Analysis of scaled-factorial-moment data

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We discuss the two standard constructions used in the search for intermittency, the exclusive and inclusive scaled factorial moments. We propose the use of a new scaled factorial moment that reduces to the exclusive moment in the appropriate limit and is free of undesirable multiplicity correlations that are contained in the inclusive moment. We show that there are some similarities among most of the models that have been proposed to explain factorial-moment data, and that these similarities can be used to increase the efficiency of testing these models. We begin by calculating factorial moments from a simple independent-cluster model that assumes only approximate boost invariance of the cluster rapidity distribution and an approximate relation among the moments of the cluster multiplicity distribution. We find two scaling laws that are essentially model independent. The first scaling law relates the moments to each other with a simple formula, indicating that the different factorial moments are not independent. The second scaling law relates samples with different rapidity densities. We find evidence for much larger clusters in heavy-ion data than in light-ion data, indicating possible *spatial* intermittency in the heavy-ion events.

I. INTRODUCTION

Recently there has been considerable interest in the idea of intermittency, or self-similar behavior, in high-energy nuclear collisions.[1–17] This phenomenon has been predicted to occur as a result of the transition from quark-gluon plasma to normal hadronic matter.[1] Data currently exist for π-p and K-p collisions,[2] p-emulsion and O-emulsion collisions,[3] and O-emulsion and S-Au collisions.[4] In this paper, we discuss and improve the current methodology[5,6] used in the search for intermittency. We also discuss similarities among most of the models[7–17] which have been proposed to account for the data.

In Sec. II, we begin by discussing the two standard constructions used in the search for intermittency, the exclusive and inclusive scaled factorial moments. We point out that the inclusive moment does not reduce to the exclusive moment when events with a fixed window multiplicity are considered. We propose the use of a new scaled factorial moment that reduces to the exclusive moment in the appropriate limit. This new moment is also free of undesirable multiplicity correlations that are contained in the inclusive moment.

There have been a large number of models proposed to explain this data.[7–17] We show that there are some similarities among most of the models, and that these similarities can be used to increase the efficiency of testing these models. We begin by calculating factorial moments from a simple independent-cluster model, which assumes only approximate boost invariance of the cluster rapidity distribution and an approximate relation among the moments of the cluster multiplicity distribution. In Sec. III we describe our model, introduce the cluster-model formalism, and calculate the single-particle rapidity density so that we can reduce the number of unknown quantities in our model.

In Sec. IV we calculate the scaled factorial moments from our model. We find two scaling laws that are essentially model independent. These laws hold in almost every model of particle collisions that reproduces existing rapidity correlation data, as most models assume that particle production occurs via some boost-invariant assortment of clusters. The first scaling law relates the moments to each other with a simple formula. The existence of this relation indicates that the different factorial moments are not independent. The best course for experimenters is then to measure the second moment, as this minimizes the statistical uncertainty. *Higher moments should then be compared to the second moment using the scaling law, in order to determine whether effects other than two-body interactions are significant.*

The second scaling law relates samples with different rapidity densities but similar cluster rapidity distributions. Using the second scaling law, we find evidence for much larger clusters in O-emulsion and S-Au collisions than in π-p and K-p collisions. The existence of these larger clusters suggests that *spatial* intermittency is actually stronger in heavy-ion collisions than in light-ion or leptonic collisions.

II. SCALED FACTORIAL MOMENTS

Intermittency in ultrarelativistic collisions is most cleanly observed using the exclusive scaled factorial moments[5]

$$F_i(M;\Delta)=\frac{1}{M}\sum_{m=1}^{M} M^i\frac{\langle k_m(k_m-1)\cdots(k_m-i+1)\rangle}{N(N-1)\cdots(N-i+1)}, \quad (1)$$

where a fixed rapidity window Δ is divided up into M equal intervals, k_m is the number of particles in the mth interval, and N is the number of particles in the rapidity window. If intermittency is presented in the rapidity distribution, the scaled factorial moments will exhibit a power-law divergence[5,18] in M as $M\rightarrow\infty$.

