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Spinning test particles and the motion of a gyroscope according to the nonsymmetric gravitation theory

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The motion of spinning test particles is derived in the nonsymmetric gravitation theory using the field equations of matter and the methods of Papapetrou. The predictions for the precession of a torque-free gyroscope in the gravitational field of the Earth, Jupiter, and the Sun are obtained and compared with the predictions of Einstein's gravitation theory.

I. EQUATIONS OF MOTION OF A SPINNING TEST PARTICLE

In previous work,[1] we have derived the motion of a test particle from the conservation laws of the nonsymmetric gravitation theory (NGT), using the methods of Fock and Papapetrou.[2,3] The motion of extended bodies with a finite mass has been derived from the NGT field equations and the conservation laws in a post-Newtonian approximation scheme, in which the field equations and the motion of bodies was obtained by expanding in powers of v/c.[4] The conservation laws in NGT take the form

$$\tfrac{1}{2}(g_{\mu\rho}\mathbf{T}^{\mu\nu}{}_{,\nu}+g_{\rho\mu}\mathbf{T}^{\nu\mu}{}_{,\nu})+[\mu\nu,\rho]\mathbf{T}^{\mu\nu}+\tfrac{1}{3}W_{[\rho,\nu]}\mathbf{S}^{\nu}=0\ , \tag{1.1}$$

where the notation is the same as in Ref. 1. $\mathbf{T}^{\mu\nu}=(-g)^{1/2}T^{\mu\nu}$ and $\mathbf{T}^{\mu\nu}$ and $\mathbf{S}^{\mu}$ are the test particle sources, and they vanish outside a narrow tube in four-dimensional space-time that surrounds the world line of some point X^{μ} of the particle. The X^{μ} coordinates of the particle are regarded as functions of the time $X^0=t$, or of the proper time s along the world line of the test particle.

The motion of a monopole test particle was obtained from the assumption that the dipole and higher moments of $\mathbf{T}^{\mu\nu}$ and $\mathbf{S}^{\mu}$ vanish, and that only the integrals

$$\int \mathbf{T}^{\mu\nu}d^3x,\quad \int \mathbf{S}^{\mu}d^3x \tag{1.2}$$

are nonvanishing along the world line of the particle. In (1.2), the integrals extend over three-dimensional space for $t=$const. The equation of motion obtained from (1.1) for a monopole test particle takes the form

$$\frac{du^{\mu}}{ds}+\begin{Bmatrix}\mu\\ \alpha\beta\end{Bmatrix}u^{\alpha}u^{\beta}=\kappa H^{\mu}{}_{\nu}u^{\nu}\ , \tag{1.3}$$

where $u^{\mu}=dx^{\mu}/ds$ is the four-velocity of the test particle,

$$\begin{Bmatrix}\mu\\ \alpha\beta\end{Bmatrix}$$

is the Christoffel symbol in NGT, defined by

$$\begin{Bmatrix}\lambda\\ \mu\nu\end{Bmatrix}=\tfrac{1}{2}\gamma^{(\lambda\rho)}(g_{(\mu\rho),\nu}+g_{(\rho\nu),\mu}-g_{(\mu\nu),\rho})\ , \tag{1.4}$$

and $\gamma^{(\mu\rho)}$ is defined by

$$\gamma^{(\mu\rho)}g_{(\rho\nu)}=\delta^{\mu}_{\nu}\ . \tag{1.5}$$

Moreover, the tensor $H^{\mu}{}_{\nu}$ is given by

$$H^{\mu}{}_{\nu}=\tfrac{1}{2}\gamma^{(\mu\rho)}R_{[\rho\nu]}(\Gamma)\ , \tag{1.6}$$

where $R_{[\mu\nu]}(\Gamma)$ is the skew part of the NGT Ricci tensor in Ref. 1. The constant κ is given by $\kappa=l_t^2/m_t$ where l_t^2 is the NGT "charge" of the test particle

$$l_t^2=\int \mathbf{S}^0 d^3x \tag{1.7}$$

and m_t is its mass given by

$$m_t=\frac{ds}{dt}\int \mathbf{T}^{00}d^3x\ . \tag{1.8}$$

For photons, $l_t=0$, and they move along geodesics

$$\frac{Du^{\mu}}{Ds}=0\ , \tag{1.9}$$

where Du^{μ}/Ds is defined by

$$\frac{Du^{\mu}}{Ds}=\frac{du^{\mu}}{ds}+\begin{Bmatrix}\mu\\ \alpha\beta\end{Bmatrix}u^{\alpha}u^{\beta}\ . \tag{1.10}$$

