We may take $\mathcal{E}^{\mu}_{a}$ to be real (as spacetime coordinates), with an inverse $\mathcal{E}^{a}_{\mu}$, while we take $\mathcal{C}^{r}_{\mu}$ to be complex ($\mathcal{C}^{r}_{\mu}=\mathcal{V}^{r}_{\mu}+i\mathcal{W}^{r}_{\mu}$). Their associated local transformations, which lead to the gauge invariance of Eq. (40), are given by

$$\mathcal{E}_{\mu}\rightarrow e^{\Omega^{*}_{r}\lambda_r/2}\mathcal{E}_{\mu}e^{\Omega_r\lambda_r/2} , \tag{42}$$

$$\mathcal{C}_{\mu}\rightarrow e^{-\Omega_r\lambda_r/2}(i\partial_{\mu}+\mathcal{C}_{\mu})e^{\Omega_r\lambda_r/2} . \tag{43}$$

The infinitesimal transformations of $\mathcal{E}_{\mu}$ are given by

$$\delta\mathcal{E}_{\mu}=\tfrac{1}{2}\{w_r\lambda_r,\mathcal{E}_{\mu}\}-\frac{i}{2}[v_r\lambda_r,\mathcal{E}_{\mu}] , \tag{44}$$

$$\delta\mathcal{E}^{0}_{\mu}=\frac{1}{n}w^{r}\mathcal{E}^{r}_{\mu}, \quad \delta\mathcal{E}^{r}_{\mu}=w^{r}\mathcal{E}^{0}_{\mu}+d^{rst}w^{s}\mathcal{E}^{t}_{\mu}+f^{rst}v^{s}\mathcal{E}^{t}_{\mu} . \tag{45}$$

Notice that the component transformations (45) are the same when we make the replacements ($w^{r}\rightarrow -w^{r};\mathcal{E}^{r}_{\mu}\rightarrow -\mathcal{E}^{r}_{\mu}$), only for the four-space SU(2,C) case ($n=1$; $d_{rst}=0$). Hence, we may introduce another (parity-conjugate) vielbein, with conjugate transformations:

$$\mathcal{F}_{\mu}=\mathcal{F}^{a}_{\mu}\bar{l}_{a}, \quad \bar{l}_{a}=(1,-\lambda_r) , \tag{46}$$

$$\mathcal{F}_{\mu}\rightarrow e^{-\Omega_r\lambda_r/2}\mathcal{F}_{\mu}e^{-\Omega^{*}_{r}\lambda_r/2} . \tag{47}$$

This is the representation which couples to (right-handed) Weyl fermions χ:

$$g^{1/2}\chi^{\dagger}i\mathcal{F}^{\mu}\nabla_{\mu}\chi+\text{H.c.}, \quad \nabla_{\mu}\chi=(\partial_{\mu}-i\mathcal{C}^{\dagger}_{\mu})\chi , \tag{48}$$

$$\chi\rightarrow e^{\Omega^{*}_{r}\lambda_r/2}\chi, \quad \chi^{\dagger}\rightarrow\chi^{\dagger}e^{\Omega_r\lambda_r/2} . \tag{49}$$

Conventional quark-lepton generations correspond to left-handed Weyl fermions ψ. In our present theory we can safely assume that right-handed (mirror) generations, described by χ, do not exist.

In order to guarantee Einstein invariance, we have also introduced $g^{1/2}$ in Eq. (40), where g is the positive determinant of the metric tensor

$$g_{\mu\nu}=\tfrac{1}{2}\text{Tr}(\mathcal{F}_{\mu}\mathcal{E}_{\nu}) . \tag{50}$$

This is invariant with respect to the local Lorentz transformations (42) and (47). Notice that $g_{\mu\nu}$ has the Minkowskian signature $(+1,-1,\ldots,-1)$.

