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Dynamical dimensional reduction induced by changing equation of state

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We study the dynamics of a homogeneous but in general anisotropic multidimensional cosmological model assuming that space-time is a product of a physical space-time M and a compact space B. We take the energy-momentum tensor in the form of a perfect fluid allowing, however, anisotropic pressure. In this case the Einstein field equations can be reduced to a two-dimensional dynamical system. We discuss the general behavior of this dynamical system and investigate conditions under which dynamical dimensional reduction takes place. We also discuss the dynamics of our model when the equation of state of matter is allowed to change. We consider slow and fast changes of the equation of state. In both cases, when the final equation of state is appropriate, the dynamical dimensional reduction takes place. It turns out that slow changes of the equation of state for a certain open set of initial conditions always create inflation in the final state. Rapid changes of the equation of state always suppress inflation if it existed in the initial state.

I. INTRODUCTION

Successes of the Weinberg-Salam-Glashow[1] theory unifying the electromagnetic and the weak interactions strengthen the belief that gauge theories provide the correct mathematical framework for the program of unification of all elementary interactions. The fact that multidimensional general relativity is capable of generating non-Abelian gauge theories places it in the focus of interest of contemporary theoretical physics. There are actually two classes of theories which use the concept of multidimensional space-time. One class contains extensions of the five-dimensional Kaluza-Klein theory which was originally designed to unify gravity and electromagnetism. It was noticed by Witten[2] that the 11-dimensional Kaluza-Klein theory seems to be almost uniquely singled out by its interesting properties. Eleven-dimensional space-time is the least dimensional space-time which could incorporate a gauge group containing SU(3)×SU(2)×U(1), and simultaneously the largest dimensional space-time accommodating supergravity theory with spins of basic fields not larger than two. The heterotic superstring theories (for details, see the recent monograph by Green, Schwarz, and Witten[3]) which describe bosonic and fermionic fields in ten-dimensional space-time is an example of another class of multidimensional theories. In both cases to obtain the physical four-dimensional space-time from the multidimensional space-time it is necessary to perform some kind of dimensional reduction or compactification. By dimensional reduction we mean a process of reduction of the multidimensional space-time to a space-time of the form of a product of a four-dimensional space-time M and a compact space of additional dimensions B (internal space) with a typical size of B comparable to the Planck length.

Several mechanism of dimensional reduction have been proposed in the literature (see, for example, Appelquist and Chodos[4]) but none is really satisfactory.

These include among others the Freund-Rubin mechanism,[5] the Casimir effect associated with matter fields or zero-point gravitational energies,[6] and the effect of higher-derivative terms in the gravitational action.[7] It was pointed out by Chodos and Detweiler[8] that the dynamics of the gravitational field might provide a natural mechanism of dimensional reduction. They considered $B(\mathrm{I})\times S^1$ space-time, where $B(\mathrm{I})$ is Bianchi type-I space, in the five-dimensional Kaluza-Klein theory and showed that the initial conditions can be chosen in such a way that during the evolution the fifth dimension shrinks to arbitrarily small size while the physical dimensions isotropically expand. Several authors have investigated cosmological dimensional reduction in the classical case and in the supergravity theory. Vacuum five-dimensional spatially homogeneous models have been investigated by Demaret and Hanquin.[9] Lorenz-Petzold,[10,11] Barrow and Stein-Schabes,[12] and independently Furusawa and Hosoya[13] have investigated the $B(\mathrm{IX})\times T^3$ model (T^3 is a three-dimensional torus). Bleyer and Liebsher[14] investigated the nonvacuum Friedmann-Robertson-Walker^d model. Spatially homogeneous world models in $d=11$ supergravity theory have been investigated by Demaret *et al.*[15] and by Lorenz-Petzold.[16] Systematic study of dynamics of multidimensional cosmological models has been undertaken by Demianski *et al.*[17]

When the idea of a multidimensional space-time is incorporated into cosmological considerations a new scenario of the very early evolution of the Universe emerges. At the very early state of evolution (at energies comparable and higher than the Planck energy $\sim 10^{19}$ GeV) it is assumed that all spacial dimensions are equivalent and the Universe is expanding. At a certain moment during the early evolution of the Universe a spontaneous dimensional reduction takes place and the space-time assumes a form of a product of a physical four-dimensional space-time and a compact manifold of additional dimensions. Typically the final size of the compactified manifold of additional dimensions is believed to be comparable with the Planck length.

In this paper we would like to study evolution of multidimensional space-time filled with matter described by a hydrodynamical energy-momentum tensor with anisotropic pressure. Our aim is to investigate dynamics of such space-times allowing for changes in the state of matter and to see under what conditions the dynamics of the gravitational field provides a natural mechanism of dimensional reduction.

In the next section we specify our model and derive equations governing its dynamics. We also study the general properties of the dynamical system and specify a region of the parameter space which corresponds to solutions in which dynamical dimensional reduction takes place creating a four-dimensional space-time modeling the observable Universe. In Sec. III we discuss dynamics of multidimensional models taking into account changes of the equation of state of the matter induced by physical processes in the early Universe. The last section contains summary of our main results and discussion of possible applications of multidimensional cosmological models.

II. MODEL OF A MULTIDIMENSIONAL UNIVERSE

For the sake of generality let us consider an $(n+1)$-dimensional space-time which is a product of a $(k+1)$-dimensional space-time M and $(k'=n-k)$-dimensional, compact if necessary, space B.

