Minijets: Cross section and energy distribution in very-high-energy nuclear collisions

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The energy spectrum from semihard partonic interactions in nucleus-nucleus collisions with c.m. energies of the order of 1 TeV per nucleon is discussed. The presence of a large number of nucleons induces incoherence among most of the partonic collisions, while the large number of partonic interactions makes the unitarization of the cross section an essential tool for a meaningful description of the processes. This goal is achieved, accounting for all semihard partonic scatterings, namely, including both disconnected collisions and rescatterings. The characteristic feature of the interaction resulting from this analysis is that it is basically a geometrical one. As a consequence of the unitarization, the energy distribution of the scattered partons turns out to be a regular function of the cutoff $p_t^{\min}$ which separates semihard events from soft ones.

I. INTRODUCTION

In very-high-energy hadronic collisions one faces the problem of discussing large-cross-section physics by means of perturbation theory. One will in fact expect that, as the scale for the perturbative coupling constant is fixed, when the c.m. energy becomes increasingly large, most of the interactions will be in a perturbative regime, so that a large fraction of all the inelastic events can be discussed using perturbative QCD.[1] Analyzing minimum-bias events at the CERN $p\bar{p}$ Collider, one has evidence of an increasingly large hard component in the interaction when increasing the c.m. energy,[2] which is consistent with this expectation.

In order to make a separation between soft and hard events one needs to introduce a cutoff $p_t^{\min}$ in the transverse momentum of the scattered partons (that can eventually be observed as minijets in the final state). Keeping this cutoff fixed and increasing the c.m. energy, one gets a corresponding increasingly large inclusive cross section for parton production. Actually the growth is faster than the one expected for the total cross section.[3] The kinematical region in s and $p_t^{\min}$, where the corresponding integrated inclusive cross section becomes comparable with the total inelastic one, is called the region for semihard interactions and, in that regime, unitarity starts to play a major role.

In eikonal models the contribution from semihard collisions can be included, in a way consistent with unitarity, by splitting the eikonal phase into two pieces, giving, respectively, the soft and the semihard component of the interaction, and giving well-defined prescriptions in order to calculate the semihard part of the phase.[4] As a consequence the semihard contribution to the inelastic cross section σ_H (with $\sigma_{\text{inelastic}}=\sigma_{\text{soft}}+\sigma_H$) can be expressed as

$$\sigma_H=\int d^2\beta\{1-\exp[-\langle n(\beta)\rangle]\}$$
$$=\sum_{n=1}^{\infty}\int d^2\beta\frac{1}{n!}\langle n(\beta)\rangle^n\exp[-\langle n(\beta)\rangle] , \qquad (1)$$

β being the impact parameter and $\langle n(\beta)\rangle/2$ the semihard contribution to the eikonal function.

Equation (1) has an immediate probabilistic interpretation and shows how σ_H is constructed with the incoherent sum of an infinitely large number of multiple parton collisions characterized by a Poisson distribution with an average number given by $\langle n(\beta)\rangle$. The connection with the usual QCD–parton-model expression for the inclusive large-p_t parton production cross section σ_{incl} is obtained by noticing that the latter is in fact given by the average number of partonic collisions multiplied by the hard cross section,[5] so that one has

$$\sigma_{\text{incl}}=\langle n\rangle\sigma_H=\int d^2\beta\langle n(\beta)\rangle . \qquad (2)$$

The same expression for the semihard cross section σ_H can be obtained without mentioning soft physics by summing all disconnected semihard partonic collisions, having assumed a Poissonian expression for the multiparton distributions.[6]

Equation (1) shows how unitarity can provide a natural cutoff for the perturbative QCD–parton-model singularity at $x=0$. Actually, although σ_{incl}, when expressed in the QCD parton model, is divergent for $p_t^{\min}\to 0$, σ_H, as given by Eq. (1), is finite. One has, in fact, that for small values of the cutoff $\langle n(\beta)\rangle$ becomes very large, so that the exponential in Eq. (1) is practically zero. When, however, β is larger than some typical hadronic radius r, $\langle n(\beta)\rangle$ becomes zero for any value of the cutoff, because there is no overlap between the interacting hadrons. The size of σ_H is then bounded by πr^2. More explicitly we remark that σ_H acquires, for $p_t^{\min}\to 0$, its limiting value rapidly enough, i.e., for values of $\langle n(\beta)\rangle$ which are not very large and correspond, therefore, to configurations where a perturbative treatment is still trustworthy. Multiple collisions can then cure the problem of the singular dependence of the single-scattering expression given by the QCD parton model because one can find physically meaningful variables which, being regular in the limit, are not necessarily dominated by the soft processes.

The situation is essentially the same when considering high-energy nuclear collisions rather than hadronic. At a given value of the nucleon-nucleon c.m. energy $\sqrt{s}$ one has a stronger effect from unitarity in this case, since the presence of a large number of nucleons will largely increase the number of multiple collisions. Moreover the atomic mass will provide a further degree of freedom allowing one to vary the amount of multiple collisions at fixed c.m. energy per nucleon pair. The transverse-energy spectrum from semihard interactions in nucleus-nucleus collisions, given the above physical picture, is discussed in Ref. 7 and the result can be written (with the help of the central-limit theorem) as

$$\frac{d\sigma_H}{dE_t}=\int d^2\beta\{1-\exp[-\langle n(\beta)\rangle]\}$$
$$\times\frac{1}{\sqrt{D_t(\beta)\pi}}\exp\left[-\frac{[E_t-\langle E_t(\beta)\rangle]^2}{D_t(\beta)}\right], \quad (3)$$

depending on the average transverse energy $\langle E_t(\beta)\rangle$ and on its variance $D_t(\beta)$.

Given the Poissonian distribution of multiple-parton collisions [Eq. (1)] and the connection with the QCD parton model [Eq. (2)], $\langle E_t(\beta)\rangle$ and $D_t(\beta)$ are essentially computed with the single-scattering expression provided by the QCD parton model. One will then notice that, while the expression for σ_H is regular for $p_t^{\min}\to 0$, the expression for $d\sigma_H/dE_t$ obtained in this way is, on the contrary, singular because the averages which characterize the differential distributions would become singular if they were computed in the QCD parton model in the limit $p_t^{\min}\to 0$.

Our aim, in the present paper, is to gain a better insight into the problem of the singular behavior of these average quantities. We want in fact to show how, when giving a more complete account of the interaction, one can also regularize the differential cross section. We are only able to consider a simpler case with respect to that of the transverse-energy flow: We will look at the energy, rather than transverse-energy distribution of the semihard scattered partons. In our opinion this quantity is interesting because, being independent of the fragmentation (at least if the scattered partons will fragment independently), it is a property of the initial state and of the interaction only.

The basic element that will allow us to get rid of the divergence in the average energy will be the introduction of the concept of wounded parton, that is to say a parton that has suffered at least one semihard interaction. While the QCD–parton-model single-scattering expression does in fact count all the partonic collisions,[8] including all possible rescatterings of the same parton, when looking at the average energy going into semihard collisions, all rescatterings are irrelevant. It is immediately obvious that introducing the concept of wounded parton, the average energy going into semihard collisions is finite, since it is given by the energy carried by the partons that have suffered at least one semihard collision and this energy is smaller than the total energy available.

In eikonal models the semihard contribution is introduced by adding to the soft eikonal phase a term which represents the single-scattering expression as given by the QCD parton model. As this term is linear in both parton distribution and also in the elementary hard cross section all parton rescatterings are neglected: When writing the expansion in multiple parton collisions [Eq. (1)] only terms where a given number of partons from the projectile scatter an equal number of partons from the target are present.

The introduction of the concept of wounded partons requires that one takes into account also parton rescatterings. In order to achieve this goal we will make the simplifying hypothesis of incoherence among all semihard partonic collisions, so that our treatment of the interaction will be purely probabilistic. This hypothesis is consistent both with previous works on inelastic nucleus-nucleus collisions[9,10] and with a recent analysis on rescattering of partons on nuclear targets.[11] We think therefore that it is a meaningful hypothesis in the context of high-energy nuclear collisions.

The paper is organized as follows: In the next section we introduce the semihard cross section and the expression for the average number of wounded partons. In Sec. III we evaluate the average energy and the dispersion and in the final section we present some numerical calculations and our conclusions.

II. SEMIHARD CROSS SECTION AND AVERAGE NUMBER OF COLLISIONS

It is now necessary to give an explicit form for the multiparton distribution, which we assume to be Poissonian. We expect that this form is a reasonable expression for small values of x; it is obviously consistent with the assumption of an incoherent superposition of Poissonian distributions of partons at the nucleon level. The picture then consists of a nucleus made of A noninteracting nucleons: In the interaction with a high-energy parton the partonic structure of the whole nucleus is obtained by just summing up the parton composition of the individual nucleons. We are neglecting, in this way, more exotic nuclear configurations which, on the other hand, give corrections to the independent nucleon picture of the order of a few percent.[12] Information on the intermediate structure is present only in the kinematical limits of the parton momenta. (Further discussions on this point are presented in Appendix C.)

The probability density for having n partons of nucleus A (A being the nuclear mass) with fractional momenta $x_1,\ldots,x_n$ and with transverse coordinates $b_1,\ldots,b_n$ is then given by

$$\frac{1}{n!}\Gamma_A^{f_1}(x_1,\mathbf{b}_1)\cdots\Gamma_A^{f_n}(x_n,\mathbf{b}_n)$$
$$\times\exp\left[-\int\sum_f\Gamma_A^f(x,\mathbf{b})dx\,d^2b\right], \quad (4)$$

where $\Gamma_A^f(x,\mathbf{b})$ is the average number of partons in the A nucleus with longitudinal-momentum fraction x (with respect to the nucleon momentum), $\mathbf{b}$ the transverse coordinate, and the index f counts the various species of partons. The normalization of $\Gamma_A^f(x,\mathbf{b})$ is A times that of the nucleon parton distributions and the integral in Eq. (4) is regularized with a cutoff related to $p_t^{\min}$. Multiparton distributions are dimensional quantities,[13] and with our assumption dimensionality is provided by the geometrical size of the whole nucleus.

