Rare decays $B \to Kl\bar{l}$ and $B \to K^* l\bar{l}$

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Using a recently developed relativistic constituent quark model based on the light-front formalism, we evaluate the rare decays $B \to Kl\bar{l}$ and $B \to K^* l\bar{l}$. The branching ratios vary between 10^{-7} and 10^{-6} for m_t between 100 and 200 GeV. We also give the K and K^* energy distributions.

I. INTRODUCTION

Rare decays of B mesons are an important way to study some higher-order effects of the standard model, and many studies have been devoted to them.[1-3] In particular the decays $b \to s\gamma$ and $b \to sl\bar{l}$ are interesting. They are expected to occur at a rate accessible to coming B-meson facilities and exhibit new signals of the mechanisms behind the effective neutral flavor changes. Their sensitivity to the top-quark mass makes rare decays an attractive test of the three-generation standard model.[4]

The quark-level transitions $b \to s\gamma$, $b \to sl\bar{l}$ give rise to decays such as $B \to K^*\gamma$, $B \to Kl\bar{l}$, $B \to K^* l\bar{l}$. Their rates involve hadronic matrix elements whose calculation is difficult. Encouraged by the success of the quark model calculations by Jaus[5] and the availability of QCD corrections to $b \to sl\bar{l}$ (Grinstein, Wise, and Savage,[6] and Grigjanis, O'Donnell, Sutherland, and Navelet[7]) we consider the decays $B \to Kl\bar{l}$ and $B \to K^* l\bar{l}$ in this paper. These decays have been evaluated most recently by Deshpande and Trampetic[8] using the (relativistic) quark-model calculations of Bauer, Stech, and Wirbel[9] and the incomplete QCD calculations then available. In Ref. 6 the decay $B \to Kl\bar{l}$ is calculated in the nonrelativistic quark model.[10] Our evaluation yields smaller rates than those of Ref. 8. This is also evident from Refs. 6 and 7 and has also been noted elsewhere.[11] We also give the q^2 distributions for the decays, where q^2 is the invariant mass of the two leptons. We have not considered $B \to K^*\gamma$ since for this process our results would agree with those obtained using Ref. 9 (see Refs. 2 and 3).

In the next section we present the quark model used and the calculations of the necessary matrix elements; Sec. III details the calculation of the decay distributions and rates. A brief discussion of the results is given in Sec. IV.

II. CALCULATION OF HADRONIC MATRIX ELEMENTS IN THE CONSTITUENT QUARK MODEL

It has been demonstrated in Ref. 5 that the form factors, which are commonly used to parametrize hadronic matrix elements, can be calculated in simple constituent quark models. The method used in Ref. 5 is based upon the light-front formalism originally due to Dirac[12] and used, for example, in Ref. 13 for the calculation of the charge form factor of the pion. The advantage of this approach is that the quark-antiquark wave functions of the constituent quark model, which have been derived in the analysis of the meson spectrum, can serve as an input to determine in a consistent relativistic treatment internal properties of bound states, such as form factors.

The light-front formalism which we shall use here is specified by the kinematic subgroup which is the symmetry group of the light-front $x^+ \equiv x^0 + x^3 = 0$. Light-front vectors will be denoted by letters with arrows $\vec{\mathbf{P}} = (P^+, \mathbf{P}_\perp)$ where, with $\mathbf{n} = (0,0,1)$,

$$P^+ = P^0 + P^3, \quad \mathbf{P}_\perp = \mathbf{P} - (\mathbf{P}\cdot\mathbf{n})\mathbf{n} \; . \tag{2.1}$$

It is crucial for the description of bound states of a quark and an antiquark to establish the appropriate variables for the internal motion of the constituents, whose momenta we shall denote by k_1 and k_2.

The total light-front momentum $\vec{\mathbf{P}} = \vec{\mathbf{k}}_1 + \vec{\mathbf{k}}_2$ is conserved and the momenta of the constituents are usually represented in the following way:

$$\begin{aligned} &k_1^+ = \xi P^+ , \quad k_2^+ = (1-\xi)P^+ \; , \\ &\mathbf{k}_{1\perp} = \xi \mathbf{P}_\perp + \mathbf{p}_\perp , \quad \mathbf{k}_{2\perp} = (1-\xi)\mathbf{P}_\perp - \mathbf{p}_\perp \; . \end{aligned} \tag{2.2}$$

Since $P^2 = P^+P^- - \mathbf{P}_\perp^2 = M^2$, the "Hamiltonian" P^- is related to the mass operator M for the bound state:

$$H = \frac{M^2 + P_\perp^2}{P^+} \; .$$

The dynamical structure of the front form can be exhibited, if the mass operator is expressed in terms of internal variables and the interaction operator $\hat{W}$:

$$M^2 = M_0^2 + \hat{W}, \quad M_0^2 = \frac{p_\perp^2 + m_1^2}{\xi} + \frac{p_\perp^2 + m_2^2}{1-\xi} \; , \tag{2.3}$$

where m_1, m_2 are the masses of the constituent quarks. It is convenient to introduce a vector $\mathbf{p} = (p_1, p_2, p_3)$ whose transverse part is $\mathbf{p}_\perp$ and the longitudinal component is defined in terms of $p_\perp$ and ξ:

$$p_3 = (\xi - \tfrac{1}{2})M_0 - \frac{m_1^2 - m_2^2}{2M_0} \; . \tag{2.4}$$

In terms of this new variable M_0 is simply given by

$$M_0=E_1+E_2\ , \tag{2.5}$$

where $E_i=(m_i^2+\mathbf{p}^2)^{1/2}$.