The line element assumes the form

$$ds^2=dt^2-g_{ab}(x^c,t)dx^a dx^b-g_{ij}(x^k,t)dx^i dx^j , \qquad (2.1)$$

where $a,b=1,2,\ldots,k$; $i,j=k+1,\ldots,n$ (we set $c=1$). We write the Einstein field equations in the form

$$R_{\mu\nu}=8\pi\bar{G}\left[T_{\mu\nu}-\frac{1}{n-1}g_{\mu\nu}T\right] , \qquad (2.2)$$

where $\mu,\nu=0,1,2,\ldots,n$,

$$T^{\mu}_{\nu}=\mathrm{Diag}\|\epsilon,-p,-p,\ldots,-p,-p',-p',\ldots,-p'\| \qquad (2.3)$$

is the hydrodynamical energy-momentum tensor, ϵ the energy density, p and p' are, respectively, the pressure in the physical space and in the space of additional dimensions, $\bar{G}$ the generalized gravitational constant, and T is the trace of the energy-momentum tensor. We assume that the matter content of the multidimensional space-time can be described by a hydrodynamical energy-momentum tensor and that

$$p=\alpha\epsilon, \quad p'=\alpha'\epsilon , \qquad (2.4)$$

where α and α' are constants such that $|\alpha|\le 1$, $|\alpha'|\le 1$.

We restrict our considerations to a simple model assuming that the space-time M and the space B are homogeneous and isotropic and the line element has the form

$$ds^2=dt^2-R^2(t)dL^2-r^2(t)dl^2 , \qquad (2.5)$$

where dL^2 and dl^2 are, respectively, Euclidean line elements of k- and k'-dimensional spaces. In general, because $\alpha\neq\alpha'$, this multidimensional space-time as a whole is homogeneous but anisotropic.

The Einstein field equations are conveniently written down with the help of Hubble parameters defined by

$$H=\frac{\dot{R}}{R}, \quad h=\frac{\dot{r}}{r} \qquad (2.6)$$

and they assume the form

$$\dot{H}+H(kH+k'h)=\kappa\epsilon\left[\alpha+\frac{1}{n-1}(1-k\alpha-k'\alpha')\right] , \qquad (2.7)$$

$$\dot{h}+h(kH+k'h)=\kappa\epsilon\left[\alpha'+\frac{1}{n-1}(1-k\alpha-k'\alpha')\right] , \qquad (2.8)$$

where $\kappa=8\pi\bar{G}$. H, h, and ϵ are related by a constraint equation [00 component of the Eq. (2.2)]

$$(kH+k'h)^2-kH^2-k'h^2=2\kappa\epsilon . \qquad (2.9)$$

Assuming that the expansion is adiabatic we have

$$-\frac{\dot{\epsilon}}{\epsilon}=(1+\alpha)kH+(1+\alpha')k'h . \qquad (2.10)$$

When α and α' are constants Eqs. (2.7)–(2.9) describe two-dimensional dynamical system. General properties of this system can be studied in detail. It turns out that such analysis is very useful for qualitative investigation of behavior of more general and more realistic models with α and α' allowed to change in time according to physical processes taking place in the early Universe (see Sec. III).

In order to simplify the system of Eqs. (2.7), (2.8), and (2.10) we substitute for h a new variable s defined by

$$h=sH . \qquad (2.11)$$

Then Eqs. (2.7), (2.8), and (2.10) can be rewritten in the form

$$\dot{H}+H^2(k+sk')=-\frac{k'}{n-1}\kappa\epsilon(\alpha'-\beta_1) , \qquad (2.12)$$

$$\dot{s}-\frac{H}{2(n-1)}[sk'(\alpha'-\beta_1)+(k-1)(\alpha'-\beta_2)]\times k'(k'-1)(s-s_+)(s-s_-)=0 , \qquad (2.13)$$

$$\kappa\epsilon=\frac{H^2}{2}k'(k'-1)(s-s_+)(s-s_-) , \qquad (2.14)$$

where $\beta_1=[1+(k'-1)\alpha]/k'$, $\beta_2=(k\alpha-1)/(k-1)$, and $s_\pm$ are the roots of the equation $\epsilon=0$ and they are equal to

$$s_\pm=-\frac{k}{k'-1}(1\mp\gamma)\ , \tag{2.15}$$

where $\gamma=\sqrt{(n-1)/kk'}$. We notice that for $k'>1$, $\gamma<1$, and therefore $s_-<s_+<0$. Our attention will concentrate on Eq. (2.13) which can be conveniently rewritten in the form

$$\frac{ds}{d\tau}=\frac{1}{2}\left[\frac{k'}{\gamma}\right]^2(1-\gamma^2)(\alpha'-\beta_1)(s-s_0)(s-s_+)\times(s-s_-)\ , \tag{2.16}$$

where we have introduced a new time variable τ related to t by

$$d\tau=\frac{1-\gamma^2}{\gamma^2}\frac{n-1}{k'-1}H\,dt \tag{2.17}$$

and

$$s_0=-\frac{k-1}{k'}\frac{\alpha'-\beta_2}{\alpha'-\beta_1}\ . \tag{2.18}$$

Equation (2.16) can be explicitly integrated and we obtain

$$|s-s_+|^{q_+}|s-s_-|^{q_-}|s-s_0|^{-2q_0}=Ce^\tau\ , \tag{2.19}$$

where

$$q_\pm=\frac{1\pm\gamma}{\alpha'-\beta_\pm}\ , \tag{2.20}$$

$$q_0=\frac{\alpha'-\beta_1}{(\alpha'-\beta_+)(\alpha'-\beta_-)}\ , \tag{2.21}$$

$$\beta_\pm=\frac{\gamma\mp\beta_1}{\gamma\mp1}\ , \tag{2.22}$$

and C is a constant of integration. Now returning to Eq. (2.12) and replacing t by τ according to (2.17) and using (2.16) we obtain

$$\frac{d\ln H}{ds}=-\frac{1}{(\alpha'-\beta_1)(s-s_0)}-\frac{2(n-1)}{k'^2(k'-1)}\frac{k+sk'}{(s-s_0)(s-s_+)(s-s_-)}\ . \tag{2.23}$$