One will notice that, in Eq. (4), there is no sharp constraint on the total energy of the nucleus: One is in fact introducing a variance in the total energy proportion to A. Since the multiparton distribution has a Poissonian form the total energy can vary by amounts proportional to $A^{1/2}$ around its average value that is proportional to A. Therefore, although writing Eq. (4) one is satisfying energy conservation only on average, the fluctuation around the averaged value is subleading as a function of the atomic mass.

To write the semihard cross section we make the simplifying assumption of complete incoherence between different semihard partonic collisions. That amounts to assuming incoherence not only between disconnected semihard collisions, but also between different rescatterings of the same parton. Moreover all partonic collisions will be treated on the same footing, so that we will not distinguish between multiple collisions on the same nucleon and multiple collisions on different nucleons. The semihard cross section is then expressed as

$$\sigma_H^{AB}=\int d^2\beta \sum_{n=1}^{\infty}\sum_{f_1\cdots f_n}\frac{1}{n!}\Gamma_A^{f_1}(x_1,\mathbf{b}_1)\cdots\Gamma_A^{f_n}(x_n,\mathbf{b}_n)\exp\left[-\int\sum_f \Gamma_A^f(x,\mathbf{b})dx\,d^2b\right]$$
$$\times\sum_{l=1}^{\infty}\sum_{f_1'\cdots f_l'}\frac{1}{l!}\Gamma_B^{f_1'}(x_1',\mathbf{b}_1'-\boldsymbol{\beta})\cdots\Gamma_B^{f_l'}(x_l',\mathbf{b}_l'-\boldsymbol{\beta})$$
$$\times\exp\left[-\int\sum_{f'}\Gamma_B^{f'}(x',\mathbf{b}')dx'd^2b'\right]\left[1-\prod_{i=1}^{n}\prod_{j=1}^{l}\prod_{f_i,f_j'}(1-\hat\sigma_{ij}^{f_if_j'})\right]$$
$$\times dx_1d^2b_1\cdots dx_nd^2b_ndx_1'd^2b_1'\cdots dx_l'd^2b_l' \,, \tag{5}$$

where $\hat\sigma_{ij}^{f_if_j'}\equiv\hat\sigma_{ij}^{f_if_j'}(x_ix_j,\mathbf{b}_i-\mathbf{b}_j')$ is the probability for the parton f_i from nucleus A to have a semihard interaction with parton f_j' from nucleus B, so $\hat\sigma$ will depend on x_ix_j, on the difference of the transverse relative distance $\mathbf{b}_i-\mathbf{b}_j'$ and the indices f_i,f_j'.

The term in large square brackets in Eq. (5) represents the probability of having at least one semihard partonic interaction between nucleus A and nucleus B, and the cross section is constructed summing over all possible partonic configurations of the two nuclei and integrating on the nuclear impact parameter β. One will notice that in Eq. (5) all possible interactions between partons of nucleus A and partons of nucleus B are taken into account, so that also all possible semihard rescatterings in nuclear matter are included.

The semihard cross section can be conveniently expressed introducing, as an intermediate step, the probability that the parton f_j' from nucleus B has at least one semihard interaction with a given configuration of n partons of nucleus A. This probability is given by

$$P_{f_1\cdots f_n}^{f_j'}\equiv P_{f_1\cdots f_n}^{f_j'}(x_j',\mathbf{b}_j';x_1,\mathbf{b}_1\cdots x_n,\mathbf{b}_n)\equiv 1-\prod_{i=1}^{n}\prod_{f_i}(1-\hat\sigma_{ij}^{f_if_j'}) \,. \tag{6}$$

One can then write

$$\sigma_H^{AB}=\int d^2\beta \sum_{n=1}^{\infty}\sum_{f_1\cdots f_n}\frac{1}{n!}\Gamma_A^{f_1}(x_1,\mathbf{b}_1)\cdots\Gamma_A^{f_n}(x_n,\mathbf{b}_n)\exp\left[-\int\sum_f \Gamma_A^f(x,\mathbf{b})dx\,d^2b\right]$$
$$\times\sum_{l=1}^{\infty}\sum_{f_1'\cdots f_l'}\frac{1}{l!}\Gamma_B^{f_1'}(x_1',\mathbf{b}_1'-\boldsymbol{\beta})\cdots\Gamma_B^{f_l'}(x_l',\mathbf{b}_l'-\boldsymbol{\beta})\exp\left[-\int\sum_{f'}\Gamma_B^{f'}(x',\mathbf{b}')dx'd^2b'\right]$$
$$\times\left[1-\prod_{j=1}^{l}\prod_{f_j'}[1-P_{f_1\cdots f_n}^{f_j'}(x_j',\mathbf{b}_j';x_1,\mathbf{b}_1\cdots x_n,\mathbf{b}_n)]\right]$$
$$\times dx_1d^2b_1\cdots dx_nd^2b_ndx_1'd^2b_1'\cdots dx_l'd^2b_l' \,. \tag{7}$$

The sums over l and f_l' can be evaluated, giving

$$\sigma_H^{AB}=\int d^2\beta \sum_{n=1}^{\infty}\sum_{f_1\cdots f_n}\frac{1}{n!}\Gamma_A^{f_1}(x_1,\mathbf{b}_1)\cdots\Gamma_A^{f_n}(x_n,\mathbf{b}_n)\exp\left[-\int\sum_f \Gamma_A^f(x,\mathbf{b})dx\,d^2b\right]$$
$$\times\left[1-\exp\left[-\int\sum_{f'}\Gamma_B^{f'}(x',\mathbf{b}'-\boldsymbol{\beta})P_{f_1\cdots f_n}^{f'}dx'd^2b'\right]\right]dx_1d^2b_1\cdots dx_nd^2b_n\ . \tag{8}$$

The number of partonic collisions can grow very rapidly with the atomic mass, but the number of partons involved in the interaction will grow much less. It is then convenient to introduce the concept of wounded parton (analogously to that of wounded nucleon[9]) as a parton that has suffered at least one semihard interaction. Equation (8) is a convenient expression to estimate the average number of wounded partons. One may in fact expand Eq. (8) in terms of the number of wounded partons of nucleus B:

$$\sigma_H^{AB}=\int d^2\beta \sum_{n=1}^{\infty}\sum_{f_1\cdots f_n}\frac{1}{n!}\Gamma_A^{f_1}(x_1,\mathbf{b}_1)\cdots\Gamma_A^{f_n}(x_n,\mathbf{b}_n)\exp\left[-\int\sum_f \Gamma_A^f(x,\mathbf{b})dx\,d^2b\right]$$
$$\times\sum_{k=1}^{\infty}\frac{1}{k!}\left[\int\sum_{f'}\Gamma_B^{f'}(x',\mathbf{b}'-\boldsymbol{\beta})P_{f_1\cdots f_n}^{f'}dx'd^2b'\right]^k$$
$$\times\exp\left[-\int\sum_{f'}\Gamma_B^{f'}(x',\mathbf{b}'-\boldsymbol{\beta})P_{f_1\cdots f_n}^{f'}dx'd^2b'\right]dx_1d^2b_1\cdots dx_nd^2b_n\ , \tag{9}$$

so that one may write

$$\sigma_H^{AB}=\sum_{k=1}^{\infty}\sigma_{AB}^{(k)}\ . \tag{10}$$

The average number of wounded partons is then easily obtained,

$$\langle k\rangle\sigma_H^{AB}=\sum_{k=1}^{\infty}k\sigma_{AB}^{(k)}\ , \tag{11}$$

that can be immediately written as

$$\langle k\rangle\sigma_H^{AB}=\int d^2\beta \sum_{n=1}^{\infty}\sum_{f_1\cdots f_n}\frac{1}{n!}\Gamma_A^{f_1}\cdots\Gamma_A^{f_n}\exp\left[-\int\sum_f \Gamma_A^f(x,\mathbf{b})dx\,d^2b\right]$$
$$\times\sum_{f'}\Gamma_B^{f'}(x',\mathbf{b}'-\boldsymbol{\beta})P_{f_1\cdots f_n}^{f'}dx'd^2b'dx_1d^2b_1\cdots dx_nd^2b_n\ . \tag{12}$$

Using Eq. (6) and introducing the definition

$$\eta_A^f(x,\mathbf{b})\equiv 1-\exp\left[-\int\sum_f \Gamma_A^{f'}(x',\mathbf{b})\hat{\sigma}^{ff'}(xx')dx'\right]\ , \tag{13}$$

one gets

$$\langle k\rangle\sigma_H^{AB}=\int d^2\beta\,dx'd^2b'\sum_{f'}\Gamma_B^{f'}(x',\mathbf{b}'-\boldsymbol{\beta})\eta_A^{f'}(x',\mathbf{b}')\ . \tag{14}$$

To obtain Eq. (14) the probability of a semihard interaction $\hat{\sigma}^{ff'}(xx',\mathbf{b}-\mathbf{b}')$ has been treated as a δ function in $\mathbf{b}-\mathbf{b}'$ in comparison with the much smoother $\mathbf{b}$ dependence of the average number of partons $\Gamma(x,\mathbf{b})$. The cross section $\hat{\sigma}^{ff'}(xx')$ is then the usual parton-parton cross section integrated on the polar c.m. angle with the cutoff provided by $p_t^{\min}$. The integral in the exponent is also regularized with the same cutoff.