To the complex connection $\mathcal{C}_{\mu}$, there corresponds the complex curvature tensor $\mathcal{C}_{\mu\nu}$, which is defined and transforms homogeneously according to

$$\mathcal{C}_{\mu\nu}=\partial_{\mu}\mathcal{C}_{\nu}-\partial_{\nu}\mathcal{C}_{\mu}-i[\mathcal{C}_{\mu},\mathcal{C}_{\nu}]=\tfrac{1}{2}\lambda_r\mathcal{C}^{r}_{\mu\nu} , \tag{51}$$

$$\mathcal{C}^{r}_{\mu\nu}=\partial_{\mu}\mathcal{C}^{r}_{\nu}-\partial_{\nu}\mathcal{C}^{r}_{\mu}+f^{rst}\mathcal{C}^{s}_{\mu}\mathcal{C}^{t}_{\nu} , \tag{52}$$

$$\mathcal{C}_{\mu\nu}\rightarrow e^{-\Omega_r\lambda_r/2}\mathcal{C}_{\mu\nu}e^{\Omega_r\lambda_r/2} . \tag{53}$$

The generalized higher-dimensionality gravidynamic action, which is Einstein invariant as well as local Lorentz invariant, takes the form

$$\mathcal{L}=\tfrac{1}{2}g^{1/2}\text{Tr}(i\mathcal{F}^{\mu}\mathcal{E}^{\nu}\mathcal{C}_{\mu\nu}+\text{H.c.}) \tag{54}$$

$$=g^{1/2}\text{Tr}[i\mathcal{F}^{\mu}\mathcal{E}^{\nu}(\partial_{\mu}\mathcal{C}_{\nu}-i\mathcal{C}_{\mu}\mathcal{C}_{\nu})+\text{H.c.}] . \tag{55}$$

Varying Eqs. (40) and (55) with respect to $\mathcal{C}_{\mu}$, one obtains the generalized torsional equations by which it is possible to express the connection in terms of the vielbein and the fermion fields. Varying with respect to the vielbeins, one obtains our generalized Einstein equations. Both sets of equations must be solved in order to obtain the reduced $D=4$ physical theory. The associated solution would describe the vacuum symmetry-breaking scenario of the theory: from a $D>4$, SU($2n$,C)-symmetric state down to a $D=4$ theory with an internal gauge sector serving the conventional gravitational and electronuclear interactions.

Tied to the symmetry-breaking scenario is the mechanism of generating masses to the fermions and the extra bosons (other than the graviton and the photon). In this regard, one must also introduce a generalized Yukawa coupling of the fermions to a Higgs field, of the form

$$\mathcal{H}^{[\alpha\beta]}\psi_{\alpha}\psi_{\beta}+\text{H.c.} \tag{56}$$

Here, $\mathcal{H}^{[\alpha\beta]}$ is a complex field in the antisymmetric representation of SU($2n$,C). This field is the only bosonic field which couples to the fermions, in addition to the vielbeins. Details of the symmetry-breaking scenario, involving a compactification mechanism for the internal dimensions, will be presented in a future work. In the remainder of this section we discuss our scheme for accommodating fermions, with particular reference to the description of (one or more) quark-lepton generations. We also discuss the spectrum of bosons described by our theory.

To accommodate a set of 15 (or 16) Weyl fermions, representing a quark-lepton generation, we must specialize our framework to the case of SU (30 or 32, C). In that case, spacetime is described by a manifold of (900 or 1024) dimensions. The underlying spin-containing symmetry SU (30 or 32) decomposes into an SU(2) spin factor, and an internal SU (15 or 16) counterpart. The latter corresponds to the maximal grand unification gauge symmetry of electronuclear interactions.[16] Extensions to several generations is straightforward. For instance, three generations would require an SU (90 or 96, C).