This equation leads to

$$H=C'|s-s_0|^{-2p_0}|s-s_+|^{p_+}|s-s_-|^{p_-}\ , \tag{2.24}$$

where C' is an integration constant and

$$p_\pm=\mp\frac{2(n-1)}{k'^2(k'-1)(\alpha'-\beta_1)}\frac{k+k's_\pm}{(s_\pm-s_0)(s_+-s_-)}\ , \tag{2.25}$$

$$2p_0=1+\frac{n-1}{k'^2(k'-1)(\alpha'-\beta_1)}\frac{k+k's_0}{(s_--s_0)(s_+-s_0)}\ . \tag{2.26}$$

Combining (2.16), (2.17), and (2.24) we obtain the following relation between t and s:

$$t-t_{\rm in}=\frac{2\gamma^4}{k'^2(1-\gamma^2)^2}\frac{k'-1}{n-1}\frac{1}{(\alpha'-\beta_1)}\frac{1}{C'}\times\int_{s_{\rm in}}^{s}|s'-s_0|^{2p_0-1}|s'-s_+|^{-p_+-1}\times|s'-s_+|^{-p_--1}ds'\ , \tag{2.27}$$

where $t_{\rm in}$ is an arbitrary constant of integration and $s_{\rm in}=s(t_{\rm in})$.

The general behavior of our system depends on relations between the roots $s_0,s_\pm$ of the right-hand side of Eq. (2.16) and the sign of $(\alpha'-\beta_1)$. Straightforward calculation leads to

$$s_\pm-s_0=\pm\gamma|s_\pm|\frac{\alpha'-\beta_\pm}{\alpha'-\beta_1}\ . \tag{2.28}$$

For a given k and k' relations between roots and the sign of s_0 are determined by values of the parameters α and α'. The physically acceptable values of α and α' are restricted to a square $|\alpha|\le1$, $|\alpha'|\le1$. This square can be divided into separate regions corresponding to different behavior of solutions of Eq. (2.16) (see Fig. 1). From Eq. (2.28) we see that relations between s_0 and $s_\pm$ are determined by relations between β_1, β_2, and $\beta_\pm$. Let us notice that from the definitions of the parameters β_1, β_2, and $\beta_\pm$ we have ($k>1$, $k'>1$),

$$\beta_1=\beta_2=\beta_+=\beta_-=1\quad\text{when }\alpha=1\ , \tag{2.29}$$

and in general

$$\beta_1-\beta_2=\frac{n-1}{k'(k-1)}(1-\alpha)\ge0\ , \tag{2.30}$$

$$\beta_1-\beta_+=\frac{\gamma}{1-\gamma}(1-\beta_1)\ge0\ , \tag{2.31}$$

$$\beta_1-\beta_-=-\frac{\gamma}{1+\gamma}(1-\beta_1)\le0\ , \tag{2.32}$$

$$\beta_2-\beta_+=\frac{(1-\alpha)(1-\gamma)}{k'(k-1)}\ge0\ . \tag{2.33}$$

From (2.29)–(2.33) it follows that for all $k>1$, $k'>1$, and all $\alpha\le1$,

$$\beta_-\ge\beta_1\ge\beta_2\ge\beta_+\ . \tag{2.34}$$

Let us now discuss the qualitative behavior of solutions of Eq. (2.16) for different values of the parameters α and α'. In region I on Fig. 1 ($\alpha'>\beta_-$),

$$s_-<s_0<s_+<0\ , \tag{2.35}$$

and the qualitative behavior of solutions of Eq. (2.16) is shown in Fig. 2(a). If initially the Universe expands in all directions then $s(t_{\rm in})=s_{\rm in}>0$. Solutions for which $s_{\rm in}>s_+$ correspond to the Universe which is asymptotically, for $t\to\infty$, expanding in all directions even if initially additional dimensions were contracting ($s_+<s_{\rm in}<0$). If $s_{\rm in}=s_\pm$ then s always remains equal to $s_\pm$ and these solutions describe empty space-time ($\epsilon=0$). The region $s_-<s<s_+$ is physically forbidden since there

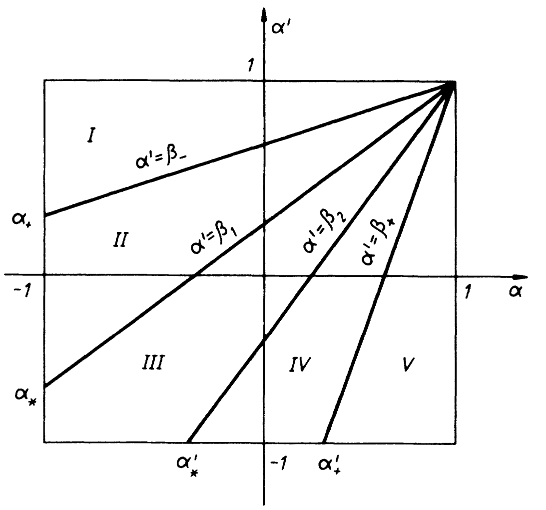

FIG. 1. Physically acceptable values of α and α'. The square $|\alpha|\le 1$, $|\alpha'|\le 1$ is divided into five regions corresponding to different behavior of the dynamical system representing the multidimensional Universe. In region I $\beta_-(\alpha)\le\alpha'\le 1$; in region II $\beta_1(\alpha)\le\alpha'\le\beta_-(\alpha)$; in region III $\beta_2(\alpha)\le\alpha'\le\beta_1(\alpha)$; in region IV $\beta_+(\alpha)\le\alpha'\le\beta_2(\alpha)$; and in region V $\beta_+(\alpha)\le\alpha'\le -1$. The intersection points α_+, α_*, α'_*, and α'_+ are given by

$$\alpha_+=\frac{2\left[\dfrac{(n-1)k'}{k}\right]^{1/2}-1-k'}{k'-1},$$

$$\alpha_*=-\frac{k-2}{k},\quad \alpha'_*=-\frac{k'-2}{k'},$$

$$\alpha'_+=\frac{\left[\dfrac{(n-1)k'}{k}\right]^{1/2}+2-k'}{\left[\dfrac{(n-1)k'}{k}\right]^{1/2}+k'}.$$

It turns out that for $k'>1$ always $\alpha_+>\alpha_*$ and $\alpha'_+>\alpha'_*$.

