Semileptonic decays of B and D mesons in the light-front formalism

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The light-front formalism is used to present a relativistic calculation of form factors for semileptonic D and B decays in the constituent quark model. The quark-antiquark wave functions of the mesons can be obtained, in principle, from an analysis of the meson spectrum, but are approximated in this work by harmonic-oscillator wave functions. The predictions of the model are consistent with the experimental data for B decays. The Kobayashi-Maskawa (KM) matrix element $|V_{cs}|$ is determined by a comparison of the experimental and theoretical rates for $D^0 \rightarrow K^- e^+ \nu$, and is consistent with a unitary KM matrix for three families. The predictions for $D \rightarrow K^*$ transitions are in conflict with the data.

I. INTRODUCTION

The study of semileptonic decays of heavy pseudoscalar mesons is of great interest, since it is our main source of information on the Kobayashi-Maskawa (KM) matrix elements for heavy quarks. This is quite similar to the situation in the past, when the pseudoscalar mode provided corresponding information for light and strange quarks. The semileptonic decay modes $D \rightarrow K, K^*$ (Refs. 1 and 2) and $B \rightarrow D, D^*$ (Ref. 3) have been clearly identified. The theoretical analysis requires knowledge of the transition form factors in terms of which the relevant weak current matrix elements are represented in a Lorentz-covariant way. The form factors are a manifestation of nonperturbative QCD processes, and various phenomenological models have been used to obtain some information on them. The relevant literature has been quoted in Ref. 4, where the form-factor dependence of semileptonic decays of D and B mesons has been investigated.

Of particular interest is the constituent quark model supplemented with dynamics suggested by QCD, which was quite successful in describing the mass spectrum of hadrons.[5] Mesons are treated as bound quark-antiquark states. The dominant effects of nonperturbative gluon dynamics can be accounted for by a scalar confining potential, a Coulombic potential, and constituent quark masses. In Ref. 6 a one-gluon-exchange potential and various relativistic effects were taken into account.

However, a successful model of hadrons must go beyond this and probe the internal structure of hadrons. In particular, it must be able to predict form factors for decay processes. To this end an analysis of strong and electroweak couplings has been performed in Refs. 6 and 7, using partly the wave functions of Ref. 6 and partly the less predictive harmonic-oscillator functions but based upon nonrelativistic approximation methods for the relevant matrix elements.[8] Grinstein, Isgur, and Wise[9] (GIW) adopted essentially the same approach to calculate transition form factors for D and B decays. Their nonrelativistic treatment utilized harmonic-oscillator functions which lead to an exponential Q^2 dependence of the form factors, where Q is the four-momentum transfer. Altomari and Wolfenstein[10] made use of the nonrelativistic approach only to determine the form factors for vanishing three-momentum transfer $\mathbf{Q}=0$ and postulated monopole forms with a pole at the mass of the nearest resonance. A first step towards a relativistic treatment of the constituent quark model was taken by Bauer, Stech, and Wirbel[11] (BSW), who calculated the transition form factors in the infinite-momentum frame for maximum three-momentum transfer; i.e., for $Q^2=0$, while the Q^2 dependence was again postulated to be determined by the dominance of the nearest pole. Körner and Schuler[12] (KS) followed the method of BSW but have used monopole and dipole form factors according to the power-counting rules of QCD.

It is evident that a complete calculation of the transition form factors in the framework of the constituent quark model has not been accomplished as yet, mainly due to the fact that the relevant weak-current matrix elements are—except for the BSW approach—derived in a nonrelativistic formalism. It is the purpose of this paper to solve this problem by applying and expanding the light-front formalism originally due to Dirac,[13] and used, for example, by Chung, Coester, and Polyzou[14] (CCP) for the calculation of the charge form factor of the pion. The equations for the wave functions of two constituents, employing Hamiltonian light-front dynamics, were derived by Terent'ev and Berestetsky.[15] The application of this method is more transparent if the diagrams of the perturbation theory on the light plane[16] are considered. It can be seen that it is possible to eliminate diagrams involving quarks created out of or annihilating into the vacuum, thus leading to a relativistic quark model which retains the usual $q\bar{q}$ structure for mesons.

The great advantage of this method is that the quark-antiquark wave functions of the constituent quark model which have been derived in the analysis of the mass spectrum of mesons,[5] can serve as an input to determine, in a consistent, relativistic treatment, internal properties of bound states, such as form factors. The relativistic features of the model lead to essentially different results as compared with the nonrelativistic approach of GIW

(Ref. 9).

In Sec. II we present a brief review of the light-front formalism for bound states of a quark and an antiquark, which is completed by a detailed analysis of the spin structure of S-state mesons in Appendix A, and a presentation of the quasipotential approach to covariant light-front perturbation theory in Appendix B. Section III contains the general formalism for semileptonic decays. In Sec. IV we give the details of our calculation of the form factors in terms of which the matrix elements for the transitions $D \to K, K^*$ and $B \to D, D^*$ are represented. In order to avoid extensive numerical calculations we have approximated the meson wave functions by harmonic-oscillator wave functions. Section V contains our results for the semileptonic decay rates for the transitions $D \to K, K^*$ and $B \to D, D^*$ and our conclusions.

II. LIGHT-FRONT FORMALISM FOR $q\bar{q}$ BOUND STATES

The dynamics of a many-particle system is specified completely by expressing the ten generators of the Poincaré group P_μ, $M_{\mu\nu}$ in terms of dynamical variables. The operator P_μ is the four-momentum, and $M_{\mu\nu}$ is related to the angular momentum J_μ and the boosts K_i. The dependence of the generators upon the interaction between the particles is not unique, but there exists always a kinematic subgroup—the set of generators which are independent of the interaction. Dirac[13] observed that a relativistic system can be described by different forms of dynamics, depending upon the way the kinematic subgroup is chosen. The kinematic subgroup is uniquely determined by the particular choice of a Hamiltonian among the components of the four-momentum operator P_μ.

In the usual form of dynamics a physical state is given at an instant of time $x^0 = \text{const}$. The instant plane is left invariant by the associated kinematic subgroup of transitions and rotations. This form of dynamics has been called instant form by Dirac.

The light-front formalism which we shall use here is specified by the kinematic subgroup which is the symmetry group of the light front

$$x^+ \equiv x^0 + x^3 = \text{const} .$$

Its generators are the three translations which are components of the light-front vector (denoted by arrows over the letters throughout) $\vec{P} = (P^+, \mathbf{P}_\perp)$ where

$$P^+ \equiv P^0 + P^3, \quad \mathbf{P}_\perp \equiv \mathbf{P} - (\mathbf{P}\cdot\mathbf{n})\mathbf{n} , \tag{2.1}$$

and the generators of the kinematic Lorentz transformations

$$\begin{aligned} &K_n = \mathbf{K}\cdot\mathbf{n}, \quad J_n = \mathbf{J}\cdot\mathbf{n} , \\ &\mathbf{E}_\perp = \mathbf{K}_\perp + \mathbf{n}\times\mathbf{J}, \quad \mathbf{n} = (0,0,1) . \end{aligned} \tag{2.2}$$

The remaining three dynamic operators are the Hamiltonian $H \equiv P^- \equiv P^0 - P^3$ and $\mathbf{F}_\perp = \mathbf{K}_\perp - \mathbf{n}\times\mathbf{J}$.