The description of the motion of the constituents in terms of the inner momentum vector $\mathbf{p}$ is effectively that of free particles, and is independent of the motion of the system as a whole. The wave function of the bound state must be a simultaneous eigenfunction of the mass operator (2.3), and the angular momentum operators j^2 and j_3, and depends only on the inner momentum vector $\mathbf{p}$ and the spin variables $\lambda,\bar{\lambda}=\pm\frac{1}{2}$ of the quarks. The rotational invariance of the wave function for a $q\bar{q}$ state with spin J and orbital angular momentum zero requires a spin dependence, given by

$$\psi(\mathbf{p},\lambda\bar{\lambda},JJ_3)=R(\lambda\bar{\lambda},JJ_3)\varphi(p)\ ,$$
$$R(\lambda\bar{\lambda},JJ_3)=\sum_{\lambda'\bar{\lambda}'}\langle\lambda|\mathcal{R}_M^\dagger(\mathbf{p},m_1)|\lambda'\rangle\langle\bar{\lambda}|\mathcal{R}_M^\dagger(-\mathbf{p},m_2)|\bar{\lambda}'\rangle \times\langle\tfrac{1}{2}\tfrac{1}{2},\lambda'\bar{\lambda}'|JJ_3\rangle\ , \tag{2.6}$$

where the spin of a quark is rotated by an amount which depends on its momentum. This Melosh rotation is specified by[14]

$$\langle\lambda'|\mathcal{R}_M(\mathbf{p},m)|\lambda\rangle=\chi_{\lambda'}^\dagger\frac{m+\xi M_0-i\boldsymbol{\sigma}(\mathbf{n}\times\mathbf{p})}{\sqrt{(m+\xi M_0)^2+p_\perp^2}}\chi_\lambda\ , \tag{2.7}$$

where χ_λ is the usual Pauli spinor and $\mathbf{n}=(0,0,1)$.

The application of the light-front formalism is more transparent in the framework of the quasipotential method[5] which allows a systematic treatment of gluon exchange in terms of the diagrams of the perturbation theory. We shall treat only the one-loop approximation, and shall derive explicit expressions for matrix elements of currents between meson states, which correspond to the Feynman diagram of Fig. 1.

For definiteness, we consider the matrix element of a general current, which couples to the quarks of the $q\bar{q}$ states:

$$\langle\vec{\mathbf{P}}'',J''J_3''|\bar{q}''\Gamma_\mu q'|\vec{\mathbf{P}}',J'J_3'\rangle\sqrt{4P'^+P''^+}=M_\mu\ . \tag{2.8}$$

The vertices of the Feynman diagram of Fig. 1 can be represented in terms of the light-front wave function (2.6) only if the correct $q\bar{q}$ structure of the vertex is maintained. It is a remarkable property of the light-front formalism that those parts of the one-loop diagram which involve quarks created out of or annihilating into the vacuum can be eliminated exactly in the component M^+ of the matrix element M_μ, if furthermore $q^+=0$, where $q=P''-P'$. The one-loop approximation for M^+ is given exactly by

$$M^+=\frac{1}{(2\pi)^3}\int d^3p'\frac{1}{\xi}\left[\frac{E_1''E_2''M_0'}{E_1'E_2'M_0''}\right]^{1/2} \times\sum_{\lambda'\lambda''\bar{\lambda}}\psi^\dagger(\mathbf{p}'',\lambda''\bar{\lambda},J''J_3'')\bar{u}(k_1'',\lambda'') \times\Gamma^+u(k_1',\lambda')\psi(\mathbf{p}',\lambda'\bar{\lambda},J'J_3')\ . \tag{2.9}$$

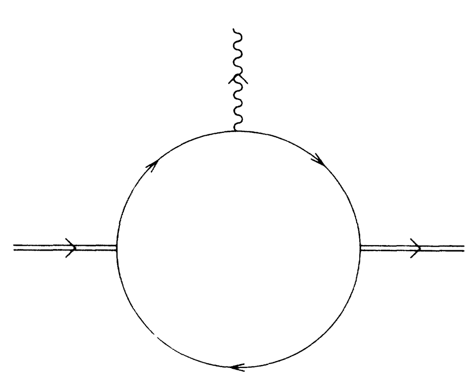

FIG. 1. One-loop Feynman diagram representing the transition of the heavy meson to the light one.

All double primed variables in Eq. (2.9) can be expressed in terms of the integration variable $\mathbf{p}'$ and $\mathbf{q}_\perp$, using Eqs. (2.2)–(2.4) and

$$k_1''^+=\xi P'^+,\quad \mathbf{k}_{1\perp}''=\mathbf{k}_{1\perp}'+\mathbf{q}_\perp\ ,$$
$$\mathbf{p}_\perp''=\mathbf{p}_\perp'+(1-\xi)\mathbf{q}_\perp\ , \tag{2.10}$$
$$p_3''=(\xi-\tfrac{1}{2})M_0''-\frac{m_1''^2-m_2^2}{2M_0''}\ .$$

For the discussion of the next section, we shall need the following hadronic matrix elements first for the transition $B\to K$:

$$J_\mu\equiv\langle p_K|\bar{s}\gamma_\mu Lb|p_B\rangle\sqrt{4E_KE_B} =\tfrac{1}{2}(F_+P_\mu+F_-q_\mu)\ , \tag{2.11}$$

$$J_\mu^T\equiv\langle p_K|\bar{s}i\sigma_{\mu\nu}q^\nu Rb|p_B\rangle\sqrt{4E_KE_B} =\frac{1}{2(m_B+m_K)}[P_\mu q^2-(m_B^2-m_K^2)q_\mu]F_T\ , \tag{2.12}$$

where $P=p_B+p_K$ and $q=p_B-p_K$. (Our convention is $\sigma_{\mu\nu}=(i/2)[\gamma_\mu,\gamma_\nu]$.) The form factor $F_-(q^2)$ in Eq. (2.11) gives no contribution to the rate of the transition $B\to Kl\bar{l}$. (We neglect the lepton masses throughout, since we only consider $l=e,\mu$.) The other form factors of Eqs. (2.11) and (2.12) can be calculated using Eq. (2.9). The relevant matrix elements of the effective couplings of the quarks are given by[5]

