Deformations of conformal field theories and symmetries of the string

Mark Evans
The Rockefeller Univeirsity, 1230 York Avenue, New York, New York 10021-6399
Burt A. Ovrut
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104-6396
(Received 17 April 1989)

Gauge symmetries in closed-string theory, and their relationship to world-sheet conformal invariance, are investigated. By studying the deformations of conformal field theories in the operator formalism, it is proved that to every primary field of dimension one there corresponds a gauge transformation. An infinite class of such gauge transformations is constructed, most of which relate string states of differing mass. It is argued that these symmetries are spontaneously broken by the space-time metric, but that this breaking becomes unimportant for high-energy scattering, thus shedding light on the results of Gross and Mende.

I. INTRODUCTION

It is a widely held view that the foundations of string theory, as currently formulated, are unsatisfactory. The physical picture of a string propagating through space-time is intuitive and appealing, but as soon as we attempt to introduce nontrivial dynamics the tension between unitarity and relativistic invariance appears to be strong enough to fix uniquely the interactions.[1] Precisely the same situation holds for gauge field theories. In the field-theoretic case, unitarity and Lorentz invariance are reconciled by gauge invariance, a space-time symmetry of the theory which facilitates the discard of ghostly states in a consistent manner. In string theory it is a *world-sheet* symmetry, conformal invariance, that performs the analogous task, but it is natural to ask whether there is a *space-time* symmetry, more closely analogous to field-theoretic gauge invariance, that also effects the reconciliation. If so, how is it related to world-sheet conformal invariance?

It seems likely that such a symmetry does indeed exist. After all, garden-variety gauge invariances (including, of course, general covariance) do appear miraculously from string theory, and scattering at high energies[2,3] (where one might expect symmetry breaking to be unimportant) seems to yield universal behavior that is strongly suggestive of a symmetry much larger than we yet know.[4] Given the infinite number of states of a string, and its exponential growth with mass level, such a symmetry would surely be immense. It might satisfy the frequently expressed need for a new fundamental principle for string theory.

A better understanding of symmetry can also be expected to yield important physical insights. Symmetry is a powerful tool for probing the properties of a theory with complicated dynamics. The reason is that symmetries are manifested in the classical theory, where they can be analyzed with relative ease, but have consequences which hold exactly (modulo anomalies) in the full quantum theory. Most of our understanding of low-energy QCD, for example, comes from our knowledge of approximate global symmetries and the manner in which they are realized. At shorter distances, much the same can be said about the standard model itself. What we "understand" about the standard model are its symmetries and the way in which they are realized. The action is the most general one renormalizable and consistent with this symmetry information, and so has a status somewhat analogous to, say, current algebra for QCD.

In the same spirit, a better understanding of the symmetries of string theory may provide a useful guide to solving one of its principal problems—choosing the correct vacuum. By examining each symmetry in turn, and asking how it should be realized for felicitous phenomenology, many constraints will be placed on the vacuum. For gauge symmetries, this program is already heavily used, but a more systematic understanding of their origin must surely help. The large discrete symmetry which the string appears to possess[5] (and about which we shall say little in this paper) provides a potentially rich additional source of information.

The problem of the symmetries of string theory has usually been addressed in the context of string field theory,[6,7] a subject beset with technical obstacles that do not have any obvious connection to the question of symmetry. In the first-quantized formulation, on the other hand, there are no such problems of principle in computing tree amplitudes on shell. This, as we remarked above, should provide enough information to deduce all the symmetries of the theory. We shall therefore work exclusively in the first-quantized formalism.

We shall argue that symmetries imply the existence of physically indistinguishable solutions to the string equations of motion, which should correspond to isomorphic conformal field theories. To understand such isomorphisms, we first study, in Sec. III, the deformation of conformal field theories in the operator formalism. In Sec. IV we are then able to prove the main result of the paper, that to every dimension-one primary field on the world sheet there corresponds a gauge transformation of the string theory. In Sec. V we explain that this is only a subset of all possible gauge transformations, specifically those preserving an analogue of the Landau gauge. In Sec. VI we show that there is an infinite number of such gauge symmetries, and argue that most of them mix excitations at differing mass levels. These symmetries are broken by the existence of a space-time metric, but this breaking becomes unimportant at high energies, shedding light on the results of Refs. 2 and 4. Section VII is devoted to a discussion of these results. Brief summaries of aspects of this work will appear elsewhere.[8]

II. THE PROBLEM

In previous papers[9,10] (related ideas have been discussed by others[11]), we described a framework for proving the existence of symmetries in string theory. We shall start by describing that method, although in a somewhat different language.

To each solution of the string equations of motion there corresponds a conformally invariant two-dimensional field theory.[12,13] This theory will be completely defined by specifying its energy-momentum tensor as a local function of world-sheet fields and their canonical momenta. The space-time fields appear as the couplings of this two-dimensional field theory. Thus the space-time field configurations Φ provide a system of local coordinates on the space of conformal field theories, and determine the two light-cone components of the energy-momentum tensor, which we shall denote T_Φ and $\bar{T}_\Phi$.

If there is a transformation of the space-time fields that is a string-theoretic symmetry, then to every solution of the equations of motion there will be a new solution, where the fields take their transformed values. Thus, to every conformal field theory there will correspond another conformal field theory with transformed couplings. Furthermore, since they are related by a symmetry, these two solutions should be physically indistinguishable. In particular, this means that all physical scattering amplitudes should be the same in the two backgrounds, so that the corresponding conformal field theories are isomorphic. Two conformal field theories are isomorphic if they are isomorphic as operator algebras. That is, they are isomorphic if there exists a bijective map

$$\rho: \mathcal{A}_1 \to \mathcal{A}_2$$

between the operator algebras $\mathcal{A}_1$ and $\mathcal{A}_2$ of the two conformal field theories that maps stress tensor to stress tensor, and preserves the equal-time commutation relations. That is, for any two operators a, b belonging to $\mathcal{A}_1$,

$$[\rho(a),\rho(b)]=\rho([a,b]) \ . \tag{2.1}$$

Note that this is sufficient to preserve the equations of motion, since time translation is generated by commuting with the Hamiltonian.