We shall now specialize the formalism for the description of bound states of a quark and an antiquark, and it is crucial to establish the appropriate variables for the internal motion of the constituents, whose momenta we shall denote by k_1 and k_2.

The total light-front momentum $\vec{P} = \vec{k}_1 + \vec{k}_2$ is conserved and the momenta of the constituents are usually represented in the following way:

$$\begin{aligned} &k_1^+ = \xi P^+, \quad k_2^+ = (1-\xi)P^+ , \\ &\mathbf{k}_{1\perp} = \xi \mathbf{P}_\perp + \mathbf{p}_\perp, \quad \mathbf{k}_{2\perp} = (1-\xi)\mathbf{P}_\perp - \mathbf{p}_\perp . \end{aligned} \tag{2.3}$$

Since $P^2 = P^+P^- - P_\perp^2 = M^2$, the Hamiltonian is related to the mass operator M for the bound state

$$H = \frac{M^2 + P_\perp^2}{P^+} .$$

The dynamical structure of the front form can be demonstrated, if the mass operator is expressed in terms of internal variables and the interaction operator $\hat{W}$:

$$M^2 = M_0^2 + \hat{W} , \tag{2.4a}$$

$$M_0^2 = \frac{p_\perp^2 + m_1^2}{\xi} + \frac{p_\perp^2 + m_2^2}{1-\xi} , \tag{2.4b}$$

where m_1, m_2 are the masses of the constituent quarks. Poincaré invariance is satisfied if and only if M^2 commutes with all generators of the kinematic subgroup.

The mass operator can be given a more transparent meaning if the momentum fraction ξ is replaced by a new variable p_3 defined by

$$\xi = \frac{E_1 + p_3}{E_1 + E_2}, \quad 1-\xi = \frac{E_2 - p_3}{E_1 + E_2} , \tag{2.5}$$

where $E_i = (m_i^2 + \mathbf{p}^2)^{1/2}$ and $\mathbf{p} = (p_1, p_2, p_3)$. In terms of this new variable M_0 is simply given by

$$M_0 = E_1 + E_2 . \tag{2.6}$$

We shall frequently use the kinematic Lorentz transformation $L_f(P)$ (Refs. 17 and 18), which transforms the four-momentum P according to

$$L_f(P)(M, M, \mathbf{0}) = (P^-, P^+, \mathbf{P}_\perp) . \tag{2.7}$$

The transformation properties of any other four-vector A are

$$\begin{aligned} &[L_f(P)A]^- = (A^- + 2\,\mathbf{A}_\perp\cdot\mathbf{P}_\perp/M + P_\perp^2 A^+/M^2)M/P^+ , \\ &[L_f(P)A]^+ = A^+P^+/M , \\ &[L_f(P)A]_\perp = \mathbf{A}_\perp + \mathbf{P}_\perp A^+/M . \end{aligned} \tag{2.8}$$

It is evident from this transformation law that the light-front momenta $\vec{k}_1, \vec{k}_2$ are components of four-vectors k_1, k_2 with $k_i^2 = m_i^2$ which transform covariantly under the kinematic Lorentz transformations $L_f(P)$ (Ref. 18). This is an essential property of the light-front form which is absent in the instant form, and is the reason for some difficulties in the conventional relativistic treatment of a bound state. Another consequence is the invariance of ξ which follows from Eq. (2.8). The invariance of $p_\perp^2$ follows from the equation $p_\perp^2 = -[(1-\xi)k_1 - \xi k_2]^2$. Therefore, p_3 and finally M_0 are invariant under kinematic

Lorentz transformations.

The description of the motion of the constituents in terms of the inner momentum vector $\mathbf{p}$ is effectively that of free particles, and is independent of the motion of the system as a whole.

It is possible to construct an angular momentum operator $\mathbf{j}$ such that is commutes with all kinematic generators (except j_3) and satisfies the commutation relations[15,17,18]

$$[j_m,j_n]=i\epsilon_{mni}j_i,\quad [M,j_i]=0 . \tag{2.9}$$

Note that j does not depend upon the interaction. These conditions are necessary and sufficient for a Poincaré-invariant formulation of the dynamics of the $q\bar{q}$ system. j can be represented as a vector sum of the contributions of angular momentum and the individual spins of the constituents:[15,18]

$$\mathbf{j}=i\nabla_p\times\mathbf{p}+\mathcal{R}_M(\mathbf{p},m_1)\mathbf{s}^{(1)}+\mathcal{R}_M(-\mathbf{p},m_2)\mathbf{s}^{(2)} , \tag{2.10}$$

where the spin of a quark is rotated by an amount that depends upon its momentum. This Melosh-type rotation is given in the appropriate spinor basis by[15]

$$\langle\lambda'|\mathcal{R}_M(\mathbf{p},m)|\lambda\rangle=\chi^\dagger_{\lambda'}\frac{m+\xi M_0-i\sigma(\mathbf{n}\times\mathbf{p})}{\sqrt{(m+\xi M_0)^2+p_\perp^2}}\chi_\lambda , \tag{2.11}$$

where χ_λ is the usual Pauli spinor and $\mathbf{n}=(0,0,1)$.

The wave function of the bound state of mass M_B is defined as an eigenfunction of the mass operator M^2, given in Eqs. (2.4) and (2.6):

$$M^2\psi=M_B^2\psi . \tag{2.12a}$$

Written in full the symbolic Eq. (2.12a) means

$$(M_B^2-M_0^2)\psi(\mathbf{p})=(2\pi)^{-3}\int d^3p'\,\hat{W}(\mathbf{p},\mathbf{p}')\psi(\mathbf{p}') \tag{2.12b}$$

and spin variables have been suppressed in (2.12b). A short review of the derivation of Eq. (2.12) in the framework of the quasipotential formalism can be found in Appendix B.

The wave function can be chosen to be a simultaneous eigenfunction of the angular momentum operators j^2 and j_3. It has been emphasized in Ref. 19 that in order to satisfy the commutation relations in Eq. (2.9), the interaction operator $\hat{W}$ must be a function of scalar products of momenta and spin operators, but otherwise is not restricted at all.