Several important physical consequences of the motion of test particles in the static, spherically symmetric field of NGT have been studied in Ref. 1, using the line element

$$ds^2=g_{(\mu\nu)}dx^{\mu}dx^{\nu}\ . \tag{1.11}$$

The choice of the metric tensor $g_{(\mu\nu)}$, in (1.11), is the only one consistent with the equations of motion derived from the conservation laws. The equations of motion of particles, including those for extended massive bodies up to the post-Newtonian order, can be derived from the variation of the action[5]

$$\delta\int ds=\delta\int[(g_{(\mu\nu)}u^{\mu}u^{\nu})^{1/2}+\tfrac{1}{6}\kappa W_{\mu}u^{\mu}]ds=0\ . \tag{1.12}$$

We can now apply Papapetrou's methods[3] to derive the motion of a spinning test particle. The spin angular momentum is defined by

$$S^{\mu\nu}=\int(x^{\mu}-X^{\mu})\mathbf{T}^{\nu0}d^3x-\int(x^{\nu}-X^{\nu})\mathbf{T}^{\mu0}d^3x\ , \tag{1.13}$$

which can be shown to have the transformation properties of a tensor. We assume that the test particle is a pole-dipole particle, whereby the pole and dipole contributions of the integrated test particle sources $\mathbf{T}^{\mu\nu}$ and $\mathbf{S}^{\mu}$ do not vanish. The equation of motion of the spin is found to be

$$\begin{aligned}\frac{DS^{\mu\nu}}{Ds}&+u^{\mu}u_{\gamma}\frac{DS^{\nu\gamma}}{Ds}-u^{\nu}u_{\gamma}\frac{DS^{\mu\gamma}}{Ds}\\&+u^{\mu}u_{\gamma}(H^{\gamma}{}_{\sigma}C^{\nu\sigma}-H^{\nu}{}_{\sigma}C^{\gamma\sigma})\\&-u^{\nu}u_{\gamma}(H^{\gamma}{}_{\sigma}C^{\mu\sigma}-H^{\mu}{}_{\sigma}C^{\gamma\sigma})=0\ ,\end{aligned} \tag{1.14}$$

where

$$C^{\mu\nu}=u^0\int\delta x^{\mu}\mathbf{S}^{\nu}d^3x\ . \tag{1.15}$$

Here $\delta x^{\mu}=x^{\mu}-X^{\mu}$ and $\delta x^0=0$. It follows from (1.15) that $C^{0\nu}=0$. Moreover, in (1.14) we have

$$\frac{DS^{\mu\nu}}{Ds}\equiv\frac{dS^{\mu\nu}}{ds}+\begin{Bmatrix}\mu\\ \alpha\beta\end{Bmatrix}S^{\alpha\nu}u^{\beta}+\begin{Bmatrix}\nu\\ \alpha\beta\end{Bmatrix}S^{\mu\alpha}u^{\beta}\ . \tag{1.16}$$

The equation of motion of the test particle will be a modification of (1.3) up to terms of order $S^{\mu\nu}$. The effect of the spin on the orbital motion of the particle will be negligible and will not be of current interest in experimental physics. If we set $\nu=0$ in (1.14) and multiply by u^{ν}/u^0, and then set $\mu=0$ in (1.14) and multiply by u^{μ}/u^0, then add the resulting two equations and substitute into (1.14), we get the noncovariant form of the equations of spin motion

$$\frac{DS^{\mu\nu}}{Ds}+\frac{u^{\mu}}{u^0}\frac{DS^{\nu0}}{Ds}-\frac{u^{\nu}}{u^0}\frac{DS^{\mu0}}{Ds}=0\ . \tag{1.17}$$

If we set $\mu=i,\nu=0$ in (1.17), where $i,j=1,2,3$ denote the spatial coordinates, then we obtain a trivial identity, so only three of the six equations (1.17) are independent. We must therefore impose a supplementary condition. The Corinaldesi-Papapetrou condition is[6]

$$S^{i0}=0\ , \tag{1.18}$$

in the rest frame of the central body, so that in this system the X^i is the center of mass. The Pirani condition is[7]

$$S^{\mu\nu}u_{\nu}=0\ , \tag{1.19}$$

which leads to $u^i=0$ in the rest frame of the test particle, whereby $S^{i0}=0$ and X^i is the center of mass in the particle rest frame. This supplementary condition removes the ambiguity in the choice of the point X^i for the spinning particle. We shall find it more convenient in what follows to use the Pirani condition (1.19). From (1.19), we get

$$\frac{DS^{\mu\nu}}{Ds}u_{\nu}+S^{\mu\nu}\frac{Du_{\nu}}{Ds}=0\ . \tag{1.20}$$

With the condition (1.19) and using (1.20), the equations of motion for the spin (1.17) take the form

$$\frac{DS^{\mu\nu}}{Ds}=(u^{\mu}S^{\nu\alpha}-u^{\nu}S^{\mu\alpha})\frac{Du_{\alpha}}{Ds}\ , \tag{1.21}$$

where we have used the equation

$$\gamma^{(\mu\nu)}u_{\mu}u_{\nu}=1\ . \tag{1.22}$$

From (1.3) we have that

$$\frac{Du^{\mu}}{Ds}=\kappa H^{\mu}{}_{\nu}u^{\nu}\ , \tag{1.23}$$

or

$$\frac{Du_{\mu}}{Ds}=\kappa H_{[\mu\nu]}u^{\nu}\ , \tag{1.24}$$

where

$$\frac{Du_{\mu}}{Ds}=\frac{du_{\mu}}{ds}-\begin{Bmatrix}\rho\\ \mu\beta\end{Bmatrix}u^{\beta}u_{\rho}\ . \tag{1.25}$$