$\epsilon<0$. Solutions for which $s_{\rm in}<s_-$ correspond to a Universe in which additional dimensions contract all the time. These solutions describe reduction of additional dimensions but without an epoch when all directions were equivalent. For this reason such solutions are not interesting from the point of view of the program of dynamical dimensional reduction which assumes that initially all spacial directions were equivalent. Therefore we will disregard them in our further considerations.

In region II ($\beta_->\alpha'>\beta_1$),

$$s_0<s_-<s_+<0\ , \tag{2.36}$$

and the qualitative behavior of solutions of Eq. (2.16) is shown on Fig. 2(b). In this region the behavior of physically acceptable solutions is similar to the previous case.

In the region III ($\beta_1>\alpha'>\beta_2$),

$$s_-<s_+<0<s_0\ , \tag{2.37}$$

and the qualitative behavior of solutions of Eq. (2.16) is shown in Fig. 2(c). Solutions for which $s_{\rm in}>s_+$ including the region $s>0$ asymptotically, for $t\to 0$, describe the Universe expanding in all directions in which the ratio of the Hubble constants tends to a fixed positive value s_0. Solutions which initially satisfy our requirement of equivalence of all directions do not describe subsequent dimensional reduction.

In region IV ($\beta_2>\alpha'>\beta_+$),

$$s_-<s_+<s_0<0\ , \tag{2.38}$$

and the qualitative behavior of solutions of Eq. (2.16) is shown in Fig. 2(d). This region is very interesting from our point of view. Solutions for which $s_{\rm in}>s_0$ asymptotically, for $t\to\infty$, describe the Universe which is expanding in physical dimensions and contracting in all additional dimensions. Asymptotically the ratio of the Hubble constants approaches a constant negative value s_0. The energy density is positive and asymptotically

$$\epsilon\sim H^2(s_0-s_+)(s_0-s_-)\sim\frac{1}{t^2}\ . \tag{2.39}$$

Solutions for which $s_{\rm in}>0$ possess the required properties and they describe dynamical dimensional reduction. For such solutions, from (2.16), (2.17), and (2.23), when $s\to s_0$ we derive the asymptotic behavior

$$H\sim(s-s_0)^{-2p_0} \tag{2.40}$$

and

$$t\sim(s-s_0)^{2p_0}\ , \tag{2.41}$$

so

$$H\sim\frac{1}{t} \tag{2.42}$$

and

$$h-s_0H\sim H_0e^{-\tau/2q_0}\ , \tag{2.43}$$

where q_0 is a positive constant given by Eq. (2.21).

Finally, let us investigate the possibility of the existence of an inflationary stage. Inflation in the physical sector of space-time occurs when $H=\dot R/R=\text{const}>0$ so the scale factor $R\sim e^{Ht}$. From Eqs. (2.7) and (2.9) it follows that h is also constant which implies that $s=h/H$ is also constant. In other words, $s=s_0$ is a necessary but not a sufficient condition for inflation. From the previous discussion we see that when $s=$const the only physically acceptable solution is $s=s_0$. From Eqs. (2.12)–(2.14) we derive

$$H(k+k's)(s-s_0)=0\ , \tag{2.44}$$

so when $s=s_0$ the value of H can be arbitrary and it is specified by initial conditions. When α and α' are constants and $\alpha\neq\alpha'\neq -1$ inflation in the physical space is possible but this solution is unstable and solutions with initial conditions only slightly deviating from $s=s_0$ do not lead to inflation. See the asymptotic behavior (2.42) and (2.43).

