Bloch-Nordsieck regularization of QCD transverse-momentum distributions

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An expression for transverse-momentum distributions is presented for Drell-Yan-type processes: it is infrared finite, includes soft-gluon summation in the leading-logarithmic approximation and reproduces the first-order perturbative results at large p_T. In addition, our expression incorporates soft QCD radiative corrections to that part of the cross section where hard bremsstrahlung dominates. We compute W transverse-momentum distributions and average values for a set of different energies: $\sqrt{s} = 630$ GeV, 1.8 TeV, 6 TeV, and 18 TeV.

I. INTRODUCTION

The question of how final-state particles acquire their transverse momentum in hadron-hadron collisions is of great theoretical and phenomenological interest since it can shed light on the dynamics of interaction between hadronic constituents. A typical example is offered by the Drell-Yan process, in which the final-state muon pair is seen to acquire a Q^2-dependent transverse momentum through initial-state bremsstrahlung from the colliding quark-antiquark pair.[1,2] This mechanism was shown to hold for Drell-Yan pairs observed at the CERN ISR and at Fermilab.[3] At higher energies, the comparison between theory and experiments for the case of W and Z production is considered to be an important test of QCD at the CERN Collider.[4,5] For transverse momenta of the order of less than 10% of the W mass, the calculation involves the use of soft-gluon summation techniques, while for higher transverse-momentum values the usual perturbative expansion is quite adequate. However, since the perturbative calculation diverges at small p_T, particular care must be taken in joining soft and hard terms, so as to reproduce correctly the perturbative limit, and at the same time avoid double counting in the soft region. We would like to point out that this is a general problem, present also in the case of jet production, where the question of how to include both high- and low-E_T jets (the latter are often called, perhaps improperly, minijets) has not been completely solved. In this paper we present a formalism for regularizing the transverse-momentum divergence which is based on an infrared regulator, rather than the usual δ-function prescription. We believe this method to be numerically simpler than the ones present in the literature and to offer, in addition, a very transparent physical picture which can be applied to other prob-

lems such as minijets.

We shall start with the soft QCD bremsstrahlung formula which gives a finite distribution for W production in the low-p_T region. This formula can be derived using a semiclassical approach based on the Bloch-Nordsieck method for QED, but its validity has also been checked through perturbative calculations.

We then apply the same formula to perform soft radiative corrections to production of a W boson and one hard jet. The subtractions needed to avoid double counting in the soft region will then show how the soft bremsstrahlung distribution can act as an infrared regulator. We present results at four different energies, $\sqrt{s} = 630$ GeV, 1.8 TeV, 6 TeV, and 18 TeV, and show both the normalized transverse-momentum distributions and the absolute cross sections.

II. LEADING-LOG QCD RADIATIVE CORRECTIONS TO W PRODUCTION

The two-vector $\mathbf{p}_T$ probability distribution of the W may be written in the parton model with QCD radiation as

$$\frac{dP}{d\mathbf{p}_T^W} = \frac{dP}{d\mathbf{K}_T},$$

where

$$\mathbf{p}_T^W = -\sum \mathbf{p}^{\text{gluons}} \equiv \mathbf{K}_T \tag{1}$$

obtains its value from the transverse momentum due to initial-state bremsstrahlung. The multiple soft-gluon emission calculation (to all orders in α_s) is available in the leading-log approximation[5,6] (LLA). A calculation which includes hard-gluon emission probability is avail-

able to order α_s in Ref. 5. A Monte Carlo calculation is available to order α_s^2 in Ref. 7.

In the following we shall approach the problem using a semiclassical four-dimensional derivation.