For any algebra, we may construct another algebra, isomorphic to the first by means of an infinitesimal inner automorphism. That is, take $\mathcal{A}_1=\mathcal{A}_2$ and

$$\rho_h(a)=a+i[h,a] \ , \tag{2.2}$$

where h is any fixed, infinitesimal operator. It is straightforward to check that Eq. (2.2) is indeed an automorphism, for

$$\begin{aligned}[\rho_h(a),\rho_h(b)]&=[a+i[h,a],b+i[h,b]]\\ &=[a,b]+i[[h,a],b]+i[a,[h,b]] \quad [+O(h^2)]\\ &=[a,b]+i[h,[a,b]]=\rho_h([a,b]) \ ,\end{aligned}$$

where we used the Jacobi identity

$$[a,[h,b]]+[h,[b,a]]+[b,[a,h]]=0$$

and so proved that the map defined by Eq. (2.2) does indeed satisfy Eq. (2.1).

For any infinitesimal operator h, then, the conformal field theories specified by T_Φ and by $T_\Phi+i[h,T_\Phi]$ are isomorphic. Thus if

$$T_\Phi+i[h,T_\Phi]=T_{\Phi+\delta\Phi} \tag{2.3}$$

for some $\delta\Phi$, it follows that

$$\Phi\to\Phi+\delta\Phi \tag{2.4}$$

is a symmetry transformation of the space-time fields, which we shall say is generated by the operator h. Equation (2.3) should be thought of as a condition on the operator h, and so the problem of finding such symmetries is reduced to finding those h which satisfy the nontrivial condition

$$[h,T_\Phi]=-i(T_{\Phi+\delta\Phi}-T_\Phi) \ . \tag{2.5}$$

It is this problem that we shall address in this paper.

Finally, let us remark that while Eq. (2.2) always defines an automorphism, not every automorphism is necessarily of this form. In general, a operator algebra will possess other, outer automorphisms, and it is quite possible that these too can be interpreted as symmetry transformations on the space-time fields. It seems likely that the discrete symmetries of string theory can be understood in this way, although we shall have nothing more to say on this subject here.

III. DEFORMATIONS OF CONFORMAL FIELD THEORIES

We first have to understand the right-hand side of Eq. (2.5). How does the stress tensor of the two-dimensional world-sheet field theory change as we change the space-time fields? Here, and throughout this paper, space-time fields will be understood to be solutions of their equations of motion. Thus making such infinitesimal changes in the space-time fields will produce infinitesimal changes in the world-sheet theory that preserve the conformal invariance. Having once understood such infinitesimal deformations, it is possible, at least in principle, to iterate this procedure and build up an entire family of connected conformal field theories corresponding, roughly speaking, to the family of conformal field theories with the same topology as the starting theory.

A number of authors have studied the deformation of two-dimensional field theories *away from* a conformal fixed point, and have investigated the induced renormalization-group flows.[14] This, however, is not the problem we wish to address. Since we wish to study neighboring classical solutions of string theory, we must consider deformations of the two-dimensional field theory that *preserve* conformal invariance. Aspects of this question have been discussed in the literature,[15] but nobody seems to have answered the question posed in the preceding paragraph.

A two-dimensional field theory is conformally invariant if the Hamiltonian can be extended to an energy-momentum tensor, the two non-vanishing components (T and $\bar{T}$) of which satisfy the equal-time commutation relations

$$\begin{aligned}[T(\sigma),T(\sigma')]=&(-ic/24\pi)[\delta'''(\sigma-\sigma')+\delta'(\sigma-\sigma')]\\&+2iT(\sigma')\delta'(\sigma-\sigma')\\&-iT'(\sigma')\delta(\sigma-\sigma')\ ,\end{aligned}\qquad(3.1)$$

$$\begin{aligned}[\bar{T}(\sigma),\bar{T}(\sigma')]=&(ic/24\pi)[\delta'''(\sigma-\sigma')+\delta'(\sigma-\sigma')]\\&-2i\bar{T}(\sigma')\delta'(\sigma-\sigma')\\&+i\bar{T}'(\sigma')\delta(\sigma-\sigma')\ ,\end{aligned}\qquad(3.2)$$

$$[T(\sigma),\bar{T}(\sigma')]=0\qquad(3.3)$$

and in terms of which H, the Hamiltonian, and P, the generator of translations in σ, may be written

$$\begin{aligned}H&=\int d\sigma[T(\sigma)+\bar{T}(\sigma)]\ ,\\P&=\int d\sigma[T(\sigma)-\bar{T}(\sigma)]\ .\end{aligned}\qquad(3.4)$$

Temporal evolution is generated by the Hamiltonian, Eq. (3.4), and so operators need only be specified on some spacelike slice, which we parametrize by the real coordinate σ. A prime denotes differentiation with respect to σ. The algebra of Eqs. (3.1)–(3.3) is essentially two copies of the Virasoro algebra (or, at higher genus the Krichever-Novikov algebras[16]), as may be seen by taking moments of T:

$$L_n=\int d\sigma\, T(\sigma)e^{in\sigma},\quad \bar{L}_n=\int d\sigma\, \bar{T}(\sigma)e^{-in\sigma}\ .\qquad(3.5)$$

It will be convenient for us to specify our conformal field theory in a rather old-fashioned way, in that we assume that the energy-momentum tensor is some local function of elementary fields and their conjugate momenta, which obey fixed, canonical commutation relations. This completely specifies the conformal field theory, but it is useful to define another set of operators, also constructed from elementary fields and momenta, known as the primary fields.[13] By definition, a primary field χ of dimension $(d,\bar{d})$ satisfies

$$\begin{aligned}[T(\sigma),\chi(\sigma')]=&id\chi(\sigma')\delta'(\sigma-\sigma')\\&-(i/\sqrt{2})\partial\chi(\sigma')\delta(\sigma-\sigma')\ ,\\[\bar{T}(\sigma),\chi(\sigma')]=&-i\bar{d}\chi(\sigma')\delta'(\sigma-\sigma')\\&-(i/\sqrt{2})\bar{\partial}\chi(\sigma')\delta(\sigma-\sigma')\ .\end{aligned}\qquad(3.6)$$

We are interested in not just one, but a family of conformal field theories parametrized by the values of the space-time fields. Changing these space-time fields changes the conformal field theory, but preserves the Virasoro algebra (including the value of the central extension), Eqs. (3.1)–(3.3). Thus, to first order, the changes in T and $\bar{T}$ must satisfy

$$\begin{aligned}[T(\sigma),\delta T(\sigma')]+[\delta T(\sigma),T(\sigma')]=&2i\delta T(\sigma')\delta'(\sigma-\sigma')\\&-i\delta T'(\sigma')\delta(\sigma-\sigma')\ ,\\[\bar{T}(\sigma),\delta\bar{T}(\sigma')]+[\delta\bar{T}(\sigma),\bar{T}(\sigma')]=&-2i\delta\bar{T}(\sigma')\delta'(\sigma-\sigma')\\&+i\delta\bar{T}'(\sigma')\delta(\sigma-\sigma')\ ,\\[T(\sigma),\delta\bar{T}(\sigma')]+[\delta T(\sigma),\bar{T}(\sigma')]=&0\ .\end{aligned}\qquad(3.7)$$