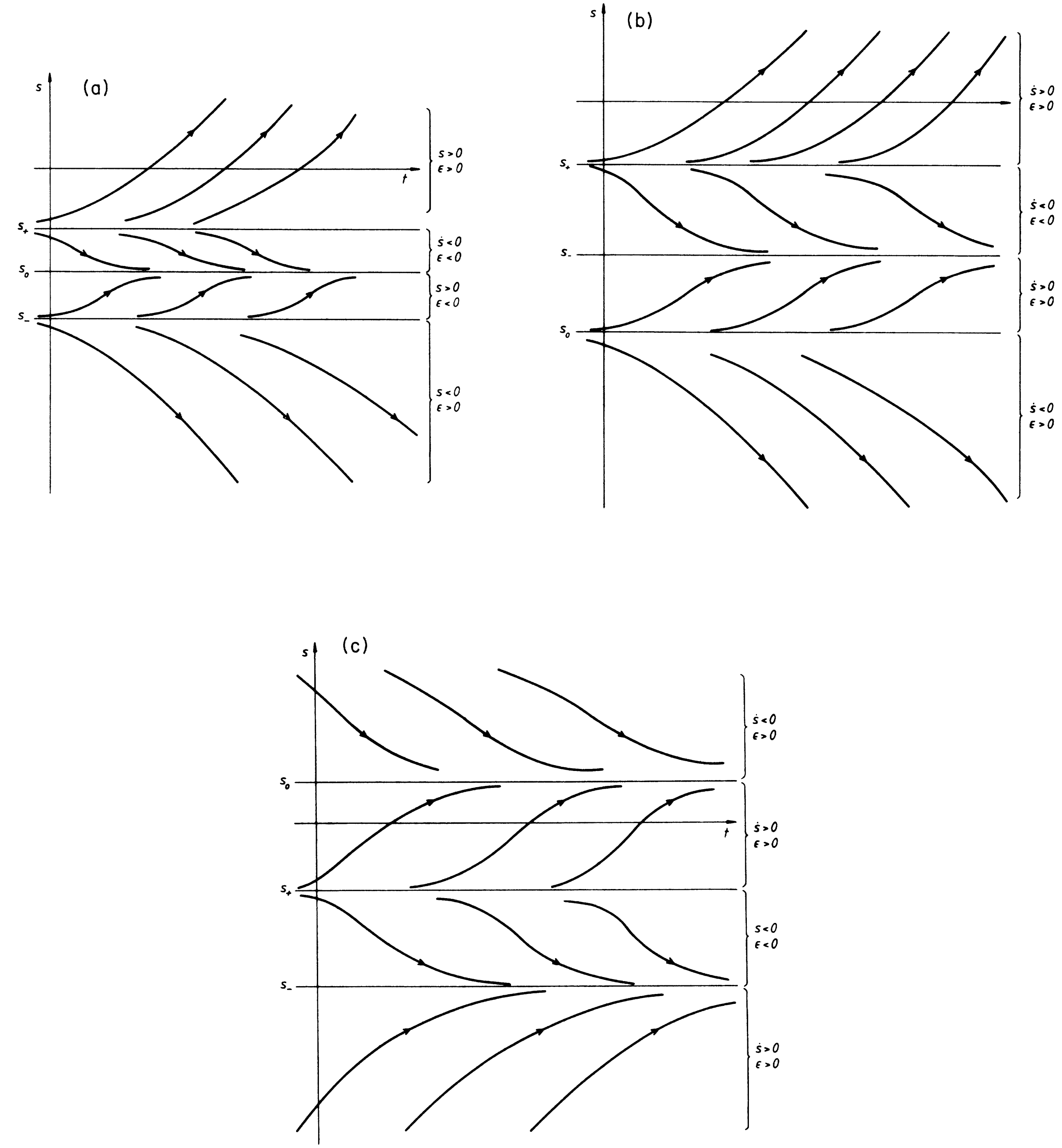

FIG. 2. Qualitative behavior of $s=h/H$ for different fixed equations of state and initial conditions. On all figures $t=0$ corresponds to the initial singularity. Physically interesting solutions correspond to curves with $s_{\text{in}}=s(t=0)\geq 0$ and $s_f=s(t\to\infty)<0$ and $s_f\neq s_+$ (asymptotically the Universe is not the Milne universe). The region of s between s_+ and s_- ($s_+>s>s_-$) is not physically allowed since in this region $\epsilon<0$. (a) Values of α and α' are confined to region I on Fig. 1. There are no physically acceptable solutions. (b) Values of α and α' are confined to region II. There are no physically acceptable solutions. (c) Values of α and α' are confined to region III. There are no physically acceptable solutions. (d) Values of α and α' are confined to region IV. There are solutions describing dynamical dimensional reduction in initially expanding multidimensional Universe. (e) Values of α and α' are confined to region V. There are solutions which describe dynamical dimensional reduction in the initially expanding multidimensional universe but asymptotically they tend to the Milne universe.

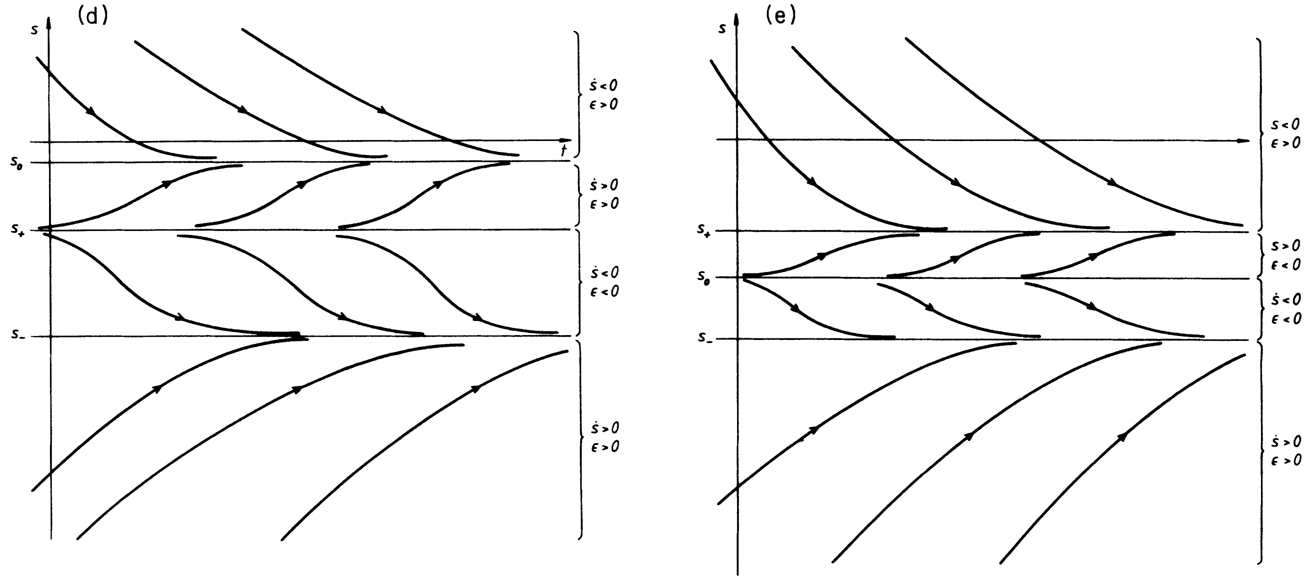

FIG. 2. (*Continued*).