Equation (1) can be rewritten in more familiar notation as

$$F_i(M;\Delta)=M^{i-1}\sum_{m=1}^{M}\frac{\left[\prod_{n=1}^{i}\int_{y_{m-1}}^{y_m}dy_n\right]\rho_N^{(i)}(y_1,\ldots,y_i)}{\left[\prod_{n=1}^{i}\int_{y_0}^{y_0+\Delta Y}dy_n\right]\rho_N^{(i)}(y_1,\ldots,y_i)}, \tag{2}$$

where $Y_m=y_0+m\,\Delta Y/M$ for rapidity window $(y_0,y_0+\Delta Y)$, and $\rho_N^{(i)}$ is the i-particle rapidity density for events with N particles in the rapidity window. We simplify Eq. (2) for our calculations by using the fact that, in our model, all rapidity densities are approximately invariant under Lorentz boosts along the collision axis. We therefore construct

$$F_i(\delta y)=\frac{\left[\prod_{n=1}^{i}\int_{y}^{y+\delta y}\frac{dy_n}{\delta y}\right]\rho_N^{(i)}(y_1,\ldots,y_i)}{\left[\prod_{n=1}^{i}\int_{y_0}^{y_0+\Delta Y}\frac{dy_n}{\Delta Y}\right]\rho_N^{(i)}(y_1,\ldots,y_i)}, \tag{3}$$

where y is now any point in the middle of the rapidity window.

The exclusive moments (3) can be measured by using a rapidity window over which the particle distribution is "relatively flat." If all particles are uncorrelated, the factorial moments are given by

$$F_{i,\mathrm{unc}}(M;\Delta)=M^{i-1}\sum_{m=1}^{M}\left[\int_{y_{m-1}}^{y_m}dy\,\bar{\rho}_N(y)\right]^i, \tag{4}$$

where

$$\bar{\rho}_N(y)=\rho_N(y)/N \tag{5}$$

is the normalized rapidity distribution. The maximum value for $F_{i,\mathrm{unc}}(M;\Delta)$ is reached when $M\to\infty$, yielding

$$F_{i,\mathrm{unc}}^{(\max)}=\Delta Y^{i-1}\int_{y_0}^{y_0+\Delta Y}dy\,[\bar{\rho}_N(y)]^i. \tag{6}$$

If $F_{i,\mathrm{unc}}^{(\max)}$ is equal to 1 within experimental error, then any significant measured value of F_i cannot be due to the shape of the single-particle distribution. In this case, we say that the single-particle distribution inside the rapidity window is "relatively flat." It is possible to correct the factorial moments for the curvature of the single-particle distribution,[6] but we will not do this as the corrections complicate the calculation of the correlations.

Unfortunately, it is often hard to obtain a large sample of events to analyze, especially when rapidity cuts (to get a relatively flat distribution) and multiplicity cuts (to eliminate effects of multiplicity correlations) are performed. In order to analyze a sample containing events with a range of multiplicities, the common practice has been to use the inclusive scaled factorial moments[6]

$$F_i(M;\Delta)=\frac{1}{M}\sum_{m=1}^{M}M^i\frac{\langle k_m(k_m-1)\cdots(k_m-i+1)\rangle}{\langle N\rangle^i}, \tag{7}$$

where $\langle N\rangle$ is the mean number of particles in the rapidity window, averaged over the ensemble of events.