$$\bar{u}(k_1'',\lambda'')\gamma^+Lu(k_1',\lambda')=\xi p_B^+\chi_{\lambda''}^\dagger(1-\sigma_3)\chi_{\lambda'}\ , \tag{2.13}$$

$$\bar{u}(k_1'',\lambda'')i\sigma^{+\nu}q_\nu Ru(k_1',\lambda') =-\xi p_B^+\chi_{\lambda''}^\dagger[i\mathbf{n}(\boldsymbol{\sigma}\times\mathbf{q})+\boldsymbol{\sigma}_\perp\cdot\mathbf{q}_\perp]\chi_{\lambda'}\ , \tag{2.14}$$

where $\mathbf{n}=(0,0,1)$, which leads to

$$F_+(q^2)=\frac{1}{(2\pi)^3}\int d^3p'\Omega(p'',p')\frac{1}{\xi(1-\xi)}\{\mathbf{p}'_\perp\cdot\mathbf{p}''_\perp+[\xi m_u+(1-\xi)m_b][\xi m_u+(1-\xi)m_s]\}\ , \tag{2.15}$$

$$F_T(q^2)=\frac{m_B+m_K}{(2\pi)^3}\int d^3p'\Omega(p'',p')\frac{1}{\xi}\left[(m_b-m_s)\frac{\mathbf{p}'_\perp\cdot\mathbf{q}_\perp}{q^2}+[\xi m_u+(1-\xi)m_s]\right]\ , \tag{2.16}$$

where

$$\Omega(p'',p')=\varphi_B^\dagger(p'')\varphi_K(p')\left[\frac{E''_1E''_2M'_0}{E'_1E'_2M''_0}\right]^{1/2}[M_0'^2-(m_s-m_u)^2]^{-1/2}[M_0''^2-(m_b-m_u)^2]^{-1/2} \tag{2.17}$$

and

$$E''_1=(m_b^2+\mathbf{p}''^2)^{1/2},\quad E''_2=(m_u^2+\mathbf{p}''^2)^{1/2}\ ,$$

$$E'_1=(m_s^2+\mathbf{p}'^2)^{1/2},\quad E'_2=(m_u^2+\mathbf{p}'^2)^{1/2}\ .$$

The relation between $\mathbf{p}''$ and $\mathbf{p}'$ is given in (2.10). The hadronic matrix elements for the transition $B\to K^*$ are given by

$$J_\mu\equiv\langle p_{K^*}|\bar{s}\gamma_\mu Lb|p_B\rangle\sqrt{4E_{K^*}E_B}$$

$$=\frac{1}{2(m_B+m_{K^*})}[\,iV\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}P^\alpha q^\beta+A_0(m_B^2-m_{K^*}^2)\epsilon^*_\mu+A_+\epsilon^*PP_\mu+A_-\epsilon^*Pq_\mu]\ , \tag{2.18}$$

$$J^T_\mu\equiv\langle p_{K^*}|\bar{s}i\sigma_{\mu\nu}q^\nu Rb|p_B\rangle\sqrt{4E_{K^*}E_B}$$

$$=\frac{1}{2}\left[ig\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}P^\alpha q^\beta+a_0(m_B^2-m_{K^*}^2)\left[\epsilon^*_\mu-\frac{1}{q^2}\epsilon^*qq_\mu\right]+a_+\epsilon^*P\left[P_\mu-\frac{1}{q^2}Pqq_\mu\right]\right]\ . \tag{2.19}$$

Again, the terms proportional to $q_\mu=(p_B-p_{K^*})_\mu$ do not contribute to the rate of the transition $B\to K^*e\bar{e}$. The polarization vector $\epsilon_\mu=\epsilon_\mu(J_3)$ of K^* has components $\epsilon(\pm1)=\mp(0,1,\pm i,0)/\sqrt{2}$ and $\epsilon(0)=(0,0,0,1)$ in the rest system of K^*. Using Eqs. (2.9), (2.13), (2.14), and (2.17) the form factors are given by

$$V(q^2)=-\frac{m_B+m_{K^*}}{(2\pi)^3}\int d^3p'\Omega(p'',p')\frac{1}{\xi}\left[(m_b-m_s)\frac{\mathbf{p}'_\perp\cdot\mathbf{q}_\perp}{q^2}+\xi m_u+(1-\xi)m_s+\frac{2}{M'_0+m_s+m_u}\left[p_\perp'^2+\frac{(\mathbf{p}'_\perp\cdot\mathbf{q}_\perp)^2}{q^2}\right]\right]\ , \tag{2.20}$$

$$A_+(q^2)=\frac{-1}{(2\pi)^3(m_B+m_{K^*})}\int d^3p'\Omega(p'',p')\frac{1}{\xi}\left[\xi m_2+(1-\xi)m_s-\frac{\mathbf{p}'_\perp\cdot\mathbf{q}_\perp}{q^2}[m_s+(1-2\xi)m_b+2\xi m_u]-\frac{2\mathbf{p}'_\perp\cdot\mathbf{q}_\perp}{(1-\xi)q^2(M'_0+m_s+m_u)}\times\{\mathbf{p}'_\perp\cdot\mathbf{p}''_\perp+[\xi m_u+(1-\xi)m_b][\xi m_u-(1-\xi)m_s]\}\right]\ , \tag{2.21}$$

$$\frac{m_B-m_{K^*}}{2m_{K^*}}\left[A_0(q^2)+\frac{m_B^2-m_{K^*}^2-q^2}{m_B^2-m_{K^*}^2}A_+(q^2)\right]\equiv A_3(q^2)\ ,$$

$$A_3(q^2)=-\frac{1}{(2\pi)^3}\int d^3p'\Omega(p'',p')\frac{1}{\xi(1-\xi)}\left[[\xi m_u+(1-\xi)m_b][\xi m_u+(1-\xi)m_s]-(1-2\xi)\mathbf{p}'_\perp\cdot\mathbf{p}''_\perp+\frac{2(1-\xi)}{M'_0+m_s+m_u}\{p_\perp'^2(m_b+m_s)-\mathbf{p}'_\perp\cdot\mathbf{q}_\perp[\xi m_u-(1-\xi)m_s]\}\right]\ , \tag{2.22}$$