For soft massless quanta emitted by a semiclassical source, the four-dimensional probability distribution is given by[8]

$$d^4P(K) = \sum P(\{n_{k'}\})\delta^4 \left[K - \sum_{k'} k'n_{k'} \right] d^4K \ . \tag{2}$$

The four-dimensional δ function selects the distributions with the correct energy-momentum loss K. The above distribution then reduces to

$$d^4P(K) = \int \frac{d^4x}{(2\pi)^4} \exp\left[iK\cdot x - \int d^3n(k)[1-\exp(-ik\cdot x)] \right] d^4K \ , \tag{3}$$

where

$$d^3n(k) = \frac{d^3k}{2k_0} |j_\mu(k)|^2 \tag{4}$$

with

$$|j_\mu(k)|^2 = c_{ij} \frac{2\alpha_s}{\pi^2} \frac{1}{k_T^2}$$

and $c_{ij} = \frac{4}{3}$ or 3 for quark or gluon source, respectively.

Thus, the distribution of the vector-boson transverse momentum due to soft initial radiation is given by

$$\frac{d^2P(K)}{d^2\mathbf{K}_T} = \int dK_0 \int dK_3 \frac{d^4P}{d^4K} \ . \tag{5}$$

Performing the integrations in Eqs. (3)–(5), one obtains for the soft radiation

$$\frac{dP^{\text{soft}}}{p_T dp_T} = \int_0^\infty b\,db\,J_0(p_T b)\exp[-h(E_W;b)] \ , \tag{6}$$

where

$$h(E_W;b) = \frac{8}{3\pi} \int_0^{E_W} \frac{dk_T}{k_T} \alpha_s(k_T^2)\ln\frac{E_W+\sqrt{E_W^2-k_T^2}}{E_W-\sqrt{E_W^2-k_T^2}}$$

$$\times [1-J_0(bk_T)]$$

$$\equiv \int_0^{E_W} dk_T(2\pi k_T)f(k_T)[1-J_0(bk_T)] \tag{7}$$

with

$$f(k_T) = \frac{4\alpha_s(k_T^2)}{3\pi^2} \frac{1}{k_T^2} \ln\frac{E_W+\sqrt{E_W^2-k_T^2}}{E_W-\sqrt{E_W^2-k_T^2}} \ . \tag{8}$$

Notice that the soft distribution depends upon the behavior of α_s as $k_T\to 0$ as well as from the upper limit E_W, which is rather arbitrary. For instance, the soft distribution discussed in Ref. 6 was obtained with a singular, but integrable, α_s given by

$$\alpha_s(k_T^2) = \frac{12\pi}{25} \frac{p}{\ln[1+p(k_T^2/\Lambda^2)^p]}$$

with $p = \frac{5}{6}$ and $E_W \approx M_W/2$.[6] This choice of α_s avoided the introduction of the intrinsic transverse momentum, a parameter needed to justify a nonzero $\langle p_T \rangle$ at very small energies. Different choices for α_s are given in Refs. 5 and 9.

The soft contribution to the differential cross section for W production can then be written as

$$\frac{d\sigma^{\text{soft}}}{d^2\mathbf{p}_T dy} = \sigma^0 \int \int F(\mathbf{Q}_T)\delta(\mathbf{k}_T)\delta(\mathbf{p}_T-\mathbf{k}_T-\mathbf{Q}_T)d^2\mathbf{Q}_T d^2\mathbf{k}_T \tag{9}$$

with

$$\sigma^0 = \frac{d\sigma^W}{dy}$$

$$= \hat{\sigma}^0 \sum_i [q_i(x_1,m_W^2)\bar{q}_i(x_2,m_W^2) + 1\leftrightarrow 2] \ , \tag{10}$$

where

$$x_1 = \sqrt{\tau}e^y, \quad x_2 = \sqrt{\tau}e^{-y} \ ,$$

$$\tau = \frac{m_W^2}{s}, \quad \hat{\sigma}^0 = \frac{\pi^2\alpha}{3\sin^2\theta_W} \frac{1}{s} \ ,$$

and with

$$F(\mathbf{Q}_T) \equiv \frac{dP^{\text{soft}}}{d^2\mathbf{Q}_T} \ . \tag{11}$$

In Eq. (9) we have folded the soft distribution $F(\mathbf{Q}_T)$ with the perturbative contribution, which in this case is just a δ function in transverse momentum.

