Are light Higgs bosons allowed?

Sally Dawson
Physics Department, Brookhaven National Laboratory, Upton, New York 11973

John F. Gunion
Department of Physics, University of California, Davis, California 95616

Howard E. Haber
Santa Cruz Institute for Particle Physics, University of California, Santa Cruz, California 95064

(Received 14 July 1989)

We examine the experimental limits on a standard-model Higgs boson available prior to the data which will be forthcoming from the Z factories. Particular attention is paid to the theoretical uncertainties involved in extracting these limits. We find that, subject to a number of theoretical assumptions, the Higgs-boson mass must be greater than $2m_\tau$.

I. INTRODUCTION

The search for the Higgs boson of the standard model (ϕ^0) is a fundamental goal of all accelerators. We explore the limits that can be placed on m_{ϕ^0} prior to the start-up of CERN LEP. In this paper we examine a variety of experimental limits on such light Higgs bosons. Many of our results are extremely sensitive to the coupling of the Higgs boson to the nucleon and to kaons and pions and we pay particular attention to the theoretical uncertainties involved. Section II contains a discussion of the limits obtained from nuclear physics. These experiments typically require $m_{\phi^0} \gtrsim 11$ MeV. Higgs-boson production in kaon decays is considered in Sec. III and limits from B physics are presented in Sec. IV. The limits obtained from these processes depend on a variety of theoretical assumptions which we discuss in detail. In Sec. V we present our conclusions. Some of the issues discussed in this paper have also recently been treated in Ref. 1.

We consider only the production of the Higgs boson contained in the minimal version of the standard model, although many of our results can easily be extended to more complicated scenarios. A recent review of the properties of the standard-model Higgs boson and a guide to the literature can be found in Ref. 2.

II. LIMITS FROM NUCLEAR PHYSICS

A very light standard-model Higgs boson ($m_{\phi^0} \lesssim 20$ MeV) may influence low-energy processes in a variety of ways. In this section we will review a selection of possibilities and the experiments which give rise to the strongest constraints and limits.[3]

The best limits on Higgs bosons with a mass less than 1 MeV come from x-ray transition rates in muonic atoms. Beltrami *et al.*[4] have measured the $3d_{5/2}$-$2p_{3/2}$ x-ray transition in ^{24}Mg and ^{28}Si. They looked for any muon-nucleon isoscalar interaction mediated by a neutral scalar or vector boson. The presence of a light scalar boson would induce a Yukawa potential for the muon-nucleon interaction and cause a change in the transition wavelength,

$$\frac{\delta\lambda}{\lambda} = -\frac{g_{\phi^0 NN} g_{\phi^0 \mu\mu}}{10\pi\alpha Z}[9f(2) - 4f(3)] , \qquad (2.1)$$

where Z is the nuclear charge, $g_{\phi^0 NN}$ and $g_{\phi^0 \mu\mu}$ are the Higgs couplings to nucleons and muons, respectively, $f(n) = (1 + n m_{\phi^0}/2\alpha Z m_\mu)^{-2n}$, and n is the principle quantum number of the muonic state. For small m_{ϕ^0}, $f(n) \to 1$ and the shift in the wavelength is independent of the scalar mass. In this limit, Beltrami *et al.*[4] find the result

$$|g_{\phi^0 NN} g_{\phi^0 \mu\mu}| < 7 \times 10^{-8} \quad \text{for } m_{\phi^0} < 1 \text{ MeV} . \qquad (2.2)$$

Using $g_{\phi^0 \mu\mu} = g m_\mu / 2 m_W$, and defining η via

$$g_{\phi^0 NN} = \frac{g M_N}{2 m_W} \eta , \qquad (2.3)$$

we find from Eq. (2.2) that this experiment requires $\eta < 0.04$ for $m_{\phi^0} < 1$ MeV. The value $\eta = 1$ would correspond to a point nucleon. This is not a realistic estimate for a nucleon made up of quarks and gluons. Following Ref. 5 one may use Higgs-boson low-energy theorems to estimate $\eta \simeq 0.3$. Using this estimate, we would conclude that these experiments exclude Higgs bosons which are lighter than about 8 MeV. The results of Ref. 4 are shown in Fig. 1, where the region above the curved line is excluded. The prediction using standard-model couplings and $\eta = 0.3$ is also shown. (For $\eta = 0.1$, this experiment excludes Higgs-boson masses less than about 3 MeV.)

In the mass region $2m_e < m_{\phi^0} < 20$ MeV certain $0^+ \to 0^+$ nuclear transitions would see the effect of a light Higgs boson. Kohler *et al.*[6] searched for ^{16}O(6.05 MeV)$\to$(ground state)$+\phi^0$, $\phi^0 \to e^+e^-$. Their results exclude $2m_e \lesssim m_{\phi^0} \lesssim 5.8$ MeV for $\eta \gtrsim 0.3$. Freedman *et al.*[7] have performed an experiment involving the excited ^{4}He decay, ^{4}He(20.1 MeV)$\to$(ground state)$+\phi^0$, $\phi^0 \to e^+e^-$. They conclude that Higgs-boson masses

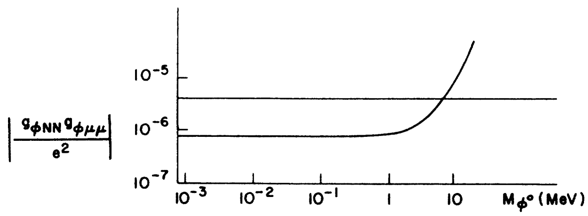

FIG. 1. Limits on light standard-model Higgs bosons from the measurement of the $3d_{5/2}$-$2p_{3/2}$ transition rates in muonic atoms of Ref. 4. The region above the curved line is excluded and the straight line is the prediction of the standard model with $\eta=0.3$ [see Eq. (2.3)].

$2.8 \lesssim m_{\phi^0} \lesssim 11.5$ MeV are excluded for $\eta \simeq 0.3$.

Numerous other mechanisms for searching for a Higgs boson with a mass less than 20 MeV have been proposed. None of them improve the limits listed above. The exact limits deriving from the experiments we discussed all depend on the assumed value of the Higgs-boson–nucleon coupling and, hence, on η. However, for $\eta \gtrsim 0.3$, all Higgs-boson masses less than 11.5 MeV are excluded by the measurements of the x-ray transitions in muonic atoms and by the $0^+ \rightarrow 0^+$ transition experiments in ^{4}He.