$$g(q^2)=-\frac{1}{(2\pi)^3}\int d^3p'\Omega(p'',p')\frac{1}{\xi(1-\xi)}\left[p_\perp'^2-2(1-\xi)\left[p_\perp'^2+\frac{(\mathbf{p}'_\perp\cdot\mathbf{q}_\perp)^2}{q^2}\right]+(1-\xi)\mathbf{p}'_\perp\cdot\mathbf{q}_\perp+[\xi m_u+(1-\xi)m_b][\xi m_u+(1-\xi)m_s]+\frac{2(1-\xi)(m_b+m_s)}{M'_0+m_s+m_u}\left[p_\perp'^2+\frac{(\mathbf{p}'_\perp\cdot\mathbf{q}_\perp)^2}{q^2}\right]\right]\ , \tag{2.23}$$

TABLE I. Values of the various $F(0)$, Λ_1, and Λ_2 defined by Eqs. (2.11), (2.12), (2.18), (2.19), and (2.26).

Parameters \ Form factor F	F_+	F	V	A_+	A_0	g	a_+	a_0
$F(0)$	0.3	−0.3	−0.35	0.24	−0.37	0.31	−0.31	0.31
Λ_1 (GeV)	4.06	4.08	4.05	4.39	5.92	4.05	4.37	6.40
Λ_2 (GeV)	5.51	5.50	5.45	5.81	8.54	5.44	5.84	8.84

$$a_+(q^2)=\frac{1}{(2\pi)^3}\int d^3p'\Omega(p'',p')\frac{1}{\xi(1-\xi)}\left[p_\perp'^2+2(1-\xi)\frac{(\mathbf{p}_\perp'\cdot\mathbf{q}_\perp)^2}{q^2}-(1-2\xi)(1-\xi)\mathbf{p}_\perp'\cdot\mathbf{q}_\perp\right.$$
$$+[\xi m_u+(1-\xi)m_b][\xi m_u+(1-\xi)m_s]$$
$$\left.-\frac{2(1-\xi)}{M_0'+m_s+m_u}\left[(m_b+m_s)\frac{(\mathbf{p}_\perp'\cdot\mathbf{q}_\perp)^2}{q^2}+[\xi m_u-(1-\xi)m_s]\mathbf{p}_\perp'\cdot\mathbf{q}_\perp\right]\right], \quad (2.24)$$

$$\frac{1}{2m_{K^*}}[(m_B^2-m_{K^*}^2)a_0(q^2)+(m_B^2-m_{K^*}^2-q^2)a_+(q^2)]\equiv a_3(q^2)\ ,$$

$$a_3(q^2)=-\frac{1}{(2\pi)^3}\int d^3p'\Omega(p'',p')\frac{1}{\xi(1-\xi)}$$
$$\times\left[(1-\xi)q^2[\xi m_u+(1-\xi)m_s]-\mathbf{p}_\perp'\cdot\mathbf{q}_\perp[(1-2\xi)\xi m_u+(1-\xi)m_s+(1-2\xi)(1-\xi)m_b]\right.$$
$$\left.+\frac{2}{M_0'+m_s+m_u}\{(1-\xi)q^2p_\perp'^2-\mathbf{p}_\perp'\cdot\mathbf{q}_\perp p_\perp'^2-\mathbf{p}_\perp'\cdot\mathbf{q}_\perp[\xi m_u-(1-\xi)m_s][\xi m_u+(1-\xi)m_b]\}\right]. \quad (2.25)$$

For convenience all form factors have been derived for transitions K $(K^*)\to B$ in the rest system of K, respectively, K^*. The expressions for the form factors given above are valid only for $q^2\leq 0$ (since $q^+=0$), while for the decay $B\to Ke\bar{e}$ the physical values for q^2 lie in the range $0\leq q^2\leq(m_B-m_K)^2$. It has been shown in Ref. 5 that the form factors for spacelike q can be continued to timelike q in the environment of $q^2=0$, if the form factor $F(q^2)$ is approximated by

$$F(q^2)\simeq\frac{F(0)}{1-q^2/\Lambda_1^2+q^4/\Lambda_2^4}\ . \quad (2.26)$$

The parameters Λ_1,Λ_2 are determined by the first and second derivative of $F(q^2)$ at $q^2=0$. In order to avoid extensive numerical calculations, we used an approach analogous to the one of Ref. 15, where harmonic-oscillator wave functions are used for the S-state wave function $\varphi(p)$ defined in Eq. (2.6). The quantity $\Omega(p'',p')$ defined in Eq. (2.17) is therefore determined in terms of oscillator wave functions of the type

$$\varphi(p)=\pi^{-3/4}\beta^{-3/2}(2\pi)^{3/2}\exp(-\mathbf{p}^2/2\beta^2)\ . \quad (2.27)$$

We used[5] constituent quark masses $m_u=0.29$ GeV, $m_s=0.42$ GeV, and $m_b=4.9$ GeV, which are close to the quark masses given in Ref. 16, and $\beta_K=\beta_{K^*}=0.3575$ GeV and $\beta_B=0.386$ GeV. The values of $F(0)$ and Λ_1,Λ_2 for the various form factors used are summarized in Table I.

III. THE DECAYS $B\to Kl\bar{l}$ AND $B\to K^*l\bar{l}$

A. Effective operator on the quark level

In this section we follow closely Refs. 6 and 7. (It has been recently shown[11] by the authors of Ref. 7 that the γ_5 procedure of Ref. 6 yields the correct QCD corrections. Also, some corrections not included in Ref. 6 yield small contributions and are not included here.)

