Pseudoscalar decay constants in the relativized quark model and measuring the CKM matrix elements

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(Received 8 November 1989)

We have calculated the pseudoscalar decay constants using the relativized quark model. We find $f_D \simeq 240 \pm 20$ MeV, $f_{D_s} \simeq 290 \pm 20$ MeV, $f_B \simeq 155 \pm 15$ MeV, $f_{B_s} \simeq 210 \pm 20$ MeV, and $f_{B_c} \simeq 410 \pm 40$ MeV where the "errors" are our estimates of the reliability of the results. We used these values to study the Cabibbo-Kobayashi-Maskawa matrix elements. In particular, we combined our theoretical estimate of f_B with the ARGUS/CLEO result for B^0-$\bar{B}^0$ mixing to study the relation between $|V_{td}|$ and the t-quark mass. Recent measurements of $|V_{bc}|$ and $|V_{bu}|$ constrain $m_t \gtrsim 70$ GeV.

I. INTRODUCTION

The discovery of B_d^0-$\bar{B}_d^0$ mixing by the ARGUS Collaboration[1] and its confirmation by CLEO (Ref. 2) has opened up a window to search for the effects of new physics. For instance, measurement of B^0-$\bar{B}^0$ mixing along with measurements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements[3] can put tight constraints on possible values of the t-quark mass.[4,5] If the value of m_t so obtained conflicts with limits from other measurements it would signify contributions from new physics outside the standard model.[5] In addition, the information one obtains from the B^0-$\bar{B}^0$ system can be used to make predictions for rare B decays such as $B \to K^* \gamma$ (Refs. 6 and 7). A big problem in using B^0-$\bar{B}^0$ mixing to constrain new physics is the uncertainty in the theoretical estimates of the hadronic matrix elements involved in these processes. The B^0-$\bar{B}^0$ matrix element is usually parametrized in such a way as to factor out $f_B^2 B$ where f_B is the pseudoscalar decay constant and B is the so-called B parameter. In this paper we estimate the pseudoscalar decay constants using the relativized quark model with the mock-meson prescription and attempt to estimate the error in our predictions.

Our motivation in performing this calculation is twofold. First, a reliable estimate of f_p is important, as they appear in many processes from which we can extract fundamental quantities in the standard model such as the Cabibbo-Kobayashi-Maskawa matrix elements and the mass of the t quark. Often the greatest theoretical uncertainties in these calculations are due to the uncertainty of the hadronic matrix elements. The quark model has been very successful in explaining hadron properties so it is only natural that we attempt to use it to make reliable estimates of f_p. Our other reason for studying f_p is that we wish to understand the limitations of our model in a relatively simple system before applying it to a more ambitious program of calculating hadronic matrix elements relevant to other weak processes such as semileptonic decays.

We begin in Sec. II by defining the mock-meson approach to calculating hadronic matrix elements and give the *relativized* quark-model expressions for f_p. We then take the nonrelativistic limit of the expressions and examine its sensitivity to different prescriptions and different meson wave functions which have appeared in the literature. We then repeat the exercise using the relativized quark model expression to obtain what we regard as our best estimate of f_p along with estimates of the theoretical uncertainties in our calculation. We end the section by comparing our results to those obtained in other models. In Sec. III we give several examples of physics that is dependent on a reliable estimate of f_p, extracting $|V_{cd}|$ from $D^+ \to \mu^+ \nu_\mu$, and studying the constraints on m_t and V_{td} that can be obtained from B^0-$\bar{B}^0$ mixing. We summarize our results in the final section.

II. f_p IN THE RELATIVIZED QUARK MODEL

We use the mock-meson approach to calculate the hadronic matrix elements. It is a prescription for relating quark-model matrix elements to the corresponding physical amplitudes.[8-10]

A. The mock-meson approach

The basic assumption of the mock-meson approach is that physical hadronic amplitudes can be identified with the corresponding quark-model amplitudes in the weak-binding limit of the valence-quark approximation. We stress that this correspondence is exact but only in the limit of zero binding and in the hadron rest frame. Away from this limit the amplitudes are not in general Lorentz invariant by terms of order p_i^2/m_i^2. In this approach the mock meson, which we denote by $\tilde{M}$, is defined as a state of a free quark and antiquark with the wave function of the physical meson M:

$$|\tilde{M}(\mathbf{K})\rangle=\sqrt{2\tilde{M}_M}\int d^3p\;\Phi_M(\mathbf{p})\chi_{s\bar{s}}\phi_{q\bar{q}}|q[(m_q/\mu)\mathbf{K}+\mathbf{p},s]\bar{q}[(m_{\bar{q}}/\mu)\mathbf{K}-\mathbf{p},\bar{s}]\rangle\;, \tag{1}$$

where $\Phi_M(\mathbf{p})$, $\chi_{s\bar{s}}$, and $\phi_{q\bar{q}}$ are momentum, spin, and flavor wave functions, respectively, $\mu=m_q+m_{\bar{q}}$, $\tilde{M}_M$ is the mock-meson mass, and $\mathbf{K}$ is the mock-meson momentum. To calculate the hadronic amplitude, the physical matrix element is expressed in terms of Lorentz covariants with Lorentz-scalar coefficients A. In the simple cases when the mock-meson matrix element has the same form as the physical meson amplitude we simply take $A=\tilde{A}$.