The wave function depends upon the inner momentum $\mathbf{p}$ and the spin variables $\lambda,\bar{\lambda}=\pm\frac{1}{2}$ of the quarks. The structure of the $q\bar{q}$ state is therefore determined by a relativistic Schrödinger equation. When relativistic equations are used in constituent quark models[5,6] they are usually of the form

$$(E_1+E_2+V_{12})\psi=M_B\psi . \tag{2.13}$$

Equations (2.12) and (2.13) are equivalent if $\hat{W}=M_0V_{12}+V_{12}M_0+V_{12}^2$. The rotational invariance of the wave function for a $q\bar{q}$ state with spin J and orbital angular momentum zero, requires a spin dependence given by[18]

$$\psi(\mathbf{p})\rightarrow\psi(\mathbf{p},\lambda\bar{\lambda},JJ_3)=R(\lambda\bar{\lambda},JJ_3)\phi(p) , \tag{2.14}$$

$$R(\lambda\bar{\lambda},JJ_3)\equiv\sum_{\lambda'\bar{\lambda}'}\langle\lambda|\mathcal{R}_M^\dagger(\mathbf{p},m_1)|\lambda'\rangle\langle\bar{\lambda}|\mathcal{R}_M^\dagger(-\mathbf{p},m_2)|\bar{\lambda}'\rangle \times\langle\tfrac{1}{2}\tfrac{1}{2},\lambda'\bar{\lambda}'|JJ_3\rangle .$$

Explicit expressions for the spin structure function R for singlet and triplet states are given in Appendix A.

The wave functions are normalized (see Appendix B):

$$\frac{N_c}{(2\pi)^3}\int d^3p\,|\phi(p)|^2=1 , \tag{2.15}$$

where N_c is the number of colors.

III. GENERAL FORMALISM FOR SEMILEPTONIC DECAYS

For the semileptonic decay of a meson $M'\rightarrow M''e\bar{\nu}$ the T matrix is given by

$$T=\frac{G_F}{\sqrt{2}}V_{q'q''}L^\mu\langle P'',J''J_3''|\bar{q}''\gamma_\mu(1-\gamma_5)q'|P',J'J_3'\rangle , \tag{3.1}$$

where the leptonic current is

$$L^\mu=\bar{u}_e\gamma^\mu(1-\gamma_5)u_\nu$$

and $V_{q'q''}$ is the element of the Kobayashi-Maskawa mixing matrix.

We represent the hadronic matrix element in terms of appropriate form factors, whereby we follow the conventions of Altomari and Wolfenstein:[10]

$$\langle P'',00|\bar{q}''\gamma_\mu q'|P',00\rangle\sqrt{4P_0'P_0''}=f_+P_\mu+f_-Q_\mu , \tag{3.2a}$$

$$\begin{aligned}\langle P'',1J_3|\bar{q}''\gamma_\mu(1-\gamma_5)q'|P',00\rangle\sqrt{4P_0'P_0''}&=ig\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}P^\alpha Q^\beta\\&\quad+f\epsilon_\mu^*+a_+(\epsilon^*\cdot P)P_\mu+a_-(\epsilon^*\cdot P)Q_\mu ,\end{aligned} \tag{3.2b}$$

$$P=P'+P'',\quad Q=P'-P'' , \tag{3.3}$$

where $\epsilon=\epsilon(J_3)$ is the polarization vector of the vector meson with $\epsilon\cdot P''=0$.

In the limit where the electron mass is neglected, the from factors f_- and a_- do not contribute to the differential decay rate, which is given by

$$\frac{d\Gamma}{dQ^2}=|V_{q'q''}|^2\frac{G_F^2M'^6}{32\pi^3}|\mathbf{Q}|\left[\alpha\frac{Q^2}{M'^4}+\beta_{++}\frac{4\mathbf{Q}^2}{2M'^2}\right] . \tag{3.4}$$

For $P'=(M',0,0,0)$ we find

$$\mathbf{Q}^2=\frac{1}{4M'^2}[(M'^2-M''^2)^2+Q^4-2Q^2(M'^2+M''^2)] . \tag{3.5}$$

We copy the expressions for α and β_{++} from Ref. 9: For $0^- \to 0^-$ transitions they are given by

$$\alpha=0, \quad \beta_{++}=f_+^2 \ . \tag{3.6}$$

For $0^- \to 1^-$ transitions we will look at the polarization of the vector meson and must separate into the contributions due to longitudinal (L) states and transverse (T) states

$$\alpha^{(L)}=0, \quad \alpha^{(T)}=f^2+4M'^2g^2\mathbf{Q}^2 \ , \tag{3.7a}$$

$$\beta^{(L)}_{++}=\frac{M'^2}{16\mathbf{Q}^2M''^2}\left[1-\frac{Q^2}{M'^2}-\frac{M''^2}{M'^2}\right]^2 f^2 + \frac{M'^2}{2M''^2}\left[1-\frac{Q^2}{M'^2}-\frac{M''^2}{M'^2}\right] f a_+ + \frac{M'^2}{M''^2}\mathbf{Q}^2 a_+^2 \ , \tag{3.7b}$$

$$\beta^{(T)}_{++}=\left[\frac{1}{4M''^2}-\frac{M'^2}{16\mathbf{Q}^2M''^2}\left[1-\frac{Q^2}{M'^2}-\frac{M''^2}{M'^2}\right]^2\right] f^2 - Q^2g^2 \ . \tag{3.7c}$$

IV. CALCULATION OF FORM FACTORS IN THE CONSTITUENT QUARK MODEL

The hadronic matrix elements of the charged weak current given in Eqs. (3.2) can be calculated in the light-front formalism, following either a Hamiltonian approach, a method used, e.g., by CCP (Ref. 14) in their calculation of the charge form factor of the pion, or a covariant perturbation theory.[16,20] The elements of the diagrammatic approach to light-front perturbation theory that are required for the evaluation of form factors have been derived in Appendix B. The light-front formalism permits the exact calculation of the one-loop Feynman diagram of Fig. 1. At the present stage of calculation we neglect all gluon-exchange diagrams.

The basic formula for the + component of the hadronic matrix element, under the condition $Q^+=0$, is given in (B10). For $0^- \to 0^-$ transitions we can determine in this manner just the relevant form factor f_+. Using the representation (A1) or (A7) for the spin structure function $R(\lambda\bar{\lambda},00)$ the result for $Q^2 \le 0$ is

$$f_+(Q^2)=\frac{N_c}{(2\pi)^3}\int d^3p'\,\Omega(p'',p')\frac{1}{\xi(1-\xi)}\{\mathbf{p}'_\perp\cdot\mathbf{p}''_\perp+[\xi m_2+(1-\xi)m'_1][\xi m_2+(1-\xi)m''_1]\} \ , \tag{4.1}$$

where

$$\Omega(p'',p')\equiv \phi^\dagger_{M''}(p'')\phi_{M'}(p')\left[\frac{E''_1E''_2M'_0}{E'_1E'_2M''_0}\right]^{1/2} \times[M'^2_0-(m'_1-m_2)^2]^{-1/2} \times[M''^2_0-(m''_1-m_2)^2]^{-1/2} \ . \tag{4.2}$$

For $m'_1=m''_1$ the normalization condition for state vectors requires that $f_+(0)=1$. Since

$$\mathbf{p}'^2_\perp+[\xi m_2+(1-\xi)m'_1]^2=\xi(1-\xi)[M'^2_0-(m'_1-m_2)^2]$$

the normalization condition of the form factor is identical with the normalization condition for the wave function (2.15). For $m'_1=m''_1=m_2$ Eq. (4.1) agrees with the expression for the charge form factor of the pion.[14]

For $0^- \to 1^-$ transitions the polarization vectors for a vector meson of mass M with light-front momentum $\vec{\mathbf{P}}=(P^+,\mathbf{P}_\perp)$ have to be known. Using the transformation properties of a four-vector $A=(A^-,A^+,\mathbf{A}_\perp)$ under the kinematic Lorentz transformation (2.8) we find

$$\epsilon(\pm1)=\hat{\epsilon}(\pm1) \ , \qquad \epsilon(0)=\frac{1}{M}\left[\frac{-M^2+P_\perp^2}{P^+},P^+,\mathbf{P}_\perp\right] \ , \tag{4.3}$$

where $\hat{\epsilon}$ is given in Eq. (A9). The polarization vectors are seen to have the standard form in the rest system of the meson.