Turning to the bosonic sector, we analyze the spin content of our vielbein components. Whereas a proper description of the particle spectrum would have to be done in the symmetry-breaking background which involves the compactification of internal dimensions, we shall be satisfied here with a description of the lowest modes (in the limit of a flat manifold). The dynamic components of the tensor fields $\phi^{\pm}_{\{\mu\nu\}}$, the symmetric parts of the (parity-conjugate) vielbeins, provide a spectrum which must be classified according to the SU(2)×SU(16) decomposition of spin-containing SU(32). Hence, we obtain

$$\phi^{\pm}_{\{\mu\nu\}}\Longrightarrow(\phi^{\pm}_{\{ij\}},\phi^{\pm}_{\{ix,jy\}},\phi^{\pm}_{\{xy\}}) . \tag{57}$$

Here, (ij) are indices of three-space, while (xy) are those of the extra dimensions corresponding to the adjoint representation of SU(16). Hence, $\phi^{\pm}_{\{ij\}}$ is a spin-2 field (describing two parity-conjugate gravitons), and $\phi^{\pm}_{\{xy\}}$ is a set of scalars [in the symmetric product of two SU(16) adjoints]. Moreover, the irreducible parts of

$$\phi^{\pm}_{\{ix,jy\}}=\phi^{\pm}_{\{ij\}\{xy\}}+\phi^{\pm}_{[ij][xy]} \quad (58)$$

would describe a set of (presumably massive) spin-2 tensor mesons $\phi^{\pm}_{\{ij\}\{xy\}}$, as well as a set of spin-1 vector mesons $\phi^{\pm}_{[ij][xy]}$. On the other hand, the SU(2)×SU(16) decomposition of the Higgs field, utilized in Eq. (56), describes scalar as well as vector Higgs particles. Mass generation for fermions, for scalar, vector, and tensor mesons,[39,40] which is tied to the geometrical and Higgs aspects of symmetry breaking, will be described in our future work.

V. DISCUSSION

Our basic result of Secs. II and III is that the leptonic $SU(3)_{EW}$ gauge theory predicts two new particles $\mathcal{U}^{\pm}$ and $\mathcal{V}^{\pm\pm}$. On the other hand, the incorporation of quarks via a separate high-energy SU(2)×U(1) electroweak symmetry, requires an additional charged particle $\mathcal{W}^{H}_{\pm}$, and two neutrals $\mathcal{W}^{H}_{3}$ and $\mathcal{B}^{H}$. All these particles may perhaps be found in the few hundred GeV range.

Further consolidation of the quark sector could proceed via an SU(12)-color-electroweak symmetry. A recently proposed model,[9,10,36] based on an SU(15) quark-lepton unification symmetry, incorporates our leptonic $SU(3)_{EW}$, while associating an SU(12) symmetry with the quarks. This model, in contrast with the minimal SU(5) scheme,[13] relegates proton decay to high orders. It is thus remarkable that the mere consolidation of the leptonic sector, prior to grand unification (as we had proposed ten years ago[8]), implies that the grand unification symmetry cannot be the otherwise elegant scheme of minimal SU(5), but rather the full symmetry associated with the kinetic terms of quarks and leptons. The latter is SU(15) or SU(16).

It is also remarkable that the same internal grand symmetry is suggested by our elaborate higher-dimensionality gravidynamic framework of Sec. IV. Whereas the attraction of our present higher-dimensionality theory is that of not requiring $V+A$ mirror generations (in contrast with conventional higher-dimensionality Einstein-Dirac schemes), it would be reasonable to ask whether the latter might be needed to cancel the chiral anomalies.[41] Our answer is that, in the conventional approach to the quantization of Weyl fermions, this seems to be of necessity. However, we should draw the reader's attention to our work in this regard[42] where, via a special treatment of the functional integral over Weyl fermions, a manifestly gauge-invariant and regular effective action could be derived.

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[38]The spin-containing SU($2n$) matrices may be formed by the outer product of the Pauli matrices associated with spin SU(2), and the Gell-Mann matrices associated with the internal SU(n) algebra. They satisfy the product rule $\lambda_r \lambda_s = (1/n)\delta_{rs} + (d_{rst} + i f_{rst})\lambda_t$, where d_{rst} and f_{rst} are totally symmetric and antisymmetric, respectively. We also have the normalization $\mathrm{Tr}(\lambda_r \lambda_s) = 2\delta_{rs}$.

[39]Our SU($2n$,C) gauge symmetry is distinct from that of the SL(6,C) theory of hadronic gravity with tensor mesons (Ref. 40), since we work with generalized Weyl fermions, and our theory is realized in $D > 4$ rather than $D = 4$ spacetime.

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