To solve Eqs. (3.7) for δT it is useful first to examine some simple examples. Consider first the classical energy-momentum tensor for the bosonic string with massless background fields.

$$\begin{aligned}T(\sigma)&=\tfrac{1}{2}G^{\mu\nu}(X)\partial X_\mu\partial X_\nu\ ,\\\bar{T}(\sigma)&=\tfrac{1}{2}G^{\mu\nu}(X)\bar{\partial}X_\mu\bar{\partial}X_\nu\ ,\end{aligned}\qquad(3.8)$$

where

$$\begin{aligned}\partial X_\mu(\sigma)&=(1/\sqrt{2})\{\pi_\mu+[G_{\mu\nu}(X)+B_{\mu\nu}(X)]X'^{\nu}\}\ ,\\\bar{\partial}X_\mu(\sigma)&=(1/\sqrt{2})\{\pi_\mu+[-G_{\mu\nu}(X)+B_{\mu\nu}(X)]X'^{\nu}\}\ ;\end{aligned}\qquad(3.9)$$

$$[\pi_\mu(\sigma),X^\nu(\sigma')]=-i\delta_\mu^\nu\delta(\sigma-\sigma')\ .\qquad(3.10)$$

Varying the space-time fields G and B, *including* the implicit dependence described in Eq. (3.9), one finds that

$$\delta T(\sigma)=\delta\bar{T}(\sigma)=-\tfrac{1}{2}[\delta G_{\mu\nu}(X)+\delta B_{\mu\nu}(X)]\partial X^\mu\bar{\partial}X^\nu\ .\qquad(3.11)$$

The reason that we must also vary the space-time fields that appear in ∂X and $\bar{\partial}X$ is that it is the commutation relations for π and X that are fixed, while those for ∂X and $\bar{\partial}X$ change as the conformal field theory is deformed. One may do a similar calculation for the heterotic string, and these results are all consistent with the hypothesis that both components of the energy-momentum tensor change by the addition of the same infinitesimal primary field of dimension (1,1). Since the physical space-time fields of the string are in direct correspondence with the primary fields of dimension (1,1), the only reasonable interpretation of this ansatz is that the deformation results from turning on the corresponding background field.

Using the definition of a primary field, Eq (3.6), it is straightforward to see that this ansatz does satisfy Eq. (3.7), and so is indeed a conformal deformation. Also, the fact that the changes in T and $\overline{T}$ are equal means that the operator P [Eq. (3.4)] is unchanged, and so continues to generate translations. This condition is both necessary and nontrivial.

It is convenient to give this type of deformation a name, and so let us define a *canonical* deformation of a conformal field theory by

$$\delta T(\sigma)=\delta\overline{T}(\sigma)=\chi_{(1,1)}(\sigma)\ , \tag{3.12}$$

where $\chi_{(1,1)}(\sigma)$ is a primary field of dimension (1,1).

In the literature[15] one sometimes sees additional conditions on the deformation, requirements on the commutator of the deformation with itself. We will therefore take some time to explain why such conditions need not concern us for the purposes of this paper. First, one can demand that a *finite* deformation preserve conformal invariance, or, more restrictively, that one can add the deformation with an *arbitrary* coefficient (the latter case implies that the β function vanishes order by order in perturbation theory, a much stronger condition than simply vanishing). However, we are interested in infinitesimal deformations (because we are interested in infinitesimal symmetries), and so may ignore these conditions.

The second way in which such a condition can occur is when dealing with a *compact* conformal field theory. In this case, the spectrum of primary fields is discrete, and will deform continuously as the field theory is deformed. It would then be possible (even likely) that a primary field that had been (1,1) would cease to be so after deformation. The infinitesimal deformations corresponding to such primary fields could not, therefore, be built up into finite deformations. That is, they would not be *integrable.* The requirement of integrability therefore imposes additional conditions on the deformations, which are equivalent to the condition that space-time equations of motion should admit solutions on compact manifolds (integrable solutions are usually called *moduli*).

Since we are interested in the problem of local symmetries rather than the question of the existence of global solutions, we avoid this problem by always considering noncompact conformal field theories. Specifically, we always consider conformal field theories which correspond to some number of space-time dimensions with the topology of R^n. The result is a *continuous* spectrum of primary fields (e.g., of the form $e^{ik\cdot X}$, which has arbitrary dimension $k^2/2$), which means that the number of (1,1) primary fields is preserved under continuous deformation. Thus, for us, there is no integrability problem.

Are all conformal deformations canonical? This is an important question for the following reason. It is widely believed that string theories have no continuous adjustable parameters. However, a string theory can be built on any conformal field theory with the correct central extension. If string theories do indeed lack free parameters, we must be able to interpret all conformal deformations which preserve the central charge as changes in the space-time fields of the theory. This strongly suggests that all conformal deformations should be equivalent to a canonical deformation. It is fairly easy to see that there are conformal deformations which are not canonical, but we conjecture that all deformations *are* canonical *up to an algebra isomorphism.* The reason is that, as we shall see below, a canonical deformation preserves an analogue of "Landau gauge." It is plausible that all other deformations correspond to the same physics in a different gauge. We shall discuss other gauges below, but will not attempt to prove this conjecture, which is not necessary for the argument we present in this paper.

It is well known that we can add vertex operators corresponding to massless states to the *action* of a conformal field theory in order to deform it.[17] What we have just shown is that adding *any* infinitesimal vertex operator to both components of the energy-momentum tensor also produces a deformation that preserves the conformal invariance. When dealing with massless space-time fields it is easy to see in particular examples that these results are equivalent, but when discussing massive fields the formalism presented here avoids a number of pathologies. For massive modes of the string the vertex operators contain higher derivatives, or are not quadratic in first derivatives. This makes it hard to interpret these actions in a conventional way. Solving for momenta in terms of time derivatives becomes much harder, the usual form of the functional integral ceases to be related to a canonical quantization in the usual way and the equations of motion are higher-order differential equations which admit ghostly, exponentially growing solutions. Making a conformal deformation of the energy-momentum tensor, as described above, avoids these sicknesses since the theory is already canonically quantized and temporal evolution is generated by the Hamiltonian through a first-order differential equation. The energy-momentum tensor is not quadratic in the momenta, but this is no more disturbing than the fact that it is not quadratic in the fields. In this language, then, massive and massless fields can be handled on an equal footing.