III. LIMITS FROM π AND K DECAYS

A. Higgs bosons in pion decays

The rate for Higgs-boson production in charged-pion decay can be easily obtained using chiral Lagrangian techniques.[8] The result is[9,1,10]

$$\frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e \phi^0)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = \frac{\sqrt{2}\, G_F m_\pi^4 f(x)}{96\pi^2 m_\mu^2 (1 - m_\mu^2/m_\pi^2)^2} \left[1 - \frac{2N_H}{3b}\right]^2 = 1.9 \times 10^{-9} f(x)\ , \quad (3.1)$$

where $f(x) \equiv (1 - 8x + x^2)(1 - x^2) - 12x^2 \ln(x)$ with $x \equiv m_{\phi^0}^2 / m_\pi^2$, and we have taken $N_H = 3$ and $b = 11 - \frac{2}{3} n_L$ with $n_L = 3$ light fields. ($N_H = 3$ corresponds to the three heavy flavors: c, b, and t.) The N_H-independent piece is due to the tree-level graph in which the Higgs boson is emitted from the virtual W. The N_H-dependent piece of Eq. (3.1) arises from the inclusion of graphs in which the Higgs boson couples to the gluons in the pion via heavy-quark intermediate loops.

The SINDRUM Collaboration[11] has recently searched for decays of the type $\pi^+ \rightarrow e^+ \nu_e e^+ e^-$. A Higgs boson in the mass range $2m_e < m_{\phi^0} \lesssim m_\pi$ would yield events of this type, since it would decay primarily to an e^+e^- pair. In Ref. 11 a 90%-confidence-level upper limit on $B(\pi^+ \rightarrow e^+ \nu_e \phi^0)$ is obtained as a function of m_{ϕ^0}. This limit is roughly a factor of 3 below the theoretical prediction quoted above[12] over the range of Higgs-boson masses from 10 to 100 MeV, which clearly excludes a standard-model Higgs boson in this mass range.

B. Higgs bosons in kaon decays

The decays $K^\pm \rightarrow \pi^\pm l^+ l^-$ and $K_L^0 \rightarrow \pi^0 l^+ l^-$ can be used to place limits on a Higgs boson with a mass less than $m_K - m_\pi$, which could contribute through the decay sequence $K \rightarrow \pi\phi^0$, $\phi^0 \rightarrow l^+ l^-$. Similarly, $K \rightarrow \pi\gamma\gamma$ limits have the potential for placing limits on the Higgs boson in the m_{ϕ^0} range where the ϕ^0 decays significantly to two photons. In addition, dedicated searches have been made for the decay $K^+ \rightarrow \pi^+ \phi^0$, $\phi^0 \rightarrow e^+ e^-$ (Refs. 13 and 14) and $K^+ \rightarrow \pi^+ \phi^0$, $\phi^0 \rightarrow \mu^+ \mu^-$ (Ref. 15). Hence, if the rate for $K \rightarrow \pi\phi^0$ can be reliably calculated, we may be able to rule out the existence of a very light Higgs boson.

There are two classes of diagrams which contribute to a quark model calculation of $K^\pm \rightarrow \pi^\pm \phi^0$. The first class consists of spectator diagrams corresponding to the two-quark operators which contribute to the transition $s \rightarrow d\phi^0$. The second class consists of nonspectator diagrams corresponding to four-quark operators. The calculation of the amplitude for $s \rightarrow d\phi^0$ is straightforward and has been checked by several groups:[16–18,9]

$$\mathcal{L}_{sd\phi^0} = \frac{G_F^{3/2}}{2^{1/4}} \frac{3}{16\pi^2} \sum_i m_i^2 V_{id}^* V_{is} [m_s \bar{d}(1+\gamma_5)s + m_d \bar{d}(1-\gamma_5)s]\phi^0 + \text{H.c.}\ , \quad (3.2)$$

where V_{ij} are the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and m_i are the corresponding quark masses. Relating the two-quark operator of Eq. (3.2) to the SU(3) currents and using an isospin rotation and the measured rate for K_{l3} decay, we find the contribution of the two-quark operator to the matrix element for $K^- \rightarrow \pi^- \phi^0$:

$$\mathcal{M}_1(K^- \rightarrow \pi^- \phi^0) \simeq \frac{G_F^{3/2}}{2^{1/4}} \frac{3}{16\pi^2} m_K^2 \sum_i m_i^2 V_{id}^* V_{is}\ . \quad (3.3)$$

The simplest nonspectator diagram for $s + u \rightarrow d + u + \phi^0$ gives rise to the (low-energy) effective four-fermion–Higgs-boson interaction:

$$\mathcal{L} = \frac{2G_F^{3/2}}{2^{1/4}} V_{ud}^* V_{us} \bar{d}\gamma^\mu(1-\gamma_5)u\bar{u}\gamma_\mu(1-\gamma_5)s\phi^0 + \text{H.c.} \quad (3.4)$$

Gluonic corrections introduce additional four-quark operators. One could compute the matrix elements of the four-quark operators between an on-shell kaon and pion state using the vacuum-insertion approximation. However, this method is known not to be very reliable; for example, it does not reproduce in detail the observed $\Delta I = \frac{1}{2}$ rule. Instead, we prefer to use the chiral-Lagrangian technique.[8] Following Ref. 19, the $\Delta I = \frac{1}{2}$ piece of the $\Delta S = 1$ chiral Lagrangian is parametrized by a parameter λ whose magnitude (but not its sign) is determined by K nonleptonic decay data to be $|\lambda| = 3.2 \times 10^{-7}$. In our convention the sign of λ is negative in the vacuum-insertion approximation; we adopt this sign for our computations. Second, there is a $\Delta I = \frac{3}{2}$ contribution which is parametrized by a parameter a. However, we know from experiment that $|a| \ll |\lambda|$, so we will henceforth neglect effects due to the $\Delta I = \frac{3}{2}$ contribution. A third parameter A appears which parametrizes weak K-π mixing. However, the A parameter can be rotated away by mass diagonalization in the absence of Higgs-boson interactions. From the chiral Lagrangian, the Higgs-boson interactions with the pseudoscalar mesons can be obtained by applying Higgs low-energy theorems, reviewed in Refs. 2 and 20. Following a similar procedure, the authors of Ref. 9 obtained the contribution of the nonspectator diagrams to $K^- \to \pi^- \phi^0$. In addition to λ, the amplitude for $K \to \pi\phi^0$ also depends on the parameter A, referred to above. When one performs the mass diagonalization to rotate away the K-π mixing, a contribution to the $K \to \pi\phi^0$ amplitude, proportional to A, remains. For $N_H = 3$ heavy quarks and $b = 11 - 2n_L = 9$ (for $n_L = 3$ light quarks), we obtain[2,20]

$$\mathcal{M}_2(K^- \to \pi^- \phi^0) = G_F^{1/2} 2^{1/4} \left[\frac{7 A m_K^2}{9} - \frac{7\lambda(m_K^2 + m_\pi^2 - m_{\phi^0}^2)}{18} \right] . \tag{3.5}$$

The parameter A is expected to be roughly of the same order as λ, but no further theoretical information on its magnitude or sign is available at present.[21]