In the case of interest, the pseudoscalar decay constants are usually expressed as

$$\langle 0|\bar{q}\gamma^{\mu}(1-\gamma_5)q|M(\mathbf{K})\rangle=\frac{i}{(2\pi)^{3/2}}f_P K^{\mu}\;. \tag{2}$$

To calculate the left-hand side of Eq. (2) we first calculate

$$\langle 0|\bar{q}\gamma^{\mu}(1-\gamma_5)q|q[(m_q/\mu)\mathbf{K}+\mathbf{p},s]\bar{q}[(m_{\bar{q}}/\mu)\mathbf{K}-\mathbf{p},\bar{s}]\rangle \tag{3}$$

using free quark and antiquark wave functions and weighting the result with the meson's momentum-space wave function.

There are a number of ambiguities in this approach that we must deal with. The first regards the definition of the meson mass $\tilde{M}_M$ appearing in Eq. (1). Several different prescriptions exist in the literature. On the one hand, to be consistent with the mock-meson prescription, we should use the mock-meson mass, which is defined as $\langle E_q\rangle+\langle E_{\bar{q}}\rangle$. However, because it is introduced to give the correct relativistic normalization of the meson's wave function, to give the correct kinematics of the process being studied, we feel it more appropriate to use the physical mass rather than the mock-meson mass. The second ambiguity is the question of which component of the four-vector in Eq. (2) we should use to obtain f_p. In principle, it should not matter as both the left- and right-hand sides of Eq. (2) are Lorentz four-vectors. This is true in the weak-binding limit where binding effects are totally neglected, but in practice, this is not the case. Although the quark level expressions of Eq. (3) are the components of a four-vector, the momentum-space wave function Φ_M used in evaluating Eq. (2) is not relativistically covariant. This leads to an ambiguity in the interpretation of f_p so derived. To understand the origin of the ambiguity, consider the zero and spatial components of Eq. (3) in the nonrelativistic limit, $\mathbf{p}\to 0$, and take the limit $\mathbf{K}\to 0$. In this limit the zero component is proportional to $m_q+m_{\bar{q}}$ while the spatial component is proportional to $\mathbf{K}$. This is what one would expect for the components of a four-vector at the quark level which emphasizes the ambiguity in defining the mock-meson mass $\tilde{M}_M$. We choose to extract f_p using the spatial components of Eq. (2) in the limit $\mathbf{K}\to 0$ but we will also evaluate other possibilities to check the sensitivity of the results to details of the specific prescription.

Using this prescription for f_p leads to

$$f_P\mathbf{K}=2\sqrt{3}\sqrt{M_P}\int\frac{d^3p}{(2\pi)^{3/2}}\Phi_P(\mathbf{p})\times\left[\left(\frac{E_q+m_q}{2E_q}\right)\left(\frac{E_{\bar{q}}+m_{\bar{q}}}{2E_{\bar{q}}}\right)\right]^{1/2}\times\left(\frac{\mathbf{p}_{\bar{q}}}{E_{\bar{q}}+m_{\bar{q}}}+\frac{\mathbf{p}_q}{E_q+m_q}\right)\;, \tag{4}$$

where

$$E_q=\sqrt{\mathbf{p}_q^2+m_q^2},\quad E_{\bar{q}}=\sqrt{\mathbf{p}_{\bar{q}}^2+m_{\bar{q}}^2},\quad \mathbf{p}_q=(m_q/\mu)\mathbf{K}+\mathbf{p}\;,$$

and

$$\mathbf{p}_{\bar{q}}=(m_{\bar{q}}/\mu)\mathbf{K}-\mathbf{p}\;.$$

B. The nonrelativistic limit

The goal of this paper is to give a reliable estimate of the pseudoscalar decay constants and understand the uncertainties in the calculation. One way of estimating the reliability of our results is to test their sensitivity by varying different aspects of the model, keeping within the general assumptions described in the previous section. In this spirit we begin by taking the nonrelativistic limit using different wave functions which have appeared in the literature.

Taking the nonrelativistic limit of Eq. (3) we obtain

$$f_p=\frac{2\sqrt{3\tilde{M}}\,\psi_p(0)}{m_q+m_{\bar{q}}}\;, \tag{5}$$

where $\psi_p(0)$ is the wave function at the origin and $\tilde{M}=M_{\text{phys}}$ is the physical mass of the meson. The resulting pseudoscalar decay constants using some of the nonrelativistic wave functions which have appeared in the literature are given in Table I. To gauge the sensitivity of our results to the specific prescription used, we also calculated f_p using the zero component of Eq. (3) and the mock-meson prescriptions of Refs. 9, 10, and 13. The results vary by the amounts given in the final row of the table. The heavier mesons, D, D_s, B, B_s, and B_c, are not very sensitive to the prescription used in calculating f_p. This is because in the nonrelativistic limit the different prescriptions reduce to different definitions of the mock-meson mass and the expression for the mass appearing in the denominator of Eq. (5). As $m_q+m_{\bar{q}}$ becomes heavier, relativistic effects and binding energies become less important so $\tilde{M}\to M_{\text{phys}}$ regardless of how $\tilde{M}$ is defined. On the other hand, for f_π and f_K, $m_q+m_{\bar{q}}$ differs considerably from M_{phys} making them very sensitive to the prescription used.