IV. GAUGE INVARIANCES OF THE STRING

We now know how the world-sheet energy-momentum tensor deforms as we change the space-time fields (or, at the least we know a set of deformations which do indeed correspond to such changes). We have understood, therefore, the right-hand side of Eq. (2.5), and must now do the same for the left.

In a previous paper,[10] we discussed the massless bosonic gauge invariances of the heterotic string, and found that each symmetry was generated by an operator h that was a field of (naive) dimension one integrated over the world-sheet spatial coordinate σ, so that h itself was dimensionless. The family of energy-momentum tensors was the most general such consistent with (super-)Poincaré invariance and constructed out of operators of dimension two. Commuting with an operator of dimension zero preserves both these properties. Super-Poincaré invariance is preserved because we are dealing with an algebra automorphism, while the fact that h is dimensionless preserves the dimension of the terms in the Hamiltonian. Thus all such operators h generate symmetries.

In conformal field theory, the notion of the dimension of an operator can be made a little more precise. What the above discussion suggests is that any operator of the form

$$h=\int d\sigma\,\Psi(\sigma)\ , \tag{4.1}$$

where Ψ is a primary field of dimension (1,0) or (0,1), will satisfy Eq. (2.5), and hence generate a symmetry.

We shall prove this result for Ψ a (1,0) primary operator, the proof for the (0,1) case follows straightforwardly, mutatis mutandis:

$$\begin{aligned}\delta T(\sigma)&=-i\int d\sigma'[T(\sigma),\Psi(\sigma')]\\&=-i\int d\sigma' i\Psi(\sigma')\delta'(\sigma-\sigma')-(i/\sqrt{2})\partial\Psi(\sigma')\delta(\sigma-\sigma')\\&=\Psi'(\sigma)-(1/\sqrt{2})\partial\Psi(\sigma)=-(1/\sqrt{2})\bar\partial\Psi(\sigma)\ .\end{aligned} \tag{4.2}$$

Similarly,

$$\delta\bar T(\sigma)=-i\int d\sigma'[\bar T(\sigma),\Psi(\sigma')]=-i\int d\sigma'-(i/\sqrt{2})\bar\partial\Psi(\sigma')\delta(\sigma-\sigma')=-(1/\sqrt{2})\bar\partial\Psi(\sigma)\ . \tag{4.3}$$

Thus we have a canonical deformation if $\bar\partial\Psi$ is a primary field of dimension (1,1). That this is indeed the case we show as follows. By definition

$$\bar\partial\Psi(\sigma')=(i\sqrt{2})[\bar L_0,\Psi(\sigma')] \tag{4.4}$$

so that

$$\begin{aligned}[T(\sigma),\bar\partial\Psi(\sigma')]&=(i\sqrt{2})[T(\sigma),[\bar L_0,\Psi(\sigma')]]\\&=-(i\sqrt{2})\{[\bar L_0,[\Psi(\sigma'),T(\sigma)]]+[\Psi(\sigma'),[T(\sigma),\bar L_0]]\}\\&=(i\sqrt{2})[\bar L_0,i\Psi(\sigma')]\delta'(\sigma-\sigma')+(i\sqrt{2})[\bar L_0,-(i/\sqrt{2})\partial\Psi(\sigma')]\delta(\sigma-\sigma')\\&=i\bar\partial\Psi(\sigma')\delta'(\sigma-\sigma')-(i/\sqrt{2})\partial\bar\partial\Psi(\sigma')\delta(\sigma-\sigma')\ ,\end{aligned} \tag{4.5}$$

where we used the Jacobi identity and the definition of a (1,0) primary field, Eq. (3.6). We also used the fact that

$$\bar\partial\partial\Psi(\sigma)=\partial\bar\partial\Psi(\sigma) \tag{4.6}$$

which follows from the Jacobi identity and the fact that L_0 and $\bar L_0$ commute. The result, Eq. (4.5), proves that $\bar\partial\Psi$ is primary and of dimension one with respect to T [as defined by Eq. (3.6)].

A similar argument proves that $\partial\Psi$ is primary and of dimension one with respect to $\bar T$. From Eq. (4.4) and the Jacobi identity we find that

$$\begin{aligned}[\bar T(\sigma),\bar\partial\Psi(\sigma')]=-(i\sqrt{2})\{&[\bar L_0,[\Psi(\sigma')\bar T(\sigma)]]\\&+[\Psi(\sigma'),[\bar T(\sigma),\bar L_0]]\}\ .\end{aligned} \tag{4.7}$$

Now using Eq. (3.6) for Ψ a (1,0) primary field and the fact that

$$[\bar T(\sigma),\bar L_0]=-i\bar T'(\sigma)$$

which comes from integrating Eq. (3.2), Eq. (4.7) becomes

$$\begin{aligned}[\bar T(\sigma),\bar\partial\Psi(\sigma')]=&-i\bar\partial\Psi(\sigma')\delta'(\sigma-\sigma')\\&-(i/\sqrt{2})\bar\partial\,\bar\partial\Psi(\sigma')\delta(\sigma-\sigma')\ ,\end{aligned}$$

which completes the proof that $\bar\partial\Psi$ is a (1,1) primary field. Thus every primary field of dimension (1,0) or (0,1) generates a symmetry. Note that adding $\bar\partial\Psi$ to a vertex operator does not change the values of S-matrix amplitudes.[18]

Let us summarize the argument.

(i) The conformal field theories with energy-momentum tensors $T(\sigma)$ and $T(\sigma)+i[h,T(\sigma)]$ are isomorphic for any operator h.

(ii) The conformal field theories with energy-momentum tensors $T(\sigma)$ and $T(\sigma)+\chi(\sigma)$ differ by an infinitesimal change in the space-time field that corresponds to the (1,1) primary field $\chi(\sigma)$.

(iii) If h is the integral of a (1,0) or (0,1) primary field, then $[h,T(\sigma)]$ is a (1,1) primary field and, by (ii) is interpretable as a change in the space-time fields. By (i) this change is a symmetry transformation of the theory.