The above distribution applies to the case when the emitted gluon energy is not larger than 20–30 % of the W mass. For really-hard-gluon emission, when $p_T^W \approx m_W$, the p_T distribution is given by the first- and eventually second-order perturbative expression. There is however an intermediate range such as

$$\frac{m_W}{4} \leq p_T^W \leq \frac{m_W}{2} \ ,$$

where the W boson acquires its transverse momentum through both soft- and hard-gluon emission, and we shall concentrate in that region in what follows.

III. LEADING-LOG QCD RADIATIVE CORRECTIONS TO W + JET PROCESSES

Consider the process drawn in Fig. 1 to which the subprocess diagrams of Fig. 2 contribute. We now apply the formalism of the preceding section to the process of Fig. 1 and write
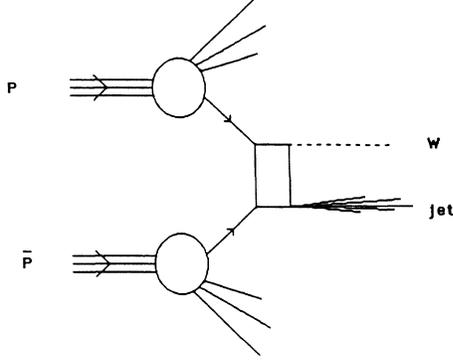
FIG. 1. W+jet production in $p\bar{p}$ collisions.

$$\frac{d\sigma^{W+\text{jet}}}{d^2\mathbf{p}_T dy}$$

$$= \int \frac{dL^{(1)}}{d^2\mathbf{k}_T dy} F(\mathbf{Q}_T) d^2\mathbf{k}_T d^2\mathbf{Q}_T \delta^2(\mathbf{p}_T - \mathbf{k}_T - \mathbf{Q}_T)$$

$$= \int \frac{dL^{(1)}}{d^2\mathbf{k}_T dy} F(\mathbf{p}_T - \mathbf{k}_T) d^2\mathbf{k}_T , \qquad (12)$$

where $dL^{(1)}/d^2\mathbf{k}_T dy$ corresponds to the cross section for W+jet emission and $F(\mathbf{Q}_T)$ is the probability distribution for soft QCD radiation of total momentum $\mathbf{Q}_T$. In this way the total observed p_T^W appears as the convolution of two contributions: soft initial-state radiation together with hard scattering Compton and $q\bar{q}$ annihilation in Wg. However, of the two distributions, while $F(\mathbf{p}_T)$ is finite as $\mathbf{p}_T \to 0$, $dL^{(1)}/d^2\mathbf{k}_T dy$ has to be defined. To do that, we start by considering the differential cross section for the process shown in Fig. 1, i.e., order-α_s corrections to W
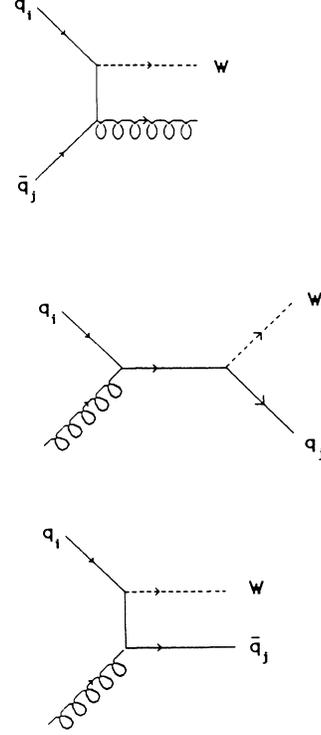
FIG. 2. Feynman diagrams for W production through $q\bar{q}$ annihilation and Compton scattering processes.

production.