The inclusive moments are equal to the exclusive moments when the particle multiplicity follows a Poisson distribution. In general, however, the multiplicity does not follow a Poisson distribution, but follows some other distribution which depends on the cuts made on the event sample as well as physical parameters such as masses and energies of the colliding particles. In particular, if we take a sample of events with fixed window multiplicity N and calculate both the inclusive and exclusive moments, we find that

$$F_i^{(\mathrm{inclusive})}=\frac{N(N-1)\cdots(N-i+1)}{N^i}F_i^{(\mathrm{exclusive})}. \tag{8}$$

This difference between the inclusive and exclusive moments is similar to the "long-range" correlations, due only to multiplicity effects, that were observed in studies of two-particle rapidity correlations.[19]

We propose that, instead of constructing the inclusive moments for event samples with a range of multiplicities, experiments construct the scaled factorial moments

$$F_i(M;\Delta)=\frac{1}{M}\sum_{m=1}^{M}M^i\frac{\langle k_m(k_m-1)\cdots(k_m-i+1)\rangle}{\langle N(N-1)\cdots(N-i+1)\rangle}. \tag{9}$$

These moments contain information on rapidity correlations that are independent of multiplicity correlations. They also clearly reduce to the exclusive moments when constructed for event samples with fixed N.

The scaled factorial moments (9) can also be expressed in terms of i-particle rapidity densities:

$$F_i(M;\Delta)=\frac{M^{i-1}\sum_{m=1}^{M}\left[\prod_{n=1}^{i}\int_{y_{m-1}}^{y_m}dy_n\right]\rho^{(i)}(y_1,\ldots,y_i)}{\left[\prod_{n=1}^{i}\int_{y_0}^{y_0+\Delta Y}dy_n\right]\rho^{(i)}(y_1,\ldots,y_i)}. \tag{10}$$

Using the boost invariance of the rapidity densities, we obtain

$$F_i(\delta y;\Delta Y)=\frac{\left[\prod_{n=1}^{i}\int_{y}^{y+\delta y}\frac{dy_n}{\delta y}\right]\rho^{(i)}(y_1,\ldots,y_i)}{\left[\prod_{n=1}^{i}\int_{y_0}^{y_0+\Delta Y}\frac{dy_n}{\Delta Y}\right]\rho^{(i)}(y_1,\ldots,y_i)}, \tag{11}$$

where y is again some arbitrary point in the rapidity window. We will use Eq. (11) for our calculations.

III. CLUSTER MODEL

We begin by constructing a very general cluster model for high-energy collisions using two assumptions.

(1) During the collisions, clusters of hot matter form. These clusters are randomly distributed in rapidity, with a distribution function $\pi(y_c)$ which is nearly invariant under boosts along the beam direction.

(2) These clusters emit pions in a random manner, with distribution function $p(y-y_c)$.

The first assumption, which was introduced by Bjorken,[20] is widely believed to be true for ultrarelativistic collisions. The second assumption is almost certain to be realistic for large clusters, and is fairly accurate as a description of the decay of any cluster into more than two pions.

The formalism that we develop, based on these two assumptions, holds for a wide range of collision models. The "clusters" could be droplets of cold[7] or hot[8] quark-gluon plasma, hadronic resonances,[12] unspecified hadronic matter,[9–11] or partonic strings.[13–16] The clusters could even be jets;[17] the pion rapidity distribution would then depend on the angle of the jet axis, but many of the features of the cluster model would be unaffected by this change. In all of these proposed models of intermittent behavior,[7–17] the intermittency is generated by the appearance of some type of clusters of matter which have a roughly boost-invariant distribution.

The exact cluster rapidity distribution $\pi(y_c)$ is not crucial, as we only calculate quantities in the central rapidity region of a collision where the particle densities are flat. To simplify our calculations we use the flat cluster rapidity distribution

$$\pi(y_c)=\begin{cases}1/2L, & -L<y_c<L,\\ 0, & \text{otherwise.}\end{cases} \tag{12}$$

The number of clusters N_c is given by the distribution $\mathcal{P}(N_c)$. We will not specify this distribution; however, we assume that for $\langle N_c\rangle \gg 1$,

$$\langle N_c^i\rangle=\langle N_c\rangle^i+O(\langle N_c\rangle^{i-1}) , \tag{13}$$

where

$$\langle N_c^i\rangle=\sum_{N_c=1}^{\infty}\mathcal{P}(N_c)N_c^i . \tag{14}$$

The distribution function $\mathcal{P}(N_c)$ can depend on the projectile and target masses, on the center-of-momentum energy of the collision, and on cuts made on the event sample.