V. GAUGES

We stated above that the symmetries we have been discussing preserve an analogue of Landau gauge and it is time to explain that remark. For simplicity, consider the case of the bosonic string in a flat 26-dimensional space-time, so that the appropriate conformal field theory is free. The energy-momentum tensor is given by

$$\begin{aligned}T(\sigma)&=\tfrac{1}{2}\eta^{\mu\nu}\partial X_\mu\partial X_\nu(\sigma)\ ,\\\bar T(\sigma)&=\tfrac{1}{2}\eta^{\mu\nu}\bar\partial X_\mu\bar\partial X_\nu(\sigma)\ ,\end{aligned} \tag{5.1}$$

where

$$\partial X_\mu(\sigma)=(1/\sqrt{2})[\pi_\mu(\sigma)+\eta_{\mu\nu}X'^\nu(\sigma)] ,$$
$$\bar{\partial} X_\mu(\sigma)=(1/\sqrt{2})[\pi_\mu(\sigma)-\eta_{\mu\nu}X'^\nu(\sigma)] . \quad (5.2)$$

The vertex operator corresponding to the graviton is

$$\chi(\sigma)=h^{\mu\nu}(X)\partial X_\mu \bar{\partial} X_\nu(\sigma) . \quad (5.3)$$

It is primary and of dimension (1,1) only if

$$\Box h^{\mu\nu}=0 , \quad (5.4)$$

$$\partial_\mu h^{\mu\nu}=0 . \quad (5.5)$$

Equation (5.4) is an equation of motion for the graviton, something that we expect to be unavoidable for conformal invariance. However, Eq. (5.5) is a gauge condition, the appearance of which is a little surprising since we would expect to be able to make sense of a conformal field theory with the space-time fields in any gauge. We shall refer to the gauge of Eq. (5.5) as the Landau gauge, since it is similar to the condition

$$\partial_\mu A^\mu=0 \quad (5.6)$$

which appears in the same way in, for example, the heterotic string.

The operators which generate coordinate transformations and two-form gauge transformations are[10]

$$h_1=\int d\sigma\, \xi^\mu(X)\partial X_\mu , \quad (5.7)$$

$$h_2=\int d\sigma\, \zeta^\mu(X)\bar{\partial} X_\mu . \quad (5.8)$$

Again, for the integrands to be primary and of dimension (1,0) and (0,1) we need ξ and ζ to satisfy

$$\Box\xi^\mu=\Box\zeta^\mu=0 , \quad (5.9)$$

$$\partial_\mu\xi^\mu=\partial_\mu\zeta^\mu=0 . \quad (5.10)$$

Since ξ and ζ are the parameters of gauge transformations, Eqs. (5.9) and (5.10) are not equations of motion. Rather they preserve the Landau-gauge condition, Eq. (5.5), and the tracelessness of the graviton.

How are we to make sense of string theory in other gauges? (This question has been addressed from other points of view.[19]) We can give a partial answer to this question. Recall that

$$T(\sigma)\to T(\sigma)+i[h,T(\sigma)] \quad (5.11)$$

is an algebra automorphism (and hence does not change the physics) for *arbitrary* operators h. This is a consequence solely of the Jacobi identity. We imposed restrictions on the form of h merely to ensure our ability to interpret the changes in the energy-momentum tensor straightforwardly as changes in the space-time fields.

Equation (5.9) is necessary to make the integrand in Eqs. (5.7) and (5.8) operators of dimension one. Relaxing this condition makes the integrand a superposition of primary fields of varying dimension, but leaves the operators h perfectly well behaved. Such operators generate symmetries, which clearly correspond to a more general set of gauge transformations.

Such gauge transformations induce deformations of the conformal field theory which are not canonical, but which are isomorphic to a canonical deformation (after all, one example of a canonical deformation is no deformation at all). This motivates the conjecture that all deformations are isomorphic to some canonical deformation.

These observations suggest a more refined interpretation of the results of this paper: the integrals of primary fields of dimension one generate the transformations which preserve Landau gauge. The full gauge invariance of string theory is some larger set of inner automorphisms, possibly those generated by all primary fields.

VI. HIGHER SYMMETRIES

So far, we have discussed only examples involving unbroken symmetries and massless particles, but the methods of this paper apply equally well to spontaneously broken symmetries involving massive particles. Indeed, we shall see that there is a very large spontaneously broken symmetry in string theory. All we need to do is exhibit the relevant primary fields of dimension one. Equations (5.7) and (5.8) were the simplest examples where the full quantum-mechanical dimension coincided with the naive classical result [this was ensured by the conditions of Eqs. (5.9) and (5.10)]. However it is possible to construct an infinite class of fields with large naive dimension, but with a quantum-mechanical dimension that is unity. This is not unfamiliar, since exactly the same mechanism permits the construction of an infinite number of vertex operators, primary fields of dimension (1,1).

Once again, let us consider the simple example of the free bosonic conformal field theory specified by Eqs. (5.1) and (5.2). The first massive level corresponds to a four index tensor field that is symmetric on both the first and second pairs of indices.[1] We shall exhibit a symmetry transformation that mixes this and massless fields. We therefore need to understand the vertex operator corresponding to the first mass level, as well as primary fields of dimension one that yield operators of this form when commuted with the energy-momentum tensor.[18] To this end consider first the operator

$$\Theta(\sigma)=\theta_{\mu\nu}(X)\partial X^\mu\partial X^\nu+\partial_\mu\theta^\mu_\nu(X)\partial^2X^\nu . \quad (6.1)$$

It is primary of dimension $(2-\alpha,-\alpha)$ if

$$\theta^\mu_\mu+2\partial_\mu\partial_\nu\theta^{\mu\nu}=0 , \quad (6.2)$$

$$\Box\theta_{\mu\nu}=2\alpha\theta_{\mu\nu} . \quad (6.3)$$

From this information it is straightforward to construct the vertex operator for the field at the first mass level of the bosonic string. The (1,1) primary field is given by

$$\begin{aligned}\Phi(\sigma)=&\phi_{\mu\nu\lambda\kappa}(X)\partial X^\mu\partial X^\nu\bar{\partial} X^\lambda\bar{\partial} X^\kappa\\&+\partial_\mu\phi^\mu{}_{\nu\lambda\kappa}(X)\partial^2X^\nu\bar{\partial} X^\lambda\bar{\partial} X^\kappa\\&+\partial_\lambda\phi_{\mu\nu}{}^\lambda{}_\kappa(X)\partial X^\mu\partial X^\nu\bar{\partial}^2X^\kappa\\&+\partial_\mu\partial_\lambda\phi^\mu{}_\nu{}^\lambda{}_\kappa(X)\partial^2X^\nu\bar{\partial}^2X^\kappa ,\end{aligned} \quad (6.4)$$

where

$$\phi^{\mu}{}_{\mu\lambda\kappa}+2\partial_{\mu}\partial_{\nu}\phi^{\mu\nu}{}_{\lambda\kappa}=0\ , \tag{6.5}$$

$$\phi_{\mu\nu}{}^{\lambda}{}_{\lambda}+2\partial_{\lambda}\partial_{\kappa}\phi_{\mu\nu}{}^{\lambda\kappa}=0\ , \tag{6.6}$$

$$\Box\phi_{\mu\nu\lambda\kappa}=2\phi_{\mu\nu\lambda\kappa}\ . \tag{6.7}$$

From our experience with the massless fields we conjecture that Eqs. (6.5) and (6.6) are gauge conditions, while Eq. (6.7) is the linearized equation of motion in that gauge.