This cross section is given by[10]

$$\frac{d^2\sigma^{(1)}}{dp_T dy} = \frac{d\sigma^{q\bar{q}}}{dp_T dy} + \frac{d\sigma^{qg}}{dp_T dy} \qquad (13)$$

with

$$\frac{d\sigma^{q\bar{q}}}{dp_T dy} = \frac{4\pi\alpha}{9\sin^2\theta_W} \frac{\alpha_s(p_T^2)}{s} \frac{1}{p_T} \int_{x_{\min}}^1 dx_1 \sum_i [q_i(x_1,m_W^2)\bar{q}_i(x_2,m_W^2) + 1\leftrightarrow 2] \left[\frac{1 + \tau^2/x_1^2 x_2^2 - x_T^2/2x_1 x_2}{x_1 - \frac{1}{2}\bar{x}_T e^y} \right] \qquad (14)$$

and

$$\frac{d\sigma^{qg}}{dp_T dy} = \frac{\pi\alpha}{3\sin^2\theta_W} \alpha_s(p_T^2) \frac{p_T}{s^2} \int_{x_{\min}}^1 \frac{dx_1}{x_1 - \frac{1}{2}\bar{x}_T e^y} \frac{1}{x_1^2 x_2^2} \sum_i \left[q_i(x_1,m_W^2)G(x_2,m_W^2)\frac{(x_1 x_2 - \tau)^2 + \frac{1}{4}(x_1 x_2 + \tau + V)^2}{x_1 x_2 - \tau + V} \right.$$

$$\left. + (1\leftrightarrow 2, V\leftrightarrow -V) \right] \qquad (15)$$

and with

$$\tau = \frac{m_W^2}{s}, \quad x_T^2 = \frac{4p_T^2}{s}, \quad \bar{x}_T^2 = x_T^2 + 4\tau .$$

We also have the definitions

$$x_2 = \frac{\frac{1}{2}\bar{x}_T e^{-y} x_1 - \tau}{x_1 - \frac{1}{2}\bar{x}_T e^y} ,$$

$$x_{\min} = \frac{\frac{1}{2}\bar{x}_T e^y - \tau}{1 - \frac{1}{2}\bar{x}_T e^{-y}} ,$$

$$V = x_1 x_2 + \tau - x_1 \bar{x}_T e^{-y} ,$$

where $\sqrt{s}$ is the proton-antiproton center-of-mass energy, G and q_i are the (singular) gluon and quark densities, respectively. We shall omit the explicit y dependence in the following. All the calculations will be done for $y=0$.

In order to get the overall W differential cross section, we have to add the soft contribution of Eq. (9) to the hard one of Eq. (12) without double counting. Double counting may occur unless the hard contribution in the soft radiation region is subtracted. Let q_{max} be the maximum value for such soft-gluon emission defined as the k_T-integration limit E_W in Eq. (7). Then one can avoid double counting by writing[11]

$$\frac{d\sigma}{d^2\mathbf{p}_T} = \sigma^0 F(\mathbf{p}_T) + \int_0^{q_{max}} \frac{dL^{(1)}}{d^2\mathbf{k}_T} [F(\mathbf{p}_T - \mathbf{k}_T)$$
$$- F(\mathbf{p}_T)] d^2\mathbf{k}_T$$
$$+ \int_{q_{max}}^{\infty} \frac{dL^{(1)}}{d^2\mathbf{k}_T} F(\mathbf{p}_T - \mathbf{k}_T) d^2\mathbf{k}_T , \qquad (16)$$

where, as k_T becomes soft, the contribution of $L^{(1)}$ is reduced, and we have an expression for the differential cross section that joins smoothly the soft and the hard contributions.

This distribution has to be such that the average value of p_T^2 verifies

$$\langle p_T^2 \rangle = \langle p_T^2 \rangle^{\text{1st order}}$$

and this constraint will define $dL^{(1)}/d^2\mathbf{k}_T$.