The transverse decay mode determines the vector form factor g of Eq. (3.2b). Again using Eqs. (B10) and the spin structure functions given in Eqs. (A1), (A2), or (A7), (A8) leads to (for $Q^2 \le 0$)

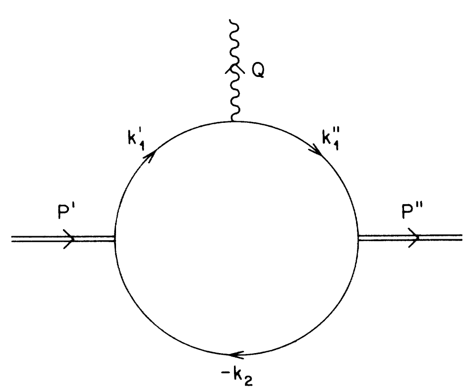

FIG. 1. One-loop Feynman diagram, which represents the transition of the meson state with four-momenta P', mass M' to the meson state with four-momentum P'', mass M''. $Q=P'-P''$.

$$g(Q^2)=-\frac{N_c}{(2\pi)^3}\int d^3p'\Omega(p'',p')\frac{1}{\xi}\left[\xi m_2+(1-\xi)m_1'+\frac{m_1'-m_1''}{Q^2}\mathbf{p}_\perp'\cdot\mathbf{Q}_\perp+\frac{2}{M_0''+m_1''+m_2}\left[p_\perp'^2+\frac{(\mathbf{p}_\perp'\cdot\mathbf{Q}_\perp)^2}{Q^2}\right]\right]. \tag{4.4}$$

Similarly, the transverse decay mode determines the axial-vector form factor a_+ [of Eq. (3.2b)]:

$$a_+(Q^2)=\frac{N_c}{(2\pi)^3}\int d^3p'\Omega(p'',p')\frac{2}{\xi}\left[(\xi-\tfrac{1}{2})[\xi m_2+(1-\xi)m_1']-\frac{\mathbf{p}_\perp'\cdot\mathbf{Q}_\perp}{2Q^2}[2\xi m_2+m_1''+(1-2\xi)m_1']\right.$$
$$\left.-\frac{\dfrac{\mathbf{p}_\perp'\cdot\mathbf{Q}_\perp}{(1-\xi)Q^2}+1}{M_0''+m_1''+m_2}\{\mathbf{p}_\perp'\cdot\mathbf{p}_\perp''+[\xi m_2+(1-\xi)m_1'][\xi m_2-(1-\xi)m_1'']\}\right] \tag{4.5}$$

and the longitudinal mode determines the following combination of axial-vector form factors:

$$\frac{1}{2M''}[f(Q^2)-(M''^2-M'^2+Q^2)a_+(Q^2)]=-A_0(Q^2)\,, \tag{4.6a}$$

$$A_0(Q^2)=\frac{N_c}{(2\pi)^3}\int d^3p'\Omega(p'',p')\frac{1}{\xi}\left[2\xi M_0''[\xi m_2+(1-\xi)m_1']\right.$$
$$\left.+\frac{(2\xi-1)M_0''+m_1''-m_2}{(1-\xi)(M_0''+m_1''+m_2)}\{\mathbf{p}_\perp'\cdot\mathbf{p}_\perp''+[\xi m_2+(1-\xi)m_1'][\xi m_2-(1-\xi)m_1'']\}\right]. \tag{4.6b}$$

The expressions for the form factors given in Eqs. (4.1)–(4.6) are valid only for $Q^2\le 0$, while for semileptonic decays the physical values of Q^2 lie in the range $0\le Q^2\le (M'-M'')^2$. We now make the crucial assumption that the form factors are continuously differentiable functions of Q^2, and use the derivatives at $Q^2=0$ to continue the form factors for spacelike Q to timelike Q. Since we need to know the form factors only in the environment of $Q^2=0$, a simple approximation procedure proved to be quite efficient. A form factor $F(Q^2)$ is parametrized in the following way:

$$F(Q^2)\simeq\frac{F(0)}{1-Q^2/\Lambda_1^2+Q^4/\Lambda_2^4-s_3Q^6/\Lambda_3^6}\,, \tag{4.7}$$

where $s_3=\pm 1$ and the parameters Λ_n are determined by the calculation of the appropriate derivatives of $F(Q^2)$ at $Q^2=0$, using the formulas (4.1)–(4.6). The range of validity of Eq. (4.7) can be determined for $Q^2\le 0$ by comparison with the exact expressions, and it is expected that the approximation is useful for a similar range of values $Q^2>0$ below the resonance region.

For the actual calculation of form factors meson wave functions that are solutions of the relativistic Schrödinger equation (2.13) would be ideally suited, but these are not readily available. However, in order to explore the quality and power of our approach we shall adopt a method proposed by GIW (Ref. 9), who used a nonrelativistic quark potential model which gave quite reasonable spin-averaged meson spectra. To avoid extensive numerical calculations variational solutions of this Schrödinger problem, based on harmonic-oscillator wave functions, are used. For S-state mesons the wave function $\phi(p)$ defined in Eq. (2.14) is therefore approximated by

$$\phi(p)=\pi^{-3/4}\beta^{-3/2}\left[\frac{(2\pi)^3}{3}\right]^{1/2}\exp(-\mathbf{p}^2/2\beta^2) \tag{4.8}$$

in which the β's are employed as variational parameters. The values for constituent quark masses and parameters β used by GIW are given in Table I.

For an easy comparison with the work of BSW (Ref. 11) we rewrite the form factors defined in Eqs. (3.2) in the following way:

$$\begin{aligned}V(Q^2)&=-(M'+M'')g(Q^2)\,,\\ A_1(Q^2)&=-(M'+M'')^{-1}f(Q^2)\,,\\ A_2(Q^2)&=(M'+M'')a_+(Q^2)\,,\\ A_0(Q^2)&=\frac{1}{2M''}\left[(M'+M'')A_1(Q^2)+\frac{M''^2-M'^2+Q^2}{M'+M''}A_2(Q^2)\right].\end{aligned} \tag{4.9}$$

The form factors have been calculated with Eqs. (4.1),

TABLE I. Table of the parameters of the constituent quark model (quark masses and harmonic-oscillator parameter β). Parameter set 1 from Ref. 9, parameter set 2 from Ref. 6.