In contrast, the pseudoscalar decay constants are sensitive to the wave functions. To gain some insight into why this is so and guidance as to which models best describe nature, consider the nonrelativistic Schrödinger

TABLE I. Pseudoscalar decay constants in the nonrelativistic limit using harmonic-oscillator wave functions. The $L=0$ ground-state harmonic-oscillator wave function is $\Phi(\mathbf{p}) = (\sqrt{\pi}\beta)^{-3/2}\exp(-p^2/2\beta^2)$ with $\beta=[2m_q m_{\bar{q}}\kappa/(m_q+m_{\bar{q}})]^{1/4}$. The meaning of the sensitivities are described in the text. Because our results for f_p are not always equal to the central values of all prescriptions surveyed, the sensitivities should not be used to obtain bounds on f_p.

Parameter set	f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}	f_{B_c}
Set 1[a]	60	92	89	96	58	67	84
Set 2[b]	100	153	149	160	96	111	141
Set 3[c]	206	290	221	252	136	167	226
Set 4[d]	144	232	227		162		
Sensitivity	$\pm>100\%$	$\pm>100\%$	$\pm15\%$	$\pm10\%$	$\pm5\%$	$\pm5\%$	$\pm7\%$

[a]Standard nonrelativistic quark-model parameters: $\kappa=0.0027$ GeV^3, $m_u=m_d=0.33$ GeV, $m_s=0.55$ GeV, $m_c=1.628$ GeV, and $m_b=4.977$ GeV.
[b]From Ref. 11 with $\kappa=0.0106$ GeV^3, $m_u=m_d,m_s,m_c,m_b$ as above.
[c]From Ref. 9 with $\kappa=0.037$ GeV^3, $m_u=m_d=0.22$ GeV, $m_s=0.43$ GeV, m_c,m_b as above.
[d]Effective oscillator parameters from Ref. 12. $\beta_\pi=0.31$ GeV, $\beta_K=0.34$ GeV, $\beta_D=0.39$ GeV, $\beta_B=0.41$ GeV, $m_u=m_d=0.33$ GeV, $m_s=0.55$ GeV, $m_c=1.82$ GeV, and $m_b=5.12$ GeV.

equation with a potential of the form

$$V(r)=\lambda r^\nu . \tag{6}$$

Simple scaling arguments show that the wave function at the origin is described by[14]

$$\psi(0)\sim(\mu|\lambda|)^{3/[2(2+\nu)]} . \tag{7}$$

Thus,

$$\psi(0)\sim\begin{cases}(\mu|\lambda|)^{3/2} & \text{Coulomb potential}\sim 1/r,\\ (\mu|\lambda|)^{1/2} & \text{linear potential}\sim r,\\ (\mu|\lambda|)^{3/8} & \text{harmonic oscillator}\sim r^2 .\end{cases} \tag{8}$$

Using these relations we plot $\psi(0)$ as a function of the reduced mass μ in Fig. 1 where we have adjusted the parameter λ such that for each case, Eq. (5) reproduces the experimental value of f_K with $m_u=m_d=0.33$ GeV and $m_s=0.55$ GeV using the weak-binding limit where $\tilde{M}_p=m_q+m_{\bar{q}}$. We indicate the values of the reduced mass for some of the mesons being considered in this paper. One sees that although $\psi(0)$ is relatively flat for the linear and commonly used harmonic-oscillator potentials, it rises sharply for the Coulomb potential. Thus, for the QCD-motivated Coulomb plus linear potential, mesons with both a heavy quark and antiquark will fall into the potential well and have a large wave function at the origin which can more than compensate for the $\sqrt{m_q+m_{\bar{q}}}$ term in the denominator of Eq. (5). For example, for $m_q=m_{\bar{q}}$, as m_q becomes heavy, the potential becomes Coulombic and $f_p\sim m_q$. On the other hand, for one light quark and one heavy quark the meson sits in the linear region of the potential and as $m_Q\to\infty$, $f_p\propto\sqrt{m_q/m_Q}$.