Substituting (1.24) into (1.21), we get

$$\frac{DS^{\mu\nu}}{Ds}=(u^{\mu}S^{\nu\alpha}-u^{\nu}S^{\mu\alpha})f_{\alpha}\ , \tag{1.26}$$

where

$$f_{\alpha}=\kappa H_{[\alpha\gamma]}u^{\gamma}\ , \tag{1.27}$$

and we understand that $f^{\alpha}=\gamma^{(\alpha\nu)}f_{\nu}$. From (1.27) it follows that

$$u^{\alpha}f_{\alpha}=0\ . \tag{1.28}$$

We see that in NGT, the new NGT tensor force, proportional to the skew part of the Ricci tensor $R_{[\mu\nu]}$, acts as an external force that makes $DS^{\mu\nu}/Ds$ nonvanishing. Since we expect that the NGT "charge density," S^0, will be nonvanishing for the test particle, such as a gyroscope orbiting the Earth, and that this force cannot be removed by shielding, we must include the contribution of f_{α} in our calculations.

II. EXPERIMENTAL PREDICTION FOR THE GYROSCOPE

We shall be concerned with the equation of motion for the spin tensor S^{ij} obtained from (1.26):

$$\frac{DS^{ij}}{Ds}=(u^iS^{j\alpha}-u^jS^{i\alpha})f_\alpha \ . \tag{2.1}$$

Following Schiff's approach,[8] we can define the purely spatial vector $\mathbf{S}$ in rectangular coordinates:

$$\mathbf{S}=(S_x,S_y,S_z)=(S^{23},S^{31},S^{12}) \ . \tag{2.2}$$

Since $\mathbf{T}^{i0}$ is the momentum density in the i direction, it follows from (1.13) that $\mathbf{S}$ is the spin angular momentum vector with respect to the point $\mathbf{r}=(X^1,X^2,X^3)$. An alternate approach is to construct the vector density

$$S^\mu=\tfrac{1}{2}\epsilon^{\mu\nu\lambda\tau}u_\nu S_{\lambda\tau} \tag{2.3}$$

and define $\mathbf{S}=(S^1,S^2,S^3)$. In the rest frame of the gyroscope ($u^i=0$) both approaches are equivalent. We choose Schiff's approach so that we may make use of the transformation properties discussed in detail in Ref. 8.

Now, from (1.28) it follows that $u^0f_0=-(u^1f_1+u^2f_2+u^3f_3)$, and, therefore, the terms in (2.1) involving f_i are of order $v|\mathbf{S}||\mathbf{f}|$, while the f_0 terms in (2.1) are of order $v|\mathbf{M}|f_0\simeq v^3|\mathbf{S}||\mathbf{f}|$ and hence can be neglected. Also, we have that $f_i\simeq -f^i$. Thus, using (2.2), we can write (2.1) for a slowly moving gyroscope

$$\frac{D\mathbf{S}}{Dt}=\mathbf{S}(\mathbf{v}\cdot\mathbf{f})-\mathbf{f}(\mathbf{v}\cdot\mathbf{S}) \ . \tag{2.4}$$

The line element associated with the static, spherically symmetric solution, in NGT, has the form[1]

$$ds^2=\left[1+\frac{l_s^4}{r^4}\right]\left[1-\frac{2m_s}{r}\right]dt^2-\left[1-\frac{2m_s}{r}\right]^{-1}dr^2-d\Omega^2 \ , \tag{2.5}$$

where

$$d\Omega^2=r^2d\theta^2+r^2\sin^2\theta\, d\phi^2 \ , \tag{2.6}$$

and l_s^2 denotes the NGT "charge" of the central singularity and m_s its mass. In isotropic coordinates, for large values of r, the line element takes the form

$$ds^2=\left[1-\frac{2m_s}{r}+\frac{l_s^4}{r^4}\right]dt^2 - \left[1+\frac{2m_s}{r}\right](dx^2+dy^2+dz^2) \ , \tag{2.7}$$

where $r=(x^2+y^2+z^2)^{1/2}$. The skew tensor $g_{[\mu\nu]}$ has the nonvanishing components

$$g_{[i0]}=\frac{l_s^2x^i}{r^3} \ . \tag{2.8}$$

A calculation of the components of the skew Ricci tensor in isotropic coordinates gives[1]

$$\tfrac{1}{2}\gamma^{(ij)}R_{[j0]}=-\frac{2l_s^2m_sx^i}{r^6} \ , \tag{2.9}$$

$$\tfrac{1}{2}\gamma^{(00)}R_{[0i]}=-\frac{2l_s^2m_sx^i}{r^6} \ , \tag{2.10}$$

where

$$\gamma^{(ij)}=-\left[1-\frac{2m_s}{r}\right]^{-1}\delta_{ij}, \quad \gamma^{00}=\left[1-\frac{2m_s}{r}+\frac{l_s^4}{r^4}\right]^{-1} . \tag{2.11}$$