Without any *a priori* knowledge of the value of A, the sign of $\mathcal{M}_2$ can be positive or negative. Thus, there is no definite theoretical prediction for $B(K \to \pi\phi^0)$. However, it was recently pointed out in Ref. 22 that there is a prediction for a *lower* limit for $B(K^\pm \to \pi^\pm \phi^0)$ due to the fact that the amplitude for $K^\pm \to \pi^\pm \phi^0$ is complex. This suggestion would make sense only if the dominant source of the imaginary part is due to the heavy quark mixing angles which appear in $\mathcal{M}_1$. In fact, we believe that this is the case. The two-quark operator is special in that the amplitude contains an explicit factor of the quark mass squared [see Eq. (3.3)]. The t-quark contribution in the sum is greatly enhanced, which permits $\mathrm{Im}\mathcal{M}_1/\mathrm{Re}\mathcal{M}_1$ to be of order 1. On the other hand, simple dimensional analysis suggests that the contributions to the amplitude from the four-quark operators can at best approach a constant in the large-m_t limit. It follows that the ratio of the imaginary to the real part of the four-quark amplitudes can be no larger than (roughly) $\mathrm{Im} V_{td}^* V_{ts} / (V_{ud} V_{us})$ which is of order 10^{-3}. Thus, it follows that

$$\mathrm{Im}\mathcal{M}_2 \sim 10^{-3} \mathrm{Re}\mathcal{M}_2 . \tag{3.6}$$

Since we expect the real parts of $\mathcal{M}_1$ and $\mathcal{M}_2$ to be roughly of the same order of magnitude, we conclude that $\mathrm{Im}\mathcal{M}_1$ should be the dominant source of the imaginary part of the total amplitude.

We therefore obtain a lower limit by setting the real part of the total amplitude to zero (which would happen for some choice of the parameter A). It is convenient to use the Wolfenstein parametrization[23] of the CKM matrix, where

$$\begin{aligned} V_{ud}^* V_{us} &\simeq -V_{cd}^* V_{cs} \simeq \sin\theta_c \cos\theta_c , \\ V_{td}^* V_{ts} &\simeq -A_w^2 \sin^5\theta_c (1 - \rho + i\eta) . \end{aligned} \tag{3.7}$$

Experimentally, $\sin\theta_c \simeq 0.22$, the parameter A_w is close to unity ($A_w = 1.05 \pm 0.17$ according to Ref. 24), and $\rho \lesssim 0$ (based on the observed B-$\bar{B}$ mixing). In addition, the CERN measurement of ϵ'/ϵ may be used to determine a value of η. Values in the range $0.1 \lesssim \eta \lesssim 0.6$ have been obtained in the literature.[22,25] In the analysis below we shall take a somewhat conservative value, $\eta = 0.2$. Thus, taking the imaginary part of the amplitude as determined from Eq. (3.3), we get

$$\begin{aligned} |\mathrm{Im}\mathcal{M}(K^\pm \to \pi^\pm \phi^0)| &\gtrsim \frac{G_F^{3/2}}{2^{1/4}} \frac{3 m_K^2 m_t^2 \eta A_w^2 \sin^5\theta_c}{16\pi^2} \\ &\gtrsim 7.9 \times 10^{-11}\ \mathrm{GeV} , \end{aligned} \tag{3.8}$$

where we have taken $m_t \approx 80$ GeV, as required in order to have a very light standard-model Higgs boson.[27] Using $\Gamma(K \to \pi\phi^0) = B_{\phi^0} |\mathcal{M}|^2 / (16\pi m_K)$, where $B_{\phi^0} = 2p_{\phi^0}/m_K$, and normalizing to the total decay rate, $\Gamma(K^\pm) = 5.32 \times 10^{-17}$ GeV, we find

$$B(K^\pm \to \pi^\pm \phi^0) \gtrsim 4.3 \times 10^{-6} B_{\phi^0} . \tag{3.9}$$

Of course, the limit of Eq. (3.9) depends crucially on the assumed value of η derived from ϵ'/ϵ. If it should turn out to be that ϵ'/ϵ is very near 0 (which is consistent with the results of Ref. 28) then clearly no limit on $K^\pm \to \pi^\pm \phi^0$ can be derived from the imaginary part of the amplitude.

Let us next turn to the decay $K_L^0 \to \pi^0 \phi^0$. The one important change in the analysis results from the fact that the matrix element for this process is real.[29] In particular, we have

$$\begin{aligned} \mathcal{M}_1(K_L^0 \to \pi^0 + \phi^0) &= -\frac{G_F^{3/2}}{2^{1/4}} \frac{3}{16\pi^2} m_K^2 \sum_i m_i^2 \mathrm{Re}(V_{id}^* V_{is}) , \\ \mathcal{M}_2(K_L^0 \to \pi^0 + \phi^0) &= -\mathrm{Re}\mathcal{M}_2(K^- \to \pi^- \phi^0) . \end{aligned} \tag{3.10}$$

Thus, we cannot obtain such a definitive theoretical bound for $K_L^0 \to \pi^0 \phi^0$, since, unlike in the case above, there exists a particular value of A for which this amplitude vanishes. Nevertheless, we can examine the branching ratio as a function of the unknown parameter A in $\mathcal{M}_2$. We do this by considering several values for A (which is presumably of order λ) and computing the decay width as a function of $m_t^2 |\mathrm{Re} V_{td}^* V_{ts}|$. The resulting width is normalized with respect to the total width $\Gamma(K_L^0) = 1.27 \times 10^{-17}$ GeV. For $m_t \approx 80$ GeV we expect $m_t^2 |\mathrm{Re} V_{td}^* V_{ts}| \gtrsim 2.5\ \mathrm{GeV}^2$, with large uncertainty. Thus, we plot in Fig. 2, $B(K_L^0 \to \pi^0 \phi^0)$ vs $m_t^2 |\mathrm{Re} V_{td}^* V_{ts}|$ for three different values of A. We see that only for a very narrow range of $m_t^2 |\mathrm{Re} V_{td}^* V_{ts}|$, and only for positive values of A does the branching ratio for $K_L^0 \to \pi^0 \phi^0$ ever fall below 10^{-5}. Therefore, barring a very unlikely conspiracy of parameters, we obtain a predicted branching ratio which satisfies