Expression (14) has a transparent physical interpretation: The factor η represents the probability for a parton of nucleus B to have at least one semihard interaction with nucleus A, so that the average number of wounded partons of B is given by the average number of partons of B multiplied by the interaction probability.

If one wanted to look rather at the average number of semihard partonic collisions $\langle\nu\rangle$ then (as discussed in Appendix A) one will have to replace the square brackets in Eq. (5) with $\sum_{p=1}^{nl}\hat{\sigma}_p$. Since the sum over p can be replaced with $nl\hat{\sigma}_l^{ff'}$, all partons being identical for the present purpose, one is able to carry out the sums over l and m so that the result takes the simple expression

$$\langle\nu\rangle\sigma_H^{AB}=\sum_{ff'}\int d^2\beta\,\Gamma_A^f(x_1,\mathbf{b}_1)\Gamma_B^{f'}(x_1',\mathbf{b}_1-\boldsymbol{\beta})\hat{\sigma}^{ff'}(x_1x_1')dx\,dx'd^2b_1\ , \tag{15}$$

that is the single-scattering expression given by the QCD parton model.

One has then checked that the approach is consistent with the cancellation involving the average number of collisions.[8] On the other hand, quantities that are more directly accessible experimentally, as will be discussed in the next section, are related rather to averages involving wounded partons.

III. AVERAGE ENERGY AND DISPERSION

The energy produced by semihard partonic interactions is the energy carried by the wounded partons (since a parton is wounded when it suffers at least one semihard interaction). Also the dispersion in the energy produced will be related to various averages involving wounded partons. We then start discussing these averages.

A further way to obtain the average number of wounded partons of nucleus B, which will be convenient later, is the following: One may sum over the partons of each given partonic configuration of B (the sum over the index s in the following expression) and then for each term in the sum one will ask for the probability that the parton of B taken into consideration will have at least one semihard interaction with A (the square brackets):

$$\begin{aligned}\langle k\rangle\sigma_H^{AB}=\int d^2\beta\sum_{n=1}^{\infty}\sum_{f_1\cdots f_n}\frac{1}{n!}\Gamma_A^{f_1}(x_1,\mathbf{b}_1)\cdots\Gamma_A^{f_n}(x_n,\mathbf{b}_n)\\ \times\sum_{k=1}^{\infty}\sum_{f_1'\cdots f_k'}\frac{1}{k!}\sum_{s=1}^{k}\sum_{f_s'}\Gamma_B^{f_1'}(x_1',\mathbf{b}_1'-\boldsymbol{\beta})\cdots\Gamma_B^{f_s'}(x_s',\mathbf{b}_s'-\boldsymbol{\beta})\cdots\Gamma_B^{f_k'}(x_k',\mathbf{b}_k'-\boldsymbol{\beta})\\ \times\exp\left[-\int\sum_f\Gamma_A^f(x,\mathbf{b})dx\,d^2b\right]\\ \times\exp\left[-\int\sum_{f'}\Gamma_B^{f'}(x',\mathbf{b}')dx'd^2b'\right]\left[1-\prod_{l=1}^{n}\prod_{f_l}(1-\hat\sigma_{ls}^{f_lf_s'})\right]\\ \times dx_1d^2b_1\cdots dx_nd^2b_ndx_1'd^2b_1'\cdots dx_s'd^2b_s'\cdots dx_k'd^2b_k'\ .\end{aligned}\tag{16}$$

With a little algebra Eq. (16) will give back Eq. (14).

One may now easily obtain $\langle k^2\rangle$ and $\langle kn\rangle$ (namely, an average involving both nuclei A and B). A natural extension of Eq. (16) will lead to

$$\begin{aligned}\langle kn\rangle\sigma_H^{AB}=\int d^2\beta\sum_{n=1}^{\infty}\sum_{f_1\cdots f_n}\frac{1}{n!}\sum_{j=1}^{n}\sum_{f_j}\Gamma_A^{f_1}(x_1,\mathbf{b}_1)\cdots\Gamma_A^{f_j}(x_j,\mathbf{b}_j)\cdots\Gamma_A^{f_n}(x_n,\mathbf{b}_n)\exp\left[-\int\sum_f\Gamma_A^f dx\,d^2b\right]\\ \times\sum_{k=1}^{\infty}\sum_{f_1'\cdots f_k'}\frac{1}{k!}\sum_{s=1}^{k}\sum_{f_s'}\Gamma_B^{f_1'}(x_1',\mathbf{b}_1'-\boldsymbol{\beta})\cdots\Gamma_B^{f_s'}(x_s',\mathbf{b}_s'-\boldsymbol{\beta})\cdots\Gamma_B^{f_k'}(x_k',\mathbf{b}_k'-\boldsymbol{\beta})\\ \times\exp\left[-\int\sum_{f'}\Gamma_B^{f'}dx'd^2b'\right]\\ \times\left[1-\prod_{l=1,l\neq j}^{n}\prod_{f_l}(1-\hat\sigma_{ls}^{f_lf_s'})\right]\left[1-\prod_{i=1,i\neq s}^{k}\prod_{f_i'}(1-\hat\sigma_{ji}^{f_jf_i'})\right]\\ \times dx_1d^2b_1\cdots dx_nd^2b_ndx_1'd^2b_1'\cdots dx_k'd^2b_k'\ .\end{aligned}\tag{17}$$

In Eq. (17) a simplification was made by neglecting the case $l=j$ and $i=s$ that correspond to the possibility of interaction for the parton s of B with the parton j of A, s and j being the partons taken into consideration in evaluating the average. (The relevance and validity of this simplification is further discussed in Appendix B.) Given the very large amount of partonic interactions in the regime considered here, this simplification will make only marginal corrections to an exact result. On the other hand, the final expression obtained in this way has a very simple and transparent physical meaning. With some manipulations one gets, in fact,

$$\langle kn\rangle\sigma_H^{AB}=\int d^2\beta\,dx'd^2b'dx\,d^2b\sum_{ff'}\Gamma_A^f(x,\mathbf{b})\eta_B^f(x,\mathbf{b}-\boldsymbol{\beta})\Gamma_B^{f'}(x',\mathbf{b}'-\boldsymbol{\beta})\eta_A^{f'}(x',\mathbf{b}')\ ,\tag{18}$$

where the probability for a parton to have a semihard interaction with a nucleus is bounded by one also in the limit of $p_t^{\min}$ close to zero: At a given transverse coordinate $\mathbf{b}$ one has, in fact, some average number of partons of the kind f: $\Gamma_A^f(x,\mathbf{b})$ from nucleus A multiplied by the interaction probability with nucleus B: $\eta_B^f(x,\mathbf{b}-\boldsymbol{\beta})$. The same operation is done with B and finally one has to sum over all possibilities.

The quantities that we are more interested in are, however, the averages involving the energy of the wounded partons. The reason is that one can then estimate the semihard energy spectrum, or better its average value and the dispersion. Apart from a trivial rescaling we are then interested in $\langle x_B \rangle$, $\langle x_B^2 \rangle$, $\langle x_A \rangle$, $\langle x_A^2 \rangle$, and $\langle x_A x_B \rangle$. Let us then look for $\langle x_B \rangle$:

$$\langle x_B \rangle \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1 \cdots f_n} \frac{1}{n!} \Gamma_A^{f_1}(x_1,\mathbf{b}_1) \cdots \Gamma_A^{f_n}(x_n,\mathbf{b}_n) \sum_{k=1}^{\infty} \sum_{f_1' \cdots f_k'} \frac{1}{k!} \Gamma_B^{f_1'}(x_1',\mathbf{b}_1'-\boldsymbol{\beta}) \cdots \Gamma_B^{f_k'}(x_k',\mathbf{b}_k'-\boldsymbol{\beta})$$
$$\times (x_1' P_{f_1 \cdots f_n}^{f_1'} + \cdots + x_k' P_{f_1 \cdots f_n}^{f_k'})$$
$$\times \exp\left[-\int \sum_f \Gamma_A^f(x,\mathbf{b}) dx\, d^2b\right] \exp\left[-\int \sum_{f'} \Gamma_B^{f'}(x',\mathbf{b}') dx' d^2b'\right]$$
$$\times dx_1 d^2b_1 \cdots dx_n d^2b_n dx_1' d^2b_1' \cdots dx_k' d^2b_k' \tag{19}$$

that will give

$$\langle x_B \rangle \sigma_H^{AB} = \int d^2\beta \sum_{n=1}^{\infty} \sum_{f_1 \cdots f_n} \frac{1}{n!} \Gamma_A^{f_1}(x_1,\mathbf{b}_1) \cdots \Gamma_A^{f_n}(x_n,\mathbf{b}_n) \exp\left[-\int \sum_f \Gamma_A^f(x,\mathbf{b}) dx\, d^2b\right]$$
$$\times \sum_{f'} x' \Gamma_B^{f'}(x,\mathbf{b}'-\boldsymbol{\beta}) P_{f_1 \cdots f_n}^{f'} dx_1 d^2b_1 \cdots dx_n d^2b_n dx' d^2b' \tag{20}$$

and finally

$$\langle x_B \rangle \sigma_H^{AB} = \int d^2\beta\, dx' d^2b' \sum_{f'} x' \Gamma_B^{f'}(x',\mathbf{b}'-\boldsymbol{\beta}) \eta_A^{f'}(x',\mathbf{b}') \tag{21}$$

that can be compared with Eq. (14).

