Quark-lepton-symmetric model

Robert Foot
Department of Physics, University of Wisconsin–Madison, Madison, Wisconsin 53706

Henry Lew
Research Center for High Energy Physics, School of Physics, University of Melbourne, Parkville 3052, Australia
(Received 12 October 1989)

The notion that there exists a fundamental quark-lepton symmetry is proposed. To implement this symmetry, leptons (as well as quarks) are assumed to come in three "colors," i.e., the gauge group of the world is assumed to be $SU(3)_q \otimes SU(3)_l \otimes SU(2)_L \otimes U(1)_{y'}$. From the consistency considerations of gauge invariance and anomaly cancellation we "derive" the form of a Z_2 discrete symmetry which transforms quarks to leptons (and leptons to quarks). We show that such a theory can reproduce the standard model at low energies (i.e., $\sqrt{s} \leq 100$ GeV), and hence quark-lepton symmetry may be realized in nature.

One of the mysteries of particle physics is the apparent similarity of quarks and leptons. The quarks and leptons have the same $SU(2)_L$ structure, with the left-handed fields transforming as doublets while the right-handed fields transform as singlets. However, the charges and masses of the quarks and leptons differ, and also, quarks transform as triplets under an unbroken $SU(3)_c$-color group. The purpose of this paper is to entertain the speculative notion that there is a fundamental quark-lepton symmetry in nature.

To see how this may come about, let us first recall the basic structure of the standard model (SM). The SM is a Yang-Mills theory with gauge group

$$SU(3)_q \otimes SU(2)_L \otimes U(1)_y \ . \tag{1}$$

Note that $SU(3)_q$ is the usual color group. The subscript q is used to emphasize that this group acts only on quarks. Under the gauge group Eq. (1), the quarks and leptons of each generation transform as

$$\begin{aligned} &f_L \sim (1,2,-1),\ e_R \sim (1,1,-2)\ , \\ &Q_L \sim (3,2,\tfrac{1}{3}),\quad d_R \sim (3,1,-\tfrac{2}{3}),\quad u_R \sim (3,1,\tfrac{4}{3})\ . \end{aligned} \tag{2}$$

In order to obtain a theory with quark-lepton symmetry, we should first define what we mean by this tentative notion. We are looking to define a Lagrangian with a Z_2 discrete symmetry. The Z_2 will transform the quark fields to the lepton fields (and the lepton fields to the quark fields). It is natural then to extend the theory so that there are equal numbers of quark and lepton degrees of freedom. Thus we expand the gauge group of Eq. (1) to

$$SU(3)_q \otimes SU(3)_l \otimes SU(2)_L \otimes U(1)y' \ . \tag{3}$$

The group $SU(3)_l$ will have to be broken spontaneously if the theory is to be experimentally consistent. Also, the $U(1)_{y'}$ charges of the quarks and leptons should be related. In order to cancel the gauge anomalies, the y' charges of the leptons will have to be opposite those of the quarks. These considerations suggest the quark and lepton fields transform under Eq. (3) as

$$\begin{aligned} &(f_L)^c \sim (1,3,2,y),\quad (e_R)^c \sim (1,3,1,y+1)\ , \\ &(\nu_R)^c \sim (1,3,1,y-1),\quad Q_L \sim (3,1,2,y)\ , \\ &u_R \sim (3,1,1,y+1),\quad d_R \sim (3,1,1,y-1)\ , \end{aligned} \tag{4}$$

where the superscript c denotes charge conjugation, and we have normalized the hypercharge of the SM Higgs doublet to one:

$$\phi \sim (1,1,2,1)\ . \tag{5}$$

The $U(1)_{y'}$ quantum numbers in Eq. (4) have been derived from the cancellation of anomalies, and also from the existence of the Yukawa term

$$\begin{aligned} L_{\text{Yuk}} = {} & \lambda_1 \bar{f}_L \phi e_R + \lambda_2 \bar{f}_L \phi^c \nu_R + \lambda_1 \bar{Q}_L \phi^c u_R + \lambda_2 \bar{Q}_L \phi d_R \\ & + \text{H.c.} \end{aligned} \tag{6}$$

