Composite Higgs bosons in the Nambu–Jona-Lasinio model

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If more than one doublet of composite Higgs bosons is formed with quarks and antiquarks alone in the Nambu–Jona-Lasinio model, their couplings to quarks are subject to interesting constraints. The Higgs doublets orthogonal to the linear combination responsible for mass generation almost decouple with the right-handed t quark, but couple with one or more of the right-handed u, d, s, c, and b quarks with Yukawa coupling constants of order 1. If there are only two light doublets, the doublet orthogonal to the mass-generating combination is likely to couple with the right-handed b quark with a Yukawa coupling constant $\simeq m_t/v$.

I. INTRODUCTION

As the experimental lower limit on the t-quark mass has been inching up, theorists have been prompted to attempt to generate composite Higgs bosons with quarks alone.[1-3] Since the t-quark mass may be close to the electroweak scale $(\sqrt{2}G_F)^{-1/2}=246$ GeV, the only missing ingredient for this scenario is a strong enough force which binds quark-antiquark states tightly. Unlike the technicolor model,[4] the recent attempts[1-3] all introduce a new four-fermion interaction which becomes strong at a cutoff energy Λ. Beyond that, there is a considerable difference in the dynamical details among the proposed models. Since the model of Nambu[2] or the interpretation of the model of Bardeen *et al.*[3] is straightforward and simpler than that of Miransky *et al.*,[1] we will work in the framework of Refs. 2 and 3 in this paper. We will hereafter refer to this model as the Nambu–Jona-Lasinio[5] (NJL) model of composite Higgs bosons or of electroweak interactions instead of a bootstrap symmetry-breaking model.[2]

In order to generate a light Higgs doublet in the NJL model, we must fine-tune a cutoff Λ of fermion loops. Otherwise the Higgs-boson mass would be of $O(\Lambda)$. Once Λ is fine-tuned and $\ln\Lambda$ is expressed in terms of the electroweak scale $v=(\sqrt{2}G_F)^{-1/2}$, the standard model (SM) is reproduced as an effective theory at low energies. By rendering physical significance to Λ, we can compute the t-quark mass m_t as a function of v and Λ. Furthermore, as a prediction specific to the approximation of the NJL model, the Higgs-boson mass m_H turns out to be $2m_t$ when one ignores all other quark masses. However, when loop corrections due to gauge interactions and self-interactions of the Higgs bosons are included,[3] the physical t-quark mass at the electroweak scale shifts from the value predicted in the bubble approximation of the NJL model and the relation $m_H=2m_t$ is modified. The predicted value of m_t may be compatible with the experimental upper limit on m_t deduced from weak isospin-breaking effects[6,7] only when Λ is chosen to be sufficiently large. A renormalization-group calculation was done by treating the composite Higgs fields as local up to energies of $O(\Lambda)$. According to this calculation,[3] as Λ varies from the lower limit 10^{10} to 10^{19} GeV, m_t changes from 257 to 220 GeV and m_H from 298 to 241 GeV. Since the model is equivalent to the SM at low energies except for the predictability of m_t and m_H in terms of v and one floating parameter Λ, the only convincing test of the NJL model against the SM will be in the relation between m_H and m_t. Nevertheless, the basic idea underlying this model is so fascinating that it deserves further theoretical study.

The novel feature of the NJL model consists in the Higgs sector. When the model is extended to accommodate more than one Higgs doublet, it reveals some interesting characteristics that are hidden in the single-doublet version of the model. The purpose of this paper is to examine the Higgs sector of the NJL model, in particular, its multiple-doublet extension. We find that *if* a second Higgs doublet exists its couplings are tightly constrained and can be quite different from those of elementary Higgs bosons. Specifically, the doublets orthogonal to the mass-generating one couple not with the right-handed t quark but with one or more of the right-handed u, d, s, c, and b quarks, most likely with the right-handed b quark, with a Yukawa coupling constant ~ 1. We will discuss the structure of the Higgs sector and related subjects in Sec. II and will argue in Sec. III that it is possible to accommodate more than one Higgs doublet in the NJL model. In Sec. IV, relations and conditions will be derived for Higgs-boson couplings in the case where more than one doublet exists, in particular, in the case where two light doublets exist.

II. HIGGS BOSONS WITH MULTIPLE GENERATIONS OF QUARKS

When n generations of fermions are introduced in the NJL model, a large number of four-fermion interactions are allowed with $SU(3)_C\times SU(2)_L\times U(1)_Y$ symmetry. For notational simplicity, we leave out leptons in the following since including them is trivial. Then the relevant couplings of interaction are parametrized in the form

$$L_{\text{int}} = \sum G_{ijkl}(\bar{\psi}^{\alpha}_{iL} u_{j\alpha R})(\bar{u}^{\beta}_{lR}\psi_{k\beta L}) + \sum G'_{ijkl}(\bar{\psi}^{c}_{i\alpha L} d^{c\alpha}_{jR})(\bar{d}^{\,c}_{l\beta R}\psi^{c\beta}_{kL}) + \sum [G''_{ijkl}(\bar{\psi}^{\alpha}_{iL} u_{j\alpha R})(\bar{d}^{\,c}_{l\beta R}\psi^{c\beta}_{kL}) + \text{H.c.}] , \tag{2.1}$$