Most of our results are independent of the exact form of the pion rapidity distribution $p(y-y_c)$. However, the most likely distribution in high-energy collisions is

$$p(y)=\frac{1}{2\cosh^2(y)} \tag{15}$$

for pions produced in the rest frame of a cluster. This distribution, used in Refs. 21 and 8, results from isotropic emission of relativistic pions. We use $p(y)$ from Eq. (15) in order to estimate the cluster size required to reproduce the factorial-moment data.[8]

Finally, we assume that the number of pions per cluster n_π is given by the distribution function $P(n_\pi)$. We define

$$\langle n_\pi^i\rangle=\sum_{n_\pi=1}^{\infty}P(n_\pi)n_\pi^i , \tag{16}$$

as we did for the number of clusters. We assume that $P(n_\pi)$ can depend on projectile mass, target mass, and collision energy, but not on any cuts performed on the event sample.

We can now predict the scaled factorial moments as a function of the multiplicity and rapidity distributions for clusters and pions. However, this will not be very useful, as we have no general way of predicting the average cluster or droplet multiplicities, or the width of the cluster distribution. We determine these by first calculating the single-pion rapidity density and comparing to experimental results. We present a detailed calculation below in order to demonstrate the cluster formalism. We do not do this for other quantities because of the length of the equations.

The single-particle rapidity density is

$$\rho(y)=\sum_{N_c=1}^{\infty}\mathcal{P}(N_c)\prod_{N=1}^{N_c}\int_{-\infty}^{\infty}dy_N\pi(y_N)\sum_{n_N=1}^{\infty}P(n_N)\prod_{n=1}^{n_N}p(y_{N,n}-y_N)\left[\prod_{N'=1}^{N_c}\prod_{n'=1}^{n_{N'}}\int_{-\infty}^{\infty}dy_{N',n'}\right]\sum_{N=1}^{N_c}\sum_{n=1}^{n_N}\delta(y-y_{N,n}) . \tag{17}$$

The first half of Eq. (17) is the probability density for events with different numbers of clusters, varying cluster size and rapidity. The second half is an operator which is equal to the single-particle rapidity density for a fixed event, where the δ is the Dirac δ function.

We first simplify Eq. (17) by using the two normalization equations

$$\int_{-\infty}^{\infty}dx\,p(x)=1 , \tag{18}$$

$$\int_{-\infty}^{\infty}dx\,\pi(x)=1 . \tag{19}$$

After integrating over the δ function, we obtain

$$\rho(y)=\sum_{N_c=1}^{\infty}\mathcal{P}(N_c)\sum_{N=1}^{N_c}\sum_{n_N=1}^{\infty}P(n_N)\sum_{n=1}^{n_N}\int_{-\infty}^{\infty}dy_N\pi(y_N)p(y-y_N) . \tag{20}$$

We then use the definitions of $\langle N_c\rangle$ and $\langle n_\pi\rangle$, Eqs. (14) and (16), to obtain the single-particle density:

$$\rho(y)=\langle N_c\rangle\langle n_\pi\rangle\int_{-\infty}^{\infty}dx\,\pi(x)p(y-x) . \tag{21}$$

We can simplify Eq. (21) by taking advantage of the approximate boost invariance of our initial state. Since $\pi(y_c)$ is constant over a distance $2L$ which is greater than the width of the pion distribution $p(y-y_c)$ (typically about 1 unit of rapidity), we can replace Eq. (12) with $\pi(y_c)=1/2L$ everywhere as long as y is well within the interval $(-L,L)$. The integration is then trivial, and we obtain

$$\rho(y)=\frac{\langle N_c\rangle\langle n_\pi\rangle}{2L} \tag{22}$$

for the central region of an ultrarelativistic collision. Equation (22), which is independent of the precise shape of the rapidity distributions $\pi(y_c)$ and $p(y-y_c)$, is used to determine the cluster rapidity density (in the central region) $\langle N_c\rangle/(2L)$ for given mean cluster size $\langle n_\pi\rangle$.