The nonvanishing Christoffel symbols are

$$\begin{Bmatrix} i \\ 00 \end{Bmatrix}=-\tfrac{1}{2}\gamma^{(ik)}g_{(00),k}, \quad \begin{Bmatrix} 0 \\ i0 \end{Bmatrix}=\tfrac{1}{2}\gamma^{00}g_{(00),i} \ ,$$
$$\begin{Bmatrix} j \\ ik \end{Bmatrix}=\tfrac{1}{2}\gamma^{(jl)}(g_{(il),k}+g_{(lk),i}-g_{(ik),l}) \ . \tag{2.12}$$

From (1.3), we have

$$\frac{du^i}{ds}=-\begin{Bmatrix} i \\ \alpha\beta \end{Bmatrix}u^\alpha u^\beta+\kappa H^i{}_\nu u^\nu \ . \tag{2.13}$$

A straightforward calculation now yields

$$\frac{d\mathbf{v}}{dt}=-\left[\frac{m_s}{r^3}-\frac{2l_s^4}{r^6}+\frac{2l_s^2l_t^2m_s}{m_tr^6}\right]\mathbf{r} \ . \tag{2.14}$$

From (1.16), we have

$$\frac{DS^{ij}}{Ds}=\frac{dS^{ij}}{Ds}+\begin{Bmatrix} i \\ \mu\nu \end{Bmatrix}S^{\mu j}u^\nu+\begin{Bmatrix} j \\ \mu\nu \end{Bmatrix}S^{i\mu}u^\nu \ , \tag{2.15}$$

which gives

$$\frac{D\mathbf{S}}{dt}=\frac{d\mathbf{S}}{dt}-\frac{2m_s}{r^3}(\mathbf{r}\cdot\mathbf{v})\mathbf{S}-\frac{m_s}{r^3}(\mathbf{r}\times\mathbf{v})\times\mathbf{S} - \left[\frac{m_s}{r^3}-\frac{2l_s^4}{r^6}\right](\mathbf{r}\times\mathbf{M}) \ , \tag{2.16}$$

where we define

$$\mathbf{M}=(S^{10},S^{20},S^{30}) \ . \tag{2.17}$$

In (2.16), we have that

$$(\mathbf{r}\times\mathbf{v})\times\mathbf{S}=\mathbf{v}(\mathbf{r}\cdot\mathbf{S})-\mathbf{r}(\mathbf{v}\cdot\mathbf{S}) \ . \tag{2.18}$$

The supplementary condition (1.19) can be written as

$$u^\rho g_{(\rho\nu)}S^{\mu\nu}=0 \ , \tag{2.19}$$

or

$$S^{10}\simeq\left[1+\frac{4m_s}{r}-\frac{l_s^4}{r^4}\right](v_yS_z-v_zS_y) \ . \tag{2.20}$$

From this result, we obtain

$$\mathbf{M}\simeq\left[1+\frac{4m_s}{r}-\frac{l_s^4}{r^4}\right](\mathbf{v}\times\mathbf{S})\simeq\mathbf{v}\times\mathbf{S} \ . \tag{2.21}$$

It now follows that

$$\mathbf{r}\times\mathbf{M}\simeq\mathbf{r}\times(\mathbf{v}\times\mathbf{S})=\mathbf{v}(\mathbf{r}\cdot\mathbf{S})-\mathbf{S}(\mathbf{r}\cdot\mathbf{v}) \ . \tag{2.22}$$

We now find that, to this order of approximation,

$$\frac{D\mathbf{S}}{dt}=\frac{d\mathbf{S}}{dt}-\frac{m_s}{r^3}[\mathbf{S}(\mathbf{r}\cdot\mathbf{v})+2\mathbf{v}(\mathbf{r}\cdot\mathbf{S})-\mathbf{r}(\mathbf{v}\cdot\mathbf{S})]$$
$$+\frac{2l_s^4}{r^6}[\mathbf{v}(\mathbf{r}\cdot\mathbf{S})-\mathbf{S}(\mathbf{r}\cdot\mathbf{v})] \ . \qquad (2.23)$$