where $\psi=(u,d)^T$, $\psi^c=(d^c,-u^c)^T$ with the superscript c denoting the charge conjugate, (R,L) chiralities, (i,k) generation indices, (j,l) flavor indices, (α,β) color indices, and the summation $\sum$ is over i, j, k, l, α, and β. There are $2n^2$ real and $n^2(2n^2-1)$ complex coupling constants after the Hermiticity conditions

$$G^{*}_{ijkl} = G_{klij}, \quad G'^{*}_{ijkl} = G'_{klij} \tag{2.2}$$

are imposed. All dynamical calculations are done to leading order in $1/N_c$ of SU(3) colors. However, we believe that many features of the model are valid beyond the leading $1/N_c$ approximation.

When the force between quark and antiquark is attractive, doublet Higgs bosons are formed of $q\bar{q}$ composite states (see Fig. 1). As the binding force is increased, the symmetric mass of the Higgs doublet becomes smaller and eventually zero at some critical strength of the force. Beyond the critical strength, quarks acquire mass and three modes of the complex doublet fields turn into the Nambu-Goldstone bosons to be absorbed by W and Z, leaving the fourth mode as a massive physical Higgs boson. To study the Higgs sector in the NJL model it is convenient to diagonalize quark-antiquark scattering due to the four-fermion interaction of Eq. (2.1) into its eigenchannels. We start by relabeling a $\bar{q}_i q_j$ state with a single index instead of a pair of indices, e.g., $a=(i,j)$. L_{int} now reads

$$L_{\text{int}} = \sum_{a,b=1}^{2n^2} G_{ab} J_a^{\dagger} J_b , \tag{2.3}$$

where

$$J_a = \begin{cases} \bar{\psi}^{\alpha}_{iL} u_{j\alpha R} & (a=1,\ldots,n^2), \\ \bar{\psi}^{c}_{i\alpha L} d^{c\alpha}_{jR} & (a=n^2+1,\ldots,2n^2). \end{cases} \tag{2.4}$$

By diagonalizing the $2n^2 \times 2n^2$ Hermitian coupling matrix G_{ab} by a unitary rotation matrix K, we obtain

$$L_{\text{int}} = \sum_{A=1}^{2n^2} G_A J_A^{\dagger} J_A , \tag{2.5}$$

where G_A $(A=1,\ldots,2n^2)$ are real eigenvalues of the matrix G_{ab} and J_A are related to J_a by

$$J_A = \sum_{a=1}^{2n^2} K_{Aa} J_a \tag{2.6}$$

with $K^{\dagger}K = I$. It is in the eigenchannel of the largest G_A, namely, the strongest attractive force,[8] that the composite Higgs doublet is generated. By tuning the value of Λ such that

$$(N_c/8\pi^2) G_1 \Lambda^2 - 1 = O(m_t^2/\Lambda^2) > 0 , \tag{2.7}$$

where $G_1 = \max G_A$ $(A=1,\ldots,2n^2)$, we ensure that the Higgs-boson mass is light $(m_H^2 \ll \Lambda^2)$. Equation (2.7) requires that the natural scale of G_{ab} be of $O(1/\Lambda^2)$. The presence of the quadratic divergence in the fermion loop and its fine-tuning are the two essential ingredients of the NJL model of Higgs bosons, as emphasized by Bardeen *et al.*[3] Thanks to the fine-tuning, there is no light composite boson other than a Higgs doublet, which guarantees the equivalence of the NJL model to the SM.

Once we have identified the most attractive channel, which we will continue to label with $A=1$, we follow the familiar procedure taken in the SM: Diagonalize the neutral component of J_1 of the most attractive channel by unitary rotations of the R and L components of up and down quarks separately.[9] Then

$$(J_1^{\dagger})_{\text{neutral}} = \sum_{j=1}^{2n} k_j \bar{q}_{jL} q_{jR} = \sum_{j=1}^{n} k_j \bar{u}_{jL} u_{jR} + \sum_{j=n+1}^{2n} k_j \bar{d}^{\,c}_{jL} d^{c}_{jR} , \tag{2.8}$$

where

$$\sum_{j=1}^{2n} k_j^2 = 1 \quad (k_j^* = k_j) \tag{2.9}$$

by unitarity of the quark rotation matrices and by the normalization condition

$$\sum_{i=1}^{n} \sum_{j=1}^{2n} |K_{1ij}|^2 = 1 , \tag{2.10}$$

which results from unitarity of the $2n^2 \times 2n^2$ matrix K_{Aa}. Since J_1 is the scalar density which couples with the Higgs boson, the eigenvalues k_j are proportional to quark masses m_j. Incorporating their normalization given by Eq. (2.9), we find

$$k_j = m_j \Big/ \left[\sum_{l=1}^{2n} m_l^2 \right]^{1/2} \quad (j=1,\ldots,2n) . \tag{2.11}$$