One will notice that in the limit of small $p_t^{\min}$ values Eq. (21) is well defined, since the interaction probability is explicitly less than one and the average momentum carried by the partons of nucleus B is regular as a function of the cutoff. In order to estimate $\langle x_B^2 \rangle$ one will rewrite Eq. (19) replacing the sum of x''s with a sum squared. One will then get two contributions:

$$(x_1' + \cdots + x_k')^2 = \sum_{i=1}^{k} (x_i')^2 + \sum_{i,j=1, i \neq j}^{k} x_i' x_j' \ .$$

The first one will give

$$\int d^2\beta\, dx' d^2b' \sum_{f'} (x')^2 \Gamma_B^{f'}(x',\mathbf{b}'-\boldsymbol{\beta}) \eta_A^{f'}(x',\mathbf{b}') \tag{22a}$$

and the second one

$$\int d^2\beta \left[dx' d^2b' \sum_{f'} x' \Gamma_B^{f'}(x',\mathbf{b}'-\boldsymbol{\beta}) \eta_A^{f'}(x',\mathbf{b}')\right]^2 \tag{22b}$$

(after having neglected the possibility for the two partons of B carrying the fractional momentum x_i' and x_j' to interact with the same parton of A).

When looking at $\langle x_A x_B \rangle$ one will get, analogously with $\langle kn \rangle$,

$$\langle x_A x_B \rangle \sigma_H^{AB} = \int d^2\beta\, dx' d^2b' dx\, d^2b$$
$$\times \sum_{ff'} x \Gamma_A^f(x,\mathbf{b}) \eta_B^f(x,\mathbf{b}-\boldsymbol{\beta})$$
$$\times x' \Gamma_B^{f'}(x',\mathbf{b}'-\boldsymbol{\beta}) \eta_A^{f'}(x',\mathbf{b}') \ , \tag{23}$$

which, as in the previous case, is a regular function of the cutoff. For the dispersion at fixed impact parameter $D(\beta)$ one will have

$$D(\beta) \equiv \langle (x_A + x_B)^2 \rangle_\beta - \langle x_A + x_B \rangle_\beta^2$$
$$= \int \sum_f x^2 \Gamma_A^f(x,\mathbf{b}) \eta_B^f(x,\mathbf{b}-\boldsymbol{\beta}) dx\, d^2b$$
$$+ \int \sum_{f'} x^2 \Gamma_B^{f'}(x,\mathbf{b}-\boldsymbol{\beta}) \eta_A^{f'}(x,\mathbf{b}) dx\, d^2b \ . \tag{24}$$

The energy spectrum is then easily written down with the help of the central-limit theorem:

$$\frac{d\sigma_H}{dE} = \int d^2\beta \{1 - \exp[-\langle n(\beta) \rangle]\}$$
$$\times \frac{1}{\sqrt{D(\beta)\pi}} \exp\left[-\frac{[E - \langle E(\beta) \rangle]^2}{D(\beta)}\right] . \tag{25}$$

In Eq. (25) $\langle n(\beta) \rangle$ is defined implicitly by Eq. (5) giving the semihard cross section σ_H^{AB}.

The feature of Eq. (25) we want to stress is that it is a regular function in the limit of small values of the cutoff $p_t^{\min}$, since the average values entering in Eq. (25) are well defined also in this limiting case.

IV. QUANTITATIVE ESTIMATES AND CONCLUSIONS

In the present paper we have tried to gain a better insight into the problem of the divergence of the QCD–parton-model cross section at $x = 0$. This singular behavior is no longer present in the expression for the integrated semihard cross section after having included in the interaction an infinite class of multiple parton collisions, namely, disconnected (or parallel) multiple parton collisions.[6] It is, however, still present in average quantities. To overcome this problem a more satisfactory description of the interaction is needed; we have then given an expression for the semihard cross section where all semihard partonic interactions have been taken into account. To achieve this goal we have basically made the assumption of complete incoherence among different semihard partonic collisions, so that the picture of the interaction is a purely probabilistic one and the semihard cross section is given by Eq. (5).

Given the expression for the semihard cross section we have been able to estimate the mean value and the variance of the energy going into semihard partonic collisions. These quantities are conveniently evaluated introducing the concept of wounded partons and are regular at small values of x. The semihard energy spectrum can then be estimated, the expression being Eq. (25) in the preceding paragraph. To have some qualitative feelings on these issues we find it interesting to comment on the limiting situation of Eq. (25) for $A \to \infty$ and later to compare the limiting distribution to a more realistic case.

One may in fact notice that, at a fixed impact parameter, the average energy produced $\langle E(\beta)\rangle$ and the dispersion $D(\beta)$ are linear in the atomic mass A [see Eqs. (21) and (24)] in the same way as the total energy. It is then convenient to introduce a new variable ϵ defined as the energy divided by the total energy available. In the limit of $A \to \infty$ one will notice that the Gaussian energy distribution at fixed impact parameter β in Eq. (25) will become a delta function as a function of ϵ:

$$\frac{1}{\sqrt{D(\beta)\pi}} \exp\left[-\frac{[E-\langle E(\beta)\rangle]^2}{D(\beta)}\right] \to \delta(\epsilon-\epsilon(\beta)) .$$

For large values of A the dispersion in the semihard energy produced in the interaction is then negligible at a given impact parameter. The consequence is that a measure of the semihard energy is also a measure of the impact parameter: if A is very large the energy carried by the nucleons in the overlap region between the two interacting nuclei will be wholly released by semihard scatterings. Measuring that amount of energy will then give the amount of overlap and therefore the value of the impact parameter. One will notice that, in such a situation, in order to measure the energy produced in the semihard scatterings, it would not be really necessary to measure all the energy carried by all the final-state minijets. It would be enough to be able to measure the energy carried by the spectator nucleons (the ones that will not happen to be in the overlap region).

In order to have a more quantitative feeling on these remarks, and without claiming to perform detailed predictions, we have performed some numerical calculations. The relevant quantities to be evaluated to be able to estimate $d\sigma_H/dE$ [Eq. (25)] are $\langle n(\beta)\rangle$, $\langle E(\beta)\rangle$, and $D(\beta)$. To this purpose explicit expressions for $\Gamma^f(x,\mathbf{b})$ and for $\hat{\sigma}^{ff'}(xx')$ have to be given. The interaction probability $\hat{\sigma}^{ff'}(xx',\mathbf{b}-\mathbf{b}')$, that enters in the general expression for σ_H [Eq. (5)], appears only convoluted with the average number of partons $\Gamma^f(x,\mathbf{b})$ and since the range in $\mathbf{b}$ of Γ is of the order of the nuclear dimension while the range in $\mathbf{b}-\mathbf{b}'$ of $\hat{\sigma}$ is rather of the order of $(p_t^{\min})^{-1}$ one can use the approximation

$$\int \sum_{f'} \Gamma^{f'}(x',\mathbf{b}')\hat{\sigma}^{ff'}(xx',\mathbf{b}-\mathbf{b}')dx'd^2b' \approx \sum_{f'} \Gamma^{f'}(x',\mathbf{b})\hat{\sigma}^{ff'}(xx') .$$

For the average number of partons $\Gamma^f(x,\mathbf{b})$ we have used the factorized expression

$$\Gamma^f(x,\mathbf{b}) = AG^f(x)\frac{3}{2\pi R^3}(R^2-b^2)^{1/2}\theta(R^2-b^2) ,$$

where $G^f(x)$ is the average number of partons with flavor f and fractional momentum x in a nucleon, A is the atomic mass number, and the dependence on $\mathbf{b}$ corresponds to a uniform spherical distribution.

For $\hat{\sigma}^{ff'}(xx')$ we have used the expression given in Ref. 3 of the gluon-gluon cross section integrated in the polar c.m. scattering angle with the cutoff $p_t^{\min}$. Effectively we have considered the case of two flavors only, gluons and quarks, the elementary cross sections being equal apart from a relative scale factor.

$\langle E(\beta)\rangle$ and $D(\beta)$ are then estimated using Eqs. (21) and (24). The quantity $\langle n(\beta)\rangle$ is obtained writing the cross section σ_H, as given explicitly in Eq. (5), in the form of Eq. (1). Since $d\sigma_H/d\beta$ depends weakly on $\langle n(\beta)\rangle$ when the number of collisions is large, we have approximated $\langle n(\beta)\rangle$ with $\langle \nu(\beta)\rangle$ that can be obtained from Eq. (15). The approximation corresponds to include in the evaluation of $\langle n(\beta)\rangle$ disconnected semihard partonic collisions only, as in the eikonal models.

The kinematical region of interest is that one of small values for the fractional momentum x: We find that with a cutoff $p_t^{\min}$ around 3 GeV and for energies of 2 TeV in the nucleon-nucleon c.m. system one is not very far from the limiting situation where all the energy in the overlap region is released by means of semihard collisions. Typical values of x are therefore $x \simeq 10^{-3}$. For these values of x the usual evolution equation is no longer adequate[14] and one has to resume the logarithms of x. The behavior of the parton distributions as a function of x in the small-x limit gets therefore changed in a sizable way: a behavior of x^{-J} with J effectively of the order of 1.5 has been argued.[15] In performing our calculations we have used the set 3 parton distributions from Ref. 16 where the behavior of the gluon distributions was obtained requiring the behavior $x^{-1.5}$. As a scale factor in the parton distributions we have used $Q^2=(p_t^{\min})^2$. In the region of interest, both for x and Q^2, the values provided by these gluon distributions are not dramatically different from those obtained in the recent analysis of Ref. 17. The semihard cross section, that appears explicitly as dimensional factor in the multiparton distributions, has been assumed to be the geometrical cross section, namely, $\sigma_H^{AB}=40\times4\times A^{2/3}$ mb. In the elementary partonic interaction a $k=2$ factor has been assumed.