At this stage, the Lagrangian terms

$$L_{\text{kin}} + L_{\text{Yuk}}\ , \tag{7}$$

where L_{kin} represents the fermion, scalar, and gauge-boson kinetic terms (with covariant derivative), contain the Z_2 quark-lepton symmetry defined by

$$(f_L)^c \leftrightarrow Q_L,\quad (e_R)^c \leftrightarrow u_R,\quad (\nu_R)^c \leftrightarrow d_R, G_q^a \leftrightarrow G_l^a\ , \tag{8}$$

where the other fields (such as ϕ) transform trivially under Z_2. The fields G_q^a and G_l^a are the gauge bosons of $SU(3)_q$ and $SU(3)_l$, respectively. Note that quark-lepton symmetry requires that the bare $SU(3)_q$ and $SU(3)_l$ coupling constants be equal. Also note that the precise form of the Z_2 discrete symmetry [defined in Eq. (8)] has been derived from the quantum numbers in Eq. (4). These quantum numbers are given by the consistency conditions of anomaly cancellation and the gauge invariance of the Yukawa Lagrangian.

The quark-lepton-symmetric Lagrangian is also left-right symmetric, the parity partners of the left-handed (right-handed) quarks being right-handed (left-handed) leptons. Thus the quark-lepton-symmetric model represents an alternative realization of the hypothesis

 ©1990 The American Physical Society

that parity violation is spontaneous in origin.[1,2] This observation obviously gives a geometric interpretation of the Z_2 quark-lepton discrete symmetry.

Having now defined precisely the notion of quark-lepton discrete symmetry, we proceed to examine the consequences. Imposing quark-lepton symmetry as a fundamental law of nature on the Lagrangian has apparently led to many problems. They are the following.

(1) Why do the quarks and leptons have different electric charges?

(2) Why is electric charge quantized in this theory?

(3) Why is it that the following mass relations do not appear to be satisfied experimentally:

$$m_{\nu_e}=m_d,\quad m_{\nu_\mu}=m_s,\quad m_{\nu_\tau}=m_b\ , \tag{9a}$$

$$m_e=m_u,\quad m_\mu=m_c,\quad m_\tau=m_t\ . \tag{9b}$$

(4) Why do the quarks form bound states, while the leptons are unconfined? Indeed, where are the extra lepton degrees of freedom?

Remarkably, by introducing a new Higgs multiplet, we can break $SU(3)_l$, give the exotic leptons a large mass, and rotate the unbroken electric charge generator to a form which gives ordinary quarks and leptons the correct electric charges provided $y=\frac{1}{3}$ [see Eq. (4)]. Thus we can solve problems (1) and (4). Problem (2) translates into the following question: Why does $y=\frac{1}{3}$ in Eq. (4)? We will return to this problem later. In the following discussion we assume that in Eq. (4) $y=\frac{1}{3}$.

To break $SU(3)_l$ symmetry, we introduce a new Higgs multiplet. This Higgs multiplet will also give the exotic leptons a large mass, and the Higgs multiplet will rotate $U(1)_{y'}\to U(1)_y$. To proceed, we add a new bit to the usual Yukawa Lagrangian of Eq. (6):

$$\begin{aligned} L'_{\rm Yuk}=&\lambda'_1\bar{f}_L\chi_1(f_L)^c+\lambda'_2\bar{\nu}_R\chi_1(e_R)^c\\ &+\lambda'_1\bar{Q}_L\chi_2^\dagger(Q_L)^c+\lambda'_2\bar{u}_R\chi_2^\dagger(d_R)^c+\text{H.c.} \end{aligned} \tag{10}$$

The Lagrangian $L'_{\rm Yuk}$ retains the quark-lepton symmetry of Eq. (8) (with $\chi_1\leftrightarrow\chi_2$), and gives Dirac mass to the two exotic lepton degrees of freedom [when χ_1 develops a vacuum expectation value (VEV)[3]]. Furthermore, from gauge invariance, the existence of the Lagrangian term in Eq. (10) implies that the quantum numbers of χ_1,χ_2 satisfy

$$\chi_1\sim(1,3,1,-\tfrac{2}{3}),\quad \chi_2\sim(3,1,1,-\tfrac{2}{3})\ . \tag{11}$$