The mass and couplings of the neutral Higgs boson can be computed easily by the diagrams of Fig. 1. In the leading-lnΛ approximation, we find, for the mass,

$$m_H^2 = 4 \sum_{j=1}^{2n} k_j^2 m_j^2 \tag{2.12}$$

and, for the Yukawa couplings,

$$f^2_{\bar{q}_j q_l H} = \delta_{jl} k_j^2 \Big/ \left[(N_c/8\pi^2) \sum_{q=1}^{2n} k_q^2 \ln(\Lambda^2/m_q^2) \right] . \tag{2.13}$$

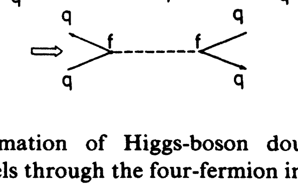

FIG. 1. Formation of Higgs-boson doublets in quark-antiquark channels through the four-fermion interaction.

It will be very important for a discussion later in Sec. IV that the Yukawa couplings of the $q\bar{q}$ composite states be independent of the strength G_1 of the binding force. This feature is fairly general and insensitive to details of binding forces. We will add a few remarks on this point later.

The electroweak scale v can be obtained from the W and Z masses. Again in the leading-$\ln\Lambda$ approximation, it is given by

$$v^2=(N_c/8\pi^2)\sum_{j=1}^{2n} m_j^2\ln(\Lambda^2/m_j^2)\ . \tag{2.14}$$

Actually the values of v computed from M_W and M_Z differ from each other to nonleading order because of the weak isospin breaking in quark masses.[10] The reason is that v in the NJL model automatically includes its quark-loop contributions of the SM. Combining Eqs. (2.11)–(2.14), we can reaffirm the relation $f_{\bar{q}qH}=m_q/v$. In the world where the t quark is much larger than all other quark masses ($k_3\simeq 1$ and $k_j\simeq 0$ for $j\neq 3$), Eq. (2.12) reduces to $m_H=2m_t$, in agreement with the relation pointed out previously.[1–3]

It is possible to compute the energy of the asymmetric vacuum. From the expectation value of the Hamiltonian including the zero-point oscillation energy of $q\bar{q}$ pairs, we obtain

$$\begin{aligned}\langle H\rangle &=\langle\bar{\psi}(i\nabla+m)\psi\rangle+\langle H_{\rm int}\rangle-\langle m\bar{\psi}\psi\rangle\\ &=-\sum_{j=1}^{2n}\sum_{\mathbf{p}}2[E_j(\mathbf{p})-E_0(\mathbf{p})]\\ &\quad-\tfrac{1}{4}G_1\left[\sum_{j=1}^{2n}k_jm_jI(m_j^2)\right]^2+\sum_{j=1}^{2n}m_j^2I(m_j^2)\ ,\end{aligned} \tag{2.15}$$

where $E_j(\mathbf{p})=(m_j^2+\mathbf{p}^2)^{1/2}$, $E_0(\mathbf{p})=|\mathbf{p}|$, and $m_jI(m_j^2)=-\langle\bar{\psi}_j\psi_j\rangle$. By using the relation

$$(\partial/\partial m_j)\sum_{\mathbf{p}}[E_j(\mathbf{p})-E_0(\mathbf{p})]=m_jI(m_j^2)\ ,$$

we find that the minimization condition $\partial\langle H\rangle/\partial m_j=0$ is nothing other than the self-consistency condition or the gap equation

$$m_j=\tfrac{1}{2}G_1k_j\sum_{l=1}^{2n}k_lm_lI(m_l^2)\ . \tag{2.16}$$

The asymmetric vacuum energy of the NJL model can be computed from Eq. (2.15):[11]

$$\langle H\rangle_{\rm min}=\tfrac{1}{4}\sum_{j=1}^{2n}m_j^4\partial I(m_j^2)/\partial m_j^2\ (<0)\ , \tag{2.17}$$

to leading-$\ln\Lambda$ order. $\partial I(m^2)/\partial m^2$ is logarithmically divergent and negative. When we replace $\ln\Lambda$ terms with v, $\langle H\rangle_{\rm min}$ coincides with the tree-level formula in the SM:

$$\langle H\rangle_{\rm min}=-\tfrac{1}{8}m_H^2v^2\ . \tag{2.18}$$

The finite terms of $\langle H\rangle_{\rm min}$ suppressed in Eq. (2.17) are precisely equal to the finite portion of the quark-loop corrections to the vacuum energy of the SM.[12] In fact, we can construct the nonlinear Higgs potential of the NJL model from the vacuum energy of Eq. (2.15) through the replacement of $m_q^2=(\sigma^2+\pi^2)f_{\bar{q}qH}^2$. If we keep only the $\ln\Lambda$ terms in the potential and express them in terms of v and m_H, this part of the potential is identical to the renormalizable potential of the SM. Differentiating this Higgs potential twice with respect to the scalar field σ at the minimum of the potential, we can reproduce the Higgs-boson mass Eq. (2.12) from the curvature. The finite terms of the nonlinear Higgs potential are those which are obtained from the familiar quark-loop corrections in the SM.[12]