The results are presented in four figures.

In Fig. 1 one is plotting the average fraction of energy released by means of semihard collisions as a function of the ratio β/R, with R the nuclear radius. The case taken into consideration is that of $A=B=208$ and of 1 TeV per nucleon c.m. energy. The dashed-dotted curve refers

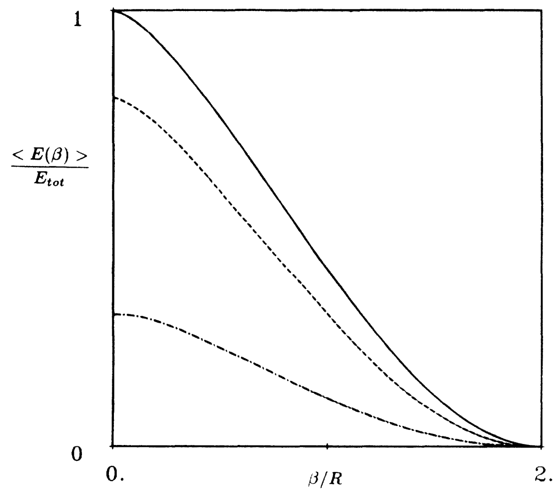

FIG. 1. Average fraction of energy released by means of semihard collisions as a function of the ratio β/R for $A=B=208$ and c.m. energy of 1 TeV per nucleon. The dashed-dotted curve refers to a cutoff $p_t^{\min}=4$ GeV, the dashed one to $p_t^{\min}=2.5$ GeV, while the solid curve is the limiting case where all the energy in the overlap region has been released.

to a cutoff $p_t^{\min}=4$ GeV, the dashed one to $p_t^{\min}=2.5$ GeV, while the solid curve is the limiting case where all the energy in the overlap region is released by means of semihard collisions.

In Fig. 2 one is plotting $(1/\sigma_H^{AB})d\sigma_H^{AB}/d\epsilon$ as a function of $\epsilon\equiv E/E_{\rm tot}$ for Pb+Pb collisions with c.m. energies of 1 TeV per nucleon. The dashed-dotted curve corresponds to a cutoff $p_t^{\min}=4$ GeV, the dashed curve to a cutoff of 2.5 GeV and the solid curve to the limiting case.

In Fig. 3 one is looking at the effect on $(1/\sigma_H^{AB})d\sigma_H^{AB}/d\epsilon$ of changing the atomic mass A, having kept fixed the cutoff ($p_t^{\min}=2.5$ GeV) and the c.m. energy per nucleon ($=1$ TeV). The dashed-dotted curve refers to $A+A$ collisions with $A=100$, the dashed one to $A=208$, and the solid one is the limiting case $A\rightarrow\infty$

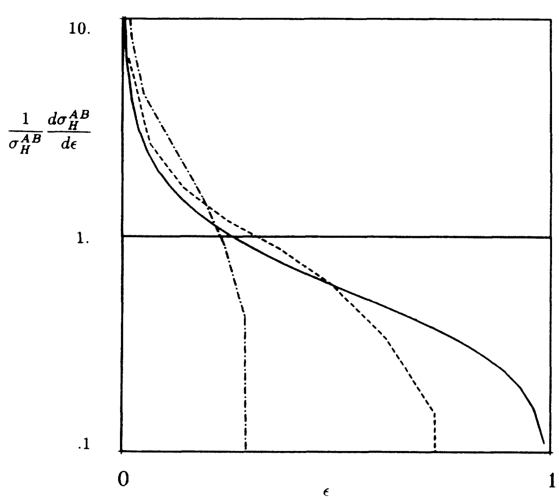

FIG. 2. Differential cross section as a function of $\epsilon\equiv E/E_{\rm tot}$ for Pb+Pb collisions with c.m. energy of 1 TeV per nucleon. The different curves are as in Fig. 1.

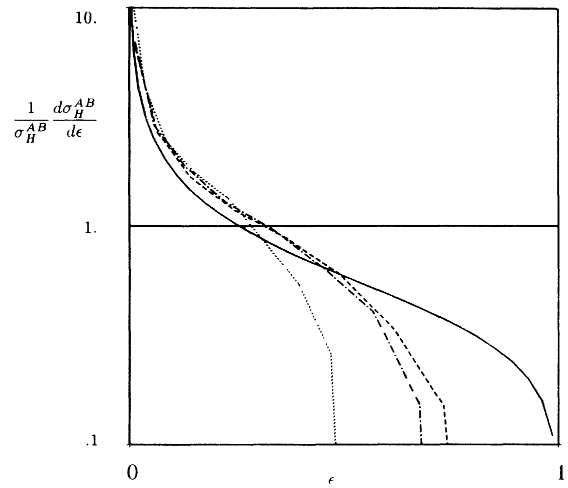

FIG. 3. Differential cross section as a function of ϵ for different atomic masses in $A+A$ collisions with $p_t^{\min}=2.5$ GeV and with 1 TeV per nucleon c.m. energy. Dotted curve, $A=10$; dashed-dotted, $A=100$; dashed, $A=208$. The solid curve refers to the limiting case that can be understood as $A\rightarrow\infty$.

(all the energy in the overlap region is released). Although the calculation is no longer reliable for small atomic numbers, we have also included in the figure the case $A=10$ (dotted curve).

Figure 4 shows the effect on $(1/\sigma_H^{AB})d\sigma_H^{AB}/d\epsilon$ of changing the c.m. energy per nucleon keeping fixed both the atomic masses ($A=B=100$) and the cutoff ($p_t^{\min}=2.5$ GeV). The dashed-dotted curve refers to a c.m. energy of 0.5 TeV per nucleon, the dashed curve to a c.m. energy of 1 TeV per nucleon, and the solid curve refers to the limiting case that can now be understood as the limiting situation where one has an infinitely large amount of energy per nucleon in the c.m. system.

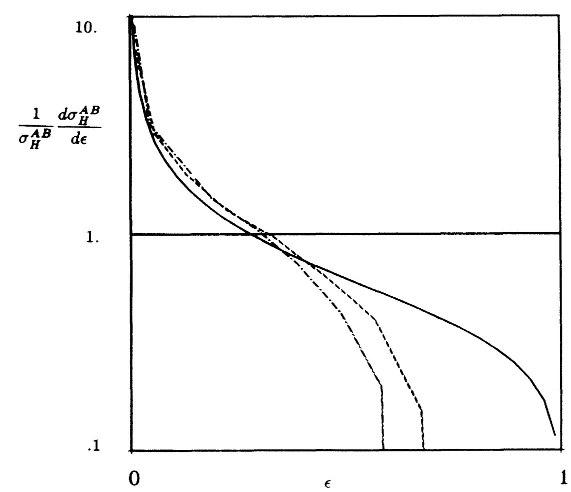

FIG. 4. Differential cross section as a function of ϵ for different c.m. per nucleon energies and with fixed atomic masses ($A=B=100$) and cutoff ($p_t^{\min}=2.5$ GeV). The dashed-dotted curve refers to a c.m. energy of 0.5 TeV per nucleon and the dashed curve to a c.m. of 1 TeV per nucleon. The solid curve is the limiting case that can be understood as the case of an infinitely large amount of energy per nucleon in the c.m. system.

The indication from the numerical calculation is that this limiting case is not as far as one could perhaps have expected. Very roughly, at these c.m. energies, the basic description of the semihard interaction between the two nuclei is represented by the simple geometrical picture where most of the constituents in the overlap region take part in the interaction, all the others being spectators. When moving from the hard to the semihard region unitarity will switch on different multiple-scattering processes. In the regime where $\sigma_{\rm incl}\simeq\sigma_{\rm inelastic}$ one has disconnected multiparton scattering. When $\eta^f_A(x,\mathbf{b})$ is sizable one has parton rescattering. The first regime will start at higher p_t with respect to the second because of the different dependence on A. One will notice in fact that, when it is possible to keep only the term linear with $\hat\sigma$ in the expansion of η, the average quantities are those computed with the single-scattering expression given in the QCD parton model. Similar conclusions have been obtained in the framework of extremely high-energy hadronic collisions.[18] The large variation of the output semihard energy distribution as a function of the cutoff $p_t^{\rm min}$ shows that the parton distributions at values of $x\simeq10^{-3}$ or less are the really critical parameter.

Our analysis is based mainly on two different assumptions. The first is the Poissonian expression for the multiparton distributions. The second is the assumption of incoherence between different multiple-parton collisions. Our justification for the first is that we expect parton-parton correlations to be relatively small in a large nucleus. Because of purely combinatorial reasons, it is in fact easier for a parton which suffers two or more collisions to hit partons originating from different nucleons than partons originating from the same nucleus. We believe that a good support for our second hypothesis is the present understanding of the high-energy inelastic nucleus-nucleus cross section in terms of the inelastic nucleon-nucleon cross section. In our opinion, however, both hypotheses can find a sounder justification and we are presently investigating this possibility.

APPENDIX A

We wish to show here how one can derive the average number of partonic collisions, from the expression for the semihard cross section [Eq. (5)]. For simplicity we will indicate here all the degrees of freedom of the interaction probability $\hat\sigma_{ij}^{f_if_j'}$ with only one index.