Starting from the standard-model[4] interaction

$$\mathcal{L}_{\rm int}=\frac{g}{\sqrt{2}}W_\mu(\bar{u},\bar{c},\bar{t},\dots)\gamma^\mu LV\begin{pmatrix}d\\ s\\ b\\ \vdots\end{pmatrix}, \quad (3.1)$$

where $V=\{V_{ij}\}$ is the Cabibbo-Kobayashi-Maskawa[4] (CKM) matrix and $L=\frac{1}{2}(1-\gamma_5)$, an effective Hamiltonian for the process $b\to sl\bar{l}$ is derived by integrating out successively heavy particles. Since we expect the top-quark mass m_t to be as large as M_W, $t,t',\dots$ disappear together with the W, leaving an effective Hamiltonian which only depends on light fields.

For our processes, a sufficient[6,7] Hamiltonian is (the notation of Ref. 6 is used)

$$\mathcal{H}=\frac{4G_F}{\sqrt{2}}\sum_{\substack{j=1,2,7,8,9\\ i=u,c,t,\dots}}\lambda_i c_j^i(\mu)O_j(\mu)$$
$$\equiv\frac{4G_F}{\sqrt{2}}\sum_j c_j(\mu)O_j(\mu)\ , \tag{3.2}$$

where

$$\begin{aligned}
O_1&=(\bar{s}\gamma_\mu Lb)(\bar{c}\gamma^\mu Lc)\ ,\\
O_2&=(\bar{s}\gamma_\mu Lb)(\bar{c}\gamma^\mu Lc)\ ,\\
O_7&=\frac{e}{16\pi^2}m_b(\bar{s}\sigma_{\mu\nu}Rb)F^{\mu\nu}\ ,\\
O_8&=\frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu Lb)(\bar{e}\gamma^\mu e)\ ,\\
O_9&=\frac{e^2}{16\pi^2}(\bar{s}\gamma_\mu Lb)(\bar{e}\gamma^\mu\gamma_5 e)\ ,
\end{aligned} \tag{3.3}$$

and

$$\lambda_i=V_{ib}V_{is}^*,\quad i=c,t,t',\dots\ . \tag{3.4}$$

For the operators O_7, O_8, O_9, $i=t,t',\dots$ (all heavy internal particles) while for O_1, O_2, $i=c$. The up quark is not included because $\lambda_u=V_{ub}V_{us}^*$ is tiny.

In (3.2), μ is the "scale;" and if $\mu\simeq m_b$, one does not expect the matrix elements of the O_i to contain large logarithms, $\ln(M_W/m_b)$. These are absorbed into the $c_j^i(m_b)$ whose value is determined by the renormalization-group equation from the perturbative value $c_j^i(M_W)$. As shown in Refs. 6 and 7, one has

$$\begin{aligned}
c_1(m_b)&=-\tfrac{1}{2}(\rho^{-6/23}-\rho^{12/23})\lambda_c\ ,\\
c_2(m_b)&=-\tfrac{1}{2}(\rho^{-6/23}+\rho^{12/23})\lambda_c\ ,
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
c_8&=\sum_i\lambda_i c_8^i(m_b)\\
&=\sum\lambda_i c_8^i(M_W)\\
&\quad-\frac{4\pi\lambda_c}{\alpha_s(M_W)}[-\tfrac{4}{33}(1-\rho^{-11/23})+\tfrac{8}{87}(1-\rho^{-29/23})]\ ,
\end{aligned} \tag{3.6}$$

$$c_9=\sum_i\lambda_i c_9^i(m_b)=\sum_i\lambda_i c_9^i(M_W)\ , \tag{3.7}$$

$$\begin{aligned}
c_7&=\sum_i\lambda_i c_7^i(m_b)\\
&=\rho^{-16/23}\left[\sum_i\lambda_i c_7^i(M_W)-\lambda_c[\tfrac{58}{135}(\rho^{10/23}-1)\right.\\
&\qquad\left.+\tfrac{29}{189}(\rho^{28/23}-1)]\right]\ ,
\end{aligned} \tag{3.8}$$

where

$$\rho=\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\simeq 1.9\ .$$

The $c_j^i(M_W)$ are given by[4,6,7]

$$\begin{aligned}
c_7^i(M_W)&=\tfrac{1}{2}A(x_i)\ ,\\
c_8^i(M_W)&=\frac{1}{\sin\theta_W}[B(x_i)-C(x_i)]\\
&\quad+4C(x_i)+D(x_i)-\tfrac{4}{9}\ ,\\
c_9^i(M_W)&=\frac{1}{\sin\theta_W}[C(x_i)-B(x_i)]\ ,
\end{aligned} \tag{3.9}$$

where

$$x_i=\frac{m_i^2}{M_W^2} \tag{3.10}$$

and the functions A,B,C,D are

$$A(x)=x\left[\frac{2x^2/3+5x/12-7/12}{(x-1)^3}-\frac{3x^2/2-x}{(x-1)^4}\ln x\right]\ , \tag{3.11}$$

$$B(x)=\frac{1}{4}\left[-\frac{x}{x-1}+\frac{x}{(x-1)^2}\ln x\right]\ , \tag{3.12}$$

$$C(x)=\frac{x}{4}\left[\frac{x/2-3}{x-1}+\frac{3x/2+1}{(x-1)^2}\ln x\right]\ , \tag{3.13}$$

$$\begin{aligned}
D(x)&=\frac{-19x^3/36+25x^2/36}{(x-1)^3}\\
&\quad+\frac{-x^4/6+5x^3/3-3x^2+16x/9-4/9}{(x-1)^4}\ln x\ .
\end{aligned} \tag{3.14}$$