In similar vein we may construct operators of dimension one:

$$\Psi(\sigma)=\Psi_{\mu\nu\lambda}(X)\partial X^{\mu}\partial X^{\nu}\bar{\partial}X^{\lambda}+\partial_{\mu}\Psi^{\mu}{}_{\nu\lambda}(X)\partial^2X^{\nu}\bar{\partial}X^{\lambda} \tag{6.8}$$

is primary and of dimension (1,0) if

$$\Psi^{\mu}{}_{\mu\lambda}+2\partial_{\mu}\partial_{\nu}\Psi^{\mu\nu}{}_{\lambda}=0\ , \tag{6.9}$$

$$\partial_{\lambda}\Psi_{\mu\nu}{}^{\lambda}=0\ , \tag{6.10}$$

$$\Box\Psi_{\mu\nu\lambda}=2\Psi_{\mu\nu\lambda}\ . \tag{6.11}$$

Drawing once again on our experience with the massless fields, Eqs. (6.9)–(6.11) presumably preserve the gauge condition of Eqs. (6.5)–(6.7). An operator of dimension (0,1) may be constructed by everywhere interchanging ∂ and $\bar{\partial}$.

With these primary fields we may construct the symmetry transformation. From Eq. (4.2),

$$\begin{aligned}\delta T(\sigma)&=-i\int d\sigma'[T(\sigma),\Psi(\sigma')]\\&=-(1/\sqrt{2})\bar{\partial}\Psi(\sigma)\\&=-(1/\sqrt{2})(\partial_{\kappa}\Psi_{\mu\nu\lambda}\partial X^{\mu}\partial X^{\nu}\bar{\partial}X^{\lambda}\bar{\partial}X^{\kappa}+\Psi_{\mu\nu\lambda}\partial X^{\mu}\partial X^{\nu}\bar{\partial}^2X^{\lambda}+\partial_{\mu}\partial_{\kappa}\Psi^{\mu}{}_{\nu\lambda}\partial^2X^{\nu}\bar{\partial}X^{\lambda}\bar{\partial}X^{\kappa}+\partial_{\mu}\Psi^{\mu}{}_{\nu\lambda}\partial^2X^{\nu}\bar{\partial}^2X^{\lambda})\ .\end{aligned} \tag{6.12}$$

Comparing Eqs. (6.12) and (6.4) and using Eqs. (6.9)–(6.11) we see that this change in the energy-momentum tensor corresponds to adding the vertex operator for a first mass level state, with the change in the space-time field given by

$$\delta\phi_{\mu\nu\lambda\kappa}=(1/\sqrt{2})(\partial_{\kappa}\Psi_{\mu\nu\lambda}+\partial_{\lambda}\Psi_{\mu\nu\kappa})\ . \tag{6.13}$$

The (0,1) operator generates the transformation

$$\delta\phi_{\mu\nu\lambda\kappa}=(1/\sqrt{2})(\partial_{\mu}\Psi_{\nu\lambda\kappa}+\partial_{\nu}\Psi_{\mu\lambda\kappa})\ . \tag{6.14}$$

Clearly an infinite number of symmetries can be constructed in similar vein. For example, generalizing Eqs. (6.8)–(6.11), there is a class of (1,0) operators that may be written schematically:

$$\Psi(\sigma)=\Psi(X)(\partial X)^{n}(\bar{\partial}X)^{n-1}\ , \tag{6.15}$$

$$h=\int d\sigma\,\Psi(\sigma)\ . \tag{6.16}$$

Here Ψ is a $(2n-1)$-index space-time field that is symmetric on the first n and last $n-1$ indices. It must be divergence-free on each index (by the symmetry properties of Ψ this is just two independent conditions), and traceless on any pair of the first n, or any pair of the second $n-1$, indices (again, two conditions). Equation (6.15) is not the most general (1,0) primary field; when $n=2$, for example, it produces only a subset of the operators of Eq. (6.8), but it is an infinite class and so demonstrates the existence of an infinite class of symmetry transformations.

The interpretation of these higher symmetries is complicated by the fact that we are presumably preserving a gauge condition, and by our ability to consider only one particularly tractable conformal field theory that corresponds to one configuration of the space-time fields. To clarify the situation let us make some general remarks. Recall that in Sec. III we described how turning on each space-time field corresponded to adding a term to the two-dimensional stress tensor. Thus there is a correspondence between space-time fields and terms in T. Under a symmetry transformation, the change in a space-time field will depend upon other fields in the same multiplet, and it is natural to ask which fields appear. Since

$$\delta T=i[h,T] \tag{6.17}$$

the fields that enter the transformation on the right-hand side of Eq. (6.17) are those for which the corresponding terms in T do *not* commute with h, the generator of the symmetry.

Note that this means that if we are seeking a classical vacuum for string theory in which a certain gauge symmetry is not spontaneously broken, the corresponding two-dimensional field theory will also exhibit a symmetry, since the condition for an unbroken symmetry is $\delta\Phi_0=0$, where Φ_0 are the vacuum values for the fields Φ. Thus,

$$\delta T_{\Phi_0}=0$$

which implies that

$$i[h,T_{\Phi_0}]=0$$

and we see that h, the generator of the space-time symmetry, also generates a two-dimensional symmetry of the *vacuum* conformal field theory. This fact has been much discussed in the context of unbroken supersymmetry in four dimensions.[20] Correspondingly, if the space-time symmetry is spontaneously broken by the expectation values of some fields, the world-sheet symmetry will be broken explicitly, by precisely the terms in the stress tensor corresponding to these fields.

In the example discussed in the beginning of this section, h failed to commute with the term in T that corresponds to the flat metric for space-time. Thus this symmetry has the change in a field at the first mass level, ϕ, depending on massless fields (in this case the space-time metric).