Meson flavor	$u\bar{q}$	$u\bar{d}$	$u\bar{s}$	$u\bar{c}$	$u\bar{b}$
Set 1	m_q (GeV)	0.33	0.55	1.82	5.12
	β (GeV)	0.31	0.34	0.39	0.41
Set 2	m_q (GeV)	0.29	0.42	1.6	4.9
	β (GeV)	0.2905	0.3575	0.368	0.386

TABLE II. Table of the parameters of the various form factors for $D\to K,K^*$ transitions for the two parameter sets of Table I. Also shown are the overlap factors $F(0)$ of BSW (Ref. 11).

	F	f_+	V	A_2	A_0
Set 1	$F(0)$	0.73	0.79	0.44	0.74
	Λ_1 (GeV)	2.01	2.00	2.29	2.26
	Λ_2 (GeV)	2.80	2.80	3.11	3.15
	$s_3\Lambda_3$ (GeV)	4.16	−3.92	−4.35	−4.33
Set 2	$F(0)$	0.74	0.92	0.42	0.74
	Λ_1 (GeV)	1.87	1.86	2.34	2.19
	Λ_2 (GeV)	2.69	2.68	3.01	3.09
	$s_3\Lambda_3$ (GeV)	3.35	−3.97	4.11	−5.06
BSW	$F(0)$	0.76	1.27	1.15	0.73

TABLE III. Table of the parameters of the various form factors for $B\to D,D^*$ transitions for the two parameter sets of Table I. Also shown are the overlap factors $F(0)$ of BSW (Ref. 11).

	F	f_+	V	A_2	A_0
Set 1	$F(0)$	0.67	0.69	0.56	0.69
	Λ_1 (GeV)	4.84	4.84	5.03	5.03
	Λ_2 (GeV)	6.52	6.52	6.76	6.77
	$s_3\Lambda_3$ (GeV)	8.75	−8.46	−8.78	−9.03
Set 2	$F(0)$	0.64	0.71	0.58	0.66
	Λ_1 (GeV)	4.61	4.61	4.81	4.80
	Λ_2 (GeV)	6.26	6.27	6.51	6.51
	$s_3\Lambda_3$ (GeV)	8.48	−7.63	−8.16	−8.81
BSW	$F(0)$	0.69	0.71	0.69	0.62

and (4.4)–(4.6) with the GIW parameters of Table I. The results in terms of the representation (4.7) have been collected in Tables II and III.

We have also taken into consideration the quark masses given by Godfrey and Isgur[6] (GI). Since appropriate approximations for the wave functions of the GI model are not available we have again used the harmonic-oscillator wave functions of Eq. (4.8) and determined values for the parameter β by means of two different methods. (a) With the values given in Table I for β_π and β_K the predictions of our model for the weak and electromagnetic properties of the light mesons π,ρ,K,K^* are in excellent agreement with experiment. This agreement is only possible if the mass of the lightest quark is larger than the value given by GI (Ref. 6), and is impossible for the parameter set of GIW (Ref. 9). (b) A similar comparison with experimental data for D and B mesons is not possible. Therefore we used the procedure of GIW (Ref. 9) and calculated variational solutions on the basis of the new masses to determine at least approximate values for β_D and β_K. The results are given in Tables I–III.

For comparison we have also included in Tables II and III the results of BSW (Ref. 11) of the calculation of form factors at $Q^2=0$ (overlap factors). The overlap factors $V(0)$ and $A_2(0)$ are calculated in the infinite-momentum frame[11] by taking matrix elements of "bad" operators. Their determination in the BSW approach is considered by Bauer and Wirbel[11] to be uncertain, and this possibly explains the discrepancy with the results of our model, where all form factors are uniquely determined by the underlying constituent quark model.

Finally we note that the results quoted in Tables II and III suggest that the form factor of Eq. (4.7) can be approximated roughly in terms of only Λ_2 by means of a dipole form factor

$$F(Q^2)\simeq\frac{F(0)}{(1-Q^2/\Lambda_2^2)^2}\ .$$

Dipole form factors have been used in the approach of Ref. 12.

V. RESULTS AND CONCLUSIONS

The decay rates for D and B decays can be determined by means of Eq. (3.4) together with Eqs. (3.6) and (3.7). The numerical results given in Tables IV and V have been obtained using the parametrization (4.7) for the form factors. However, the quoted numerical results do not change if s_3 is put equal to zero in Eq. (4.7), which means that it is sufficient to approximate the transition form factors in terms of only Λ_1,Λ_2.

The rates do not depend very sensitively upon the parameters of the constituent quark model (quark masses and values for β), as the comparison between the results for the two sets of parameters already indicates. Nevertheless, more reliable parameters are needed.

In Tables IV and V we also quote results for decay

TABLE IV. Table of semileptonic decay rates for $D\to K,K^*$ transitions. Listed are total decay rates Γ, transverse and longitudinal decay rates Γ_T and Γ_L, the ratios Γ_L/Γ_T and $R=\Gamma(D-K^*)/\Gamma(D-K)$. Results are quoted for the light-front formalism with two sets of parameters (see Table I), for the approaches of GISM (Refs. 9 and 21) and of BSW (Ref. 11).

	$\Gamma(D\to K)$	$\Gamma(D\to K^*)$ ($\lvert V_{cs}\rvert^2\times10^{11}$ s^{-1})	Γ_T	Γ_L	Γ_L/Γ_T	R
Set 1	0.81	0.57	0.24	0.33	1.36	0.70
Set 2	0.87	0.58	0.25	0.33	1.34	0.67
GISW		0.96	0.46	0.50	1.09	
BSW	0.83	0.95	0.50	0.45	0.90	1.15
Expt.					$2.4^{+1.7}_{-0.9}\pm0.2$	$0.45\pm0.09\pm0.7$

TABLE V. Table of semileptonic decay rates for $B \to D, D^*$ transitions. Listed are total decay rates Γ, transverse and longitudinal decay rates Γ_T and Γ_L, the ratios Γ_L/Γ_T and $R=\Gamma(B\to D^*)/\Gamma(B\to D)$. Results are quoted for the light-front formalism with two sets of parameters (see Table I), for the approaches of GISW (Refs. 9 and 21) and of BSW (Ref. 11).

	$\Gamma(B\to D)$ ($\lvert V_{bc}\rvert^2\times 10^{13}$ s^{-1})	$\Gamma(B\to D^*)$	Γ_T	Γ_L	Γ_L/Γ_T	R
Set 1	0.88	2.17	1.00	1.17	1.17	2.48
Set 2	0.83	2.17	1.03	1.14	1.11	2.61
GISW	1.11	2.52	1.28	1.24	0.97	2.27
BSW	0.81	2.17	1.05	1.12	1.07	2.69
Expt.					0.85±0.43	$3.3^{+3.7}_{-1.1}$

rates, obtained in a nonrelativistic treatment by GISW (Refs. 9 and 21), but a fair comparison is possible only between our results and those of BSW (Ref. 11). Both models predict similar rates for B decays, while the results for D decays are quite different, which is mainly due to the fact that the overlap factors $V(0)$ and $A_2(0)$ cannot be determined reliably in the BSW approach, and disagree most for D decay (see Table II).