$$B(K_L^0 \to \pi^0 \phi^0) \gtrsim 10^{-5} B_{\phi^0} . \tag{3.11}$$

We also note that if $\epsilon'/\epsilon \simeq 0$, then the result for

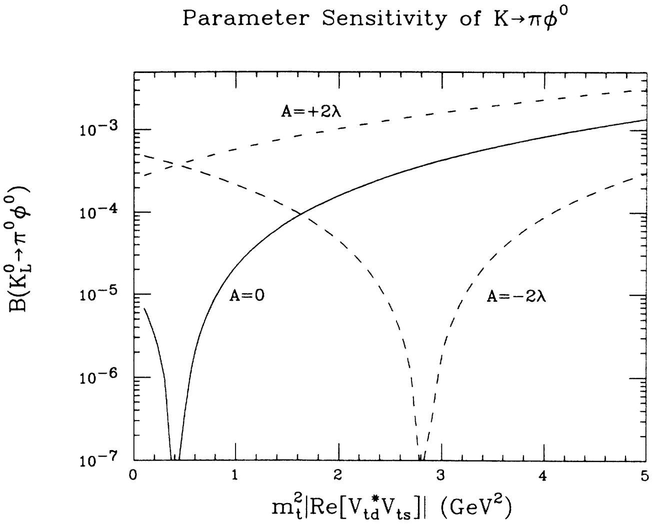

FIG. 2. The $K_L^0\to\pi^0\phi^0$ branching ratio as a function of $m_t^2|\mathrm{Re}V_{td}^*V_{ts}|$, assuming $m_{\phi^0}\ll m_K$. The three curves shown are $A=0$ (solid), $A=2\lambda$ (dot-dashed), and $A=-2\lambda$ (dashes), where $\lambda=-3.2\times10^{-7}$.

$K^\pm\to\pi^\pm\phi^0$ corresponding to Eq. (3.11) is

$$B(K^\pm\to\pi^\pm\phi^0)\gtrsim 2.5\times10^{-6}B_{\phi^0}\ . \tag{3.12}$$

Limits on m_{ϕ^0} obtained on the basis of Eq. (3.12) are generally not very different from those based on Eq. (3.9).

Similarly, we can calculate the amplitude for the *CP*-violating decay $K_S^0\to\pi^0\phi^0$. If we can neglect $\mathrm{Im}A$ and $\mathrm{Im}\lambda$ as suggested above (and neglecting small *CP*-violating effects in the kaon state mixing), then $\mathcal{M}_2$ can be ignored. $\mathcal{M}_1$ is modified by replacing $\mathrm{Re}V_{id}^*V_{is}$ with $\mathrm{Im}V_{id}^*V_{is}$ in Eq. (3.10). Using the numbers quoted above we find

$$B(K_S^0\to\pi^0\phi^0)\simeq10^{-8}B_{\phi^0}\ . \tag{3.13}$$

We now turn to a discussion of the extent to which currently available experimental data can be used to constrain the existence of a light Higgs boson.

Potentially relevant experimental data are of two types: first, there are limits on the branching ratio for K decay to a variety of final states to which production of a ϕ^0 might contribute—these typically must be examined critically in order to determine the extent to which they lead to limitations on the Higgs boson; and second, there have been a number of direct searches for Higgs bosons in many of the same final states. We begin by reviewing data of the first type. Available limits are the following.[30]

$L1$: $B(K_L^0\to\pi^0\mu^+\mu^-)<1.2\times10^{-6}$ (Ref. 31);

$L2$: $B(K_L^0\to\pi^0e^+e^-)<2.3\times10^{-6}$ (Ref. 31), with recent limits of $<3.2\times10^{-7}$ (Ref. 32) and $<4.2\times10^{-8}$ (Ref. 33);

$L3$: $B(K^+\to\pi^+e^+e^-)=(2.7\pm0.5)\times10^{-7}$ (Ref. 34) and $<2.7\times10^{-7}$ (Ref. 35);

$L4$: $B(K^+\to\pi^+\mu^+\mu^-)<2.4\times10^{-6}$ (Ref. 36) with a recent experimental limit reported of $<2\times10^{-6}$ (Ref. 37);

$L5$: $B(K^+\to\pi^+\gamma\gamma)<8\times10^{-6}$ (Ref. 38), and $B(K^+\to\pi^+\gamma\gamma)<1.4\times10^{-6}$ (Ref. 39); and

$L6$: $B(K^+\to\pi^++\text{nothing})<3.8\times10^{-8}$ (Ref. 40), and a new limit of $<3\times10^{-8}$ (Ref. 41).

Some discussion concerning the above limits is necessary. In particular, to ascertain the implications of the above limits for the standard-model Higgs boson, we must know the branching ratios for $\phi^0\to\gamma\gamma$, $\phi^0\to e^+e^-$, and $\phi^0\to\mu^+\mu^-$. We must also consider the expected lifetimes for the ϕ^0 when these various decay modes are dominant. Finally, we must examine the extent to which the quoted limits are restricted in applicability to the ϕ^0 search by limitations coming from experimental acceptance, cuts and backgrounds. For example, the lepton pair final-state limits have typically assumed a phase-space distribution over a certain invariant-mass range, while the $\gamma\gamma$ and *nothing* final-state limits can only be applied to Higgs-boson decays after accounting for the Higgs-boson lifetime.

First, consider the limits $L2$ and $L3$ for the e^+e^- final state. These limits will be relevant for $m_{\phi^0}<2m_\mu$, where the Higgs boson decays almost exclusively to e^+e^-. There is a question as to how far down in $M(e^+e^-)$ they can be applied. Unfortunately, the recent very strong limit of Ref. 32 does not apply for $M(e^+e^-)<2m_\mu$ (the region of interest for ϕ^0 decays). In the case of Ref. 32 one finds that their branching-ratio limit deteriorates very rapidly as $M(e^+e^-)$ decreases, rising to 1.2×10^{-6} at their lowest quoted point of $M(e^+e^-)=240$ MeV. A similar deterioration[42] occurs in the case of Ref. 31 (see also Ref. 43). In both cases the limit deterioration is due to the accumulation of backgrounds in this region.

The limit on $K_L\to\pi e^+e^-$ from the Fermilab experiment E731 (Ref. 33) is also of dubious utility because of the cuts made in obtaining the explicit acceptance plots as a function of $M(e^+e^-)$ that are presented. While these show good acceptance over the region 50 MeV $\lesssim M(e^+e^-)\lesssim350$ MeV, their current method of analysis is such that the e^+e^- would have to emerge within roughly 1 m of the initial $K_L^0\to\pi^0$ decay vertex. Since their initial K_L^0 has momentum of ~60 GeV on average, the ϕ^0 emerging in $K_L^0\to\pi^0\phi^0$ would be quite energetic. For $m_{\phi^0}\le2m_\mu$, where the decay $\phi^0\to e^+e^-$ is dominant, the ϕ^0 will have a long lifetime and its path length generally exceeds 1 m. For instance, for $m_{\phi^0}=140$ MeV, $c\tau\sim0.8$ cm; to first approximation the π and ϕ^0 will share the K_L^0 momentum in proportion to their masses yielding a boost factor of order 220 and a path length for the ϕ^0 of order 1.6 m. Even for $m_{\phi^0}\sim200$ MeV, the path length is only just below 1 m.