More generally, the energy-momentum tensor corresponding to arbitrary metric and torsion is

$$T=\tfrac{1}{2}G^{\mu\nu}(X)\partial X_\mu \partial X_\nu \ , \tag{6.18}$$

where

$$\partial X_\mu=(1/\sqrt{2})\{\pi_\mu+[G_{\mu\lambda}(X)+B_{\mu\lambda}(X)]X'^{\lambda}\} \tag{6.19}$$

and, of course, G and B satisfy some equation of motion and Landau-gauge condition. Equation (6.18) consists purely of terms of naive dimension two. Commuting T with a dimension-one operator that contains terms of *naive* dimension $2n-2$ [as in Eq. (6.16)] yields terms, at least some of which are of naive dimension $2n$, and which depend on G and B. These terms cannot correspond to the vertex operator for a massless field (which have naive dimension two), and so must include massive vertices, presumably of mass level $2n-2$. Once again, this implies that the variation of a massive field is depending on the massless fields, placing these fields in the same multiplet.

In principle, we can repeat the arguments leading to the transformations of Eqs. (6.13) and (6.14), but this time with the stress tensor given by Eq. (6.18) and the generator given by the corresponding deformation of the (1,0) primary field of Eq. (6.8). The transformations of Eqs. (6.13) and (6.14) will be altered, and, on the basis of general covariance, must be of the form

$$\begin{aligned} \delta\phi_{\mu\nu\lambda\kappa}&=(1/\sqrt{2})(\mathcal{D}_\kappa\Psi_{\mu\nu\lambda}+\mathcal{D}_\lambda\Psi_{\mu\nu\kappa})+\cdots \ , \\ \delta\phi_{\mu\nu\lambda\kappa}&=(1/\sqrt{2})(\mathcal{D}_\mu\Psi_{\nu\lambda\kappa}+\mathcal{D}_\nu\Psi_{\mu\lambda\kappa})+\cdots \ , \end{aligned} \tag{6.20}$$

where $\mathcal{D}_\mu$ is the covariant derivative constructed from $G_{\mu\nu}$, $B_{\mu\nu}$ and we have neglected possible terms proportional to the curvature and torsion. The dependence of the covariant derivative on the metric and two form explicitly demonstrates that the variation of a mass level one field depends upon massless fields.

We may also imagine turning on backgrounds at mass level k, by adding a vertex operator of naive dimension $2k+2$ to the stress tensor. Again commuting with an operator of naive dimension $2n-2$ yields the vertex for a field at mass level $k+n-1$, implying that the transformation of a mass level $(k+n-1)$ field depends on mass level k fields. All of this suggests that the symmetry algebra that we are discussing may be graded by the integers, the grading being associated with the naive dimension of the operator that generates the transformation. A generator of grade r would have fields at mass level k rotating into fields at mass level $k+r$.

The higher symmetries that we are discussing relate fields of different mass, and so they must be spontaneously broken. We are also dealing with gauge symmetries, so no Goldstone bosons appear in the physical spectrum because they can be gauged away. Presumably the Higgs mechanism is operating, and the Goldstone bosons become the longitudinal components of (higher spin) gauge fields which acquire mass. Indeed, it is an attractively economical hypothesis that all the states of the string are gauge fields for the infinite gauge symmetry that we are discussing in this paper.

If the symmetry is broken, it is natural to ask what order parameter does the breaking. We explained earlier in this section that the fields which break a symmetry are those for which the corresponding terms in the world-sheet stress tensor fail to commute with the generator of the symmetry h. The energy-momentum tensor that describes a string propagating on a flat manifold contains only one term, corresponding to the space-time metric. It is this term which fails to commute with the generators of the higher symmetries [given by Eq. (6.16)], and so we may say that the symmetry is spontaneously broken by the metric for space-time. There is no value for the metric which restores these higher symmetries, indicating that there is no space-time interpretation for the symmetric phase of string theory (if such a phase exists). The Coleman-Mandula theorem[21] (if it can be generalized to string theory) forbids having these higher symmetries, which relate fields of very different spin, as symmetries of the S matrix. The fact that a symmetric phase of string theory would not correspond to a space-time (and so has no associated S matrix) thus neatly avoids any inconsistency.

The absence of a field configuration invariant under all the symmetries of a theory may, at first sight, seem bizarre, but there are well-known field-theoretic analogues. For example, a nonlinear sigma model with an n-sphere for its target manifold has an obvious $O(n+1)$ symmetry, but no classical configuration is invariant under the whole group. Nevertheless, above a critical temperature the system demagnetizes, and symmetry is restored. It would be nice to understand a similar process in string theory, and perhaps make contact with the work of Atick and Witten.[22]

Other authors have attempted to shed light on the higher symmetries of string theory by investigating high-energy collisions.[2-4] In particular, Gross and Mende investigated fixed angle, high-momentum collisions, and found that as the momentum exceeded all relevant scales in the problem (Planck and particle masses) the scattering amplitude came to be dominated by one Riemann surface at each genus. This universality, and linear relations between S-matrix elements that hold in the high-energy limit,[4] are taken to be an indication that some symmetry is being restored at high energy.

We may argue that the higher symmetries that we have been discussing in this paper are indeed being restored. The high-momentum limit is the limit $\alpha'\to\infty$, where α' is the slope parameter of the Regge trajectories. However the world-sheet stress tensor has as its coefficient (heretofore suppressed) a factor of $1/\alpha'$. Thus $\alpha'\to\infty$ is equivalent to the stress tensor $T\to 0$. Recall from the discussion surrounding Eq. (6.17) that the condition for a symmetry to be unbroken by a given configuration of the space-time fields is that the generator of the symmetry, h, commute with the energy-momentum tensor of the corresponding conformal field theory. $T=\overline{T}=0$ clearly satisfies this condition for any operator h, indicating the restoration of all symmetries in the high-energy limit.

What are we to make of the two-dimensional field theory $T=\bar{T}=0$? Since

$$T_{\alpha\beta}=\delta S/\delta g^{\alpha\beta},$$

where g and S are the world-sheet metric and action, respectively, we see that the world-sheet action must be independent of the world-sheet metric; it is a topological field theory,[23] a result suggested by Witten on other grounds.[24]

VII. DISCUSSION

Let us summarize what we have done in this paper. We have discussed the deformation of conformal field theories, and used the understanding gained to give a fairly general discussion of the gauge invariances of closed strings (while we have always worked with the bosonic string, only trivial changes are needed to discuss any other closed-string theory). In particular, we have proved that to each primary field of dimension one there corresponds a gauge transformation. This includes known gauge symmetries as well as an infinite class of symmetries that, we argued, relate fields of differing mass level. These symmetries appear to be broken by the space-time metric, but the symmetry breaking becomes unimportant at high energies.