Finally, we compare our results for decay rates with the experimental data. While we find reasonable agreement for B decays, the quark model predictions seem to be in conflict with the data for D decays. Though the light-front formalism predicts decay rates which are close to the experimental data for D decays, even drastic variations of the parameters of the constituent quark model do not improve the predictions for the ratios Γ_L/Γ_T and R given in Table IV. For illustration we have quoted results for D decays in Table VI which have been calculated with the second set of parameters of Table I, but admitting a large range of variation for the harmonic-oscillator parameter β_D. The results listed in Table VII have been calculated in the same way, except that we have put $m_c=1.4$ GeV.

A similar conclusion has been reached by Scora and Isgur,[21] who state that it appears difficult for quark potential models to produce ratios Γ_L/Γ_T for D and B decays very different from the value $\frac{5}{4}$ of the Shifman-Voloshin limit.

In spite of these uncertainties one can attempt to derive values for the KM matrix elements. For a variation of β_D within the range 0.36 GeV $\le \beta_D \le 0.46$ GeV we find from Table VI a rate

$$\Gamma(D\to K)=(0.87\pm0.01)\lvert V_{cs}\rvert^2\times 10^{11}\ \mathrm{s}^{-1}\ .$$

We can neglect the small theoretical error, due to the uncertainty of the wave function, in the following. Taking the measured value from the E691 experiment,[2]

$$\Gamma(D^0\to K^-)=(0.91\pm0.11\pm0.14)\times 10^{11}\ \mathrm{s}^{-1}\ ,$$

we find

$$\lvert V_{cs}\rvert=1.02\pm0.06\pm0.08\ ,$$

which is consistent with the value $\lvert V_{cs}\rvert=0.975$, obtained by assuming three families and unitarity of the KM matrix.

Similar arguments give a rate

$$\Gamma(B\to D)=(0.86\pm0.04)\lvert V_{bc}\rvert^2\times 10^{13}\ \mathrm{s}^{-1}\ ,$$

where the error accounts for a variation of β values within the range 0.36 GeV $\le \beta_D, \beta_B \le 0.46$ GeV. Again, it can be neglected in the following. Taking the measured value from the ARGUS Collaboration,[3]

$$\Gamma(B^0\to D)=(0.0016\pm0.0007)\times 10^{13}\ \mathrm{s}^{-1}\ ,$$

we find

$$\lvert V_{bc}\rvert=0.043\pm0.009\ .$$

We wish now to discuss possible sources of the disagreement for $D\to K^*$ transitions. The most obvious possibility is our method of approximating meson wave functions, which are solutions of wave equations (2.12) or (2.13), by harmonic-oscillator wave functions (4.8). However, we emphasize first that our model predicts weak and electromagnetic properties of the light mesons in agreement with experiment. While the values of the harmonic-oscillator parameter β for D and B mesons cannot be constrained as yet, we emphasize next, that the decay rates are not very sensitive to variations of β_D, as an inspection of Table VI shows. The ratios are more sensi-

TABLE VI. Table of semileptonic decay rates for $D\to K, K^*$ transitions for different values of β_D in GeV. Symbols and units are the same as in Table IV. The calculation has been performed for the parameter set 2 of Table I.

β_D	0.33	0.35	0.37	0.39	0.41	0.43	0.45	0.47	0.49
$\Gamma(D\to K)$	0.83	0.85	0.87	0.88	0.88	0.88	0.87	0.86	0.85
$\Gamma(D\to K^*)$	0.55	0.57	0.58	0.59	0.59	0.59	0.59	0.59	0.58
R	0.66	0.67	0.67	0.67	0.67	0.68	0.68	0.68	0.69
Γ_L/Γ_T	1.35	1.34	1.34	1.33	1.33	1.33	1.32	1.32	1.31

TABLE VII. Same as Table VI, but with $m_c = 1.4$ GeV.

β_D	0.33	0.35	0.37	0.39	0.41	0.43	0.45	0.47	0.49
$\Gamma(D\to K)$	1.02	1.04	1.06	1.07	1.07	1.07	1.06	1.04	1.02
$\Gamma(D\to K^*)$	0.63	0.65	0.67	0.68	0.68	0.68	0.68	0.67	0.66
R	0.62	0.63	0.63	0.63	0.63	0.64	0.64	0.64	0.65
Γ_L/Γ_T	1.29	1.28	1.28	1.27	1.26	1.26	1.25	1.25	1.24

tive to variations of β_K, but this parameter was fixed in the evaluation of K properties. Therefore, it seems that the simplified treatment of the meson wave function does not create serious problems.

Since the light-front formalism used in this work permits, in principle, a systematic and consistent derivation of the predictions of the constituent quark model, it may eventually be necessary to face the possibility that the constituent quarks, which are dressed quarks, are more complex than what we assumed in the beginning, namely, massive, but structureless quarks. An analogous calculation of form factors of quark model nucleons seems to point to a definite structure of constituent quarks.[22]

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APPENDIX A: SPIN STRUCTURE FUNCTIONS FOR MESONS

The spin structure function $R(\lambda\bar{\lambda},JJ_3)$ for S-state mesons has been defined in Eq. (2.14) and the matrix element of the Melosh rotation operator $\mathcal{R}_M(\mathbf{p},m)$ has been given in Eq. (2.11). For pseudoscalar mesons one finds

$$R(\lambda\bar{\lambda},00)=\frac{1}{\sqrt{2\xi(1-\xi)}[M_0^2-(m_1-m_2)^2]^{1/2}}\times\chi_\lambda^\dagger\{[\xi m_2+(1-\xi)m_1]i\sigma_2-p_1+i\sigma_3 p_2\}\chi_{\bar{\lambda}}\ . \quad \text{(A1)}$$

For vector mesons one finds

$$R(\lambda\bar{\lambda},11)=\frac{1}{2\sqrt{\xi(1-\xi)}[M_0^2-(m_1-m_2)^2]^{1/2}}\chi_\lambda^\dagger\Bigg[[\xi m_2+(1-\xi)m_1](1+\sigma_3)+(p_1+ip_2)[i\sigma_2+(2\xi-1)\sigma_1] + \frac{2(p_1+ip_2)}{M_0+m_1+m_2}\{p_1\sigma_3-ip_2-[\xi m_2-(1-\xi)m_1]\sigma_1\}\Bigg]\chi_{\bar{\lambda}}\ , \quad \text{(A2)}$$

$$R(\lambda\bar{\lambda},10)=\frac{1}{\sqrt{2\xi(1-\xi)}[M_0^2-(m_1-m_2)^2]^{1/2}}\chi_\lambda^\dagger\Bigg[[\xi m_2+(1-\xi)m_1]\sigma_1+(1-2\xi)(p_1\sigma_3-ip_2) + \frac{2}{M_0+m_1+m_2}\{p_\perp^2\sigma_1+[\xi m_2-(1-\xi)m_1](p_1\sigma_3-ip_2)\}\Bigg]\chi_{\bar{\lambda}}\ . \quad \text{(A3)}$$