III. POSSIBILITY FOR A SECOND DOUBLET

We study here on eigenchannels of $q\bar{q}$ with zero angular momentum other than the most attractive one. In the preceding section, the neutral component of the scalar density J_1 of the most attractive channel was diagonalized in flavor by transforming quarks into mass eigenstates. Under this transformation neutral components of other scalar density J_A ($A\neq 1$) remain nondiagonal in general so that $J_A^\dagger J_A$ induces neutral flavor-changing (NFC) processes. However, with G_A being of $O(1/\Lambda^2)$, these NFC processes are small enough to be compatible with experimental observations unless another light Higgs doublet is formed in one of these channels.

To form a second light doublet, additional fine-tuning is needed for the four-fermion interaction: $(N_c/8\pi^2)G_B\Lambda^2-1=O(m_t^2/\Lambda^2)>0$ if a composite doublet appears in the eigenchannel B ($\neq 1$). It is worth pointing out here that the fine-tuning to be made in the NJL model is no worse than that for the Higgs potential parameters in models where Higgs bosons are elementary particles: One quadratic fine-tuning is necessary for the bare mass of the Higgs bosons in the SM and if one wishes to accommodate two light Higgs doublets, two quadratic fine-tunings are called for in the extension of the SM. An important difference between composite and elementary Higgs-boson model is the lack of a clean-cut symmetry argument in the former which eliminates NFC couplings of a second Higgs doublet. In the latter, one can forbid NFC couplings by introducing a U(1) symmetry or an equivalent, assigning different U(1) charges to two Higgs doublets, and by breaking this U(1) symmetry softly. Supersymmetry motivates this symmetry argument nicely. On the other hand, the NJL model of electroweak interactions, relying on the presence of a quadratic cutoff and its fine-tuning, cannot incorporate supersymmetry. In this section, we present one example of dynamical postulates which keeps NFC couplings small for a second Higgs doublet in the NJL model.

Let us write the four-fermion couplings G_{ab} of Eq. (2.3) in a 2×2 block matrix:

$$G_{ab}=\begin{bmatrix}G^{(uu)} & G^{(ud)}\\ G^{(ud)\dagger} & G^{(dd)}\end{bmatrix}_{ab}\ , \tag{3.1}$$

where each block consists of an $n^2\times n^2$ matrix, and the first and second blocks refer to $\bar{\psi}_{iL}^{\alpha}u_{j\alpha R}$ and $\bar{\psi}_{i\alpha L}^{c}d_{jR}^{c\alpha}$, respectively [see Eq. (2.4)]. We postulate, by some dynamical reason yet to be understood at the scale Λ or above, that the entries in the off-diagonal block $G^{(ud)}$ and its conjugate be much smaller in magnitude, by a factor of $O(m_q^2/\Lambda^2)$, than those in the diagonal blocks $G^{(uu)}$ and $G^{(dd)}$. To see how this postulate works, we first diagonalize the diagonal blocks by making two unitary rotations, one among J_a with $a=1,\ldots,n^2$ and the other among J_a with $a=n^2+1,\ldots,2n^2$:

$$G_{ab}\rightarrow\begin{bmatrix}\tilde{G}_1 & 0 & & \\ & \ddots & \tilde{G}^{(ud)} & \\ 0 & \tilde{G}_{n^2} & & \\ & & \tilde{G}_{n^2+1} & 0 \\ \tilde{G}^{(ud)\dagger} & & \ddots & \\ & & 0 & \tilde{G}_{2n^2}\end{bmatrix}, \tag{3.2}$$

where the entries in the off-diagonal blocks remain to be of $O(m_q^2/\Lambda^4)$ as compared with the diagonal entries of $O(\Lambda^{-2})$. If one each of $(\tilde{G}_1,\ldots,\tilde{G}_{n^2})$ and of $(\tilde{G}_{n^2+1},\ldots,\tilde{G}_{2n^2})$ satisfies the fine-tuning conditions

$$(N_c/8\pi^2)\tilde{G}_1\Lambda^2-1=O(m_t^2/\Lambda^2)$$

and

$$(N_c/8\pi^2)\tilde{G}_{n^2+1}\Lambda^2-1=O(m_t^2/\Lambda^2)\ ,$$

two light Higgs doublets are formed by mixing of the two channels. The two eigenchannels which generate the light doublets are given up to $O(m_q^4/\Lambda^4)$ by

$$\begin{aligned}J_1&=|1\rangle\cos\theta+|n^2+1\rangle\sin\theta+\sum_{n'\neq 1,n^2+1}O(m_q^2/\Lambda^2)|n'\rangle\ ,\\ J_2&=|n^2+1\rangle\cos\theta-|1\rangle\sin\theta+\sum_{n'\neq 1,n^2+1}O(m_q^2/\Lambda^2)|n'\rangle\ ,\end{aligned}\tag{3.3}$$