This discussion raises many questions. Perhaps foremost is the question, what is the gauge algebra? It is certainly infinite, and, as we suggested above, possibly integer graded. The number of generators appears to grow with the grading, unlike the case of such simple infinite examples as the Kac-Moody and Virasoro algebras. Further understanding would seem to require the classification of all dimension-one primary fields, which may prove to be a formidable task. We would also like to understand this symmetry outside the Landau gauge. This requires an understanding of precisely what conditions on the operators h can be relaxed without spoiling the interpretation of the change in the stress tensor as a change of the space-time fields.

We have not shown that we have exhibited all the gauge symmetries of the string, although it is plausible that we have. If so, are all the states of the string gauge fields? It would be interesting to understand the connection with string field theory. There is no universally accepted version of closed-string field theory, and so comparison with the work of this paper would necessarily be rather tentative, but the methods described here should apply equally well to open strings, and in that case a direct comparison should be possible.[7,25]

We hope that some of these issues will soon be clarified.

ACKNOWLEDGMENTS

M.E. acknowledges the hospitality of the Aspen Center for Physics, where this work was begun. The work of B.A.O. was supported in part by Department of Energy Contract No. DOE-ACO2-76-ERO-3071. The work of M.E. was supported in part by Department of Energy Contract No. DE-AO2-87ER40325, task B.

[1]J. Schwarz, Phys. Rep. **89**, 223 (1982); J. Scherk, Rev. Mod. Phys. **47**, 123 (1975); M. Green, J. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, England, 1987).

[2]D. Gross and P. Mende, Nucl. Phys. **B303**, 407 (1988).

[3]D. Amati, M. Ciafaloni, and G. Veneziano, Phys. Lett. B **197**, 81 (1987).

[4]D. Gross, Phys. Rev. Lett. **60**, 1229 (1988).

[5]I. Frenkel, J. Lepowski, and A. Meurman, Proc. Nat. Acad. Sci. U.S.A. **81**, 3256 (1984); in *Unified String Theories,* proceedings of the Workshop, Santa Barbara, California, 1985, edited by M. B. Green and D. J. Gross (World Scientific, Singapore, 1986), p. 533.

[6]W. Siegel, Phys. Lett. **151B**, 391 (1985); **151B**, 396 (1985); T. Banks and M. Peskin, Nucl. Phys. **B264**, 513 (1986); W. Siegel and B. Zwiebach, *ibid.* **B263**, 106 (1986); **B282**, 125 (1987); Phys. Lett. B **184**, 325 (1987); K. Itoh, T. Kugo, H. Kunitomo, and H. Ooguri, Prog. Theor. Phys. **75**, 162 (1986).

[7]E. Witten, Nucl. Phys. **B268**, 253 (1986).

[8]M. Evans and B. Ovrut, Phys. Lett. B **231**, 80 (1989); M. Evans, in *The Storrs Meeting,* proceedings of the Annual Meeting of the Division of Particles and Fields of the APS, Storrs, Connecticut, 1988, edited by K. Haller *et al.* (World Scientific, Singapore, 1989).

[9]M. Evans and B. Ovrut, Phys. Lett. B **214**, 177 (1988); M. Evans, J. Louis, and B. Ovrut, Phys. Rev. D **35**, 3045 (1987).

[10]M. Evans and B. Ovrut, Phys. Rev. D **39**, 3016 (1989).

[11]G. Veneziano, Phys. Lett. **167B**, 388 (1986); J. Maharana and G. Veneziano, *ibid.* **169B**, 177 (1986); Nucl. Phys. **B283**, 126 (1987).

[12]C. Lovelace, Phys. Lett. **135B**, 75 (1984); C. Callan, D. Friedan, E. Martinec, and M. Perry, Nucl. Phys. **B262**, 593 (1985); A. Sen, Phys. Rev. D **32**, 2102 (1985).

[13]A. Belavin, A. Polyakov, and A. Zamolodchikov, Nucl. Phys. **B241**, 333 (1984); D. Friedan, E. Martinec, and S. Shenker, *ibid.* **B271**, 93 (1986).

[14]A. Zamolodchikov, Yad. Fiz. **44**, 821 (1986) [Sov. J. Nucl. Phys. **44**, 529 (1987)]; **46**, 1819 (1987) [**46**, 1090 (1987)]; T. Banks and E. Martinec, Nucl. Phys. **B294**, 733 (1987).

[15]J. Freericks and M. Halpern, Ann. Phys. (N.Y.) **188**, 258 (1988); R. Dijkgraaf, E. Verlinde, and H. Verlinde, Commun. Math. Phys. **115**, 649 (1988); P. Ginsparg, Nucl. Phys. **B295**, 153 (1988); L. Dixon, P. Ginsparg, and J. Harvey, *ibid.* **B306**, 470 (1988).

[16]I. Krichever and S. Novikov, Funkts. Anal. Prilozhen. **21**, No. 2, 46 (1987); **21**, No. 4, 47 (1987).

[17]C. Hsue, B. Sakita, and M. Virasoro, Phys. Rev. D **2**, 2857 (1970); E. Fradkin and A. Tseytlin, Phys. Lett. **158B**, 316 (1985); Nucl. Phys. **B261**, 1 (1985); B. Fridling and A Jevicki, Phys. Lett. B **174**, 75 (1986).

[18]E. D'Hoker and D. Phong, Phys. Rev. D **35**, 3890 (1987).

[19]P. Nelson, Phys. Rev. Lett. **62**, 993 (1989).

[20]P. Candelas, G. Horowitz, A. Strominger, and E. Witten, Nucl. Phys. **B258**, 46 (1985); W. Boucher, D. Friedan, and A.

Kent, Phys. Lett. B **172**, 316 (1986); T. Banks, L. Dixon, D. Friedan, and E. Martinec, Nucl. Phys. **B299**, 613 (1988); T. Banks and L. Dixon, *ibid.* **B307**, 93 (1988).

[21]S. Coleman and J. Mandula, Phys. Rev. **159**, 1251 (1967).

[22]J. Atick and E. Witten, IAS Report No. IASSNS-HEP-88/14, (unpublished).

[23]E. Witten, Commun. Math. Phys. **117**, 353 (1988); **118**, 411 (1988); **121**, 351 (1989); J. Labastida, M. Pernici, and E. Witten, Nucl. Phys. **B310**, 611 (1988).

[24]E. Witten, IAS Report No. IASSNS-HEP-88/55 (unpublished).

[25]P. Argyres and C. Nappi, Phys. Lett. B **224**, 89 (1989).