Turning to the other experiments we note that explicit branching-ratio limits as a function of $M(e^+e^-)$ are not available for the $K^+\to\pi^+e^+e^-$ experiments of Refs. 34 and 35. Reference 34 quotes a lower limit on accepted $M(e^+e^-)$ of 140 MeV. Reference 35 quotes a lower limit of 50 MeV. In analogy with the two analyses discussed above we expect that the actual lower limits in the region $M(e^+e^-)<2m_\mu$ are significantly worse than those quoted above based on a phase-space distribution of $M(e^+e^-)$.

For $m_{\phi^0}<2m_e$ the $\phi^0\to\gamma\gamma$ mode dominates. However, the ϕ^0 lifetime is so long that it would have escaped the apparatus used to establish the experimental limit $L5$, which is not sufficiently strong to have been useful in any case. Fortunately, because of the long ϕ^0 lifetime the limit $L6$ becomes relevant and clearly rules out $m_{\phi^0}<2m_e$, given the predicted branching-ratio lower bound of Eq. (3.9).

For $2m_e<m_{\phi^0}<2m_\mu$, the ϕ^0 decays to e^+e^- and $\gamma\gamma$, with the branching ratio for the e^+e^- mode being close to 1 over a large part of this mass range. If we adopt the conservative lower limit of Eq. (3.11) away from the cancellation dips in Fig. 2, we have $B(K_L^0\to\pi^0\phi^0)\times B(\phi^0\to e^+e^-)\gtrsim 0.8\times10^{-5}$. Unfortunately, as we have discussed, the $K_L^0\to\pi^0e^+e^-$ limit from $E731$ is not applicable. Further, the upper limits on this decay for $M(e^+e^-)<200$ MeV ($B\leq10^{-5}$) from the other experiments do not allow us to rule out any m_{ϕ^0} values below $2m_\mu$ on the basis of this decay if we are anywhere near the cancellation dips of Fig. 2.

Consider next the charged-kaon decays. The nominal limits on $B(K^+\to\pi^+e^+e^-)$ are a factor of more than 30 below that expected from $K^+\to\pi^+\phi^0(\to e^+e^-)$; see Eq. (3.9). But, as discussed above, we cannot be certain how these limits would apply as a function of $M(e^+e^-)$. Thus, a conservative conclusion (away from the cancellation dips of Fig. 2) is that the $K\to\pi e^+e^-$ experiments that are not explicitly dedicated to searching for the ϕ^0 rule out only a very small range of Higgs-boson masses, 200 MeV $\lesssim m_{\phi^0}\lesssim 2m_\mu$.

However, there is still the possibility that the $K^+\to\pi^+$ + nothing limit $L6$ in our list can be used to rule out the ϕ^0 for some range of m_{ϕ^0} above $2m_e$. The idea is that the ϕ^0 has a sufficiently long lifetime, even when the $\phi^0\to e^+e^-$ channel is allowed, that some of the decays will have been invisible to the detector. Since the detector of Ref. 40 is much smaller than that of BNL-787 (Ref. 41), only the former experiment will be considered in the following discussion. Consider, for example, $m_{\phi^0}=50$ MeV; the lifetime of such a ϕ^0 is 8.5×10^{-11} sec and the time dilation factor in the K rest frame is ~4.6, implying a decay path length of order 0.1 m. The fiducial dimension of the apparatus in the experiment of Ref. 40 was about 0.37 m. A fraction $e^{-3.7}\sim0.03$ of the ϕ^0 decays would have occurred outside the apparatus. Thus, the theoretical prediction of Eq. (3.9) would imply an effective branching ratio for $K^+\to\pi^+\phi^0$, $\phi^0\to$nothing of $\gtrsim1.3\times10^{-7}$, which is clearly excluded by limit $L6$. Even for $m_{\phi^0}=60$ MeV, where the ϕ^0 lifetime is 6.5×10^{-11}, the decay path length is of order 0.072 m, and the effective branching ratio for $K^+\to\pi^+\phi^0$, $\phi^0\to$nothing computed using Eq. (3.9) is $\sim2.3\times10^{-8}$, only slightly below the limit of Ref. 40. The apparatus size used above was supplied by Shinkawa,[44] from the collaboration of Ref. 40. In addition, Shinkawa has checked that there is a high probability that the range of the detected pion falls in a background-free region. Combining all efficiencies, he finds that the 90%-confidence-level upper limit for $B(K^+\to\pi^+\phi^0)$ is 1.3×10^{-6}, 6.3×10^{-6}, and 4.6×10^{-5} for $m_{\phi^0}=50$ MeV, 60 MeV, and 65 MeV, respectively,[44] confirming that masses up to $m_{\phi^0}\sim55$ MeV are excluded.

Shinkawa has also pointed out that their collaboration's limit (see limit $L5$ above) on the visible electromagnetic decays $K^+\to\pi^+\gamma\gamma$ can be reinterpreted to yield limits on $K^+\to\pi^+\phi^0$, where the ϕ^0 decays either to $\gamma\gamma$ or e^+e^- inside their lead-glass detector. According to Shinkawa's analysis, in order for the ϕ^0 decay to be observed, the decay must take place after 5 cm and before 2 radiation lengths short of the 37-cm lead-glass average size, i.e., before 32 cm. Including also the probability that the π^+ be in a background-free range region, he finds 90%-confidence-level upper limits on $B(K^+\to\pi^+\phi^0)$ of 15, 4.5, 2.5, 5.1, and 5.9×10^{-6} at m_{ϕ^0} values of 10, 20, 50, 75, and 80 MeV (Ref. 44). Comparing to Eq. (3.9) we conclude that m_{ϕ^0} values between about 25 and 40 MeV would be ruled out on the basis of not having seen the visible decays of the ϕ^0. The range of m_{ϕ^0} values excluded in this way would broaden to include the range from 20 to 80 MeV if there is any significant real part to the charged K decay amplitude. Of course, if $\epsilon'/\epsilon\simeq0$ and we adopt the lower bound of Eq. (3.12), the m_{ϕ^0} regions excluded via the $\pi^+\gamma\gamma$ and π^+ + nothing analysis would be slightly smaller than those quoted above on the basis of Eq. (3.9).