where $|1\rangle$ and $|n^2+1\rangle$ are the first and (n^2+1)th scalar densities in the basis chosen in Eq. (3.2). The first eigenchannel of Eq. (3.3) develops the Nambu-Goldstone bosons. Upon diagonalization of the neutral component of this channel, the physical Higgs boson from this channel conserves quark flavors strictly and the quark masses are completely diagonalized. The mixing angle θ is found to be $\tan\theta\simeq m_b/m_t$. The Yukawa couplings of neutral Higgs bosons generated in the second eigenchannel of Eq. (3.3) are diagonal in flavor up to terms of $O(m_q^2/\Lambda^2)$. Since Λ is sufficiently large, this NFC interaction does not conflict with experiment. Thus suppression of NFC couplings of additional Higgs bosons can be stated as the dynamical postulate that the four-fermion couplings in the off-diagonal blocks such as $(\bar{\psi}u)(\bar{d}^{\,c}\psi^c)$ be of higher order in $1/\Lambda^2$.

It is needless to say that this argument is guided by our experience in the elementary-Higgs-boson theory, in particular the two-doublet extension of the standard model (TDSM). The states $|1\rangle$ and $|n^2+1\rangle$ correspond to Φ_1 and Φ_2 of the TDSM. The doublets formed in the two eigenchannels are $(v_1\Phi_1+v_2\Phi_2)/v$ and $(-v_2\Phi_1+v_1\Phi_2)/v$, where $\langle\Phi_1\rangle=v_1/\sqrt{2}$ and $\langle\Phi_2\rangle=v_2/\sqrt{2}$ with $v=(v_1^2+v_2^2)^{1/2}$. The mixing angle $\theta\simeq\arctan(m_b/m_t)$ between the sources of Φ_1 and Φ_2 is equivalent to the familiar mixing angle $\alpha=\arctan(v_2/v_1)$ between the fields Φ_1 and Φ_2 because the sources carry the Yukawa couplings. If the eigenchannels correspond to pure Φ_1 and Φ_2 of the TDSM and develop separate vacuum expectation values (VEV's), $v_1/\sqrt{2}$ and $v_2/\sqrt{2}$, respectively, the NJL model would generate an axion as well as a light neutral boson with mass $\simeq 2m_b$. In the TDSM, neutral physical modes from $\Phi_1\cos\alpha+\Phi_2\sin\alpha$ and $-\Phi_1\sin\alpha+\Phi_2\cos\alpha$ further mix with each other through Higgs-boson–Higgs-boson interactions. The same mixing can occur in the composite Higgs model once we go beyond the leading $1/N_c$ expansion in the quark-bubble approximation. Nevertheless, it is interesting to see what values are obtained for the masses of the second doublet before induced Higgs-boson–Higgs-boson couplings are added. These are the values to be compared with $m_H=2m_t$ of the single-doublet model. Repeating the computation of Sec. II, we find the masses of the four modes, $\phi^{\pm}$, $\chi(0^-)$, and $\sigma(0^+)$:

$$\begin{aligned}&m_\chi^2=m_\sigma^2=m_0^2,\quad m_\phi^2=m_0^2+2m_t^2\ ,\\ &m_0^2=2(\bar{G}/\bar{G}'-1)\Lambda^2/\ln(\Lambda^2/m_t^2)-2m_t^2\ ,\end{aligned}\tag{3.4}$$

where $\bar{G}$ and $\bar{G}'$ are the eigenvalues of the coupling matrix G_{ab} in the eigenchannels of the strongest and next strongest forces, respectively. If we adopt the fine-tuning postulate discussed above, the value of m_0^2 is of $O(m_t^2)$. It is possible to make m_0^2 larger than $O(m_t^2)$ within the fine-tuning postulate. There is no light particle like an axion; the off-diagonal coupling $\tilde{G}^{(ud)}$ in Eq. (3.2), which contributes to a splitting between $\bar{G}$ and $\bar{G}'$, breaks Peccei-Quinn symmetry explicitly.

As we increase $\bar{G}'$ toward $\bar{G}$, m_0^2 decreases. Therefore, the experimental lower bound on the mass m_0^2 may not allow an excessive fine-tuning of $\bar{G}'$ and $\bar{G}$ with a single VEV. As $\bar{G}'$ approaches $\bar{G}$ further, m_0^2 becomes negative and the next leading attractive channel starts developing a VEV. One can compute in principle the VEV's from the effective Higgs potential (cf. Appendix B). Let us examine the case where the two leading eigenchannels develop VEV's. We can write the Yukawa interaction of the two composite Higgs doublets in the form

$$L_{\rm int}=-f\phi_1J_1-f\phi_2J_2 \tag{3.5}$$

in the notation of Eq. (3.3). Here the scale f is common for the two channels: As was emphasized in Sec. II, the scale f is independent of the four-quark coupling G_A and equal to $(4\pi^2/N_c\ln\Lambda^2)^{1/2}$ in the leading logarithm [see Eqs. (2.9) and (2.13)]. We then introduce two new scalar densities of quarks J_1' and J_2' by the orthogonal rotation