In the region V ($\beta_- > \alpha'$),

$$s_- < s_0 < s_+ < 0 \tag{2.45}$$

and the qualitative behavior of solutions of Eq. (2.16) is shown in Fig. 2(e). Solutions for which $s_{\rm in} > s_+$ are very similar to the previous case but there is one very important difference. Asymptotically such solutions describe an empty universe (Milne universe) in a sense that the density parameter $\Omega = \epsilon/\epsilon_{\rm crit} \to 0$ where $\epsilon_{\rm crit} = 3H^2/8\pi G$ while in the previous case $\Omega \to$ const.

We see therefore that when equations of state are fixed and they do not change during the evolution of our models dynamical dimensional reduction is possible only when $\beta_2 > \alpha' > \beta_+$ and $s_{\rm in} > 0$.

III. MODELS DESCRIBING MULTIDIMENSIONAL UNIVERSE WITH CHANGING EQUATION OF STATE

In the previous section we considered the dynamics of multidimensional cosmological models assuming that the equation of state of matter is fixed and does not change in time. In the real Universe the effective equation of state is changing because the Universe is filled with different kinds of matter and phase transitions are allowed. These physical processes and their influence on the dynamics of the real Universe can be qualitatively described by allowing the parameters α and α' to depend on time. Our general equations (2.7)–(2.9) are valid also in this case. Strictly speaking, our equations describe only slow changes of the equation of state when the time scale of change of parameters α and α' is longer than the Hubble time (H^{-1}). Such slow changes in α and α' imply conservation of entropy. But even in the case when entropy is not conserved and α and α' are changing rapidly in comparison with the time scale of the expansion of the Universe and our Eqs. (2.7)–(2.9) fail to describe this transition, they are still valid in the asymptotic regions before and after the transition where α and α' are constants. The intermediate case when the time scale of change of α and α' is comparable with the Hubble time cannot be described by our model and it requires different treatment, for example, a numerical analysis. Let us notice that since $s_\pm$ does not depend on α and α' only s_0 will be altered by changes in the values of α and α'.

Now we will discuss only two limiting cases of slow and fast changes of the equation of state. Following the general philosophy of this paper we will consider only such changes of the equation of state which lead to dimensional reduction. It means that the final values of parameters α and α' should be confined to region IV on Fig. 1. Since inflation seems to solve so many problems of the standard scenario of the evolution of the Universe it is interesting to investigate when an inflationary phase of the evolution could be created by allowing changes in the equation of state.

When the equation of state changes slowly we still have several possibilities (see Fig. 3). Let the initial values of α and α' be confined to region I or II in Fig. 1 and the final values to region IV [Fig. 3(a)]. Since we assume that the changes of α and α' are slow in comparison with the time scale of expansion, in this case if initially $s(t_{\rm in}) > s_0(\alpha(t_{\rm in}), \alpha'(t_{\rm in}))$ then always $s(t) > s_0(\alpha(t), \alpha'(t))$ and the character of the solution does not change. It means that during this transition it is not possible to create inflation which requires that $s = s_0(\alpha, \alpha')$. However the dimensional reduction is possible. All physically reasonable solutions ($s_{\rm in} > 0$) describe multidimensional Universe which is initially expanding in all directions but later the additional dimensions start to contract but in a nonexponential way.

If the values of α and α' move from region III to region IV on Fig. 1 then $s_0(\alpha(t), \alpha'(t))$ decreases and becomes negative [Fig. 3(b)]. Let us at first consider solutions for which $s(t_{\rm in}) > s_0(\alpha(t_{\rm in}), \alpha'(t_{\rm in}))$ which describe