Since the factor

$$\left[1-\prod_{i=1}^{n}\prod_{j=1}^{l}\prod_{f_i,f_j'}(1-\hat\sigma_{ij}^{f_if_j'})\right]\equiv\left[1-\prod_{\nu=1}^{Q}(1-\hat\sigma_\nu)\right] \qquad \text{(A1)}$$

in Eq. (5) is multiplied by a symmetric expression it is convenient to introduce the symmetrizing operator $\mathcal{S}$ defined as

$$\mathcal{S}x_1x_2\cdots x_k\equiv(x_1x_2\cdots x_k+x_1x_2\cdots x_{k+1}+\cdots+x_1x_2\cdots x_{k+3}+\cdots+x_2x_3\cdots x_{k+3}\cdots)\Big/\begin{bmatrix}n\\k\end{bmatrix} \qquad \text{(A2)}$$

so that one is choosing in all possible ways k elements in a set of n, summing the products and dividing by the number of combinations.

Let us first consider

$$\begin{aligned}\mathcal{S}\sum_{\nu=0}^{Q}\begin{bmatrix}Q\\\nu\end{bmatrix}(1-\hat\sigma_1)\cdots(1-\hat\sigma_\nu)\hat\sigma_{\nu+1}\cdots\hat\sigma_Q&=\mathcal{S}\sum_{\nu=0}^{Q}\begin{bmatrix}Q\\\nu\end{bmatrix}\sum_{\mu=0}^{\nu}(-1)^\mu\hat\sigma_{j_1}\cdots\hat\sigma_{j_\mu}\hat\sigma_{\nu+1}\cdots\hat\sigma_Q\begin{bmatrix}\nu\\\mu\end{bmatrix}\\&=\mathcal{S}\sum_{\nu=0}^{Q}\sum_{\mu=0}^{\nu}\frac{Q!}{(Q-\nu)!(\nu-\mu)!\mu!}(-1)^\mu\hat\sigma_{l_1}\cdots\hat\sigma_{l_{Q-\nu+\mu}}\\&=\mathcal{S}\sum_{\lambda=0}^{Q}\sum_{\mu=0}^{Q-\lambda}\frac{Q!(Q-\lambda)!}{(Q-\mu-\lambda)!\lambda!\mu!(Q-\lambda)!}(-1)^\mu\hat\sigma_{l_1}\cdots\hat\sigma_{l_{Q-\lambda}}\\&=\mathcal{S}\sum_{\lambda=0}^{A}\begin{bmatrix}Q\\\lambda\end{bmatrix}(1-1)^{Q-\lambda}\hat\sigma_{l_1}\cdots\hat\sigma_{l_{Q-\lambda}}=1\ .\end{aligned} \qquad \text{(A3)}$$

One will then write the relation

$$\mathcal{S}\sum_{\nu=0}^{Q-1}\begin{bmatrix}Q\\\nu\end{bmatrix}(1-\hat\sigma_1)\cdots(1-\hat\sigma_\nu)\hat\sigma_{\nu+1}\cdots\hat\sigma_Q=\mathcal{S}\sum_{\nu=0}^{Q}-(\nu=Q)=1-(1-\hat\sigma_1)\cdots(1-\hat\sigma_Q)\ . \qquad \text{(A4)}$$

We can now evaluate the average number of collisions:

$$\begin{aligned}\mathcal{S}\sum_{\nu=0}^{Q}\nu\begin{bmatrix}Q\\\nu\end{bmatrix}(1-\hat\sigma_1)\cdots(1-\hat\sigma_\nu)\hat\sigma_{\nu+1}\cdots\hat\sigma_Q&=\mathcal{S}Q\sum_{\rho=0}^{Q-1}\begin{bmatrix}Q-1\\\rho\end{bmatrix}(1-\hat\sigma_1)\cdots(1-\hat\sigma_{Q-\rho-1})\hat\sigma_{Q-\rho}\cdots\hat\sigma_Q\\&=\mathcal{S}Q\hat\sigma_Q\sum_{\rho=0}^{Q-1}\begin{bmatrix}Q-1\\\rho\end{bmatrix}(1-\hat\sigma_1)\cdots(1-\hat\sigma_\rho)\sigma_{\rho+1}\cdots\hat\sigma_{Q-1}\\&=\mathcal{S}Q\hat\sigma_Q=\hat\sigma_1+\hat\sigma_2+\cdots\hat\sigma_Q\ ,\end{aligned} \qquad \text{(A5)}$$

which is the relation we wanted to prove.

APPENDIX B

In this appendix we will show how to obtain the complete expression for $\langle x_A x_B\rangle\sigma_H^{AB}$ and $\langle x_B^2\rangle\sigma_H^{AB}$, including the correction term that has been neglected when writing Eqs. (22) and (23) in the text. To simplify the notation we do not write the indices identifying the various species of partons.

Following Ref. 9 we introduce the generating functions for the collisions between n partons from nucleus A and m partons from nucleus B:

$$F(x_1,\ldots,x_n;y_1,\ldots,y_m)=\prod_{i=1}^{n}\prod_{j=1}^{m}(1-\hat{\sigma}_{ij}+x_iy_j\hat{\sigma}_{ij})\ . \tag{B1}$$

The expansion of F in powers of x's and y's provides all the probabilities for multiple collisions: e.g., the coefficient of the term containing $x_{i_1}^{n_1}x_{i_2}^{n_2}\cdots y_{j_1}^{m_1}y_{j_2}^{m_2}\cdots$ is the probability for i_1 to collide n_1 times, $i_2 n_2$ times, etc. (One will notice that $\sum_r n_r=\sum_s m_s$ by construction.)

We then introduce the notation

$$\begin{aligned}
&F(1,1)=F(1,\ldots,1;1,\ldots,1)=1\ ,\\
&F_i(0,1)=F(1,\ldots,x_i=0,\ldots,1;1,\ldots,1)=\prod_{j=1}^{m}(1-\hat{\sigma}_{ij})\ ,\\
&F_j(1,0)=F(1,\ldots,1;1,\ldots,y_j=0,\ldots,1)=\prod_{i=1}^{n}(1-\hat{\sigma}_{ij})\ ,\\
&F_{ij}(0,0)=F(1,\ldots,x_i=0,\ldots,1;1,\ldots,y_j=0,\ldots,1)\\
&\qquad=(1-\hat{\sigma}_{ij})\prod_{l\neq j}(1-\hat{\sigma}_{il})\prod_{k\neq i}(1-\hat{\sigma}_{kj})\ .
\end{aligned} \tag{B2}$$

To get the probability for i to have at least one interaction one then writes

$$\sum_{k=1}^{n}\frac{1}{k!}\left[\frac{\partial}{\partial x_i}\right]^k F(x_1,\ldots,x_A;y_1,\ldots,y_B)|_{x=0,y=0}=F(1,1)-F_i(0,1)\ . \tag{B3}$$

The probability for x_i and y_j to have at least one interaction is, analogously,

$$F(1,1)-F_i(0,1)-F_j(1,0)+F_{ij}(0,0)\ . \tag{B4}$$

One then has

$$\begin{aligned}
\langle x_Ax_B\rangle\sigma_H^{AB}=\int d^2\beta&\sum_{n=1}^{\infty}\frac{1}{n!}\Gamma_A(x_1,\mathbf{b}_1)\cdots\Gamma_A(x_n,\mathbf{b}_n)\exp\left[-\int\Gamma_A(x,\mathbf{b})dx\,d^2b\right]\\
&\times\sum_{k=1}^{\infty}\frac{1}{k!}\Gamma_B(x_1',\mathbf{b}_1'-\boldsymbol{\beta})\cdots\Gamma_B(x_k',\mathbf{b}_k'-\boldsymbol{\beta})\exp\left[-\int\Gamma_B(x',\mathbf{b}')dx'd^2b'\right]\\
&\times\sum_{i=1}^{n}\sum_{j=1}^{k}x_ix_j'[F(1,1)-F_i(0,1)-F_j(1,0)+F_{ij}(0,0)]\\
&\times dx_1d^2b_1\cdots dx_nd^2b_ndx_1'd^2b_1'\cdots dx_k'd^2b_k'\ .
\end{aligned} \tag{B5}$$

The contribution from $F(1,1)$ is

$$\int d^2\beta\,\Gamma_A(x_1,\mathbf{b}_1)\Gamma_B(x_1',\mathbf{b}_1'-\boldsymbol{\beta})x_1dx_1d^2b_1x_1'dx_1'd^2b_1'\ . \tag{B6}$$

The contribution from $F_i(0,1)$ is

$$\int d^2\beta\,\Gamma_A(x_1,\mathbf{b}_1)(1-\hat{\sigma}_{11'})\Gamma_B(x_1',\mathbf{b}_1'-\boldsymbol{\beta})\exp\left[-\int\Gamma_B(x_2',\mathbf{b}_2'-\boldsymbol{\beta})\hat{\sigma}_{12'}dx_2'd^2b_2'\right]x_1dx_1d^2b_1x_1'dx_1'd^2b_1'\ . \tag{B7}$$

The contribution from $F_j(1,0)$ is obtained exchanging A and B in $F_i(0,1)$. The contribution from $F_{ij}(0,0)$ is

$$\begin{aligned}
&\int d^2\beta\,x_1dx_1d^2b_1x_1'dx_1'd^2b_1'\Gamma_A(x_1,\mathbf{b}_1)(1-\hat{\sigma}_{11'})\Gamma_B(x_1',\mathbf{b}_1'-\boldsymbol{\beta})\\
&\qquad\times\exp\left[-\int\Gamma_B(x_2',\mathbf{b}_2'-\boldsymbol{\beta})\hat{\sigma}_{12'}dx_2'd^2b_2'\right]\exp\left[-\int\Gamma_A(x_2,\mathbf{b}_2)\hat{\sigma}_{21'}dx_2d^2b_2\right]\ .
\end{aligned} \tag{B8}$$