To tie these general scaling results to a specific potential we show in Fig. 2 the QCD-motivated Coulomb plus linear potential of Ref. 13. We indicate the $q\bar{q}$ separations for mesons of interest. The mesons with the lightest reduced masses lie in the linear region of the potential but as the reduced mass becomes larger, the Coulomb part of the potential becomes increasingly more impor-

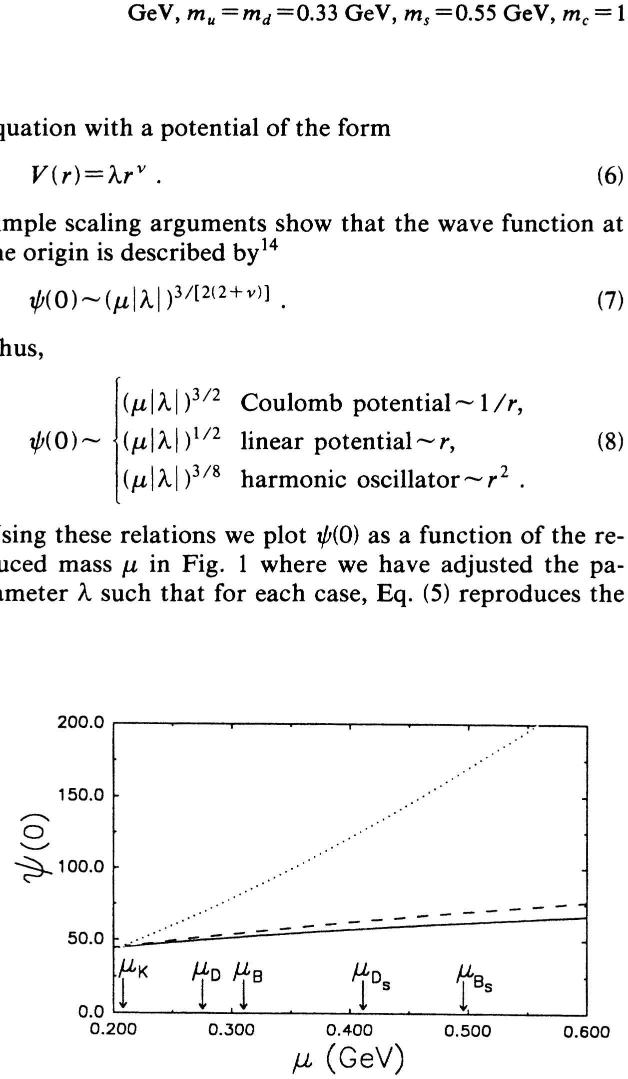

FIG. 1. $\psi(0)$ as a function of the reduced mass for the linear, harmonic-oscillator, and Coulomb potentials. The solid line is for the harmonic-oscillator potential, the dashed line for the linear potential, and the dotted line for the Coulomb potential. The reduced masses were obtained using $m_u=m_d=0.33$ GeV, $m_s=0.55$ GeV, $m_c=1.628$ GeV, and $m_b=4.977$ GeV.

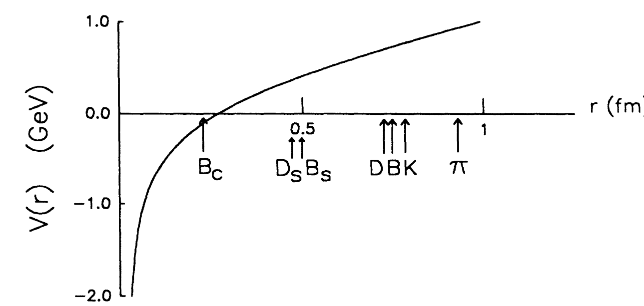

FIG. 2. QCD-motivated potential of the form $-\frac{4}{3}\alpha_s(r)/r + br$ from Ref. 13. The rms separation of some mesons of interest are shown.

tant. We conclude that if predictions for the pseudoscalar decay constants are to be reliable and not just a naive extrapolation of f_π and f_K, it is important that meson wave functions reflect the correct dynamics via the choice of $q\bar{q}$ potential.

C. The relativized quark model

We next calculate the pseudoscalar decay constants using the relativized formula of Eq. (4). One might question the importance of including relativistic corrections. However, we need only consider the importance of another such relativistic correction: QCD hyperfine interactions which give rise to the ρ-π,K^*-K, . . . ,B^*-B splittings. Although it is difficult to gauge the importance of relativistic corrections to f_p, if nothing else their inclusion acts as one more means of judging the reliability of the results. The results of our calculation are given in Table II for representative meson wave functions which have appeared in the literature. In addition to the nonrelativistic wave functions used in Table I, we also give results using the wave functions from the relativized quark model of Ref. 13. This model is very successful in describing the properties of all mesons, in particular, for our purposes, the B^*-B, D_s^*-D_s, and D^*-D mass splittings. As in the nonrelativistic limit, to gauge how sensitive our results are to the assumptions stated in Sec. II A, we calculate f_p using other prescriptions which have appeared in the literature. Again, to give a crude estimate of how these results vary with different prescriptions we give the "sensitivities" for f_p in the final row of Table II.

Comparing the results of Table II to the nonrelativistic results of Table I we find that as the masses increase the nonrelativistic and relativized results differ less and less. This should not be unexpected; the heavier the quark or antiquark the less relativistic is its motion so that relativistic corrections become less and less important and, as the quark or antiquark becomes heavier, the binding energy contributes less to the mass of the meson, making the specific definition of the mock-meson mass used by the different models less important. This observation emphasizes the conclusion of the previous section; the most important ingredient for obtaining reliable estimates of the pseudoscalar decay constants is to use meson wave functions which reflect the correct hadron dynamics. As a consequence, we feel that even though the pseudoscalar constants calculated using the relativized quark model are considerably larger than the values obtained using naive harmonic-oscillator wave functions, they more correctly reflect the true dynamics of QCD.