The operators O_1,O_2 will contribute to the $b\to sl\bar{l}$ matrix elements through c-quark loops. These result in an expression with the same structure as O_8, because the coupling of the $c\bar{c}$ pair to ll is through the (vectorial) photon. Thus, O_1,O_2 can be included in an "effective" O_8 if the coefficient $c_8(m_b)$ is replaced by

$$c_8^{\text{eff}}=\sum_i\lambda_i c_8^i(m_b)-\lambda_c[3c_1(m_b)+c_2(m_b)]g\left[\frac{m_c}{m_b},\hat{s}\right]\ , \tag{3.15}$$

$$\begin{aligned}
g(z,\hat{s})=-&\left[\frac{4}{9}\ln z^2-\frac{8}{27}-\frac{16}{9}\frac{z^2}{\hat{s}}\right.\\
&+\frac{2}{9}\left[1-\frac{4z^2}{\hat{s}}\right]^{1/2}\left[2+\frac{4z^2}{\hat{s}}\right]\\
&\left.\times\left[\ln\frac{1+\left[1-\frac{4z^2}{\hat{s}}\right]^{1/2}}{1-\left[1-\frac{4z^2}{\hat{s}}\right]^{1/2}}+i\pi\right]\right]\ ,\\
&\hat{s}>z^2\ ,
\end{aligned} \tag{3.16}$$

$$g(z,\hat{s})=-\left[\frac{4}{9}\ln z^2-\frac{8}{27}-\frac{16}{9}\frac{z^2}{\hat{s}}+\frac{4}{9}\left[\frac{4z^2}{\hat{s}}-1\right]^{1/2}\left[2+\frac{4z^2}{\hat{s}}\right]\times\arctan\left[\frac{4z^2}{\hat{s}}-1\right]^{-1/2}\right],\quad \hat{s}<z^2, \tag{3.17}$$

where

$$z=\frac{m_c}{m_b},\quad \hat{s}=\frac{(p_l+p_{\bar{l}})^2}{m_B^2}\equiv q^2/m_B^2 . \tag{3.18}$$

The second piece in (3.15) reflects the contributions of O_1 and O_2.

Similarly, O_7 will contribute to $b\to sl\bar{l}$ through photon exchange, resulting in an effective coupling $\propto c_7/q^2$. Including this, the effective transition operator is

$$\mathcal{H}=\frac{4G_F}{\sqrt{2}}\frac{e^2}{16\pi^2}\left[\left[c_8^{\text{eff}}\bar{s}\gamma_\mu Lb-\frac{2ic_7m_b}{q^2}\bar{s}\sigma_{\mu\nu}q^\nu Rb\right](\bar{l}\gamma^\mu l)+c_9(\bar{s}\gamma_\mu Lb)(\bar{l}\gamma^\mu\gamma_5 l)\right] . \tag{3.19}$$

Here, $q=p_l+p_{\bar{l}}$; c_8^{eff},c_7,c_9 are given by (3.15), (3.8), and (3.7), respectively.

B. Matrix element and rate for exclusive decays

Starting from (3.19), we can obtain the rate in a standard fashion. We write

$$M=\langle Fl\bar{l}|\mathcal{H}|B\rangle=\frac{4G_F}{\sqrt{2}}\frac{e^2}{16\pi^2}\left[\left[c_8^{\text{eff}}J_\mu-\frac{2m_b}{q^2}c_7J_\mu^T\right]\bar{l}\gamma^\mu l+c_9J_\mu\bar{l}\gamma^\mu\gamma_5 l\right], \tag{3.20}$$

where J_μ,J_μ^T are the matrix elements of $\bar{s}\gamma_\mu Lb$ and $i\bar{s}\sigma_{\mu\nu}q^\nu Rb$ [see Eqs. (2.11) and (2.12)] and the final state F is either K or K^*. After integrating over the lepton momenta and summing over their spins, keeping $q=p_l+p_{\bar{l}}$ fixed, we get

$$\sum_{\text{leptons}}\int|M|^2=\frac{4}{3\pi}G_F^2(q^\mu q^\nu-g^{\mu\nu}q^2)\times\left[\left[c_8^{\text{eff}}J_\mu-\frac{2m_b}{q^2}c_7J_\mu^T\right]\times\left[c_8^{\text{eff}}J_\nu-\frac{2m_b}{q^2}c_7J_\nu^T\right]^*+|c_9|^2J_\mu J_\nu^*\right] . \tag{3.21}$$

In obtaining (3.21) we have neglected the mass of the leptons (we only consider e and μ).

1. $B\to Kl\bar{l}$

Using the matrix elements (2.11) and (2.12) of the previous section, we get, for the total rate,

$$\Gamma(B\to Kl\bar{l})=\frac{m_B^5G_F^2}{48\pi^3}\left[\frac{e^2}{16\pi^2}\right]^2\int_{\hat{s}_{\min}}^{\hat{s}_{\max}}d\hat{s}\frac{\phi^{3/2}}{2}\left[\left|c_8^{\text{eff}}F_+-\frac{2c_7F_T}{1+\sqrt{\kappa}}\right|^2+|c_9|^2F_+^2\right]\equiv\frac{m_B^5G_F^2}{48\pi^3}\left[\frac{e^2}{16\pi^2}\right]^2\int d\hat{s}\frac{d\Gamma(\hat{s})}{d\hat{s}} . \tag{3.22}$$

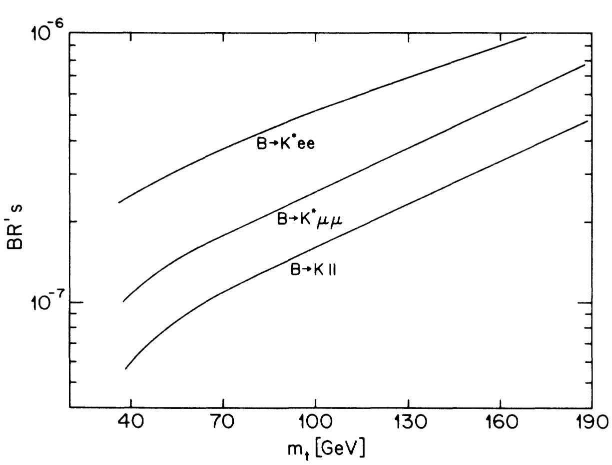

FIG. 2. Branching ratios of the rare processes $B\to Kl\bar{l}$, $B\to K^*e\bar{e}$, and $B\to K^*\mu\mu$ as a function of m_t.