For $2m_\mu<m_{\phi^0}<2m_\pi$, the ϕ^0 decays almost entirely to $\mu^+\mu^-$, and we believe that the limits $L1$ and $L4$ definitively rule out this region of m_{ϕ^0} values. For the case of $m_{\phi^0}>2m_\pi$, the Higgs-boson-mass limits will depend crucially on the relative sizes of $B(\phi^0\to\mu^+\mu^-)$ and $B(\phi^0\to\pi\pi)$. There is uncertainty in the theoretical prediction for $B(\phi^0\to\pi\pi)$ due to the possibility of strong-interaction corrections coming from final-state $\pi\pi$ interactions.[45] These tend to enhance $B(\phi^0\to\pi\pi)$ [at the expense of $B(\phi^0\to\mu^+\mu^-)$], particularly near the $f_0(975)$ resonance. There is some disagreement in the literature as to the size of the $\pi\pi$ decay mode enhancement as a function of $M_{\pi\pi}$ (see Refs. 45 and 46). Employing the benchmark branching ratio of Eq. (3.11), we require $B_{\phi^0}\times B(\phi^0\to\mu^+\mu^-)\gtrsim0.15$ in order that the prediction for $B(K_L^0\to\pi^0\phi^0)$ be clearly larger than the limit $L1$ for the $\pi^0\mu^+\mu^-$ final state. Since the kinematical factor B_{ϕ^0} is significantly smaller than 1 ($B_{\phi^0}=0.53$ for $m_{\phi^0}\sim2m_\pi$), $B(\phi^0\to\mu^+\mu^-)$ is only marginally large enough even in the absence of $\pi\pi$ decay-mode enhancements. Thus, any amount of enhancement in the $\phi^0\to\pi\pi$ decay width (whether the moderate amount found in Ref. 46 or the very large enhancement found in Ref. 45) would make it impossible to eliminate any $m_{\phi^0}>2m_\pi$ on the basis of the limit $L1$. However, note that if $B(K_L^0\to\pi^0\phi^0)>10^{-4}$ (which may be possible according to Fig. 2), then $B_{\phi^0}\times B(\phi^0\to\mu^+\mu^-)$ values as small 0.015 would still allow exclusion of a ϕ^0 using the limit $L1$. Severe suppression from B_{ϕ^0} only sets in very near threshold; for $m_{\phi^0}<340$ MeV we find $B_{\phi^0}>0.2$ and $B_{\phi^0}>0.5$ by

$m_{\phi^0}=2m_\pi$. Then, for $B(K_L^0\to\pi^0\phi^0)>10^{-4}$, a value of $B(\phi^0\to\mu^+\mu^-)>0.075$ at $m_{\phi^0}=340$ MeV would be sufficient to violate the bound $L1$. Such a branching ratio is consistent with the moderate $\pi\pi$ enhancements predicted in Ref. 46, but not with the larger enhancements predicted in Ref. 45.

In contrast, the $K^+\to\pi^+\mu^+\mu^-$ limit $L4$ cannot be used to exclude a Higgs boson with $m_{\phi^0}>2m_\pi$. Combining limit $L4$ with the lower bound of Eq. (3.9), we find that a ϕ^0 is excluded only if $B(\phi^0\to\mu^+\mu^-)B_{\phi^0}\gtrsim 0.5$, which is inconsistent with the values of B_{ϕ^0} quoted above. However, in parallel with the discussion of the preceding paragraph, the lower bound of Eq. (3.9) on $B(K^+\to\pi^+\phi^0)$ would be a substantial underestimate away from zeros of the real part of the amplitude. In this case, some restrictions on allowed m_{ϕ^0} values from limit $L4$ might be possible, depending upon the precise amount of $\pi\pi$ enhancement.

We now turn to the dedicated Higgs-boson search analyses that have been performed for K decays. We shall see that these eliminate all the gaps left by the restrictions obtained in the analysis of limits $L1$ through $L6$. Let us focus first on the dedicated searches for a Higgs boson that have been performed for $K^+\to\pi^+\phi^0$ in Refs. 13, 14, and 41. The first experiment places a limit on a light Higgs boson decaying to e^+e^- pairs which is dependent on its lifetime (see Fig. 3). To calibrate this figure we note that a Higgs boson of mass 1.8, 10, 50, 70, 100, and 200 MeV has a lifetime of approximately 4×10^{-9}, 4.3×10^{-10}, 8.5×10^{-11}, 6×10^{-11}, 4.3×10^{-11}, and 2×10^{-11} sec, respectively. From the available curves we estimate that the Baker *et al.* limit falls below the prediction of Eq. (3.9) at 60 MeV. The existing analysis, however, only applies for $m_{\phi^0}<100$ MeV (Ref. 47). Furthermore, the most recent data of Ref. 47 gives a limit

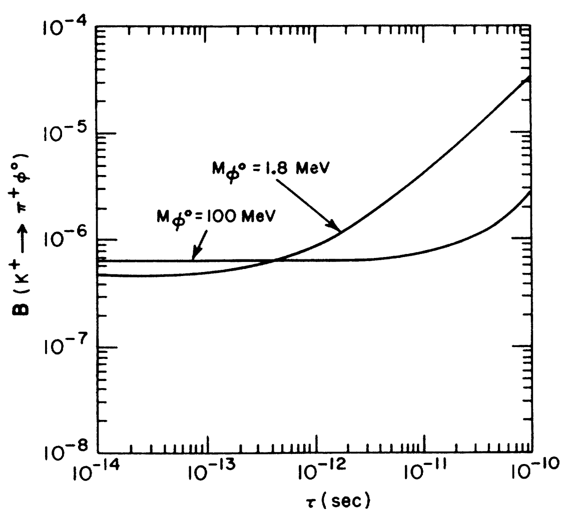

FIG. 3. The experimental limit on the kaon decay width to a light Higgs boson as a function of its lifetime from the mode $K^+\to\pi^+\phi^0,\phi^0\to e^+e^-$ from Ref. 13.

$$B(K^+\to\pi^+\phi^0)\times B(\phi^0\to e^+e^-)<10^{-8}\,, \tag{3.14}$$

for all $M(e^+e^-)$ above 120 MeV. Thus, these experiments[13,47] rule out Higgs-boson masses in the range $60<m_{\phi^0}<100$ MeV and 120 MeV $<m_{\phi^0}<2m_\mu$, respectively.

The second direct search K^+-decay experiment relies on measuring the decay spectrum of the π^+ and does not rely on detecting the ϕ^0 decay products. It is thus potentially applicable for any m_{ϕ^0} kinematically accessible in K^+ decays. From the results of Ref. 14 (see their Fig. 2) one sees that there is no sensitivity to the mass range $110<m_{\phi^0}<150$ MeV, due to the large rate for $K^+\to\pi^+\pi^0$. Above 150 MeV their limit is above 10^{-5}, which is too high to be useful for the theoretical lower bound branching ratio of Eq. (3.9), but below 100 MeV their limit is below 4×10^{-6} and $m_{\phi^0}<100$ MeV is ruled out (down to their lowest plotted point of about 10 MeV).