IV. CALCULATION OF MOMENTS

We have reduced the factorial moments $F_i(\delta y)$ to integrals of rapidity densities, so we now calculate the i-particle rapidity densities. The calculation of these densities can be very complicated in cluster models because the i particles could come from any number of clusters from one to i, and we need to sum over all of these possibilities. In practice, however, the clusters overlap very much, so we therefore make the approximation that *at most two of the i pions come from the same cluster.* The approximation is valid whenever the second moment is near one, as is true for hadronic collisions.[2–4]

In our cluster model, the i-particle rapidity densities are

$$\rho^{(i)}(y_1,\ldots,y_i)=\langle N_c(N_c-1)\cdots(N_c-i+1)\rangle\langle n_\pi\rangle^i\prod_{n=1}^{i}\bar{p}(y_n)$$
$$+\langle N_c(N_c-1)\cdots(N_c-i+2)\rangle\langle n_\pi\rangle^{i-2}\langle n_\pi(n_\pi-1)\rangle\sum_{j=1}^{i}\sum_{k\neq j}\left[\prod_{l\neq j,k}\bar{p}(y_l)\right]\bar{p}^{(2)}(y_j,y_k)+O(\langle N_c\rangle^{i-2}), \tag{23}$$

where

$$\bar{p}(y)=\int_{-\infty}^{\infty}dx\,\pi(x)p(y-x) \tag{24}$$

is the normalized single-pion distribution function, and

$$\bar{p}^{(2)}(y_1,y_2)=\int_{-\infty}^{\infty}dx\,\pi(x)p(y_1-x)p(y_2-x) \tag{25}$$

is the normalized two-pion distribution function. The first term in Eq. (23) is the contribution of pions from i different clusters, in which case the pion rapidities are uncorrelated. The second term is the contribution of pions from $i-1$ clusters, in which case two of the pions are correlated in rapidity and the rest are uncorrelated.

Using the approximations $\bar{p}(y_n)=\pi(y_c)=1/2L$, which are both valid in the central region for ultrarelativistic collisions, and performing the sums, we obtain

$$\left[\prod_{n=1}^{i}\int_a^b dy_n\right]\rho^{(i)}(y_1,\ldots,y_i)=\langle N_c(N_c-1)\cdots(N_c+i+1)\rangle\left[\frac{\langle n_\pi\rangle(b-a)}{2L}\right]^i$$
$$\times\left[1+i(i-1)\frac{2L\langle n_\pi(n_\pi-1)\rangle}{\langle N_c\rangle\langle n_\pi\rangle^2}\int_a^b\frac{dy_1}{b-a}\int_a^b\frac{dy_2}{b-a}p^{(2)}(y_1,y_2)+O(\langle N_c\rangle^{-2})\right], \tag{26}$$

where

$$p^{(2)}(y_1,y_2)=2L\bar{p}^{(2)}(y_1,y_2)=\int_{-\infty}^{\infty}dx\,p(y_1-x)p(y_2-x)\,. \tag{27}$$

After substituting $\rho^{(i)}$ from Eq. (26) into Eq. (11) and using Eq. (22) for $\rho(y)$, we find

$$F_i(\delta y)=1+i(i-1)\left[\frac{\langle n_\pi(n_\pi-1)\rangle/\langle n_\pi\rangle}{dN/dy}\right]$$
$$\times\left[\left[\int_y^{y+\delta y}\frac{dy_1}{\delta y}\int_y^{y+\delta Y}\frac{dy_2}{\delta y}p^{(2)}(y_1,y_2)\right]-\left[\int_{y_0}^{y_0+\Delta Y}\frac{dy_1}{\Delta Y}\int_{y_0}^{y_0+\Delta Y}\frac{dy_2}{\Delta Y}p^{(2)}(y_1,y_2)\right]\right]+O(\langle N_c\rangle^{-2})\,. \tag{28}$$

Equation (28) displays a scaling behavior which is independent of the distribution $p(y)$ and of the rapidity window Δ. All of the factorial moments are simply related to each other:

$$\frac{F_2(\delta y)-1}{2}=\frac{F_i(\delta y)-1}{i(i-1)}\,. \tag{29}$$

This scaling behavior is the same as that demonstrated for inclusive moments.[22] The behavior of this new factorial moment is more robust, however, as the scaling laws have a very weak dependence on the cluster and pion multiplicity distributions.