All together one gets

$$\int d^2\beta\, x_1 dx_1 d^2b_1 x_1' dx_1' d^2b_1' \Gamma_A(x_1,\mathbf{b}_1)\left[1-\exp\left[-\int \Gamma_B(x_2',\mathbf{b}_2'-\beta)\hat{\sigma}_{12'}dx_2' d^2b_2'\right]\right]$$
$$\times \Gamma_B(x_1',\mathbf{b}_1'-\beta)\left[1-\exp\left[-\int \Gamma_A(x_2,\mathbf{b}_2)\hat{\sigma}_{21'}dx_2 d^2b_2\right]\right], \quad \text{(B9)}$$

that is the expression given in Eq. (23), and the correction term

$$\int d^2\beta\, x_1 dx_1 d^2b_1 x_1' dx_1' d^2\mathbf{b}_1' \Gamma_A(x_1,\mathbf{b}_1)\hat{\sigma}_{11'}\Gamma_B(x_1',\mathbf{b}_1'-\beta)$$
$$\times\left\{1-\left[1-\exp\left[-\int \Gamma_B(x_2',\mathbf{b}_2'-\beta)\hat{\sigma}_{12'}dx_2' d^2b_2'\right]\right]\left[1-\exp\left[-\int \Gamma_A(x_2,\mathbf{b}_2)\hat{\sigma}_{21'}dx_2 d^2b_2\right]\right]\right\}. \quad \text{(B10)}$$

One will notice that, in the limit of a large number of collisions, the term in the curly brackets will get a negligible contribution from the overlap region between the two nuclei, unless $\mathbf{b}_1$ or $\mathbf{b}_1'$ are close to the nuclear border.

The correction term is then a term of order $A^{1/3}$ while the dominant term is of order $A^{2/3}$. In order to evaluate the correction term to the squared energy emitted by the partons of nucleus B we make use of Eq. (B3) twice. We get

$$\langle x_B^2\rangle \sigma_H^{AB}=\int d^2\beta \sum_{n=1}^{\infty}\frac{1}{n!}\Gamma_A(x_1,\mathbf{b}_1)\cdots\Gamma_A(x_n,\mathbf{b}_n)\exp\left[-\int \Gamma_A(x,\mathbf{b})dx\, d^2b\right]$$
$$\times\sum_{k=1}^{\infty}\frac{1}{k!}\Gamma_B(x_1',\mathbf{b}_1'-\beta)\cdots\Gamma_B(x_k',\mathbf{b}_k'-\beta)\exp\left[-\int \Gamma_B(x',\mathbf{b}')dx' d^2b'\right]$$
$$\times\left[\sum_{j=1}^{B} x_j'^2[F(1,1)-F_j(1,0)]\right.$$
$$\left.+\sum_{j=1}^{B}\sum_{i\neq j}^{1\cdots B} x_j' x_i'[F(1,1)-F_i(1,0)][F(1,1)-F_i(1,0)]\right]dx_1 d^2b_1\cdots dx_k' d^2b_k' . \quad \text{(B11)}$$

From the first term we get the expression (22a); the second term can be splitted into two pieces, the first one of which reproduces expression (22b) while the second one is

$$\int d^2\beta\, x''\Gamma_B(x'',\mathbf{b}''-\beta)V(x',x'';\mathbf{b}',\mathbf{b}'')x'\Gamma_B(x',\mathbf{b}'-\beta)$$
$$\times\exp\left[-\int \Gamma_A(x,\mathbf{b}')\hat{\sigma}(x,x')dx\right]\exp\left[-\int \Gamma_A(x,\mathbf{b}'')\hat{\sigma}(x,x'')dx\right]dx'dx''d^2b'd^2b'' , \quad \text{(B12)}$$

where V is defined as

$$V=\exp\left[\int \hat{\sigma}(x'',\mathbf{b}'';x,\mathbf{b})\Gamma_A(x,\mathbf{b})\hat{\sigma}(x,\mathbf{b};x',\mathbf{b}')dx\, d^2b\right]-1 .$$

One should notice that, since $\hat{\sigma}$ is a probability, it will never exceed 1; as a consequence the positive exponential in V will always be compensated by the negative ones in expression (B10). In the limit of a large number of partons one has then that Eq. (B12), analogously to Eq. (B10), gives a contribution of order $A^{1/3}$ to be compared with the dominant contributions that are rather of order $A^{2/3}$.

APPENDIX C

In this appendix the origin and consequences of possible deviations from a strict Poissonion distribution for the nuclear parton population are discussed. We take into consideration two possible sources of deviation: The first one is the effect of the intermediate nucleon structure, which mainly enters into the game inducing correlations; in the second case the partonic distribution deviates from a Poissonian already at nucleonic level. The treatment discussed until now relies on a partonic description of the whole nucleus [see Eq. (4)] and the only explicit remnant of the nuclear properties is in the index A appended to the functions $\Gamma_A^f(x_i,\mathbf{b}_i)$.

The effects of the intermediate nucleonic level could show up in different ways. We have chosen a well-defined nuclear effect which is certainly present; it is the correlation between nucleons and the related fluctuation in the nuclear density.

In the absence of density fluctuations, partonic Poissonian distributions from the single nucleons would result in a strictly Poissonian distribution of partons for the whole nucleus, at fixed $\mathbf{b}$; if, on the contrary, there is a fluctuation in the nucleon number, at a fixed impact parameter, then the resulting distribution cannot be Poissonian, as discussed, in general, in Ref. 9.

We will consider here the correlations rising from the Pauli principle. The nucleus will be represented by a Fermi gas (at zero temperature) in a rigid box as, e.g., in Ref. 19.

Starting from the normalized nuclear wave function one defines

$$w_A(\mathbf{r}_1\cdots\mathbf{r}_A)=|\Phi(\mathbf{r}_1\cdots\mathbf{r}_A)|^2\ ,$$
$$w^{(1)}(\mathbf{r}_1)=\int d\mathbf{r}_2\cdots d\mathbf{r}_A w_A\ , \qquad \text{(C1)}$$
$$w^{(2)}(\mathbf{r}_1,\mathbf{r}_2)=\int d\mathbf{r}_3\cdots d\mathbf{r}_A w_A\ .$$

Since, in this description, the surface effects are neglected, the one-body distribution is constant: $w^{(1)}=1/V$, with $V=(4\pi/3)R^3$ the volume.

We always work in the impact-parameter representation and therefore it turns out to be convenient to introduce the distributions

$$F_A(\mathbf{s}_1\cdots\mathbf{s}_A)=\int dz_1\cdots dz_A w_A\ ,$$
$$F^{(1)}(\mathbf{s}_1)=\int dz_1 w^{(1)}\ , \qquad \text{(C2)}$$
$$F^{(2)}(\mathbf{s}_1,\mathbf{s}_2)=\int dz_1 dz_2 w^{(2)}\ ,$$

where $r^2=s^2+z^2$.

Following Ref. 19 we get

$$w^{(2)}(\mathbf{r}_1,\mathbf{r}_2)=\frac{A}{A-1}[w(\mathbf{r}_1)w(\mathbf{r}_2)-c(\mathbf{r}_1,\mathbf{r}_2)]\ .$$

The correlation function c depends only on the relative coordinate $\mathbf{r}=\mathbf{r}_1-\mathbf{r}_2$. Actually,

$$c(r)=\frac{1}{V}\cdot\frac{9}{u^4}\left[\frac{\sin u}{u}-\cos u\right]^2$$

and

$$\int d^3r\, c(r)=\frac{1}{AV}\ ,$$

where $u=rk_F$ and the Fermi momentum is taken equal both for neutrons and protons ($N=Z=\frac{1}{2}A$), $k_F^3=\frac{3}{2}\pi^2A/V$.

An important property of c is that it differs sizably from zero only in a range of r which is, in a heavy nucleus, quite small with respect to the nuclear radius; as a consequence the integration in r can be extended to infinity.

Going to the impact-parameter representation we get

$$F^{(1)}(\mathbf{s})=f(s)=\frac{2R}{V}(1-s^2/R^2)^{1/2}\theta(R-s)\ ,$$
$$F^{(2)}(\mathbf{s}_1,\mathbf{s}_2)=f(s_1)f(s_2)+g(\mathbf{s}_1,\mathbf{s}_2)\ , \qquad \text{(C3)}$$

with

$$g(\mathbf{s}_1,\mathbf{s}_2)=\frac{1}{A-1}f(s_1)f(s_2)-\frac{A}{A-1}\int dz_1dz_2c(\mathbf{r}_1,\mathbf{r}_2)$$

and

$$\int g(\mathbf{s}_1,\mathbf{s}_2)d\mathbf{s}_1=\int g(\mathbf{s}_1,\mathbf{s}_2)d\mathbf{s}_2=0\ . \qquad \text{(C4)}$$

The analytical expression for g is rather cumbersome; in the case $\mathbf{s}_1\simeq\mathbf{s}_2$ (that is also the case where the effect of the Pauli principle is larger) one gets, however, a simple expression.

In this case $F^{(2)}$ is given by

$$F^{(2)}(\mathbf{S}-\tfrac{1}{2}\mathbf{s},\mathbf{S}+\tfrac{1}{2}\mathbf{s})=\frac{A}{A-1}\left[f^2(S)\left[1-\frac{s}{4}\frac{R^2+S^2}{(R^2-S^2)^2}\right]-f(S)\frac{6\pi}{5}\frac{1}{Vk_F}[1-\tfrac{11}{63}(k_Fs)^2]\right]\ . \qquad \text{(C5)}$$

The first part is from the factorized part (ff), the second from the correction (g).