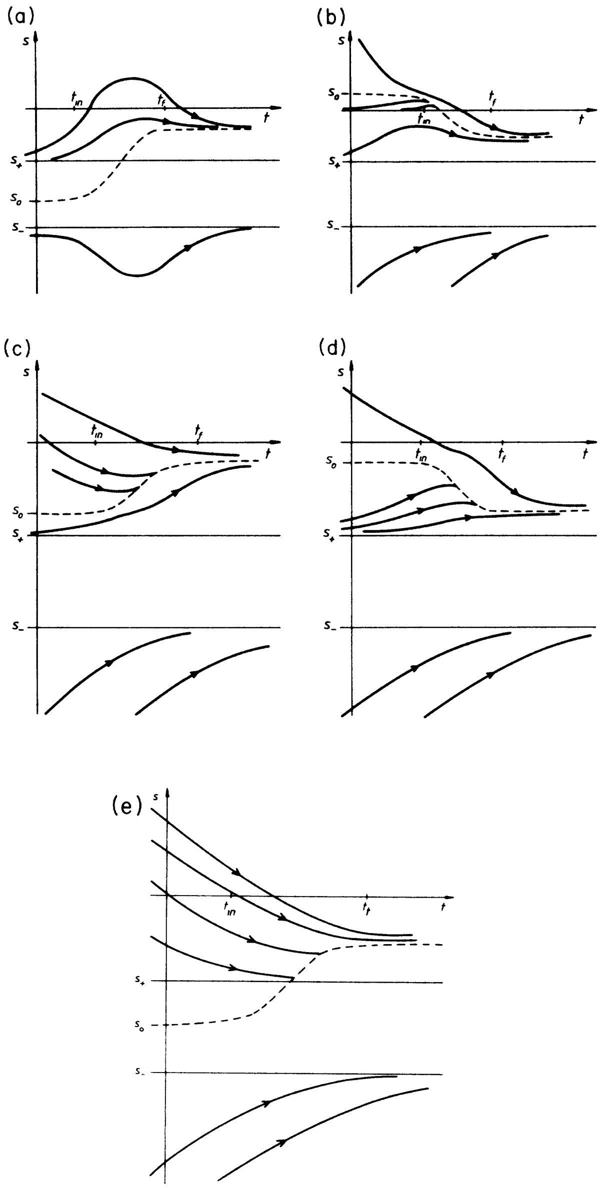

FIG. 3. Qualitative behavior of $s=h/H$ when the equation of state changes slowly. Only physically acceptable solutions are shown. Equation of state changes during the time interval $t_{\rm in},t_f$ and it is fixed at the earlier and later epochs. The final values of parameters α and α' are confined to region IV on Fig. 1. On all figures the dashed line corresponds to $s=s_0(\alpha,\alpha')$. (a) Initial values of α and α' lie in region I or II. Final state for all solutions is not inflationary. (b) Initial values of α and α' lie in region III. Solutions for which $s(t_{\rm in})>s_0$ do not lead to inflation. When $0<s(t_{\rm in})<s_0(t_{\rm in})$ solutions which initially behaved nonexponentially at a certain moment change their behavior and describe inflation in the physical space and anti-inflation in the space of additional dimensions. (c) and (d) Initial and final values of α and α' lie in region IV; (c) corresponds to increasing $s_0(\alpha,\alpha')$ and (d) to decreasing $s_0(\alpha,\alpha')$. In both cases there are solutions describing nonexponential behavior of the model (solid line) and solutions which from a certain moment describe inflation [this is possible only in (c)]. (e) Initial values of α and α' lie in region V. In this case there are also two kinds of solutions. Transition to inflation is possible as in the previous case.

multidimensional space-time expanding in all directions. When s_0 is confined to the region $0>s_0(\alpha,\alpha')>s_+$ then there are solutions which describe a universe with expanding physical space and contracting space of additional dimensions but inflation in the physical space and anti-inflation in the space of additional dimension is not possible. When $s(t_{\rm in})=s_0(\alpha(t_{\rm in}),\alpha'(t_{\rm in}))$ initially we have inflation in all dimensions but when $s_0(\alpha(t),\alpha'(t))$ becomes less than zero, inflation in the space of additional dimensions is replaced by anti-inflation (the scale factor decreases exponentially) while the physical space continues to expand exponentially. If $s_0(\alpha(t_{\rm in}),\alpha'(t_{\rm in}))>s(t_{\rm in})>0$ then initially the Universe expands but not exponentially in all directions and at a certain moment t^* which is determined by initial conditions $s(t^*)=s_0(\alpha(t^*),\alpha'(t^*))>0$ inflation sets in all directions. When s_0 becomes negative, inflation in the space of additional dimensions is replaced by anti-inflation while the physical space continues to expand exponentially.

When initially α and α' are confined to region IV in Fig. 1 then final values of α and α' remain also in this region. In this case the physically reasonable solutions describe an initially nonexponential expansion which at the moment t^* when $s(t^*)=s_0(\alpha(t^*),\alpha'(t^*))$ is replaced by inflation in the physical space and anti-inflation in the space of additional dimensions [Figs. 3(c) and 3(d)].

When the initial values of α and α' are confined to region V and the final values lie in region IV the behavior of the physically reasonable solutions is similar to the previous case. The only difference is that now there exist solutions which begin to inflate when they are in an asymptotically Milne stage [Fig. 3(e)].

We see therefore that slow changes of the parameters α and α' from regions III or V to region IV or changes within region IV in Fig. 1 always assure dimensional reduction and they also can create inflation in the physical space. Slow changes of the parameters α and α' can maintain inflation or create it but they cannot stop inflation. In our model, inflation in the physical space is always accompanied by inflation or anti-inflation of the additional dimensions.

Let us now discuss how rapid changes of α and α' influence the behavior of our system. We will consider only sudden jumps of α and α' (see Fig. 4). When the values of parameters α and α' jump from region I and II to region IV [Fig. 4(a)] there exist solutions which describe initial expansion of the multidimensional universe followed by expansion of the physical space and contraction of additional dimensions after the jump. Initially inflation is not possible. There is one particular solution with $s_{\rm in}=s_{\rm inf}$ such that $s_{\rm inf}=s_0(\alpha_f,\alpha'_f)$. This solution describes a multidimensional universe in which initially dimensional reduction takes place with the physical space expanding and the space of additional dimensions contracting nonexponentially. Let us notice that this solution does not satisfy our requirement that initially the multidimensional universe should expand in all directions. After the jump in α and α' inflation sets in the physical space and anti-inflation in the space of additional dimensions. This property is not generic and rapid changes of the parameters α and α' cannot create