These Higgs fields can be represented by 3×3 antisymmetric matrices. If χ_1 gains a VEV,

$$\langle\chi_1\rangle=\begin{bmatrix}0&0&0\\0&0&w\\0&-w&0\end{bmatrix}, \tag{12}$$

then the exotic leptons will gain mass, the group $SU(3)_l$ will break to $SU(2)'$ (this unbroken group acts only on the heavy exotic leptons), and the unbroken electric charge generator will be

$$Q=I_3+Y'/2-T/6\ , \tag{13}$$

where T is the $SU(3)_l$ generator normalized so that its fundamental representation has the form

$$T=\begin{bmatrix}-2&0&0\\0&1&0\\0&0&1\end{bmatrix}. \tag{14}$$

It is straightforward to check that $Y=Y'-T/3$ coincides with the usual hypercharges for the known fermions. The four exotic leptons per generation have charges $\pm\frac{1}{2}$. Note that by adding one additional Higgs multiplet we have broken $SU(3)_l$, given the exotic leptons a large mass (of order $\lambda'\langle\chi_1\rangle$), and rotated the electric charge so that the "light" leptons and quarks (with masses $\lambda\langle\phi\rangle$) have the correct electric charge quantum numbers to be interpreted as the familiar leptons. This feature has also appeared in models examining the possibility that the color group is larger then SU(3).[4,5] The existence of charged $\pm\frac{1}{2}$ fermions also occurs in those models. For three generations of fermions the $SU(2)'$, like the color group $SU(3)_c$, is asymptotically free and is therefore expected to be confining. Thus, the exotic charged $\pm\frac{1}{2}$ leptons are expected to be confined into new hadrons because they interact with the unbroken $SU(2)'$ subgroup of $SU(3)_l$. [The situation here is similar to the case studied in Ref. 5, where the SU(3)-color group was replaced by an SU(5) group which was then broken spontaneously to $SU(3)\otimes SU(2)'$. In that model, the $SU(2)'$ was expected to confine the exotic quarks into new hadrons.]

The Higgs potential can be written as

$$\begin{aligned} V(\phi,\chi_1,\chi_2)=&\lambda_1(\mathrm{Tr}\chi_1^\dagger\chi_1+\mathrm{Tr}\chi_2^\dagger\chi_2-2w^2)^2\\ &+\lambda_2\mathrm{Tr}\chi_1^\dagger\chi_1\mathrm{Tr}\chi_2^\dagger\chi_2+\lambda_3(\phi^\dagger\phi-u^2)^2\\ &+\lambda_4\Big[(\phi^\dagger\phi-u^2)\\ &\qquad+(\mathrm{Tr}\chi_1^\dagger\chi_1+\mathrm{Tr}\chi_2^\dagger\chi_2-2w^2)\Big]^2, \end{aligned} \tag{15}$$

where "Tr" denotes the trace and $\lambda_1,\lambda_2,\lambda_3,\lambda_4>0$. The minimum of this potential occurs when

$$\langle\phi\rangle=\begin{bmatrix}0\\u\end{bmatrix},\quad \langle\chi_1\rangle=\begin{bmatrix}0&0&0\\0&0&w\\0&-w&0\end{bmatrix},\quad \langle\chi_2\rangle=0\ . \tag{16}$$

Thus, the quark-lepton-symmetric potential breaks spontaneously the quark-lepton symmetry. The case here resembles the usual left-right-symmetric model.[1,2] In the usual left-right-symmetric model, it was shown in Ref. 2 that there exists a range of parameters of the Higgs potential where the left-right symmetry is spontaneously broken.