We now give what we believe to be reasonable estimates of f_p which are based mainly on Eq. (4), along with our estimates of the theoretical uncertainty in these results which are based on the variation of our results with the different prescriptions and wave functions described in the previous sections:

$$f_D \simeq 240\pm 20 \text{ MeV}, \quad f_{D_s} \simeq 290\pm 20 \text{ MeV} ,$$

$$f_B \simeq 155\pm 15 \text{ MeV}, \quad f_{B_s} \simeq 210\pm 20 \text{ MeV} ,$$

$$f_{B_c} \simeq 410\pm 40 \text{ MeV} .$$

D. Discussion of results

We have several comments on our results. First, we do not include values for f_π and f_K since we do not believe that they can be computed with any confidence in this approach. That is not to say that we do not obtain reasonable agreement with the measured values, rather, we do not believe that the agreement represents a success of the model. The observant reader will note that what we consider to be reliable numbers are close to those found using the effective β's of Ref. 13 with the small deviations from this model reflecting the variation we see when using different prescriptions.[15] We believe those wave functions to be the most reliable. The problem with the naive-quark-model wave functions of parameter sets 1, 2, and 3 is that they are based on harmonic-oscillator poten-

TABLE II. Pseudoscalar decay constants using the relativized mock-meson matrix elements. Parameter sets 1–4 are the same as in Table I. The meaning of the sensitivities are described in the text. Because our results for f_p are not always equal to the central values of all prescriptions surveyed, the sensitivities should not be used to obtain bounds on f_p.

Parameter set	f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}	f_{B_c}
Set 1	43	70	72	84	47	59	82
Set 2	60	101	109	129	71	91	133
Set 3	79	138	131	175	83	119	204
Set 4	74	132	148		107		
Set 5[a]	122	217	238	291	150	197	400
Set 6[b]	157	269	301	364	204	263	552
Sensitivity	$\pm>100\%$	$\pm 50\%$	$\pm 20\%$	$\pm 20\%$	$\pm 20\%$	$\pm 16\%$	$\pm 12\%$

[a]Effective oscillator parameters from Ref. 13. They were obtained by fitting harmonic-oscillator wave functions to the rms radii of the wave functions of Ref. 13 to obtain $\beta_\pi=0.623$ GeV, $\beta_K=0.613$ GeV, $\beta_D=0.601$ GeV, $\beta_{D_s}=0.651$ GeV, $\beta_{B_d}=0.580$ GeV, $\beta_{B_s}=0.636$ GeV, $\beta_{B_c}=0.929$ GeV, and $m_u=m_d=0.22$ GeV, $m_s=0.419$ GeV, $m_c=1.628$ GeV, and $m_b=4.977$ GeV.

[b]The relativized quark-model wave functions from Ref. 13 with $m_u=m_d$, m_s, m_c, and m_b as above.

tials and do not take into account the increased importance of the Coulomb piece for heavier-quark–antiquark systems. Parameter set 4 is based on effective β's using harmonic-oscillator trial wave functions in a linear plus Coulomb potential. The authors of Ref. 12 found that using a larger basis increases the wave function at the origin since the higher states mixed into the wave function can see the short-distance behavior of the potential. It appears that using the more accurate wave functions gives results more in line with ours. Finally, we did not weight the results using the exact relativized quark model wave functions very heavily since our experience has been that they tend to overestimate quantities proportional to the wave function at the origin.[13] Thus, we believe that the more realistic wave functions are more accurate since they contain information on the short-distance part of the wave function.

For completeness, in Table III we survey results of some other calculations which have appeared in the literature. We find that our results are in rough agreement with the potential model results of Krasemann[16] and of Claudson[17] who also use QCD-motivated potentials. We assume that the remaining disagreement and the disagreement with the results of Sinha[18] can be attributed to the relativized approach we used in calculating f_p. We do not feel qualified to comment on the bag model and QCD sum-rule results and only include them for completeness. Ultimately, the lattice gauge theory calculations will be the most reliable as they are a calculation from first principles. However, for the present time there are still considerable theoretical uncertainties in the results.[36] Specifically, most calculations to date are in the quenched approximation; they do not include the virtual fermion loops. We find that in general, the lattice calculations (in the quenched approximation) give values for f_p which are smaller than ours, although we note that when uncertainties in the calculations are taken into account the results are not inconsistent. It is also expected that including fermion loops will increase the ratio f_K/f_π (Ref. 36) and in fact, the only calculation that includes fermion loops[32] does predict values for f_D and f_B larger than our results. Thus, at present, the theoretical situation is sufficiently unsettled that one cannot say whether the lattice results are in conflict with those of the quark model.