Finally, we obtain from (2.4) and (2.23), to this order of approximation, the result

$$\frac{d\mathbf{S}}{dt}=\frac{m_s}{r^3}[\mathbf{S}(\mathbf{r}\cdot\mathbf{v})+2\mathbf{v}(\mathbf{r}\cdot\mathbf{S})-\mathbf{r}(\mathbf{v}\cdot\mathbf{S})]$$
$$-\frac{2l_s^4}{r^6}[\mathbf{v}(\mathbf{r}\cdot\mathbf{S})-\mathbf{S}(\mathbf{r}\cdot\mathbf{v})]+\mathbf{S}(\mathbf{v}\cdot\mathbf{f})-\mathbf{f}(\mathbf{v}\cdot\mathbf{S}) \ . \qquad (2.24)$$

We must now transform to the rest frame of the gyroscope. First, we must rescale the spatial basis vectors using the metric to maintain orthonormality, and second, we perform a Lorentz transformation. These two changes in $\mathbf{S}$ are given respectively by[8]

$$\mathbf{S}_{\text{rest}}=\left[1+\frac{2m_s}{r}\right]\mathbf{S}, \quad (\mathbf{S}_{\perp v})_{\text{rest}}=\left[1-\frac{v^2}{2}\right]\mathbf{S}_{\perp v} \ , \qquad (2.25)$$

where

$$\mathbf{S}_{\perp v}=\mathbf{S}-\hat{\mathbf{v}}(\hat{\mathbf{v}}\cdot\mathbf{S}) \ . \qquad (2.26)$$

$\mathbf{S}_{\text{rest}}$ denotes the rest-frame value of the spin vector $\mathbf{S}$. Thus, the combined change in $\mathbf{S}$ is

$$\mathbf{S}_{\text{rest}}=\left[1+\frac{2m_s}{r}\right]\mathbf{S}-\frac{v^2}{2}[\mathbf{S}-\hat{\mathbf{v}}(\hat{\mathbf{v}}\cdot\mathbf{S})]$$
$$=\left[1+\frac{2m_s}{r}\right]\mathbf{S}-\frac{v^2}{2}\mathbf{S}+\tfrac{1}{2}\mathbf{v}(\mathbf{v}\cdot\mathbf{S}) \ . \qquad (2.27)$$

Differentiating this, and using (2.14), we obtain the desired result for the time dependence of the spin vector in the rest frame of the gyroscope to lowest order:

$$\frac{d\mathbf{S}_{\text{rest}}}{dt}=\frac{3m_s}{2r^3}[\mathbf{v}(\mathbf{r}\cdot\mathbf{S}_{\text{rest}})-\mathbf{r}(\mathbf{v}\cdot\mathbf{S}_{\text{rest}})]$$
$$-\frac{l_s^2m_sa}{r^6}[\mathbf{v}(\mathbf{r}\cdot\mathbf{S}_{\text{rest}})-\mathbf{r}(\mathbf{v}\cdot\mathbf{S}_{\text{rest}})] \ , \qquad (2.28)$$

where a is given by

$$a=\frac{l_s^2}{m_s}+\frac{l_t^2}{m_t} \ . \qquad (2.29)$$

Equation (2.28) can be written in the form

$$\frac{d\mathbf{S}_{\text{rest}}}{dt}=\left[\frac{3m_s}{2r^3}-\frac{l_s^2m_sa}{r^6}\right](\mathbf{r}\times\mathbf{v})\times\mathbf{S}_{\text{rest}} \ , \qquad (2.30)$$

or

$$\frac{d\mathbf{S}_{\text{rest}}}{dt}=\boldsymbol{\Omega}\times\mathbf{S}_{\text{rest}} \ . \qquad (2.31)$$

We have

$$\boldsymbol{\Omega}=\left[\frac{3m_s}{2r^3}-\frac{L_s}{r^6}\right](\mathbf{r}\times\mathbf{v}) \ , \qquad (2.32)$$

where

$$L_s=l_s^2am_s \ . \qquad (2.33)$$

Let us take the gyroscope's orbit to be a circle of radius r with unit normal orbital angular momentum vector $\hat{\mathbf{J}}$, then the gyroscope's velocity can be calulated from (2.14) to be

$$\mathbf{v}=-\left[\frac{m_s}{r^3}-\frac{2l_s^4}{r^6}+\frac{2l_s^2l_t^2m_s}{m_tr^6}\right]^{1/2}\mathbf{r}\times\hat{\mathbf{J}} \ . \qquad (2.34)$$

We have that

$$\mathbf{r}\times(\mathbf{r}\times\hat{\mathbf{J}})=(\mathbf{r}\cdot\hat{\mathbf{J}})\mathbf{r}-(\mathbf{r}\cdot\mathbf{r})\hat{\mathbf{J}}$$
$$=-(\mathbf{r}\cdot\mathbf{r})\hat{\mathbf{J}}=-r^2\hat{\mathbf{J}} \ . \qquad (2.35)$$

The precession rate, averaged over a revolution, is given to lowest order by

$$\langle|\boldsymbol{\Omega}|\rangle=\frac{3m_s^{3/2}}{2R_s^{5/2}}\left[\frac{R_s}{r}\right]^{5/2}-\frac{L_sm_s^{1/2}}{R_s^{11/2}}\left[\frac{R_s}{r}\right]^{11/2} \ , \qquad (2.36)$$

where R_s is the radius of the central source.