Under the unbroken group $SU(3)_q\otimes SU(2)'\otimes U(1)_Q$, χ_1 transforms as

$$3=(1,2,-\tfrac{1}{2})+(1,1,0)\ . \tag{17}$$

The multiplet $(1,2,-\frac{1}{2})$ and the imaginary part of (1,1,0) are unphysical Goldstone bosons and these fields are absorbed by the five gauge bosons in the coset SU(3)/SU(2) which have become massive. In addition there are charged $\frac{1}{3}$ color-triplet massive physical Higgs bosons which belong to χ_2. Let us now summarize the situation so far. We can define a Lagrangian of the world with quark-lepton symmetry by including an extra SU(3) gauge group which acts on the leptons. This group is broken by the vacuum, so that we have the symmetry-breaking pattern

$$\begin{aligned}&\mathrm{SU}(3)_q\otimes\mathrm{SU}(3)_l\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_{y'}\\&\qquad\downarrow g_s w\\&\mathrm{SU}(3)_q\otimes\mathrm{SU}(2)'\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_y\\&\qquad\downarrow gu\\&\mathrm{SU}(3)_q\otimes\mathrm{SU}(2)'\otimes\mathrm{U}(1)_Q\ .\end{aligned} \tag{18}$$

The symmetry is broken down in such a way so as to give the two exotic lepton degrees of freedom a large mass $[O(\lambda' w)]$, while at the same time rotate the electric charge operator so that the light fermions [with mass $O(\lambda u)$] have electric charges which agree with experiment.

Let us now examine the gauge sector. Under the unbroken group $\mathrm{SU(3)}_q\otimes\mathrm{SU(2)}'\otimes\mathrm{U(1)}_Q$, the eight gauge bosons of $\mathrm{SU(3)}_l$ transform as

$$8=(1,3,0)+(1,2,\tfrac{1}{2})+(1,2,-\tfrac{1}{2})+(1,1,0)\ . \tag{19}$$

We identify the three massless gauge bosons associated with the unbroken SU(2)' group. In addition there are charged $\frac{1}{2}$ massive gauge bosons, and a massive neutral gauge boson. This neutral gauge boson can mix with the Z-gauge boson of the electroweak sector. The gauge-boson mass matrix arises from the Higgs-boson kinetic term:

$$L=(D_\mu\phi)^\dagger(D^\mu\phi)+(D_\mu\chi_1)^\dagger(D^\mu\chi_1)+(D_\mu\chi_2)^\dagger(D^\mu\chi_2)\ , \tag{20}$$

where the covariant derivative D_μ is given by

$$\begin{aligned}D_\mu=\partial_\mu+igW_\mu^0+ig'B_\mu\frac{Y'}{2}+ig_sC_\mu T_4+ig_s\sum_{a=1}^{3}G_{l\mu}^aT_a\\+ig_s\sum_{a=1}^{8}G_{q\mu}^a\frac{\lambda_a}{2}+(\text{charged-gauge-boson terms})\ ,\end{aligned} \tag{21}$$

where $T_4=T/2\sqrt{3}$ and T is normalized as in Eq. (14). The generators T_a for $a=1,\ldots,3$ are given by

$$T_a=\begin{bmatrix}0 & 0\\ 0 & \tau_a/2\end{bmatrix}\ , \tag{22}$$

where τ_a are the Pauli matrices. Substituting the VEV's of Eq. (16) into Eq. (20) we obtain the gauge-boson mass matrix. It is easy to see that the massless G_μ^a gauge bosons associated with the unbroken SU(2)' and $\mathrm{SU(3)}_q$ groups decouple from the neutral-gauge-boson mass matrix, leaving the 3×3 mass matrix

$$L_{\text{mass}}=\tfrac{1}{2}V^TM^2V\ , \tag{23}$$

where

$$V^T=(W_0,B,C) \tag{24}$$

and

$$M^2=\begin{bmatrix}g^2u^2/2 & -gg'u^2/2 & 0\\ -gg'u^2/2 & g'^2u^2/2+4g'^2w^2/9 & 4g'g_sw^2/3\sqrt{3}\\ 0 & 4g'g_sw^2/3\sqrt{3} & 4g_s^2w^2/3\end{bmatrix}\ . \tag{25}$$