The internal momentum $\mathbf{p}_\perp=(p_1,p_2)$ is defined by Eq. (2.3) and the Pauli spinors are given by

$$\chi_{1/2}=\begin{bmatrix}1\\0\end{bmatrix},\quad \chi_{-1/2}=\begin{bmatrix}0\\1\end{bmatrix}.$$

It is instructive to use the appropriate basis of Dirac spinors:

$$u(p,\lambda)=\frac{1}{\sqrt{p^+}}(\not p+m)u(\lambda)\ ,\qquad v(p,\lambda)=\frac{1}{\sqrt{p^+}}(\not p-m)v(\lambda)\ , \quad \text{(A4a)}$$

$$u(\tfrac{1}{2})=\begin{bmatrix}1\\0\\0\\0\end{bmatrix},\quad u(-\tfrac{1}{2})=\begin{bmatrix}0\\0\\0\\1\end{bmatrix}, \quad \text{(A4b)}$$

and $v(\lambda)=u(-\lambda)$. In this basis the γ matrices are represented by

$$\gamma^0=\begin{bmatrix}0&1\\1&0\end{bmatrix},\quad \boldsymbol{\gamma}=\begin{bmatrix}0&\boldsymbol{\sigma}\\-\boldsymbol{\sigma}&0\end{bmatrix}. \quad \text{(A5)}$$

The normalization is $\bar{u}(p,\lambda)u(p,\lambda)=-\bar{v}(p,\lambda)v(p,\lambda)=2m$. The Dirac spinors of Eq. (A4) are independent of p^- since

$$\not{p}=\tfrac{1}{2}p^{+}\gamma^{-}+\tfrac{1}{2}p^{-}\gamma^{+}-\gamma_{\perp}\cdot\mathbf{p}_{\perp}\ ,$$
$$\gamma^{+}=\gamma^{0}+\gamma^{3},\quad \gamma^{-}=\gamma^{0}-\gamma^{3}\ , \tag{A6}$$

and $\gamma^{+}u(\lambda)=0$. In this basis the spin structure functions can be represented in the following way:

$$R(\lambda\bar{\lambda},00)=\frac{1}{\sqrt{2}[M_0^2-(m_1-m_2)^2]^{1/2}}\bar{u}(k_1,\lambda)\times\gamma_5 v(k_2,\bar{\lambda})\ , \tag{A7}$$

where $\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3$, and

$$R(\lambda\bar{\lambda},1J_3)=\frac{-1}{\sqrt{2}[M_0^2-(m_1-m_2)^2]^{1/2}}\hat{\epsilon}_\mu(J_3)\times\bar{u}(k_1,\lambda)\left[\gamma^\mu-\frac{(k_1-k_2)^\mu}{M_0+m_1+m_2}\right]v(k_2,\bar{\lambda})\ . \tag{A8}$$

The four-vectors k_1,k_2 are given by Eq. (2.3) and by $k_1^2=m_1^2, k_2^2=m_2^2$. The polarization vectors are given by

$$\hat{\epsilon}(\pm1)=\left[\frac{2}{P^+}\boldsymbol{\epsilon}_\perp(\pm1)\cdot\mathbf{P}_\perp,0,\boldsymbol{\epsilon}_\perp(\pm1)\right]\ ,$$
$$\boldsymbol{\epsilon}_\perp(\pm1)=\mp(1,\pm i)/\sqrt{2}\ , \tag{A9}$$
$$\hat{\epsilon}(0)=\frac{1}{M_0}\left[\frac{-M_0^2+P_\perp^2}{P^+},P^+,\mathbf{P}_\perp\right]\ ,$$

where we have used the transformation properties of a four-vector $A=(A^-,A^+,\mathbf{A}_\perp)$ under kinematic Lorentz transformations as given in Eq. (2.8).

APPENDIX B: QUASIPOTENTIAL FORMALISM AND COVARIANT LIGHT-FRONT PERTURBATION THEORY

The diagrammatic approach to light-front perturbation theory is well known.[16,20] We present some of the details that may be helpful for practical calculations.

The starting point is the Bethe-Salpeter equation for the $q\bar{q}$ bound state vertex function Γ which is, in symbolic operator notation,

$$\Gamma=UG\Gamma\ , \tag{B1}$$

where U is the irreducible $q\bar{q}$ kernel and has a meaning only in the context of a field-theoretic description of the $q\bar{q}$ interaction. The Green's function G for two free quarks is well known. In order to apply Eq. (B1) to bound-state problems in light-front dynamics, which are described by a wave function at equal "time" x^+, the quasipotential method has been developed (e.g., in Ref. 23), in which the four-dimensional Eq. (B1) is reduced to a three-dimensional equation by using a new Green's function g, which restricts the component p^- of the internal momentum to a fixed value. Equation (B1) is then replaced by

$$\Gamma=Wg\Gamma \quad\text{with}\quad W=U+U(G-g)W\ . \tag{B2}$$

Among the many possible choices for a g we select the one that puts one of the constituents on its mass shell. For a $q\bar{q}$ bound state with total momentum P, whose constituents have momenta k_1 and k_2, we start with the general, covariant expression for g:

$$g(P,\mathbf{p})=2\pi i\int ds\frac{1}{P^2-s}\delta^+(k_1^2-m_1^2)\times\delta^+(k_2^2-m_2^2)(\not{k}_1+m_1)(-\not{k}_2+m_2)\ , \tag{B3a}$$

where $s=(k_1+k_2)^2$ and k_1,k_2 are restricted by $k_1^+\geq0, k_2^+\geq0$. The choice of appropriate light-front variables leads to

$$g(P,\mathbf{p})=2\pi i\delta(k_2^2-m_2^2)\theta(1-\xi)\theta(\xi)\frac{\Lambda^+(k_1)\Lambda^-(k_2)}{\xi(P^2-M_0^2)} \tag{B3b}$$

and the spin projection operators are

$$\Lambda^+(k_1)=\sum_\lambda u(k_1,\lambda)\bar{u}(k_1,\lambda)\ ,$$
$$\Lambda^-(k_2)=-\sum_{\bar{\lambda}} v(k_2,\bar{\lambda})\bar{v}(k_2,\bar{\lambda})\ . \tag{B4}$$

The relations between k_1,k_2 and $M_0,\xi,\mathbf{p}$ have been given in Eqs. (2.3)–(2.6). An explicit basis of Dirac spinors has been written down in Eq. (A4).