In Fig. 1 we show that data[2] from π-p and K-p collisions at 250 GeV obey the scaling law (29). We convert the inclusive moments (7) to the proper moments (9) by dividing all moments by $F_i(M=1)=\langle N(N-1)\cdots(N-i+1)\rangle/\langle N\rangle^i$. The scaling law holds very well for the third moment, not quite so well for the fourth moment, and seems to be significantly inaccurate for the fifth moment. The deviations from the scaling law may be a result of inaccuracies in the data, as the value of the fifth moment for large M seems to depend mainly on whether M is even or odd.

Because all of the moments contain the same information, at least to lowest order in the cluster density, the most sensible approach for experimenters is to present the second moment and to indicate the deviations from scaling behavior of the other moments. The second moment usually has the least statistical error and thus provides the most stringent test of any models. The deviations from scaling behavior are usually small enough that they are difficult to measure, so they do not contain much useful information.

Equation (28) also predicts that the factorial moments scale with rapidity density, as the inclusive moments do;[22] for event samples a and b,

$$(dN/dy)_a[F_i^a(\delta y)-1]=(dN/dy)_b[F_i^b(\delta y)-1]\ , \qquad (30)$$

as long as the droplet multiplicity factor $\langle n_\pi(n_\pi-1)\rangle/\langle n_\pi\rangle$ remains constant. This law seems to work at least to some extent, as the highest moments are observed in π-p collisions[2] where the rapidity density is lowest. This trend continues in the p-A and A-B collisions,[3,4] where the smallest moments are seen in S-Au collisions where the rapidity density is highest.

The multiplicity scaling law works very well with O-emulsion and S-Au collisions,[8] but collisions involving lighter particles have smaller moments than predicted by Eq. (30). In Fig. 2 we compare $(dN/dy)[F_2(\delta y)-1]$ for π-p and K-p collisions[2] and for S-Au collisions.[4] In order to make a direct comparison we divide the hadronic moments by $F_2(\delta y=2)$, as the S-Au moments are construct-