This matrix can be diagonalized to obtain the mass-eigenstate fields. We expand these eigenvalues in a power series assuming $g_s^2\gg g'^2,g^2$. We find

$$\begin{aligned}M_\gamma^2&=0\ ,\\ M_Z^2&=\tfrac{1}{2}(g^2+g'^2)u^2-\frac{g'^4u^2}{6g_s^2}+O\left[\frac{g'^6u^2}{g_s^4},\frac{g'^6u^4}{g_s^4w^2}\right]\ ,\\ M_{Z'}^2&=\frac{4g_s^2w^2}{3}\left[1+\frac{g'^2}{3g_s^2}+O\left[\frac{g'^4w^2}{g_s^4w^2}\right]\right]\ .\end{aligned} \tag{26}$$

Note that there are several different ways of defining the mixing angle $\cos\theta_W$. A transparent approach is to define $\cos^2\theta_W$ by

$$\cos^2\theta_W\equiv\frac{M_W^2}{M_Z^2}\ , \tag{27}$$

then one can show that $e=g\sin\theta_W$ (to order g'^4/g_s^4) and that the Z boson couples to the generator:

$$\frac{2e}{\sin2\theta_W}\left[I_3-\sin^2\theta_WQ\right]+\Delta\ . \tag{28}$$

The correction term Δ represents the deviation from the SM Z-boson coupling, and to leading order in g'^2/g_s^2, we calculate

$$\Delta=\frac{2e}{\sin2\theta_W}\left[\frac{3g'^2u^2T}{48g_s^2w^2}\right]\ . \tag{29}$$

Thus it is clear that the model reproduces the SM provided $w^2>u^2$. Note that the new Z'-gauge boson couples dominantly to leptons (and weakly to quarks through a small mixing angle). The lower bound on the heavy Z' is expected to be 300 GeV from neutral-current phenomenology. In this paper we restrict ourselves to showing that a quark-lepton-symmetric model is a theoretical possibility. Of course, it is evident that the model contains a very rich and interesting phenomenology at energy scales $O(w)$.

We now return to the two apparent problems with the assumption of quark-lepton symmetry. First, there is the charge quantization problem. Why does $y=\frac{1}{3}$ in Eq. (4)? The existence of this problem is an indication that the model is incomplete and new physics is required. The situation here resembles that of the right-handed (RH) neutrino extended SM. This extension also requires new physics in order to understand the electric charges of the fermions.[6] To constrain the U(1) charge requires a new constraint. This constraint may be either classical (e.g., some new terms in the Lagrangian), or quantum mechanical (new anomaly conditions arising from extending the gauge group), or a mixture of both of the above. For the RH neutrino extension of the SM, both types of mechanisms can be implemented.[7] The second problem of the quark-lepton-symmetric model is the prediction of the mass relations given in Eq. (9). In the neutrino sector [Eq. (9a)], the discrepancy can be accounted for by implementing the seesaw mechanism.[8] This will require the addition to the model of extra Higgs multiplets [of the form $H_1 \sim (1,\bar{6},1,\frac{4}{3})$ and $H_2 \sim (\bar{6},1,1,\frac{4}{3})$ and H_1 develops a large VEV]. These Higgs fields couple to the fermions via the terms

$$L = \lambda \bar{\nu}_R H_1 (\nu_R)^c + \lambda \bar{d}_R H_2^{\dagger} (d_R)^c \ . \tag{30}$$

When $\chi_1 H_1 \chi_1$ terms (and the corresponding terms with $1 \leftrightarrow 2$) are included in the Higgs potential, the Majorana mass terms [Eq. (30)] for the neutrino fix the hypercharge degree of freedom (i.e., solve the charge quantization problem). Thus the seesaw mechanism provides one possible solution of the electric charge quantization problem and the mass problem in the neutrino sector.[9] The mass relations Eq. (9b) implying the equality of the masses of the e-type leptons and u-type quarks, still remain a problem. There are many possible solutions to this problem.[10] One of the simplest solutions to this problem is to extend the Higgs sector.[11] Alternatively, one may hope to generate the fermion masses radiatively. Indeed, the model has some of the ingredients (e.g., the χ_2 colored Higgs bosons) of recent attempts to generate the mass spectrum of the fermions radiatively.[12]

In conclusion, the notion of quark-lepton symmetry has been elevated to a new principle. We have proposed that the Lagrangian of the world possess a Z_2 discrete symmetry which mixes quarks with leptons leaving them indistinguishable at high energies. We have derived the precise form of this symmetry from the consistency requirements of gauge invariance and anomaly cancellation. These considerations imply the form that a quark-lepton-symmetric model should take. It was then shown that it is a possibility that the SM is a remnant of such a theory. The existence of such a model underscores our lack of knowledge regarding the origin of the quarks and leptons.