We now attach appropriate spinors to the operators $\Gamma=\Gamma(k_1,k_2)$, $W=W(k_1,k_2;k_1',k_2')$, and define

$$\hat{g}(\mathbf{p})=\frac{1}{P^2-M_0^2}\ ,$$
$$\hat{\Gamma}(\mathbf{p},\lambda\bar{\lambda})=\left[\frac{M_0}{2E_1E_2}\right]^{1/2}\bar{u}(k_1,\lambda)\Gamma v(k_2,\bar{\lambda})\ , \tag{B5}$$
$$\hat{W}(\mathbf{p},\mathbf{p}';\lambda\bar{\lambda},\lambda'\bar{\lambda}')=\left[\frac{M_0'}{2E_1'E_2'}\right]^{1/2}\bar{u}(k_1',\lambda')\bar{v}(k_2,\bar{\lambda})\times Wu(k_1,\lambda)v(k_2',\bar{\lambda}')\left[\frac{M_0}{2E_1E_2}\right]^{1/2}.$$

The full statement of Eq. (B2) is

$$\hat{\Gamma}(\mathbf{p},\lambda\bar{\lambda})=(2\pi)^{-3}\int d^3p'\hat{W}(\mathbf{p},\mathbf{p}';\lambda\bar{\lambda},\lambda'\bar{\lambda}')\times\hat{g}(\mathbf{p}')\hat{\Gamma}(\mathbf{p}',\lambda'\bar{\lambda}')\ . \tag{B6}$$

Here it has been used that Poincaré invariance requires the interaction term $\hat{W}$ and, consequently, the vertex function $\hat{\Gamma}$ to be independent of the total light-front momentum $\vec{\mathbf{P}}$, to write $\hat{W}$ and $\hat{\Gamma}$ in terms of internal variables $\mathbf{p},\mathbf{p}'$. Now we define a wave function ψ in terms of

$$\hat{\Gamma}(\mathbf{p},\lambda\bar{\lambda})=(P^2-M_0^2)\psi(\mathbf{p},\lambda\bar{\lambda})\ . \tag{B7}$$

Using this definition and Eq. (2.4a), Eq. (B6) becomes the wave equation (2.12).

Again starting from the Bethe-Salpeter equation the normalization of the wave function is determined[20] by Eq. (2.15), which is consistent with the normalization of the charge form factor [see the discussion following Eq. (4.1)].

The quasipotential formalism provides in principle a framework[23] for a systematic treatment of higher-order gluon exchange. We shall limit ourselves in this work to the one-loop approximation and shall derive explicit expressions for matrix elements of currents between meson states, which correspond to the Feynman diagram of Fig. 1. For definiteness, let us consider the matrix element of the vector current, which couples to the quarks of the quark-antiquark states:

$$\langle \vec{P}'',J''J_3''|\bar{q}''\gamma_\mu q'|\vec{P}',J'J_3'\rangle\sqrt{4P'^+P''^+}\equiv M_\mu \ .$$

Single-meson states are normalized according to

$$\langle \vec{P}'',J''J_3''|\vec{P}',J'J_3'\rangle=\delta_{J''J'}\delta_{J_3''J_3'}\delta(\mathbf{P}_\perp''-\mathbf{P}_\perp')\delta(P''^+-P'^+)$$

and instant form and light-front form states are connected by

$$|\vec{P},JJ_3\rangle\sqrt{P^+}=|P,JJ_3\rangle\sqrt{P_0} \ .$$

In order to fix our notation we write the expression for M_μ in full, first in terms of the instant form momenta:

$$M_\mu=\frac{iN_c}{(2\pi)^4}\int d^4k'\mathrm{Tr}\left[\Gamma(k_1'',k_2'')\frac{\not{k}_1''+m_1}{k_1''^2-m_1^2+i\epsilon}\times\gamma_\mu\frac{\not{k}_1'+m_1}{k_1'^2-m_1^2+i\epsilon}\times\Gamma(k_1',k_2')\frac{-\not{k}_2'+m_2}{k_2'^2-m_2^2+i\epsilon}\right] . \tag{B8}$$

The momenta are given by

$$k_1'=\tfrac{1}{2}P'+k',\quad k_2'=\tfrac{1}{2}P'-k' \ ,$$

$$k_1''=\tfrac{1}{2}P''+k'',\quad k_2''=\tfrac{1}{2}P''-k'' \ ,$$

$$k''=k'-\tfrac{1}{2}Q,\quad Q=P'-P'' \ .$$

Light-front momenta can be defined in the usual way. The crucial step to generate the covariant perturbation theoretic expression of the light-front formalism is the assumption that the vertex operator Γ is independent of the component k'^-. The integral (B8) may be evaluated by contour methods in the k'^- plane. In general the correct $q\bar{q}$ structure of the vertex is maintained only in the component M^+ of the matrix element M_μ. This can be seen easily if, using the Dirac spinor basis (A4), the quark and antiquark propagators are decomposed as[20]

$$\frac{\pm\not{p}+m}{p^2-m^2}=\frac{\Lambda^\pm(p)}{p^2-m^2}+\frac{\gamma^+}{2p^+} \ , \tag{B9}$$

where the spin projections operators are given in Eq. (B4). Since $(\gamma^+)^2=0$ the second term in Eq. (B9) gives no contribution to the component M^+, if furthermore $Q^+=0$. If the contour is closed in the upper k'^- plane, the condition $Q^+=0$ ensures that M^+ is given exactly by the residue of the second quark pole, at the position $k_2'^2=m_2^2$.

Under the conditions discussed above the contribution of the one-loop diagram of Fig. 1 can be calculated exactly in the quasipotential formalism in light-front form.

We use Eqs. (B5) and (B7) to replace vertex functions by wave functions, and arrive at the result

$$M^+=\frac{N_c}{(2\pi)^3}\int d^3p'\frac{1}{\xi}\left[\frac{E_1''E_2''M_0'}{E_1'E_2'M_0''}\right]^{1/2}\sum_{\lambda'\lambda''\bar{\lambda}}\psi^\dagger(\mathbf{p}'',\lambda''\bar{\lambda},J''J_3'')\bar{u}(k_1'',\lambda'')\gamma^+u(k_1',\lambda')\psi(\mathbf{p}',\lambda'\bar{\lambda},J'J_3') \ . \tag{B10}$$

All double primed variables in Eq. (B10) can be expressed in terms of the integration variables $\mathbf{p}'=(p_1',p_2',p_3')$ and $\mathbf{Q}_\perp$:

$$(k_1'')^+=\xi P'^+,\quad (\mathbf{k}_1'')_\perp=(\mathbf{k}_1')_\perp-\mathbf{Q}_\perp \ ,$$

$$\mathbf{p}_\perp''=\mathbf{p}_\perp'-(1-\xi)\mathbf{Q}_\perp \ ,$$

$$p_3''=(\xi-\tfrac{1}{2})M_0''-(m_1''^2-m_2^2)/2M_0'' \ .$$

The quantity M_0'' can be evaluated in terms of $p_\perp''$ by means of Eq. (2.4b).

For the calculation of matrix elements of the charged weak current between meson states on the basis of Eq. (B.10) we need to know the relevant matrix elements of the current of pointlike quarks:

$$\bar{u}(k_1'',\lambda'')\gamma^+u(k_1',\lambda')=2\xi P'^+\delta_{\lambda'\lambda''} \ , \tag{B11}$$

$$\bar{u}(k_1'',\lambda'')\gamma^+\gamma_5u(k_1',\lambda')=2\xi P'^+\chi_{\lambda''}^\dagger\sigma_3\chi_{\lambda'} \ , \tag{B12}$$

where $\gamma_5=i\gamma^0\gamma^1\gamma^2\gamma^3$ and the representation (A4) and (A5) has been used.

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