Until now, we have ignored the Lense-Thirring effect, which is the precession due to the rotation of the Earth on its axis.[9] We provide a treatment here to show that the NGT Lense-Thirring result is identical to that of Einstein gravitation theory (EGT). In EGT, we can include the effect of the Earth's rotation, to first order, in the metric in rectangular isotropic coordinates as was shown by Lense and Thirring:[9]

$$g_{(\mu\nu)}^{\text{EGT}}=\begin{bmatrix}-(1+2m_s/r) & 0 & 0 & -2\mathbf{J}_sy/r^3\\ 0 & -(1+2m_s/r) & 0 & 2\mathbf{J}_sx/r^3\\ 0 & 0 & -(1+2m_s/r) & 0\\ -2\mathbf{J}_sy/r^3 & 2\mathbf{J}_sx/r^3 & 0 & 1-2m_s/r\end{bmatrix} , \qquad (2.37)$$

where the off-diagonal elements $g_{(0i)}$ account for the rotation of the Earth with angular momentum $\mathbf{J}_s$ aligned along the z axis.

The solution of the first post-Newtonian approximation to NGT is given in Ref. 4. The NGT metric can be shown to be

$$g^{\rm NGT}_{(\mu\nu)}=\begin{bmatrix} -(1+2m_s/r) & 0 & 0 & -2\mathbf{J}_s y/r^3 \\ 0 & -(1+2m_s/r) & 0 & 2\mathbf{J}_s x/r^3 \\ 0 & 0 & -(1+2m_s/r) & 0 \\ -2\mathbf{J}_s y/r^3 & 2\mathbf{J}_s x/r^3 & 0 & 1-2m_s/r+l_s^4/r^4 \end{bmatrix} . \tag{2.38}$$

To post-Newtonian order, we find that the corrections to the off-diagonal metric elements $g_{(0i)}$ are identical to the above EGT case. The corrections to the skew metric $g_{[0i]}$ are not relevant because the calculation of $d\mathbf{S}_{\rm rest}/dt$ involves only the symmetric parts of the metric. Thus, the precession due to the Lense-Thirring effect, which arises from the off-diagonal elements of the metric $g_{(\mu\nu)}$, is identical in NGT and EGT. Moreover, the inclusion of the Earth's rotation has no effect (to post-Newtonian order) on the diagonal elements of the metric. The diagonal elements lead to geodetic precession, which was calculated above.

It is straightforward to include the off-diagonal metric elements into the above calculation [(2.12) to (2.30)], yielding an additional term,

$$\left[\frac{3(\mathbf{J}_s\cdot\mathbf{r})\mathbf{r}}{r^5}-\frac{\mathbf{J}_s}{r^3}\right]\times\mathbf{S}_{\rm rest} , \tag{2.39}$$

on the right side of (2.30). We see from (2.31) that the Lense-Thirring contribution to the precession rate is

$$\mathbf{\Omega}=\frac{3(\mathbf{J}_s\cdot\mathbf{r})\mathbf{r}}{r^5}-\frac{\mathbf{J}_s}{r^3} . \tag{2.40}$$

As we are interested in observing how NGT corrects EGT predictions of precession rates, and as there are no NGT corrections to the Lense-Thirring effect, we drop this and return to (2.36). We can write

$$\langle\mathbf{\Omega}\rangle=\langle\mathbf{\Omega}\rangle_{\rm EGT}+\langle\mathbf{\Omega}\rangle_{\rm NGT} . \tag{2.41}$$

So we can separate the EGT and NGT contributions in (2.36) as follows:

$$\langle|\mathbf{\Omega}|\rangle_{\rm EGT}=\frac{3(GM_s)^{3/2}}{2c^2R_s^{5/2}}\left[\frac{R_s}{r}\right]^{5/2} , \tag{2.42}$$

$$\langle|\mathbf{\Omega}|\rangle_{\rm NGT}=\frac{L_s(GM_s)^{1/2}}{R_s^{11/2}}\left[\frac{R_s}{r}\right]^{11/2} . \tag{2.43}$$

For the Earth, we have

$$\langle|\mathbf{\Omega}|\rangle_{\rm EGT}=8.45\left[\frac{R_\oplus}{r}\right]^{5/2} \text{ arcsec/yr} . \tag{2.44}$$