R.F. would like to thank Oscar Hernandez for his comments.

---

[1] J. C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. N. Mohapatra and J. C. Pati, *ibid.* **11**, 566 (1975); **11**, 2558 (1975).

[2] G. Senjanovic and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975); R. N. Mohapatra and G. Senjanovic, *ibid.* **23**, 165 (1981).

[3] Note that the masses of the exotic leptons are vectorlike with respect to $SU(2)_L$ and hence there is no restriction on the masses of the exotic fermions from loop corrections to ρ [where $\rho \equiv M_W^2/(M_Z^2 \cos^2\theta_W)$].

[4] R. Foot, Phys. Rev. D **40**, 3136 (1989).

[5] R. Foot and O. F. Hernandez, Phys. Rev. D **41**, 2283 (1990).

[6] R. Foot, G. C. Joshi, H. Lew, and R. R. Volkas, Mod. Phys. Lett. A **5**, 95 (1990); N. G. Deshpande, Oregon University Report No. OITS-107, 1979 (unpublished).

[7] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **63**, 938 (1989); Phys. Rev. D **41**, 271 (1990); R. Foot, University of Wisconsin Report No. MAD/TH/89-6, 1989 (unpublished).

[8] T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, Japan, 1979); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1980); R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[9] Another solution to the charge quantization and mass problems may reside in the usual left-right-symmetric model. Babu and Mohapatra (in Ref. 7) have shown that Majorana mass terms for the neutrino also fix the charges of the particles in that model. This result suggests that we could consider a quark-lepton–left-right–symmetric model [with gauge group $SU(3)_q \otimes SU(3)_l \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)$]. Indeed, we find that in such a model the Majorana mass terms fix the charges. This model exhibits remarkable symmetry at high energy. See R. Foot and H. Lew, Nuovo Cimento (to be published). Other solutions/extensions include replacing the gauged $U(1)_{y'}$ group [or $SU(2)_L$] by $U(1)_{\text{quarks}} \otimes U(1)_{\text{leptons}}[SU(2)_{\text{quarks}} \otimes SU(2)_{\text{leptons}}]$. We refer to such extensions as "segregated hypercharge" or "segregated isospin." Such extensions of the SM have been discussed previously by S. Rajpoot [Mod. Phys. Lett. A **1**, 645 (1986)], X-G. He *et al.* [Phys. Lett. B **222**, 86 (1989)], and H. Georgi, E. E. Jenkins, and E. H. Simmons [Phys. Rev. Lett. **62**, 2789 (1989); **63**, 1540(E) (1989)]. Note that in the quark-lepton-symmetric model, color is already segregated so that the above extensions immediately suggest themselves. When SU(2) or/and U(1) is segregated, the gauge anomalies do not cancel. The possibility of canceling these anomalies and yet preserving the charge quantization feature of the SM was examined by R. Foot [Report No. MAD/TH/89-8 (unpublished)].

[10] Of course this is only a problem provided one assumes that the full Lagrangian has the quark-lepton symmetry. It may be that this symmetry is broken.

[11] For example, instead of the single Higgs doublet in Eq. (6), one could have two Higgs doublets ϕ_1, ϕ_2, where $\phi_1 \to \phi_1$ $\phi_2 \to -\phi_2$ under the discrete symmetry.

[12] See, for example, B. S. Balakrishna, Phys. Rev. Lett. **60**, 1602 (1988); B. S. Balakrishna, A. L. Kagan, and R. N. Mohapatra, Phys. Lett. B **205**, 345 (1988); X-G. He, R. R. Volkas, and D-D. Wu, Phys. Rev. D **41**, 1630 (1990).