$$\begin{aligned}J_1'&=J_1\cos\beta+J_2\sin\beta\ ,\\ J_2'&=-J_1\sin\beta+J_2\cos\beta\ ,\end{aligned}\tag{3.6}$$

where $\tan\beta=\langle\phi_2\rangle/\langle\phi_1\rangle$. The Yukawa interaction of Eq. (3.5) turns into

$$L_{\rm int}=-f\phi_1'J_1'-f\phi_2'J_2'\ , \tag{3.7}$$

with

$$\phi_1' = \phi_1\cos\beta + \phi_2\sin\beta\ ,$$
$$\phi_2' = -\phi_1\sin\beta + \phi_2\cos\beta\ . \tag{3.8}$$

Since $\langle\phi_1'\rangle = v/\sqrt{2}$ and $\langle\phi_2'\rangle = 0$ and orthonormality of J_1' and J_2' is maintained, Eq. (3.7) is identical in structure with the corresponding Yukawa interaction for the case of a single VEV. The crucial ingredient here is that the scale f of the Yukawa interaction is common for all eigenchannels. ϕ_1' and ϕ_2' are no longer mass eigenstates even in the leading $1/N_c$ order. J_1' and J_2' are no longer eigenchannels of the four-quark interaction forces. But this does not affect our argument to be developed in Sec. IV. The important property which we will use is that the quark-antiquark channels which couple with linearly independent Higgs doublets are *orthonormal.*

IV. TWO HIGGS DOUBLETS

Before we start computing for Yukawa couplings of nonminimal Higgs doublets, we would like to clarify one subtlety which becomes important in the nonminimal model. As we have seen in the vacuum energy at the end of Sec. III, the NJL model includes the effects which are attributed to quark-loop corrections in the SM. When we obtain some physical quantity in the NJL model beyond the leading-$\ln\Lambda$ approximation, it contains automatically in its finite part $SU(2)_L$-breaking effects due to quark mass terms. For this reason the Yukawa couplings of the Higgs bosons computed by the fully self-consistent calculation do not obey $SU(2)_L$ symmetry. For the purpose of comparing the Yukawa couplings of the NJL model with the $SU(2)_L$-symmetric ones of the SM, it is more illuminating to compute for the former in an $SU(2)_L$-symmetric form by assigning a tiny common mass to all quarks and, if we wish, later to incorporate quark-loop corrections by the standard technique of field theory for effective-Higgs-boson fields. If we follow this prescription, we can carry through the diagonalization procedure described in Sec. II for the eigenchannels of quark-antiquark scattering. Otherwise, channel mixing caused by quark mass terms would prevent us from any systematic treatment of the problem because channel mixing can no longer be dismissed as an effect of $O(m_q^2/\Lambda^2)$ when two eigenvalues of G_{ab} are nearly degenerate by fine-tuning. In the following, we will substitute all quark masses in loop diagrams with a tiny common mass $\overline{m}$ ($\to 0$). Then the eigenchannels of quark-antiquark scattering can be readily identified through the diagonalized form of the four-fermion interaction Eq. (2.5).

Starting with Eq. (3.2), we first of all postulate that the couplings G_{ab} possess some property like the one discussed in Sec. III to suppress NFC processes. Completing diagonalization of the couplings matrix Eq. (3.2), we obtain the coupling G_A ($A=1,\ldots,2n^2$) of the eigenchannels. By Hermiticity of G_{ab}, its diagonalization matrix K is unitary when it is viewed as a transformation from the space of J_a to the space of J_A:

$$\sum_{a=1}^{2n^2} K_{Aa}^* K_{Ba} = \delta_{AB}\ . \tag{4.1}$$

The case of two VEV's involves an additional 2×2 rotation, $K\to OK$, as discussed in Sec. III. Such a rotation does not change the form of Eq. (4.1). Hereafter, the channel labeled with $A=1$ refers to the channel of ϕ_1' in the case of two VEV's. Now we regard K_{1a} with $a=(ij)$ as a matrix transforming u_j to ψ_i and d_j^c to ψ_i^c. It is convenient for the purpose of the following discussion to relabel ψ_i^c by ψ_{i+n} and treat the index i and $i+n$ like a single index l for $2n$ flavors. Then in the basis where the matrix K_{1a} is diagonal ($K_{1a}=k_j\delta_{lj}$: l and $j=1,\ldots,2n$), the orthogonality conditions in Eq. (4.1) read

$$\sum_{j=1}^{2n} k_j K_{Ajj} = 0 \quad (A\neq 1)\ . \tag{4.2}$$

From the diagrams of Fig. 1, we obtain the Yukawa couplings of the Higgs fields to chiral states of quarks. The results are, for the neutral Higgs fields,

$$f^2_{\bar{q}_{jL} q_{lR}\phi_A} = |K_{Ajl}|^2/(N_c/4\pi^2)\ln(\Lambda^2/\overline{m}^2)\ , \tag{4.3}$$