For a drag-free satellite at a height of $h=600$ km above the Earth, we get from EGT the predicted gyroscope precession

$$\langle|\mathbf{\Omega}|\rangle_{\rm EGT}=6.75 \text{ arcsec/yr} . \tag{2.45}$$

To estimate the NGT contribution to the precession, we need an upper bound on $l_\oplus$. A bound for $l_\oplus$ has been obtained, in Ref. 4, using the LAGEOS artificial satellite data and the Lunar Laser Ranging data.[10] The result is $l_\oplus=(3\pm3)$ km. We know from the results of the Pound, Rebka, and Snyder experiment for the redshift of a photon in the Earth's gravitational field that $l_\oplus\le 6$ km.[11] If we use $l_\oplus=5$ km, and estimate $a\simeq 2l_\oplus^2/m_\oplus$ based on the consideration that the constituents of the Earth and the gyroscope rotor are of a similar substance, then (2.43) yields

$$\langle|\mathbf{\Omega}|\rangle_{\rm NGT}=0.0037 \text{ arcsec/yr} \tag{2.46}$$

as the expected maximum correction to the gyroscope rate of precession due to NGT.

However, we are able to calculate a particle model value of l^2 by making some basic assumptions. In general, we have, from NGT,[12]

$$l^2=f_p^2Z+f_e^2Z+f_n^2N+f_\nu^2N_\nu+f_c^2N_c , \tag{2.47}$$

where f_p^2, f_e^2, f_n^2, f_ν^2, and f_c^2 are the universal coupling constants of protons, electrons, neutrons, neutrinos, and cosmions, respectively. Z, N, N_ν, and N_c denote the number of protons, neutrons, neutrinos, and cosmions. In (2.47), we have explicitly included the electron coupling and taken into account the charge neutrality of the source: $Z=N_e$. Now, for bodies within the solar system, $N\sim Z\gg N_\nu$. Thus, we are justified in neglecting the neutrino contribution in (2.47) above. Moreover, $N\simeq m_s/2m_p$, where m_p is the proton mass. Thus, for the central source, we can write (2.47) as

$$l_s^2=\frac{m_s}{2m_p}(f_e^2+f_p^2+f_n^2+2f_c^2\eta_c) , \tag{2.48}$$

where we define

$$\eta_c=\frac{N_c}{m_s/m_p} . \tag{2.49}$$

For the gyroscope rotor, we assume no cosmion content, and by similar arguments that precede (2.48), we obtain

$$\kappa=\frac{l_t^2}{m_t}=\frac{f_e^2+f_p^2+f_n^2}{2m_p} . \tag{2.50}$$

An upper bound on the neutron coupling constant, f_n^2, was derived from stability conditions on neutron-star solutions,[13] where it was found that $f_n^2 \leq 5\times 10^{-46}$ cm^2. Moreover, a value of $f_e^2=(8.7^{+3.9}_{-8.7})\times 10^{-43}$ cm^2 has been recently obtained by an NGT fit to the pulse time-of-arrival observations of the binary pulsar PSR 1913+16,[14] where it was assumed that $f_p^2=f_n^2$. In what follows, we shall neglect f_n^2 and f_p^2, and treat the value of $f_e^2=8.7\times 10^{-43}$ cm^2 as an upper bound. The value of L_s, in (2.43), can be determined by substituting (2.50) and (2.29) into (2.33). This yields

$$\begin{aligned} L_s &= l_s^2 a m_s \\ &\leq l_s^4 + (3.545\times 10^9\ \text{cm}) m_s l_s^2 \ . \end{aligned} \tag{2.51}$$

Now, in the case of the Earth, we initially neglect the cosmion contribution in (2.48). Then, we have

$$l_\oplus^2 = \frac{M_\oplus f_e^2}{2m_p} \ , \tag{2.52}$$

which yields

$$l_\oplus \leq 0.40 \text{ km} \ . \tag{2.53}$$

We now have, from (2.43),

$$\langle |\boldsymbol{\Omega}| \rangle_{\text{NGT}} \leq 2.43\times 10^{-7} \left[\frac{R_\oplus}{r} \right]^{11/2} \text{arcsec/yr} \ . \tag{2.54}$$

The projected experimental precision of the Stanford gyroscope is approximately 0.0005 arcsec.[15] Thus, if there are no cosmions in the Earth, then the NGT correction to the precession rate would be unobservable at any altitude.

To consider the effect of cosmions in the Earth on gyroscope precession, we require at least an upper bound for the number of cosmions contained in the Earth. From an NGT fit to the data of the eclipsing binary systems DI Herculis, and AS Cam, and the solar system, a value of $f_c^2=8.75\times 10^{-30}$ cm^2 (Refs. 12 and 16) and $\eta_{c_\oplus} \leq 10^{-13}$ (Ref. 16) were predicted. We find from (2.48) that the inclusion of cosmions yields at most a value of

$$l_\oplus \leq 0.69 \text{ km} \ . \tag{2.55}$$

Substitution of this value into (2.43) gives an upper limit to the NGT correction to the gyroscope precession:

$$\langle |\boldsymbol{\Omega}| \rangle_{\text{NGT}} \leq 1.45\times 10^{-6} \left[\frac{R_\oplus}{r} \right]^{11/2} \text{arcsec/yr} \ . \tag{2.56}$$

This is also below our ability to detect.