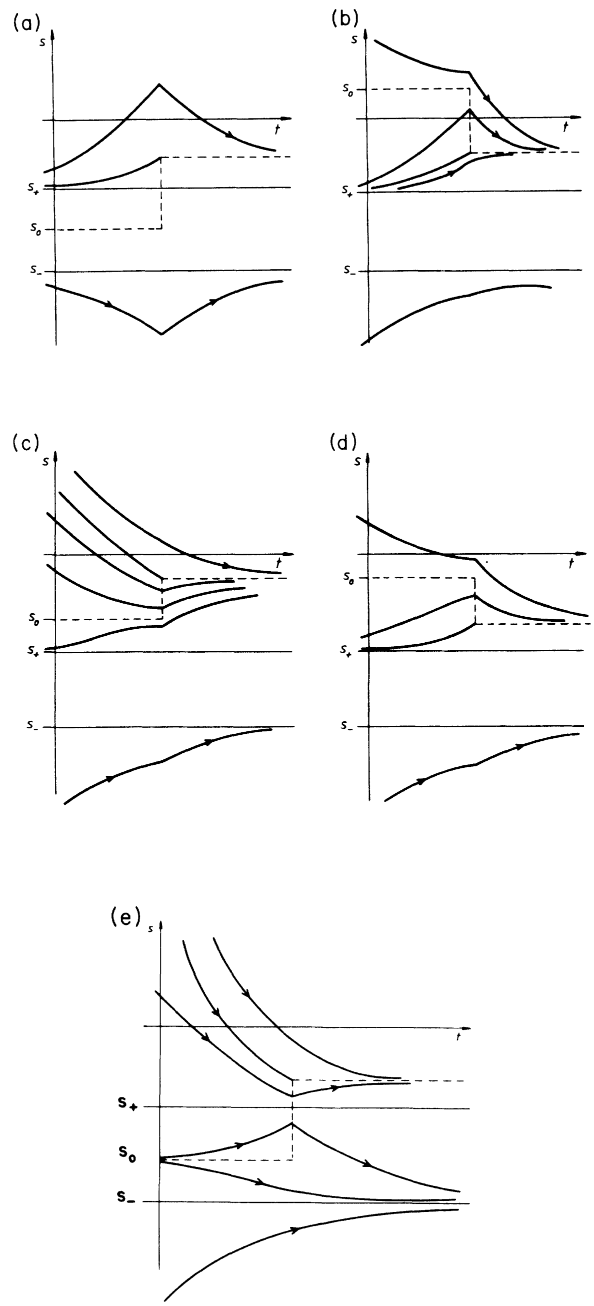

FIG. 4. Qualitative behavior of $s = h/H$ when the equation of state changes rapidly. The rapid transition is approximated by a jump in α and α'. The dashed line corresponds to $s = s_0(\alpha, \alpha')$. In all cases rapid changes of the equation of state do not create inflation except the unique solution for which at the moment of the jump $s = s_{0_f}$. (a) Transition from regions I or II to region IV. No inflation. (b) Transition from region III to region IV. Initial inflation is possible ($s = s_{0_{\rm in}}$) but the jump in α in α' turns it into a nonexponential behavior. In this case inflation in the final state is not possible. (c) and (d) Transition is confined to region IV. When s_0 increases (c) inflation is impossible except the unique solution mentioned above. When s_0 is decreasing inflation is never possible. (e) Transition from region V to region IV. Behavior of solutions is similar to (c).

inflation.

A jump of α and α' from region III to region IV [Fig. 4(b)] produces physically reasonable dimensional reduction. Let us notice that in this case there exists a solution describing initial inflation of all directions. This is possible only when $s_{\rm in} = s_0(\alpha_{\rm in}, \alpha'_{\rm in})$. After the jump of α and α' this solution describes dimensional reduction with the physical space expanding and the additional dimensions contracting nonexponentially. As in the previous case there is only one particular nonphysical value of $s_{\rm in}$ which in the final state leads to inflation of the physical space and anti-inflation of the space of additional dimensions.

As expected rapid changes of the parameters α and α' within region IV [Figs. 4(c) and 4(d)] preserve the dynamical dimensional reduction. There is however one unique solution with a special choice of physically reasonable initial conditions which after the jump of α and α' describes inflation of the physical space and anti-inflation of the space of additional dimensions. A jump of α and α' from region V to region IV leads qualitatively to the same behavior of solutions as in the previous cases. In this case there are solutions which pass through an asymptotically Milne stage before they reach another asymptotic region of nonexponential expansion of the physical space and nonexponential contraction of additional dimension. There is also one unique solution describing final inflation of the physical space and anti-inflation of the space of additional dimensions. This solution always emerges from physically reasonable conditions.

We see therefore that in general a rapid change of the equation of state cannot create inflation but can terminate inflation. To create a physically reasonable solution in the process of a rapid change of the equation of state the final values of α and α' have to be confined to region IV.

IV. CONCLUSIONS

The behavior of the multidimensional cosmological models which we have investigated in this paper allows for several scenarios of the very early evolution of the Universe. The model considered in this paper is compatible with either the symmetric beginning of expansion in the sense that all dimensions are equivalent or asymmetric initial expansion. However from the point of view of the dynamical dimensional reduction it is natural to assume that the initial state of the Universe was very symmetric. Symmetric initial expansion corresponds to $\alpha = \alpha'$ so the initial state is represented by a point lying on the diagonal in Fig. 1 which always is contained in region III. As it was shown in our previous discussion physically reasonable solutions with appropriate dimensional reduction are represented by α and α' confined to region IV. Since inflation solves so many problems of the standard scenario we are interested mostly in those solutions of our model which allow inflation. However there are many acceptable solutions without inflation.

If we assume the initial equivalence of all dimensions and the existence of inflation then we are left with two basically different possibilities of the early evolution of the Universe corresponding to a different type of changes of the equation of state. Let us stress that now we want to focus our attention on those changes of the equation of state which break the initial equivalence of all dimensions. In fact just this breaking of symmetry makes some of the dimensions physical. In our model the number of physical dimensions can be arbitrary and in particular it can coincide with the number of spatial dimensions of the observable Universe.

If the equation of state changes slowly then it is impossible to stop inflation; but in this case for a certain range of initial conditions inflation is always created regardless of whether or not the initial state of expansion was inflationary. We want to stress that when the equation of state changes slowly reduction of additional dimensions at the stage of inflation in the physical space is accomplished by anti-inflation. To stop inflation in physical space when it is already three dimensional another change of the equation of state is required. The decay of a vacuum is just one example of such a change.

If the equation of state changes rapidly then it is impossible to create inflation but it is always possible to suppress inflation. Therefore no matter what was the initial character of expansion, reduction of additional dimensions is accomplished by nonexponential contraction while the physical dimensions also expand nonexponentially. So inflation could have either existed from the very beginning of the expansion of the multidimensional universe or it had to be created by some physical processes at the stage when the universe was already three dimensional.

It is interesting to notice that there is a test which enables us in principle to distinguish between those two different possibilities. The idea of this test was presented at the Centennial Friedmann Conference.[18] Detailed analysis of this test will be presented elsewhere.

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