and, for the charged Higgs fields,

$$f^2_{\bar{u}_{jL} d_{lR}\phi_A} = |(U^*_{\rm KM}K_A)_{jl}|^2/(N_c/4\pi^2)\ln(\Lambda^2/\overline{m}^2)\ ,$$
$$f^2_{\bar{d}_{lL} u_{jR}\phi_A} = |(U^\dagger_{\rm KM}K_A)_{lj}|^2/(N_c/4\pi^2)\ln(\Lambda^2/\overline{m}^2)\ , \tag{4.4}$$

where $\overline{m}$ is the common mass assigned to quarks in loops and $U_{\rm KM}$ is the Kobayashi-Maskawa mixing matrix. According to Eq. (4.3), the matrix element K_{Ajj} is proportional to the flavor-diagonal Yukawa coupling of the neutral Higgs field which appears in the channel A ($A\neq 1$). Since k_j is proportional to the quark mass m_j, the orthogonality Eq. (4.2) means that the t_R-quark couplings of the doublets orthogonal to the mass-generating one are highly suppressed.

Let us turn to the normalization conditions in Eq. (4.1). Since the Yukawa couplings in Eqs. (4.3) and (4.4) are independent of the strength G_A of the binding force, the normalization condition $\sum_a |K_{Aa}|^2=1$ means that the overall scales of the couplings are common to all Higgs doublets. Since after diagonalization of the quark mass matrices, NFC couplings are highly suppressed in the channels of light Higgs doublets, the normalization condition holds for the second channel:

$$\sum_{j=1}^{2n} |K_{2jj}|^2 = 1 - \sum_{j\neq l} |K_{2lj}|^2 \simeq 1\ . \tag{4.5}$$

If a model accommodates more light doublets, Eq. (4.5) must apply to them as well.

Combining Eqs. (2.11), (4.2), (4.3), and (4.5), we reach interesting relations for the Yukawa couplings of the nonminimal neutral Higgs fields:

$$\sum_{j=1}^{n} m_j f_{\bar{u}_{jL}u_{jR}\phi} + \sum_{j=n+1}^{2n} m_j f_{\bar{d}_{jL}d_{jR}\phi} = 0\ , \tag{4.6}$$

$$\sum_{j=1}^{2n} |f_{\bar{q}_{jL}q_{jR}\phi}|^2 \simeq \frac{2}{v^2}\sum_{j=1}^{2n} m_j^2\ . \tag{4.7}$$

For the charged Higgs fields, corresponding relations are obtained by the replacement of the coupling f with

$U_{KM}f$ or $U_{KM}^T f$. The orthogonality relations are

$$\sum_{j=1}^{n}\sum_{l=n+1}^{2n}[m_j(U_{KM})_{jl}f_{\bar{d}_{lR}u_{jL}\phi}-m_l(U_{KM})_{jl}f_{\bar{d}_{lL}u_{jR}\phi}]=0 \quad (4.8)$$

and the normalization conditions are

$$\sum_{j=1}^{n}\sum_{l=n+1}^{2n}(|f_{\bar{d}_{lR}u_{jL}\phi}|^2+|f_{\bar{d}_{lL}u_{jR}\phi}|^2)=\frac{2}{v^2}\sum_{j=1}^{2n}m_j^2 . \quad (4.9)$$

The restrictions on the Yukawa couplings of the doublet orthogonal to the mass-generating one are the main new result of this paper.

No such restrictions are imposed on the couplings of elementary Higgs bosons: They are completely arbitrary so long as they are $SU(3)_C\times SU(2)_L\times U(1)_Y$ invariant up to Kobayashi-Maskawa mixing. By contrast composite Higgs bosons are generated in eigenchannels orthogonal to each other and, in the NJL model in particular, the overall normalization of the couplings is fixed to a value independent of the strength of the binding forces. A remark is in order on this latter point. It is special to the bubble approximation of the NJL model that the Yukawa couplings are strictly independent of the strength of the binding forces. In general, the couplings depend on the strength of the binding, but rather mildly according to our experience in the past. When one computes for a coupling constant of a composite particle to its constituents in the Bethe-Salpeter equation or the N/D method,[13] the coupling constant comes out to be independent of the strength of the binding in the first-order iteration. When the binding is strong, the first-order iteration is not a good approximation for obtaining a binding energy. Although a binding energy is sensitive to the strength of force, a coupling constant of a composite particle or a residue of a bound-state pole in a scattering amplitude is less sensitive to the strength of binding. It is quite possible that the normalization conditions Eqs. (4.7) and (4.9) are approximately valid beyond the NJL model.