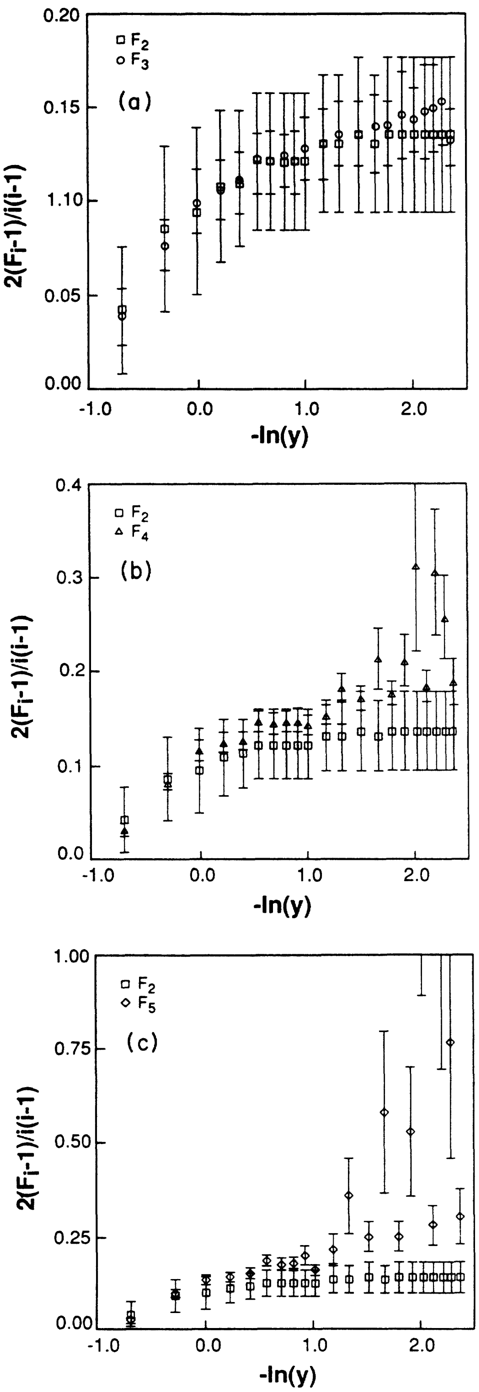

FIG. 1. $2(F_i-1)/i(i-1)$ vs $-\ln(y)$ for π-p and K-p collisions[2] at 250 GeV. The parameter y is the width, δy, of the rapidity bins. (a) The second moment (open squares) and the third moment (open circles). (b) The second moment (open squares) and the fourth moment (open triangles). (c) The second moment (open squares) and the fifth moment (open diamonds).

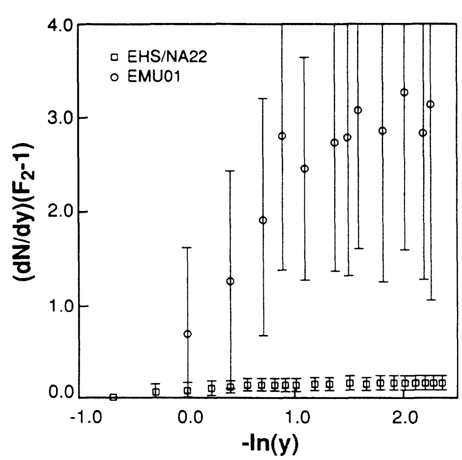

FIG. 2. $(dN/dy)(F_2-1)$ vs $-\ln(y)$ for π-p and K-p collisions[2] at 250 GeV (open squares) and for S-Au collisions[4] at 200 GeV (open circles).

ed for $\Delta Y=2$. The violation of the multiplicity scaling law suggests that the correlations observed in the S-Au collisions are produced by clusters approximately 30 times as large as the clusters produced in the hadronic collisions.

There are two possible interpretations of this violation of the multiplicity scaling law. The first possibility is that large cluster formation occurs in heavy-ion collisions but not in hadronic collisions. The second possibility is that large cluster formation occurs in both types of collisions, but is less frequent in hadronic collisions than in heavy-ion collisions. In either case, it is likely that spatial intermittency, or the formation of clusters of varying spatial extent and thus varying pion number, is actually stronger in heavy-ion collisions of lighter particles.

We further simplify Eq. (28) when the width of the rapidity window ΔY is much greater than the width of the pion distribution $p(y-y_c)$. In this case, we can perform the integrations over the rapidity window Δ, obtaining

$$F_i(\delta y)=1+i(i-1)\left[\frac{\langle n_\pi(n_\pi-1)\rangle/\langle n_\pi\rangle}{dN/dy}\right]\left[\left[\int_y^{y+\delta y}\frac{dy_1}{\delta y}\int_y^{y+\delta y}\frac{dy_2}{\delta y}p^{(2)}(y_1,y_2)\right]-\frac{1}{\Delta Y}\right]+O(\langle N_c\rangle^{-2})+O(\Delta Y^{-2})\ . \tag{31}$$

It is clear from Eq. (31) that the scaling of the factorial moments will be most accurate for $\Delta Y\gg 1$.