III. USING f_p TO EXTRACT CKM MATRIX ELEMENTS

As we stated in the Introduction our goal is not to calculate f_p to test the quark model but, rather, to use the quark model as a tool to calculate hadronic matrix elements so that we can study electroweak physics and ex-

TABLE III. Some theoretical estimates of the pseudoscalar decay constants.

f_π	f_K	f_D	f_{D_s}	f_B	f_{B_s}	f_{B_c}	
			Experimental results				
132	166	<290[a]					
			Potential models				
139	176	150	210	125	175	425	Ref. 16
281	199	260	328	153	199	449	Ref. 17
182	232	243	335	174	277	570	Ref. 13
		287	357	229	277	429	Ref. 18
		118	129	75	87		Ref. 19
		182	200	231	246		Ref. 20
		360–580	380–590	260–300		420–480	Ref. 21
			Bag model				
178	182	148	166	98			Ref. 22
		172	196	149	170	255	Ref. 17
			Sum rules				
		174±16	~220	185±19	~200		Ref. 23
		170		130			Ref. 24
		220		140			Ref. 25
	140–165	~192	~232	~241	~288		Ref. 26
		224±26	277±13	150–210			Ref. 27
		200		170±20			Ref. 28
			Lattice				
		194±15	215±17	~120	~150		Ref. 29
		174±53	234±72	105±34	155±57		Ref. 30
108	146	190	223				Ref. 31
141±21	155±21	282±28		183±28			Ref. 32
		~240	~280				Ref. 33
			Other calculations				
		~210		~185			Ref. 34

[a]Reference 35.

tract fundamental parameters of the electroweak theory from experimental measurements. In what follows we apply our results to several examples.

A. $D^+ \to \mu^+ \bar{\nu}_\mu$ and $|V_{cd}|$

A measurement of the leptonic decay of the D^+ would measure the Cabibbo-Kobayashi-Maskawa matrix element $|V_{cd}|$ provided we know f_D:

$$B(D^+ \to \mu^+ \nu) = \frac{\Gamma(D^+ \to \mu^+ \nu)}{\Gamma(D^+ \to \text{all})} = \frac{G_F^2}{8\pi} f_D^2 \tau_D m_D m_\mu^2 |V_{cd}|^2 (1 - m_\mu^2/m_D^2)^2 , \quad (9)$$

where m_D is the meson mass, m_μ is the muon mass, G_F is the Fermi constant, and τ_D the lifetime of the D^+. The Mark III Collaboration[35] has placed a 90%-confidence-level limit of 7.2×10^{-4} on the branching ratio $B(D^+ \to \mu^+ \nu)$. Using $\tau_D = (10.9 \pm 0.3 \pm 0.25) \times 10^{-13}$ s and $|V_{cd}|^2 = 0.0493$ which assumes three generations and unitarity of the CKM matrix they obtain $f_D \le 290$ MeV/c^2. If, on the other hand, we know the value of f_D, we can use a measurement of $B(D^+ \to \mu^+ \nu)$ to determine $|V_{cd}|^2$. Using our calculated value for f_D, $\tau_D = (10.69^{+0.34}_{-0.32}) \times 10^{-13}$ s, and $|V_{cd}| = 0.21 \pm 0.03$ (Ref. 37) we expect $B(D^+ \to \mu^+ \nu) = (4.63^{+2.69}_{-1.86}) \times 10^{-4}$ with the error obtained by taking the 1σ bounds of the input numbers (not by adding the errors in quadrature). This is at the threshold of being observed and would certainly be measured with reasonable precision at a τ-charm factory.[38] Assuming that $B(D^+ \to \mu^+ \nu)$ will be measured, the uncertainty in f_D would translate into an error in $|V_{cd}|$ of ± 0.02. Although this is only a slight improvement over the present value it is important to check for consistency in different measurements. Such a measurement could be useful in checking the unitarity of the CKM matrix which, in turn, can constrain new physics.[39] Using the same reasoning, observation of the decay $D_s^+ \to \mu^+ \nu$, with an expected branching ratio of $B(D_s^+ \to \mu^+ \nu) \simeq 6 \times 10^{-3}$, and $D_s^+ \to \tau^+ \nu$, with $B(D_s^+ \to \tau^+ \nu) \simeq 3 \times 10^{-2}$, would greatly improve our knowledge of $|V_{cs}|$ which is presently not well measured. In addition, knowledge of f_D will be important for using bounds on flavor-changing decays such as $D^0 \to \mu^\pm e^\mp$ to put constraints on new physics. Finally, observation of the decay $B^+ \to \tau^+ \nu$ with expected branching ratio $B(B^+ \to \tau^+ \nu) \simeq 6 \times 10^{-5}$, would give additional insight into $|V_{bu}|$.