The main observation is that, while for $\mathbf{r}_1=\mathbf{r}_2$ $w^{(2)}$ is zero, for $\mathbf{s}_1=\mathbf{s}_2$ $F^{(2)}$ is not zero. The negative correcting term is roughly 0.2 for $A\simeq 200$; the effect then is not a very relevant one unless $\mathbf{s}\to\mathbf{r}$, where, on the other hand, the surface effects are important. (If $\mathbf{s}$ is very close to $\mathbf{R}$ the expression for $F^{(2)}$ can no longer be trusted since the integration over z cannot be done over all the real axis in that case.)

As expected the z integration softens the effect of the Pauli principle in a rather efficient way; as a consequence, whenever an impact-parameter description is allowed, the effect, on large nuclei, can be treated perturbatively. We will then work out a definite example (neglecting for simplicity flavor indices). The semihard cross section [see Eq. (5)] then takes the form

$$\sigma_H^{AB}=\int d^2\beta\prod d\mathbf{s}F_A(\mathbf{s}_1\cdots\mathbf{s}_A)\prod d\mathbf{s}'F_B(\mathbf{s}_1'\cdots\mathbf{s}_B')$$
$$\times\sum_{\{n_Z\}}\frac{1}{n_Z!}\Gamma(x_{Z,1},\mathbf{b}_{Z,1}-\mathbf{s}_Z)\cdots\Gamma(x_{Z,n_Z},\mathbf{b}_{Z,n_Z}-\mathbf{s}_Z)$$
$$\times\sum_{\{l_Y\}}\frac{1}{l_Y!}\Gamma(x'_{Y,1},\mathbf{b}'_{Y,1}-\mathbf{s}'_Y)\cdots\Gamma(x'_{Y,l_Y},\mathbf{b}'_{Y,l_Y}-\mathbf{s}'_Y)$$
$$\times\exp\left[-A\int\Gamma(x,\mathbf{b})dx\,d^2b\right]\exp\left[-B\int\Gamma(x',\mathbf{b}')dx'b^2b'\right]$$
$$\times\left[1-\prod_{Z=1}^{A}\prod_{i=1}^{n_Z}\prod_{Y=1}^{B}\prod_{j=1}^{l_Y}(1-\hat{\sigma}_{Z,i;Y,j})\right]\prod dx\,dx'\prod d^2b\,d^2b'\ , \qquad \text{(C6)}$$

where $Z=1\cdots A, Y=1\cdots B$ refer to the nucleons in A and B, Γ are the parton distributions of the single nucleon, and $\{n_Z\}$ denotes the indices $n_1\cdots n_A$ that have to be summed.

The mean energy is easily obtained:

$$\langle x_B\rangle\sigma_H^{AB}=\int d^2\beta\prod ds F_A\prod ds' F_B\sum_Y x'dx'\Gamma(x',\mathbf{b}'-\mathbf{s}_Y)\left[1-\prod_Z[1-\eta(x',\mathbf{b}'-\mathbf{s}_Z)]\right]. \quad \text{(C7)}$$

η has been defined in Eq. (13); in this case, however, Γ refers to a single nucleon. The function F_B is symmetric in its arguments so the sum $\sum_Y$ gives simply a factor B; moreover, F_B can be integrated over $B-1$ arguments giving $f_B(s')$; on the contrary, all the arguments of F_A are present also in $1-\eta$ and therefore a more detailed treatment is needed in this case.

In general one may write

$$F_A(\mathbf{s})=f_A(s_1)\cdots f_A(s_A)+f_A(s_1)\cdots g_A(\mathbf{s}_j\mathbf{s}_k)\cdots f_A(s_A)$$
$$+f_A(s_1)\cdots h_A(\mathbf{s}_j\mathbf{s}_k\mathbf{s}_l)\cdots f_A(s_A)+f_A(s_1)\cdots g_A(\mathbf{s}_j\mathbf{s}_k)\cdots g_A(\mathbf{s}_m\mathbf{s}_n)\cdots f_A(s_A)+\cdots . \quad \text{(C8)}$$

Since the effect of the correlation is not large we keep (in addition to the factorized term) only the first-order term in g. Introducing this truncated expression in Eq. (C7) and using the approximation $(1-y)^A\simeq e^{-Ay}$ we get

$$\langle x_B\rangle\sigma_H^{AB}=\int d^2\beta Bf_B(s')\Gamma(x',\mathbf{b}'-\mathbf{s}')x'd\mathbf{s}'$$
$$\times\left[1-\exp\left[-A\int f_A(s)\eta(x',\mathbf{b}'-\mathbf{s})d\mathbf{s}\right]\right.$$
$$\left.-\frac{A^2}{2}\int g(\bar{\mathbf{s}},\bar{\bar{\mathbf{s}}})\eta(x',\mathbf{b}'-\bar{\mathbf{s}})\eta(x',\mathbf{b}'-\bar{\bar{\mathbf{s}}})\exp\left[-(A-2)\int f_A(s)\eta(x',\mathbf{b}'-\mathbf{s})d\mathbf{s}\right]d\bar{\mathbf{s}}d\bar{\bar{\mathbf{s}}}+\cdots\right]. \quad \text{(C9)}$$

The nuclear parton distributions are obtained convoluting f with Γ. More precisely,

$$\Gamma_B(x',\mathbf{b}'-\boldsymbol{\beta})=B\int f_0(s')\Gamma(x',\mathbf{b}'-\boldsymbol{\beta}-\mathbf{s}')d\mathbf{s}' ,$$

where, keeping into account that f_B is centered around $\boldsymbol{\beta}$, one has defined f_0 as $f_0(|\mathbf{s}'-\boldsymbol{\beta}|)\equiv f_B(s')$. The comparison with Eq. (21) shows two differences: The first is the term in g; the second is the appearance of η at the exponent. One will notice that if η is "small" then $\eta\simeq\Gamma\hat{\sigma}$ and defining

$$\Gamma_A(x,\mathbf{b}_v)=A\int f_A(s_v)\Gamma(x,\mathbf{b}-\mathbf{s}_v)d\mathbf{s}_v$$

one gets the same exponent as in Eq. (21). The correction term in (C9) looks proportional to A^2 and thus potentially large. It may, however, be recast in the form $\frac{1}{2}\int\Gamma_A\hat{\sigma}\Gamma_A\hat{\sigma}(g/ff)$ and, in accordance with the previous discussion, the term in parentheses is small.

The same procedure can be applied also in computing $\langle x_B^2\rangle$ and $\langle x_Ax_B\rangle$, and as the details are rather complicated we will only sketch the calculation.

The term $\langle x_B^2\rangle$ has three different kinds of contributions: one of kind $x_{jY}'^2$ (energy squared of one parton), one of kind $x_{jY}'x_{iY}'$ (energies of two partons of the same nucleon), and one of kind $x_{jY}'x_{iW}'$ (energies of two partons of different nucleons). The first two contributions do not involve the correlation function for the nucleus B; the third does. The term $\langle x_Ax_B\rangle$ finally will depend on the correlation functions of both nuclei that will act between the terms linear in Γ and the exponential terms.

In conclusion we see that a systematic way to treat the density fluctuations of the nucleus is available and that this kind of perturbation does not play an important role in the problems discussed in the present paper.

There is a question about the parton distribution which is complementary to the problem analyzed above, namely, that one can ask how much the Poissonian distribution is fundamental for the conclusions which have been drawn. The question in this form is too general, so we will look at a much more specialized case, where one keeps the distribution still factorized, but not Poissonian. The alternative chosen is the negative-binomial distribution, which, although in a different context, has been suggested as relevant in high-energy multiplicity phenomena.[20]

We start, therefore, with a distribution

$$\begin{bmatrix}-\alpha\\ n\end{bmatrix}(-)^n\Lambda(x_1,\mathbf{b}_1)\cdots\Lambda(x_n,\mathbf{b}_n)\left[1-\int\Lambda(x,\mathbf{b})dx\,d^2b\right]^{\alpha}.$$

From the exclusive distribution Λ one can derive the partonic density which is

$$\mathcal{D}(x,\mathbf{b})=\alpha\frac{\Lambda(x,\mathbf{b})}{1-\int\Lambda(x,\mathbf{b})dx\,d^2b} .$$

One can use the previously given formulation in order to calculate physical observables. As an example we find

$$\langle x_B'\rangle\sigma_H=\int d^2\beta\,dx'd^2b'x'\mathcal{D}(x',\mathbf{b}'-\boldsymbol{\beta})$$
$$\times\left[1-\left[1+\frac{1}{\alpha}\int\mathcal{D}(x,\mathbf{b}')\hat{\sigma}(xx')dx\right]^{-\alpha}\right] \quad \text{(C10)}$$

The geometric limit corresponds to $\int\mathcal{D}\hat{\sigma}\to\infty$ at fixed α. As is well known one can obtain the Poisson distribution and the related result in a limiting case: $\Lambda=\Gamma/\alpha$, $\alpha\to\infty$, in fact in this case one gets back, from Eq. (C10), Eq. (21).

This kind of distribution can be taken for the single nucleon, then inserted into the nuclear structure as in (C7) (which remains valid provided an obvious redefinition of η is made) and, in the same way, Eq. (C9) is obtained. When $\mathcal{D}$ is small, the approximation $\eta \simeq \mathcal{D}\hat{\sigma}$ holds and the parameter α disappears. We are induced to conclude, in this example, that all the features of the Poisson distribution are reproduced, not because of the original partonic distribution, but because of the large-A effect.

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