[Note that with our normalization, the usual phase-space factors $(2E)^{-1}$ are included in (2.11) and (2.12).] Here

$$\kappa=\frac{m_K^2}{m_B^2},\quad \phi=(\hat{s}-1-\kappa)^2-4\kappa ,\quad \hat{s}=\frac{q^2}{m_B^2},\quad \hat{s}_{\min}=\frac{4m_l^2}{m_B^2},\quad \hat{s}_{\max}=1-2\sqrt{\kappa}+\kappa . \tag{3.23}$$

Since $d\Gamma(\hat{s})/d\hat{s}$ can have a pole at $\hat{s}=0$ (see Sec. III B 2) we must include the lepton mass at this point. In obtaining (3.22) we have used $m_B=m_b$, a rather unproblematic simplification. Note that $\sqrt{\phi}/2$ is the normalized three-momentum of the kaon.

We divide (3.22) by the total width of the B meson, estimated to be[17]

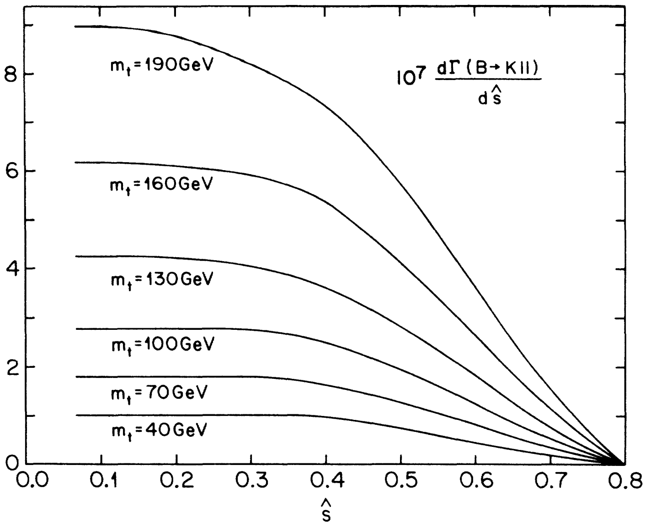

FIG. 3. Partial branching ratios of $B \to Kl\bar{l}$; $\hat{s}=(p_l+p_{\bar{l}})^2/m_B^2$.

$$\Gamma_{\text{tot}}=\frac{f m_B^5 G_F^2}{192\pi^3}|V_{cb}|^2 ,\qquad f \simeq 3.0 \tag{3.24}$$

and obtain the branching ratio

$$B=\frac{\Gamma}{\Gamma_{\text{tot}}}=\frac{4}{|V_{cb}|^2 f}\left[\frac{\alpha}{4\pi}\right]^2\int_{\hat{s}_{\min}}^{\hat{s}_{\max}} d\hat{s}\frac{d\Gamma(\hat{s})}{d\hat{s}} , \tag{3.25}$$

where $d\Gamma(\hat{s})/d\hat{s}$ is given in (3.22).

Restricting ourselves now to three generations, we need only $\lambda_t \simeq -\lambda_c$ (λ_u is small). Equation (3.22) is then proportional to λ_c which drops in (3.25); we then have

$$B=4.5\times 10^{-7}\int d\hat{s}\frac{d\Gamma(\hat{s})}{d\hat{s}} . \tag{3.26}$$

We note that the lepton masses have been neglected throughout, except in determining the lower limit of the $\hat{s}$ integration. This is irrelevant in the case of the kaon; however, in the following discussion of the K^*, the correct $\hat{s}_{\min}$ is crucial since the partial rate has a $(1/\hat{s})$ behavior near $\hat{s}=0$. All other effects of the lepton mass are tiny, except for the τ which is not considered here.

In Fig. 2 result (3.25) is given for various values of m_t. Figure 3 shows $[d\Gamma(\hat{s})/\Gamma_{\text{tot}}]/d\hat{s}$.

2. $B \to K^* l\bar{l}$

We now use (2.18) and (2.19) and obtain, instead of (3.22), the expression

$$\Gamma=\frac{m_B^5 G_F}{48\pi^3}\left[\frac{e^2}{16\pi^2}\right]^2\int_{\hat{s}_{\min}}^{\hat{s}_{\max}} d\hat{s}\frac{\phi^{1/2}}{2}\left[|G|^2 2\phi\hat{s}+|F|^2\left[2\hat{s}+\frac{(1-\kappa^*-\hat{s})^2}{4\kappa^*}\right]+|H_+|^2\frac{\phi^2}{4\kappa^*}-2\,\mathrm{Re}(FH_+^*)\frac{\phi}{4\kappa^*}(\hat{s}-1+\kappa^*)\right] , \tag{3.27}$$

where $\kappa^*=m_{K^*}^{2}/m_B^2$, and ϕ and $\hat{s}_{\max}$ are given in (3.23) but with κ replaced by κ^*; furthermore,

$$|G|^2=\left|c_8^{\text{eff}}\frac{V}{1+\sqrt{\kappa^*}}-\frac{2c_7}{\hat{s}}g\right|^2+\left|\frac{c_9 V}{1+\sqrt{\kappa^*}}\right|^2 , \tag{3.28}$$

$$|F|^2=\left|c_8^{\text{eff}}A_0(1-\sqrt{\kappa^*})-\frac{2c_7}{\hat{s}}a_0(1-\kappa^*)\right|^2+|c_9 A_0(1-\sqrt{\kappa^*})|^2 , \tag{3.29}$$

$$|H_+|^2=\left|c_8^{\text{eff}}\frac{A_+}{1+\sqrt{\kappa^*}}-\frac{2c_7}{\hat{s}}a_+\right|^2+\left|\frac{c_9 A_+}{1+\sqrt{\kappa^*}}\right|^2 , \tag{3.30}$$

$$\mathrm{Re}FH_+^*=\mathrm{Re}\left[c_8^{\text{eff}}A_0(1-\sqrt{\kappa^*})-\frac{2c_7}{\hat{s}}a_0(1-\kappa^*)\right]\left[c_8^{\text{eff}}\frac{A_+}{(1+\sqrt{\kappa^*})^{1/2}}-\frac{2c_7}{\hat{s}}a_+\right]+\mathrm{Re}\left[\frac{c_9 A_+ c_9 A_0(1-\sqrt{\kappa^*})}{1+\sqrt{\kappa^*}}\right] . \tag{3.31}$$

Here, A_0, A_+, V, a_0, a_+, g are the form factors introduced in (2.18) and (2.19).