First of all we rewrite Eq. (16) as

$$\frac{d\sigma}{d^2\mathbf{p}_T} = (\sigma^0 + \sigma^1) F(\mathbf{p}_T)$$
$$+ \int_0^{\infty} \frac{dL^{(1)}}{d^2\mathbf{k}_T} [F(\mathbf{p}_T - \mathbf{k}_T) - F(\mathbf{p}_T)] d^2\mathbf{k}_T , \qquad (17)$$

where

$$\sigma^1 \equiv \int_{q_{max}}^{\infty} \frac{dL^{(1)}}{d^2\mathbf{k}_T} d^2\mathbf{k}_T \qquad (18)$$

and we notice that $\int d\mathbf{p}_T (d\sigma/d\mathbf{p}_T) = (\sigma^0 + \sigma^1)$ because the soft distribution $F(\mathbf{p}_T)$ is normalized to 1.

Then

$$\langle p_T^2 \rangle = \int p_T^2 F(\mathbf{p}_T) d^2\mathbf{p}_T + \frac{1}{\sigma^0 + \sigma^1} \int p_T^2 \int \frac{dL^{(1)}}{d^2\mathbf{k}_T} [F(\mathbf{p}_T - \mathbf{k}_T) - F(\mathbf{p}_T)] d^2\mathbf{k}_T d^2\mathbf{p}_T$$

$$= \int p_T^2 F(\mathbf{p}_T) d^2\mathbf{p}_T + \frac{1}{\sigma^0 + \sigma^1} \int \frac{dL^{(1)}}{d^2\mathbf{k}_T} k_T^2 d^2\mathbf{k}_T$$

and expanding $F(\mathbf{p}_T)$ we have, to first order in α_s,

$$\int p_T^2 f(p_T) d^2\mathbf{p}_T + \frac{1}{\sigma^0 + \sigma^1} \int \frac{dL^{(1)}}{d^2\mathbf{k}_T} k_T^2 d^2\mathbf{k}_T \equiv \frac{1}{\sigma^0 + \sigma^1} \int p_T^2 \frac{d\sigma^1}{d^2\mathbf{p}_T} d^2\mathbf{p}_T ,$$

where $f(p_T)$ is defined in Eq. (8) and $d\sigma^1/d^2\mathbf{p}_T$ is the first-order perturbative cross section, Eqs. (13)–(15). Then,

$$\frac{dL^{(1)}}{d^2\mathbf{k}_T} \equiv \frac{d\sigma^1}{d^2\mathbf{k}_T} - (\sigma^0 + \sigma^1) f(k_T) \qquad (19)$$

and, from Eq. (18),

$$\sigma^1 = \int_{q_{max}}^{\infty} \frac{d\sigma^1}{d^2\mathbf{k}_T} d^2\mathbf{k}_T \qquad (20)$$

because $f(k_T)$ is defined only up to $k_T = q_{max}$ (the maximum transverse momentum for soft-gluon emission).

The full distribution can now be written as

$$\frac{d\sigma}{d^2\mathbf{p}_T} = \left[\sigma^0 + \int_{q_{max}}^{\infty} \frac{d\sigma^1}{d^2\mathbf{k}_T} d^2\mathbf{k}_T \right] F^{\text{soft}}(\mathbf{p}_T) + \int_0^{\infty} \left[\frac{d\sigma^1}{d^2\mathbf{k}_T} - (\sigma^0 + \sigma^1) f(k_T) \right] [F^{\text{soft}}(\mathbf{p}_T - \mathbf{k}_T) - F^{\text{soft}}(\mathbf{p}_T)] d^2\mathbf{k}_T , \qquad (21)$$

where q_{max} is the only parameter we have to specify.