The third K^+-decay result is that quoted in Ref. 15 for Brookhaven experiment BNL-787. They obtain

$$B(K^+\to\pi^+\phi^0)\times B(\phi^0\to\mu^+\mu^-)<1.5\times10^{-7}\,, \tag{3.15}$$

for $2m_\mu<m_{\phi^0}<320$ MeV. Combining the lower bound on $B(K^+\to\pi^+\phi^0)$ of Eq. (3.9) with $B_{\phi^0}\sim0.34$ at $m_{\phi^0}=320$ MeV, we see that $B(\phi^0\to\mu^+\mu^-)\gtrsim0.1$ is required in order to exclude a ϕ^0 of this mass. Since m_{ϕ^0} values below 320 MeV yield even larger values for B_{ϕ^0} than obtained at $m_{\phi^0}=320$ MeV, we conclude that the BNL-787 results exclude m_{ϕ^0} values throughout the range quoted above, provided that $B(\phi^0\to\mu^+\mu^-)$ is not unduly suppressed. Even if $\epsilon'/\epsilon\simeq0$ and we employ Eq. (3.12), the required $B(\phi^0\to\mu^+\mu^-)>0.17$ is still consistent with the moderate $\pi\pi$ mode enhancement of Ref. 46.

The final direct Higgs-boson search in K decays is that performed by the CERN NA-31 experiment on K_L^0 decays. They have obtained[48] a 90%-confidence-level upper limit of

$$B(K_L^0\to\pi^0\phi^0)\times B(\phi^0\to e^+e^-)<2\times10^{-8} \tag{3.16}$$

throughout the range $50\lesssim m_{\phi^0}\lesssim350$ MeV, except for the region of m_{ϕ^0} within 15 MeV on either side of m_π where their limit for the above B product is $<4\times10^{-8}$. Using the results of Eq. (3.11) and Fig. 2, we see that this experiment conclusively eliminates all m_{ϕ^0} values in the range 50 MeV $<m_{\phi^0}<2m_\mu$, where $B(\phi^0\to e^+e^-)$ is near 1.

In summary, even if we adopt the conservative branching ratios of Eqs. (3.9) and (3.11), we are able to conclude from current experimental information that kaon decays do not allow a standard-model Higgs boson in the mass region $m_{\phi^0}\lesssim2m_\mu$, with the strongest information coming from the limits of Refs. 14, 40, and 48. In addition the region $2m_\mu<m_{\phi^0}<320$ MeV is excluded on the basis of the results of Ref. 15 (with additional support in some mass regions from a variety of other experiments) unless the $\pi\pi$ enhancement is as large as predicted in Ref. 45. We also note that further analysis of the data from many of the existing experiments on rare K decays may yield additional restrictions on m_{ϕ^0}. We conclude by again cautioning that most of the restrictions on m_{ϕ^0} considered above are subject to potentially significant theoretical uncertainties. In addition, if we allow for the possibility of a fourth generation, the CKM matrix elements could be substantially altered, thereby invalidating some of the more marginal constraints. Certainly, the improved limits from rare K decay experiments now in progress should further reduce the impact of the theoretical uncertainties.

IV. LIMITS FROM b PHYSICS

A. Exclusive and inclusive B decays

Here we consider both inclusive decays $B\to\phi^0X$ and exclusive decay channels such as $B\to\phi^0K$. Let us focus first on the former. An important advantage of B decays over K decays is that, in the B meson case, the two-quark operator term is completely dominant because of the much larger CKM matrix elements occurring in this term ($|V_{tb}|\gg|V_{td}|$). In addition, for B meson decays the quark spectator approach is expected to be valid; in particular, uncertainties from nonperturbative terms such as those discussed in connection with the chiral Lagrangian approach to K decays should be quite small. We can obtain the $b\to\phi^0s$ branching ratio directly from a formula like that of Eq. (3.2), with appropriate substitutions. The most reliable way of predicting the rate for the decay $B\to\phi^0X$ is then to compute the ratio: $\Gamma(B\to\phi^0X)/\Gamma(B\to e\nu X)$. This is because hadronic operators which occur in the quark-level transitions $b\to s\phi^0$ and $b\to c\nu s$ can be related. Thus uncertainties in the matrix element due to strong-interaction effects will be minimized in the ratio. A convenient form for the relative widths is

$$\frac{\Gamma(B\to\phi^0X)}{\Gamma(B\to e\nu X)}=2.95\left[\frac{m_t}{m_W}\right]^4\left[1-\frac{m_{\phi^0}^2}{m_b^2}\right]^2\left|\frac{V_{st}V_{tb}}{V_{cb}}\right|^2, \quad (4.1)$$

where we have taken $m_b=4.5$ GeV. If we now employ the experimental branching ratio for $B\to e\nu X$ of 0.123 (Ref. 38), we find

$$B(B\to\phi^0X)=0.36\left[\frac{m_t}{m_W}\right]^4\left[1-\frac{m_{\phi^0}^2}{m_b^2}\right]^2\left|\frac{V_{st}V_{tb}}{V_{cb}}\right|^2. \quad (4.2)$$

If there are only three families, unitarity of the CKM matrix implies that $|V_{tb}|\simeq1$ and $|V_{ts}|\simeq|V_{cb}|$. In addition, we remind the reader that in order to have a very light Higgs boson in the standard model we must require $m_t\sim m_W$ (Ref. 27). Since the ϕ^0 will, in part, decay to lepton pairs, it is useful to now consider the implications of available experimental limits on the branching ratio for $B\to l^+l^-X$, with $l=e$ or μ.

For $m_{\phi^0}<2m_\mu$, the ϕ^0 decays to e^+e^- and $\gamma\gamma$, with the branching ratio for the e^+e^- mode being nearly 1. The Mark II Collaboration at SLAC PEP has looked for the decay $B\to K\phi^0$, with the Higgs boson decaying to e^+e^-. They exclude the region 50 MeV $<m_{\phi^0}<2m_\mu$ (Ref. 49).

Turning next to the region $2m_\mu<m_{\phi^0}<2m_\pi$, we first note that the decay $\phi^0\to\mu^+\mu^-$ has a branching ratio near 1. The most directly relevant experimental information derives from the TASSO Collaboration[50] who obtain a limit of

$$B(B\to\mu^+\mu^-X)<0.02\ . \quad (4.3)$$

This limit assumes that the muon momentum is greater than 1.2 GeV. Since the muons coming from the ϕ^0 decay would be quite energetic, the probability that they would pass the momentum cut is high. Combining this with the branching ratio of Eq. (4.2) and $B(\phi^0\to\mu^+\mu^-)\sim1$ (in the mass region under discussion), we obtain a clear conflict with the experimental limit of Eq. (4.3).