B. B^0-$\bar{B}^0$ mixing and constraints on the t-quark mass and V_{td}

B^0-$\bar{B}^0$ mixing provides a sensitive test of the standard model and a probe for new physics.[4,5] If only standard-model contributions are considered the B_q^0-$\bar{B}_q^0$ mixing parameter r_q is given by[40]

$$r_q = \frac{\Gamma(B^0 \to l^+ \nu X)}{\Gamma(B^0 \to l^- \bar{\nu} X)} \simeq \frac{x_q^2}{2 + x_q^2} \quad (10)$$

with $q = d$ or s, where we use the convention that the B^0 meson contains a b quark, and

$$x_q = \left(\frac{\Delta M}{\Gamma}\right)_{B_q} = \frac{G_F^2}{6\pi^2} m_t^2 \tau_{B_q} B_{B_q} f_{B_q}^2 M_{B_q} |V_{tq} V_{tb}^*|^2 \eta_{\text{QCD}} F(m_t^2/M_W^2) , \quad (11)$$

where η_{QCD} is a QCD-correction factor which is estimated to be about 0.85 (Ref. 41) and

$$F(Z_t) = \frac{1}{4} + \frac{9}{4} \frac{1}{1 - Z_t} - \frac{3}{2} \frac{1}{(1 - Z_t)^2} - \frac{3}{2} \frac{Z_t^2 \ln Z_t}{(1 - Z_t)^3} . \quad (12)$$

There are additional contributions involving the c and u quarks which are much smaller and so we neglect them in our calculations. The mixing parameter x_q is sensitive to several input parameters: the top-quark mass m_t, the CKM matrix element V_{td}, the B parameter B_{B_q}, and the pseudoscalar decay constant f_{B_q}. Given a reliable estimate of $(B_{B_q}^{1/2} f_{B_q})$, x_q will relate V_{tq} to m_t so if m_t is known, x_q will constrain V_{tq}. Inconsistencies between the values determined from B_q^0-$\bar{B}_q^0$ mixing and other determinations would point to the need for new physics beyond the standard model.

To calculate x_q we use $\tau_{B_d} = (1.18 \pm 0.12) \times 10^{-12}$ s (Ref. 42), $M_{B^0} = 5.2794 \pm 0.0015$ GeV (Ref. 37), $\eta_{\text{QCD}} = 0.85$, $|V_{tb}| \simeq 1$, $f_{B_d} = 155 \pm 15$ MeV, and $B_{B_d} \simeq 1$. Estimating the remaining parameter V_{td} is more problematic and has been discussed extensively in the literature.[43] In what follows we will use typical values for the parameters and do not make any claim to do justice to this important subject. Three-generation unitarity of the CKM matrix constrains $|V_{td}| \le 0.023$ (Ref. 37). However, to study the properties of the CKM matrix it is convenient to use the Wolfenstein parametrization[44] which to $O(\lambda^3)$ is given by

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3 \rho e^{i\phi} \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho e^{-i\phi}) & -A\lambda^2 & 1 \end{pmatrix} . \quad (13)$$

In this parametrization $\lambda = V_{us} = 0.220 \pm 0.003$ and, from the B lifetime τ_B and the semileptonic branching ratio, $|V_{cb}| = A\lambda^2 = 0.046 \pm 0.008$ (Ref. 45) gives $A = 0.94 \pm 0.16$.

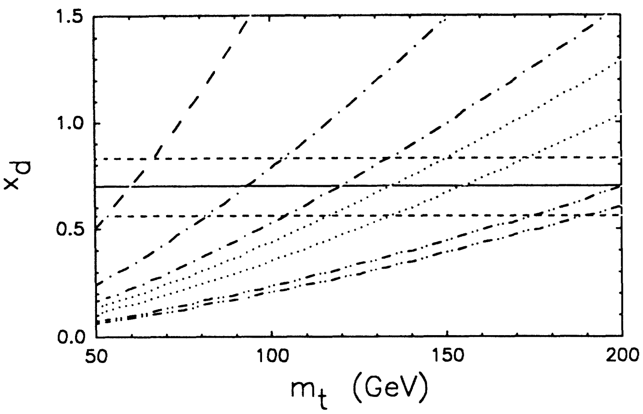

FIG. 3. The B^0-$\bar{B}^0$ mixing parameter x_d vs the t-quark mass using Eq. (11) with the parameters described in the text. The solid line is the combined ARGUS/CLEO results and the short dashed lines are the 1σ limits. Going from left to right, the long dashed line is the limit obtained using the CKM three-generation unitarity bound of $|V_{td}|<0.023$. The dotted-dashed lines are for $\cos\phi=-0.9$ with $\rho=0.6$ and 0.3. The dotted lines are for $\cos\phi=0.0$ with $\rho=0.6$ and 0.3. The dotted-dotted-dashed lines are for $\cos\phi=0.6$ with $\rho=0.3$ and 0.6

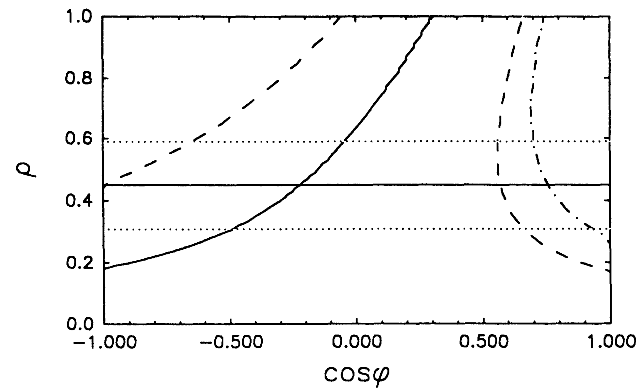

FIG. 5. Values of the Wolfenstein parameters ρ vs $\cos\phi$. The horizontal line comes from the recent measurement of $|V_{ub}/V_{cd}|$ and the horizontal dotted lines are the 1σ bounds. The solid curve is obtained by using the central values of the free parameters in Eq. (11). The dashed curves are the 1σ bounds (with the errors combined in quadrature) while the dotted-dashed curve is the bound found by combining the errors to maximize the deviation from the central value.