The maximum transverse momentum for the soft radiation is, for $y = 0$ (Ref. 3), $q_{max} = Q(1-z)/2\sqrt{z}$ with $z = Q^2/\hat{s}$, where $\hat{s} = x_1 x_2 s$ is the energy squared of the subprocess $q\bar{q} \to Wg$ and $Q^2 = m_W^2$ in the case of W production. Then the exact expression to calculate would be

$$\frac{d\sigma}{d^2\mathbf{p}_T} = \int dx_1 f(x_1, Q^2) \bar{f}(x_2, Q^2) \Bigg[[\hat{\sigma}^0 \delta(x_1 x_2 - \tau) + \hat{\sigma}^1] F^{\text{soft}}(\mathbf{p}_T, q_{max}(x_1, x_2))$$
$$+ \int_0^{\infty} d^2\mathbf{k}_T \left[\frac{d\hat{\sigma}^1}{d^2\mathbf{k}_T} - [\hat{\sigma}^0 \delta(x_1 x_2 - \tau) + \hat{\sigma}^1] f(k_T) \right]$$
$$\times [F^{\text{soft}}(\mathbf{p}_T - \mathbf{k}_T, q_{max}) - F^{\text{soft}}(\mathbf{p}_T, q_{max})] \Bigg] ,$$

TABLE I. Mean values of q_{max}, calculated by Eqs. (22) and (23).

$\sqrt{s}$ (TeV)	$q_{max}^{(a)}$ (GeV)	$q_{max}^{(b)}$ (GeV)
0.63	18.4	22.6
1.8	23.8	32.6
6	24.8	36.6
18	24.6	36.9

where

$$\hat{\sigma}^1 = \int_{q_{max}(x_1,x_2)}^{\infty} \frac{d\hat{\sigma}^1}{d^2k_T} d^2k_T \ .$$

However, in order to make the numerical calculations easier, we have calculated the mean value for q_{max} at different energies in two different ways and computed the distribution as in Eq. (21) at each energy for the two different values of q_{max}. Then we test how sensitive the result is to the value of q_{max}. For $y=0$ the averages are performed as

$$\langle q_{max}^{(a)} \rangle = \frac{\sqrt{s}}{2} \frac{\int_{\sqrt{\tau}}^{1} \frac{dx}{x^2} f_i(x)\bar{f}_j(x)(x - \tau/x)}{\int_{\sqrt{\tau}}^{1} \frac{dx}{x^2} f_i(x)\bar{f}_j(x)} \quad (22)$$

and

$$\langle q_{max}^{(b)} \rangle = \frac{\sqrt{s}}{2} \frac{\int_{\sqrt{\tau}}^{1} \frac{dx}{x} f_i(x)\bar{f}_j(x)(x - \tau/x)}{\int_{\sqrt{\tau}}^{1} \frac{dx}{x} f_i(x)\bar{f}_j(x)} \quad (23)$$

with $\tau = Q^2/s$ and $f_i(x)$ is a parton density.

In Table I we show the results for $\sqrt{s} = 0.63$, 1.8, 6, and 18 TeV. Notice that the value of q_{max} increases with energy but stabilizes around $\sqrt{s} = 6$ TeV.

We can now calculate the p_T distribution for different

FIG. 4. The differential cross section for the set of higher values of q_{max} ($q_{max}^{(b)}$) for the same energy values as in Fig. 3.

$\sqrt{s}$ values. Using the expression for $F^{soft}(p_T)$ [Eqs. (6) and (11)] we obtain

$$\frac{d\sigma}{dp_T} = (\sigma^0 + \sigma^1)\frac{dP^{soft}}{dp_T}$$

$$+ p_T \int dk_T \left[\frac{d\sigma^1}{dk_T} - (\sigma^0 + \sigma^1)(2\pi k_T)f(k_T) \right]$$

$$\times R(p_T, k_T; q_{max}) \ , \quad (24)$$

where the Bloch-Nordsieck infrared regulator has been defined as

$$R(p_T, k_T; q_{max}) = \int b \, db \, J_0(bp_T)e^{-h(b,q_{max})}[J_0(bk_T) - 1]$$

and the integration in Eq. (24) extends from zero to all the allowed values.