It has been suggested by Krisher that Jupiter provides a useful testing ground for NGT.[17] Krisher suggests using radiometric observational data of spacecraft flybys. To obtain a reasonable estimate of the cosmion content, Krisher took into account accretion during body formation and subsequent trapping, but did not consider the possible greater rate of cosmion evaporation in Jupiter than in the Sun, due to the higher density of the core of the planet than that of the Sun. The value quoted by Krisher, $\eta_{c_J}=10^{-12}$, should be treated as an upper bound.

It is therefore of interest to ask whether a gyroscope experiment in orbit about Jupiter would provide an NGT effect within our ability to detect. From (2.48), we obtain

$$l_J \leq 32.3 \text{ km} \ , \tag{2.57}$$

which, by application of (2.43), corresponds to a maximal NGT correction of

$$\langle |\boldsymbol{\Omega}| \rangle_{\text{NGT}} \leq 1.92\times 10^{-4} \left[\frac{R_J}{r} \right]^{11/2} \text{arcsec/yr} \ . \tag{2.58}$$

For Jupiter, the EGT contribution is determined from (2.42) to be

$$\langle |\boldsymbol{\Omega}| \rangle_{\text{EGT}} = 1.21\times 10^{2} \left[\frac{R_J}{r} \right]^{5/2} \text{arcsec/yr} \ . \tag{2.59}$$

For the Sun, if we choose $\eta_{c_\odot}=10^{-11}$,[12] and substitute this value into (2.48), we get

$$l_\odot \leq 3230 \text{ km} \ . \tag{2.60}$$

This value of $l_\odot$ yields a value of the quadrupole moment of the Sun: $J_2=5.3\times 10^{-6}$. By applying (2.43), we have

$$\langle |\boldsymbol{\Omega}| \rangle_{\text{NGT}} \leq 1.91 \left[\frac{R_\odot}{r} \right]^{11/2} \text{arcsec/yr} \ . \tag{2.61}$$

From (2.42), we obtain

$$\langle |\boldsymbol{\Omega}| \rangle_{\text{EGT}} = 1.30\times 10^{4} \left[\frac{R_\odot}{r} \right]^{5/2} \text{arcsec/yr} \ . \tag{2.62}$$

III. DISCUSSION OF THE RESULTS

The precession of the gyroscope will be affected by higher-order factors neglected in the above treatment. For an experiment in orbit about the Earth, the projected experimental precision of the Stanford gyroscope experiment is $\simeq$0.0005 arcsec.[15] This is about five times the approximate size of the terms predicted by EGT in the next higher order, which are down from the first-order terms by a factor of v. The effects of the Earth's oblateness and the ellipticity of the satellite orbit has been estimated by Wilkins,[18] Barker and O'Connell,[19] Hoots and Fitzpatrick,[20] and more recently by Breakwell.[21] Breakwell obtained for a polar orbit the result

$$\langle \boldsymbol{\Omega} \rangle_{\text{Earth correction}} = -0.007 \text{ arcsec/yr} \ . \tag{3.1}$$

Wilkins[17] has estimated the effect of the moon and the other planets and found that

$$\langle |\boldsymbol{\Omega}| \rangle_{\text{EGT,moon}} < 2\times 10^{-5} \text{ arcsec/yr} \ , \tag{3.2}$$

$$\langle |\boldsymbol{\Omega}| \rangle_{\text{EGT,planets}} < 4\times 10^{-6} \text{ arcsec/yr} \ . \tag{3.3}$$

The effect of the Sun was estimated by de Sitter and Roy[22] to be

$$\langle |\boldsymbol{\Omega}| \rangle_{\text{EGT,Sun}} \simeq 0.019 \text{ arcsec/yr} \ . \tag{3.4}$$

The effect of the other stars of the Milky Way is well below detection:[17]

$$\langle |\Omega| \rangle_{\mathrm{EGT,Milky\ Way}} < 10^{-8} \text{ arcsec/yr} . \tag{3.5}$$

The largest corrections to the gyroscope precession that we should concern oursevles with, caused by the ellipticity of the satellite's orbit, the oblateness of the Earth, and the gravitational field of the Sun, can be corrected for in the gyroscope experiment. The value obtained by Breakwell[21] for the former two effects can be subtracted from the experimentally observed precession, while the latter effect due to the Sun is easily accounted for, since the plane of the ecliptic will not coincide with the plane of the satellite orbit.

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