Again, we work in the three-generation model. This yields

$$B(B\to K^* l\bar{l})=3.9\times 10^{-7}\int d\hat{s}\frac{d\Gamma(\hat{s})}{d\hat{s}} , \tag{3.32}$$

where $d\Gamma(\hat{s})/d\hat{s}$ must be taken from (3.27).

The normalized partial rates are given in Fig. 4. The branching ratios are plotted against m_t in Fig. 2. The rates for $B\to Ke\bar{e}$ are larger than those of $B\to K\mu\bar{\mu}$ because of the pole at $\hat{s}=0$ discussed earlier.

IV. SUMMARY AND DISCUSSION

In this paper we have used a light-front formalism based on relativistic quark model[5] and QCD-corrected[6,7] quark-level operators to calculate the exclusive branching ratios (BR's) for the rare processes $B\to Kl\bar{l}$, $B\to K^*\mu\bar{\mu}$,

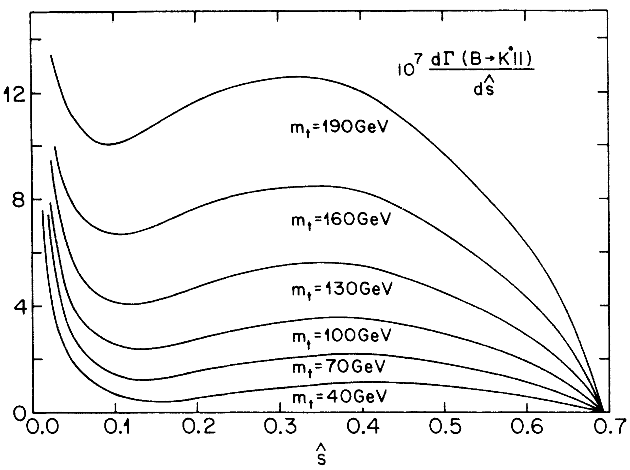

FIG. 4. Partial branching ratios of $B \rightarrow K^* l\bar{l}$; $\hat{s}=(p_l+p_{\bar{l}})^2/m_B^2$.

and $B \rightarrow K^* e\bar{e}$. We have not reevaluated $B \rightarrow K^*\gamma$ since in this case our model agrees with previous estimates.[2]

Our results are shown in Figs. 2–4 for the total and partial branching ratios, respectively. [We have not drawn the threshold effect at $\hat{s}=4m_c^2/m_B^2$ coming from the function g [Eq. (3.16)], since it gives rise to a very small jump (Refs. 6 and 7).] In the favored range for m_t between 100 and 170 GeV, these BR's are between 10^{-7} and 10^{-6}.

In comparison to previous calculations,[8] the BR's are considerably reduced. This effect of the QCD correction is already present in the inclusive calculations of Refs. 6, 7, and 11.

In the exclusive decays, in particular those with a K^*, this suppression can be partially understood from the first piece in κ [Eq. (3.19), or from Eq. (3.21)]. Depending on the signs of the form factors J_μ and J_μ^T, there is a strong cancellation between the terms proportional to c_8 and c_7, respectively. (This is not the case in the inclusive decays, where the matrix elements of γ_μ and $\sigma_{\mu\nu}q^\nu$ have a different form.) It turns out that in our model this cancellation indeed takes place, if $\hat{s}\simeq|2c_7/c_8^{\text{eff}}|\simeq 0.15$, depending on m_t. (One might be tempted to use the position of the minimum to predict m_t. However, it is more likely that top will be discovered before these curves can be determined precisely.) This effect is seen clearly in Fig. 3, where the partial rate goes through a minimum in this range of $\hat{s}$. A change of the relative signs of J_μ and J_μ^T would increase the rate by about a factor of 3–4.

This suppression is not operative for very small $\hat{s}$; as a result the rates for $K^*e\bar{e}$ and $K^*\mu\bar{\mu}$ differ more strongly than in Ref. 8. This preference of small $\hat{s}$ clearly favors the use of the relativistic quark model, since the nonrelativistic model[15,16] does not predict reliably the $\hat{s}$ dependence of form factors, and is particularly uncertain for small values of $\hat{s}$. Considering the $\hat{s}$ dependence, we note that the form factors in our model [Eq. (2.26)] deviate from the usual ansatz (Ref. 9) for the relativistic model or the nonrelativistic model used in Ref. 6 to estimate $B \rightarrow Kl\bar{l}$. Since (2.26) can be roughly represented by

$$F(q^2)\simeq\frac{F(0)}{(1-q^2/\Lambda_{\text{eff}}^2)^2} \quad \text{with } \Lambda_{\text{eff}}\simeq\Lambda_2 ,$$

the rate for larger values of $\hat{s}$ is increased compared to the ansatz $F(q^2)\simeq F(0)/(1-q^2/\Lambda^2)$ in Ref. 9. Comparing our results to those obtained with the nonrelativistic model (Ref. 6), say, for $m_t=100$ GeV, there is good agreement for $\hat{s}\gtrsim 0.6$. For smaller $\hat{s}$, the exponential damping in the nonrelativistic model reduces the rate in comparison to the relativistic model.

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