In the numerical calculation of $d\sigma^1/dk_T$ we have used $\alpha_s(k_T^2)$ rather than $\alpha_s(Q^2)$ as in Ref. 5. This has been

FIG. 3. The normalized p_T distribution for the two different values of q_{max} for $\sqrt{s} = 0.63$, 1.8, 6, and 18 TeV.
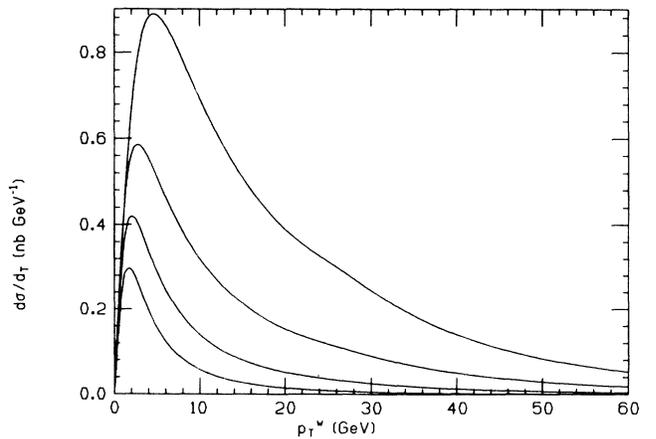
FIG. 5. As in Fig. 4, for the set of lower values of q_{max} ($q_{max}^{(a)}$).

TABLE II. Average p_T of W boson.

$\sqrt{s}$ (TeV)	$\langle p_T \rangle^{(a)}$ (GeV)	$\langle p_T \rangle^{(b)}$ (GeV)
0.63	7.27	7.36
1.8	10.96	11.15
6	14.86	15.22
18	17.8	18.6

done so as to be consistent with the argument of α_s which appears in $f(k_T)$. Then for the small-k_T limit we follow Ref. 5 and use the expression

$$\alpha_s(k_T^2) = \frac{12\pi}{27} \frac{1}{\ln[(k_T^2 + a\Lambda^2)/\Lambda^2]}$$

with $a = 2$.

In Fig. 3 we show the normalized p_T distribution for the two values of q_{max} for the same four values of the total energy as before. We have used the set of parton densities given by Eichten, Hinchliffe, Lane, and Quigg[12] (EHLQ) with $\Lambda = 0.2$ GeV.

One can see that the two values of q_{max} gives a very similar normalized distribution even at higher energies. This is an indication of how stable our regularization procedure is with regard to the choice of the arbitrary separation between soft and hard regimes.

From the above distributions one can numerically calculate the average transverse-momentum value acquired by the W boson at different energies: it is, again, rather independent of the choice of q_{max}. We show, in Table II, the relative numbers.

While the probability distribution is q_{max} independent, this is not true for the differential cross section, which depends upon the chosen q_{max} value through the quantity $\sigma_{tot} = \sigma^0 + \sigma^{(1)}$. In Table III we show the values of σ^0 [Eq. (10)] and $\sigma^1(q_{max})$ [Eq. (20)] for the four energy values.

In Figs. 4 and 5 we plot the differential cross section from Eq. (24) for the two values of q_{max}. At large-p_T values, all the cross sections correctly tend to their first-order perturbative result. An example of how our regularization procedure works in recovering the first-order perturbative result at large p_T is shown in Fig. 6, for $\sqrt{s} = 1.8$ TeV.

To further illustrate our normalization prescription, we plot separately in Fig. 7, the soft contribution, the hard one conveniently regularized once subtraction is performed, and the total contribution (solid line) to the differential cross section for $\sqrt{s} = 6$ TeV and $q_{max} = 36.6$ GeV.