Finally, in the case of small δy we use the approximation

$$\int_{-\delta y/2}^{\delta y/2}\frac{dy}{\delta y}p(y)=p(0)+\tfrac{1}{24}p''(0)\delta y^2+O(\delta y^4)\ . \tag{32}$$

We then obtain, after integration by parts,

$$F_i(\delta y)=1+i(i-1)\left[\frac{\langle n_\pi(n_\pi-1)\rangle/\langle n_\pi\rangle}{dN/dy}\right]\times\left[\left[\int_{-\infty}^{\infty}dy\,p(y)^2\right]-\tfrac{1}{12}\left[\int_{-\infty}^{\infty}dy\,p'(y)^2\right]\delta y^2-\frac{1}{\Delta Y}\right]+O(\langle N_c\rangle^{-2})+O(\Delta Y^{-2})+O(\delta y^4)\ . \tag{33}$$

We can see from Eq. (33) that, in general, the factorial moments will converge as $\delta y\to 0$, and that the deviation from the asymptotic value will be negative and proportional to δy^2. This behavior, like most of the behavior discussed in this paper, is practically independent of any specific theory of factorial moments, depending only on the assumption that pions are created in some sort of clusters. The quadratic dependence on δy is sufficient to explain the observed behavior of the factorial moments in nucleus-nucleus collisions, as shown in Fig. 3.

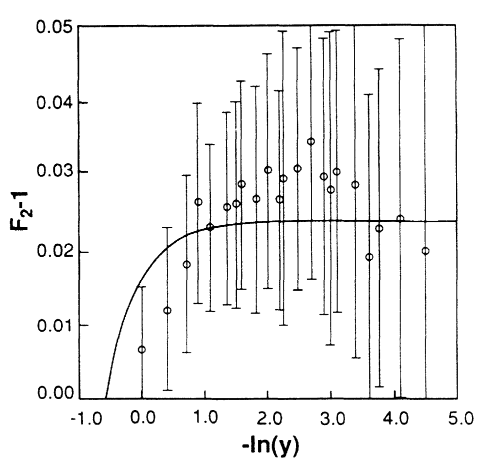

FIG. 3. F_2-1 vs $-\ln(y)$ for S-Au collisions[4] at 200 GeV (open squares). The solid curve is the best fit to the form $F_2-1=a-b\,\delta y^2$, with $a=0.024$ and $b=0.0076$.

Our calculation to $O(\langle N_c\rangle^{-1})$ is exact for the second moment. Because of this, violations of the multiplicity scaling law can be used to obtain information about the pion and cluster distributions. The scaling violations due to cluster multiplicity, occurring as a result of two-particle correlations, are usually larger than corrections due to three- and four-particle effects, and so the latter can often be neglected. Corrections due to three- and four-particle effects can be calculated from the general model given here; we leave that as an exercise for the interested reader.

V. SUMMARY

In this paper, we have discussed the current methodology used in the search for intermittency. We pointed out that the inclusive moment does not reduce to the exclusive moment when events with a fixed window multiplicity are considered. We proposed the use of a new scaled factorial moment, which reduces to the exclusive moment in the appropriate limit, and is free of undesirable multiplicity correlations which are contained in the inclusive moment (but not the exclusive moment).

We calculated factorial moments from a simple independent cluster model, which assumes only approximate boost invariance of the cluster rapidity distribution and an approximate relation between the moments of the cluster multiplicity distribution. We found two scaling laws that are essentially model independent. These laws hold in almost every model of particle collisions that reproduces existing rapidity correlation data, as most models assume that particle production occurs via some boost-invariant assortment of clusters.

The first scaling law relates the moments to each other with a simple formula, indicating that the different factorial moments are not independent. The best course for experimenters is then to measure the second moment, as this minimizes the statistical uncertainty. The second scaling law relates samples with different rapidity densities but similar cluster rapidity distributions. The violation of the second scaling law suggests that the correlations observed in the O-emulsion and Su-Au collisions are produced by clusters approximately 30 times as large as the clusters produced in the hadronic collisions. These large clusters indicate that *spatial* intermittency is actually stronger for heavy-ion collisions than for hadronic collisions.

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