To summarize, when a second light Higgs doublet is introduced in the composite Higgs model, the linear combination ϕ_2' of two doublets which does not develop a VEV must virtually decouple with the t_R quark by orthogonality. If NFC processes are avoided by the postulate suggested in Sec. III, ϕ_2' should couple dominantly with the b_R quark because the same linear combination of the down quarks $\bar{\psi}^c_{jL}d^c_{jR}$ couples with the mass-generating doublet ϕ_1' and with ϕ_2' orthogonal to ϕ_1'. Furthermore, their scales of coupling are common by the normalization condition, which results from the fact that the scale of the Yukawa interaction is independent of the magnitude of the fundamental four-quark couplings. Therefore, the $\bar{b}_R\psi_L\phi_2'^{\dagger}$ coupling is expected to be equal to the $\bar{\psi}_L t_R\phi_1'$ coupling in the limit of keeping only t and b quarks.

The Higgs-boson couplings discussed above are subject to finite loop corrections due to quarks. Furthermore, they are subject to loop corrections of gauge interactions and self-interactions of Higgs bosons as well. They are therefore identifiable with the Higgs-boson couplings in the renormalizable low-energy Lagrangian of electroweak interactions.

In conclusion, the NJL model of composite Higgs bosons predicts that quark-antiquark sources which couple with linearly independent Higgs doublets are orthonormal. In the two-doublet extension of the SM, the two linearly independent doublets may be chosen to be the linear combination which develops a VEV, $v=246$ GeV, and the other combination which develops no VEV. This orthonormality imposes tight constraints on the Yukawa couplings of the doublet orthogonal to the mass-generating one.

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APPENDIX A

Some formulas which appear in Sec. II are given here beyond the leading-$\ln\Lambda^2$ approximation. The neutral-Higgs-boson mass in the single-doublet NJL model is

$$m_H^2=4\sum_{j=1}^{2n}k_j^2m_j^2[\ln(\Lambda^2/m_j^2)-1]\Big/\sum_{l=1}^{2n}k_l^2[\ln(\Lambda^2/m_l^2)-1] . \quad (A1)$$

The Yukawa coupling constants are

$$f^2_{\bar{q}_jq_lH}=\delta_{jl}k_j^2\Big/\left[(N_c/8\pi^2)\sum_{q=1}^{2n}k_q^2[\ln(\Lambda^2/m_q^2)-1]\right] . \quad (A2)$$

The electroweak scale v can be deduced either from the W-boson mass or from the Z-boson mass. A formula obtained from the Z-boson mass reads

$$v^2=(N_c/8\pi^2)\sum_{j=1}^{2n}m_j^2[\ln(\Lambda^2/m_j^2)-1] . \quad (A3)$$

A formula for v^2 from the W-boson mass, which differs from (A3) to nonleading order because of quark mass splitting, is a little involved since it contains Cabibbo-Kobayashi-Maskawa mixing and the Feynman parameter integrals involving two different masses:

$$v^2=(N_c/8\pi^2)\sum_{j=1}^{n}\sum_{l=n+1}^{2n}|(U_{\rm KM})_{jl}|^2F(m_j,m_l,\Lambda)\ , \tag{A4}$$

where

$$F(x,y,\Lambda)=\int_0^1 d\alpha[\alpha x+(1-\alpha)y](\ln\{\Lambda^2/[\alpha x+(1-\alpha)y]\}-1)\ . \tag{A5}$$

The relation $f_{\bar{q}qH}=m_q/v$ is valid beyond the leading-$\ln\Lambda$ approximation if the v determined from the Z-boson mass is used.

APPENDIX B

It is straightforward to compute the effective Higgs potential V_H from the quark box diagrams with chains of quark bubbles attached at the four corners. The result agrees, of course, with the corresponding Coleman-Weinberg potential. Dominant contributions come from the third-generation quark loops, since their couplings are by far the strongest. If we keep only the third generation ($m_b/m_t\to 0$) and the leading-logarithmic terms, the Higgs potential for the case of a single VEV is given by

$$V_H=\lambda|\phi_1^\dagger\phi_1-\tfrac{1}{2}v^2|^2+m_0^2(\phi_2^\dagger\phi_2)+\lambda|\phi_2^\dagger\phi_2|^2+2\lambda|\phi_1^\dagger\phi_2^c|^2-\tfrac{1}{4}\lambda v^4\ , \tag{B1}$$

where

$$\lambda=16\pi^2/[N_c\ln(\Lambda^2/\bar{m}^2)]\ , \tag{B2}$$

$$\phi_1^\dagger\phi_2^c=\phi_1^0\phi_2^+-\phi_1^+\phi_2^0\ , \tag{B3}$$

and m_0^2 is defined in Eq. (3.4). This potential assures a correct, stable vacuum.

For the case of two VEV's, the convenient basis in which the ϕ^4 terms of V_H are to be written is Φ_1 and Φ_2, the two linear combinations which couple purely with t_R and b_R, respectively:

$$V_H=-\mu_1^2(\phi_1^\dagger\phi_1)-\mu_2^2(\phi_2^\dagger\phi_2)+\lambda(|\Phi_1^\dagger\Phi_1|^2+2|\Phi_1^\dagger\Phi_2^c|^2+|\Phi_2^\dagger\Phi_2|^2)\ . \tag{B4}$$

The term $|\Phi_1^\dagger\Phi_2^c|^2$ aligns the two VEV's.

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