TABLE III. Values of σ^0 and $\sigma^1(q_{max})$.

$\sqrt{s}$ (TeV)	σ^0	$\sigma^1(a)$	$\sigma^1(b)$
0.63	1.9 nb	0.15 nb	0.10 nb
1.8	3.7 nb	0.5 nb	0.3 nb
6	7.5 nb	2.0 nb	1.1 nb
18	14.6 nb	6.0 nb	3.4 nb
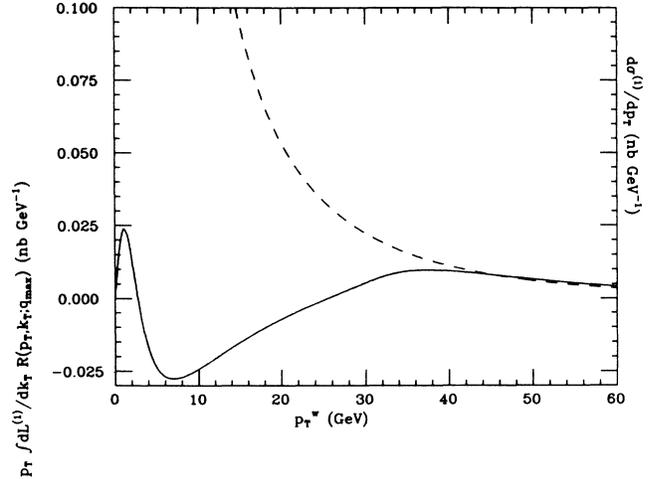
FIG. 6. The first-order perturbative distribution (dashed line) and the hard part conveniently regularized (solid line) for $\sqrt{s} = 1.8$ TeV and $q_{max} = 32.6$ GeV.

IV. CONCLUSIONS

The formalism which we have presented in this paper regularizes the infrared divergence encountered in QCD for transverse-momentum distributions, through a finite Bloch-Nordsieck-type function. This regulator is used in the cross section instead of the usual δ-function prescription, to avoid the divergence at small p_T in the first-order expansion. Its physical justification lies in the fact that the first-order calculation is meant to reproduce a physical process in which a hard jet is observed: in such a case one must also perform soft radiative corrections and it is through the latter that the Bloch-Nordsieck distribution
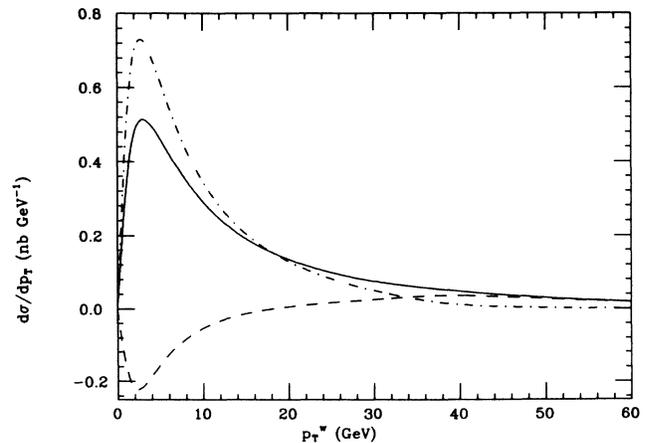
FIG. 7. The soft contribution (dotted-dashed line), the regularized hard one (dashed line), and the total differential cross section for $\sqrt{s} = 6$ TeV and $q_{max} = 36.6$ GeV.

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appears. At $\sqrt{s} = 630$ GeV our method reproduces within a few percent the calculations by Altarelli *et al.* in the soft region and it can be improved further by adding nonleading-logarithmic corrections.[13] At Fermilab Tevatron energies, our calculation predicts an average transverse momentum for the W boson lower than that in Ref. 5 (see Table II), and not very different from the one found at CERN energies.

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