Recent measurements of $|V_{ub}/V_{cb}|$ from semileptonic B decay[45] give $\rho=0.45\pm0.14$. Substituting the central values into Eq. (11) we obtain

$$x_d=(4.6\pm1.4)\times10^{-5}\ \mathrm{GeV}^2\tilde{m}_t^2(1+\rho^2-2\rho\cos\phi)\ , \qquad (14)$$

where $\tilde{m}_t$ is an effective t-quark mass which absorbs the dependence on $F(Z_t)$.

In Fig. 3 we plot x_d vs m_t for several possible values of ρ and $\cos\phi$. It can be seen that there is considerable latitude in the $m_t,\rho,\cos\phi$ parameter space although consistency with recent measurements of $|V_{bu}|$ and $|V_{bc}|$ constrains $m_t \gtrsim 70$ GeV. We can reduce this parameter space using the precision measurements of the Z^0 mass combined with low-energy neutral-current measurements to constrain the mass of the t quark. Ellis and Fogli[46] find that $m_t=132^{+31}_{-37}$ GeV. We should caution the reader that the errors quoted by Ellis and Fogli may be optimistic. A similar analysis by Langacker[47] finds the more conservative bounds that for $m_H=100$ GeV, $m_t=140^{+43}_{-53}$. In Fig. 4 we plot $|V_{td}|$ vs m_t with the upper and lower bounds allowed by varying the remaining free input parameters and the t-quark mass bounds. Substituting these values into Eq. (14) we obtain the limits on the $\rho-\cos\phi$ parameter space shown in Fig. 5. These constraints can be compared against those obtained in other processes to test for consistency of the standard model.

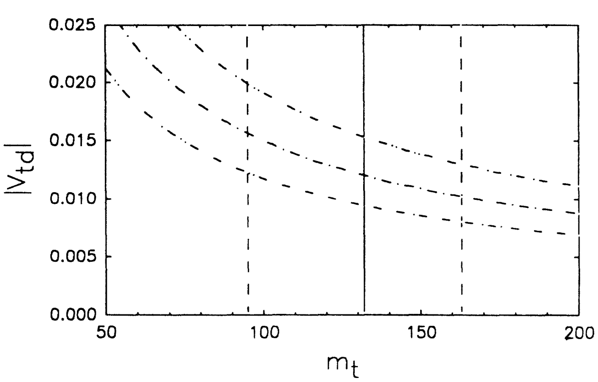

FIG. 4. $|V_{td}|$ vs m_t. The solid line is the central value of m_t obtained from combined neutral-current data and measurements of M_{Z^0} with the dashed lines the 1σ errors. The dotted-dashed curve is the value obtained using the central values of all the remaining free parameters in Eq. (11) while the dotted-dotted-dashed lines are obtained by taking the 1σ upper and lower limits of the free parameters (not combining the errors in quadrature).

IV. SUMMARY AND CONCLUSIONS

We have made what we consider to be reliable estimates of the weak decay constants and their theoretical uncertainties for all but the lightest pseudoscalar mesons. We find $f_D\simeq240\pm20$ MeV, $f_{D_s}\simeq290\pm20$ MeV, $f_B\simeq155\pm15$ MeV, $f_{B_s}\simeq210\pm20$ MeV, and $f_{B_c}\simeq410\pm40$ MeV. Our model uses a relativized prescription, and realistic wave functions which contain information about the short-distance behavior of the interquark potential. We have also surveyed the results of other models in order to establish the degree of model dependence in these estimates.

Our motivation has been that a knowledge of the pseudoscalar decay constants is crucial to extracting fundamental parameters of the standard model from experiment; the decay constants f_D and f_{D_s} are necessary if we are to find the Cabibbo-Kobayashi-Maskawa matrix elements $|V_{cd}|$ and $|V_{cs}|$, and similarly the value of f_B has implications for the size of B_q^0-$\bar{B}_q^0$ mixing, and so relates the top-quark mass to $|V_{td}|$. Present measurements of $|V_{bu}|$ and $|V_{bc}|$ imply $m_t > 70$ GeV. In order to test the standard model we need some understanding of these hadronic matrix elements, and although ultimately lattice gauge theory will give the most reliable estimates of the decay constants, at present we must rely on the quark model. Physics beyond the standard model may (soon) be present in the data but without some means to estimate the hadronic effects we will never know.

ACKNOWLEDGMENTS

This work was funded in part by the Natural Sciences and Engineering Research Council of Canada and by the University of Guelph Excellence Fund and in part by the U.S. Department of Energy (DOE) under Contract No. DE-AC02-76ER-03066. The authors thank Rob Kutschke, Steve Sharpe, and Lincoln Wolfenstein for helpful conversations.

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