In the region $m_{\phi^0}>2m_\pi$ we are again faced with the various uncertainties regarding the relative branching ratio for $\phi^0\to\pi\pi$ compared to that for $\phi^0\to\mu^+\mu^-$. The major uncertainty is due to the possibility of large final-state interactions which enhance the $\pi\pi$ over the $\mu^+\mu^-$ final state. As an example, if we neglect final-state interactions, we find $B(\phi^0\to\mu^+\mu^-)\sim0.4$–$0.2$ for 300 MeV $\lesssim m_{\phi^0}<2m_K$. However, including $\pi\pi$ enhancements can reduce the value of $B(\phi^0\to\mu^+\mu^-)$. For example, Ref. 45 obtains $B(\phi^0\to\mu^+\mu^-)\sim5\times10^{-2}$ at $m_{\phi^0}\sim300$ MeV falling to $\sim10^{-2}$ at $m_{\phi^0}\sim800$ MeV. In contrast, Ref. 46 finds $B(\phi^0\to\mu^+\mu^-)\gtrsim0.25$ in this same range. Above 800 MeV, the $\phi^0\to\mu^+\mu^-$ branching ratio precipitously drops in the vicinity of the $f_0(975)$ 0^{++} scalar resonance peak. For $m_{\phi^0}>2m_K$ final states involving kaons become the dominant Higgs-boson decay modes, and presumably there is some resonant enhancement in the 1–2 GeV region. However, for $m_{\phi^0}\gtrsim2$ GeV we expect the spectator model to be a reasonable approximation for Higgs-boson decay, implying a branching-ratio to $\mu^+\mu^-$ that is 1–2×10^{-2} up to $m_{\phi^0}\simeq2m_\tau$. Assuming perfect acceptance, the TASSO branching-ratio limit of Eq. (4.3) combined with Eq. (4.2) implies that one can only rule out a Higgs boson with $B(\phi^0\to\mu^+\mu^-)\gtrsim5.6\times10^{-2}$. In light of the above discussion, the TASSO limits may rule out the existence of a light Higgs boson with $2m_\pi\lesssim m_{\phi^0}\lesssim800$ MeV, if $B(\phi^0\to\mu^+\mu^-)$ is not too suppressed by $\pi\pi$ decay mode enhancements.

For $m_{\phi^0}>500$ MeV, various limits from the CLEO Collaboration become relevant.[51,52] Combining the best limits from Refs. 51 and 52 we find

$$B(B\to\phi^0X)\times B(\phi^0\to\mu^+\mu^-) < \begin{cases} 8\times10^{-3}, & 0.5\ \text{GeV} < m_{\phi^0} < 1\ \text{GeV}\ , \\ (1-8)\times10^{-4}, & 1\ \text{GeV} < m_{\phi^0} < 3\ \text{GeV}\ , \\ (4-6)\times10^{-5}, & 3.2\ \text{GeV} < m_{\phi^0} < 2m_\tau\ . \end{cases} \tag{4.4}$$

These branching-ratio limits (given at 90% confidence level) have been read off from a set of curves which are quite jagged due to the small number of candidate events over the relevant invariant-mass regions. We note that a clear signal is seen in $B\to\mu^+\mu^-X$ for $m_{\mu^+\mu^-}$ near 3.1 GeV, which is interpreted as being due to $B\to J/\psi+X$. Outside the J/ψ region, the branching-ratio limits can be interpreted as limits for $B\to\phi^0X$. From Eqs. (4.2) and (4.4) we see that in the range between 500 MeV and 1 GeV a Higgs boson can be ruled out if the $\mu^+\mu^-$ branching ratio is larger than 2×10^{-2}. From the discussion of the $\phi^0\to\mu^+\mu^-$ branching ratio given above, we see that some fraction of the lower part of this Higgs-boson-mass range may be ruled out. Clearly, a definitive statement depends sensitively on the magnitude of the $\pi\pi$ mode enhancement. For the mass range 1 GeV $< m_{\phi^0} < 2m_\tau$, the above limits of Eq. (4.4) combined with Eq. (4.2) imply that a Higgs boson is ruled out so long as $B(\phi^0\to\mu^+\mu^-) > 2.5\times10^{-3}$ (we exclude the J/ψ region). We have seen that the $\mu^+\mu^-$ branching ratio is larger than this value throughout this mass region, except (perhaps) near 1 GeV, where $\pi\pi$ resonant effects are very large. (Even in the J/ψ region the data is close to ruling out a standard-model Higgs boson.)

As pointed out in Ref. 18, complementary data on $B(B\to\mu^+\mu^-X)$, applicable over some of this same mass range, is available from the ARGUS Collaboration,[53] as a by-product of their measurement of $B\to\psi X$ at the $\Upsilon(4S)$. Their overall sample contained $\gtrsim 170\,000$ B's. In selecting their e^+e^- and $\mu^+\mu^-$ events, they impose cuts of $p(l^+l^-) < 2.0$ GeV and $p(l) > 0.9$ GeV as well as an acoplanarity cut. The efficiency for observing $B\to\psi(\to l^+l^-)X$ with these cuts is of order 50%. It is not possible to be certain how this efficiency varies with l^+l^- invariant mass, but it is clear that as $M(l^+l^-)$ decreases, $p(l^+l^-)$ will increase on average and the first of the above cuts will eliminate more and more events. Since the B's are produced pretty much at rest, if the spectator system X is treated as massless then once $M(l^+l^-)\lesssim 2.5$ GeV we would find $p(l^+l^-) > 2$ GeV. However, in the observed $B\to\psi X$ decays, the momentum spectrum of the ψ is quite soft, implying that the spectator system X is fairly massive on average. Indeed, their plotted $M(l^+l^-)$ spectra extend all the way down to $M(l^+l^-)=1.5$ GeV. Thus, it may not be unreasonable to assume that their acceptance for $B\to\phi^0(\to\mu^+\mu^-)X$ events remains substantial (say >0.20) over the entire plotted mass range, from 1.5 GeV up to the point where phase-space suppression becomes severe. This upper boundary is also quite sensitive to the average mass of the X system produced in $B\to\phi^0X$ decays, and also cannot be determined with precision. However, it seems apparent from the mass spectra that it probably extends up to the upper limit of interest here, namely, $2m_\tau$. For any $M(l^+l^-)$ value with efficiency greater than 20%, and using Eq. (4.2), we estimate that the 170 000 B's would produce $\gtrsim 10^4\times B(\phi^0\to\mu^+\mu^-)$ events in one of the experimental 50-MeV-wide bins of the $\mu^+\mu^-$ spectrum. Outside the J/ψ region the experimental distributions contain at most 10 to 15 events in each such bin. Since resonant enhancements are unlikely to yield $B(\phi^0\to\mu^+\mu^-) < 10^{-3}$ in the mass region of interest, we conclude that the ARGUS experiment does indeed rule out a significant, but imprecise, range of m_{ϕ^0} masses between about 1.5 GeV and $2m_\tau$.

For $m_{\phi^0} > 2m_\tau, 2m_c$, the dominant decays for the ϕ^0 are $\phi^0\to\tau^+\tau^-, c\bar{c}$, for which there are no reliable limits. In summary, the most important B inclusive decay limits are as follows.

(1) There are no constraints for $2m_e < m_{\phi^0} \lesssim 50$ MeV. The Mark II results exclude 50 MeV $< m_{\phi^0} < 2m_\mu$.

(2) Higgs-boson masses in the range $2m_\mu < m_{\phi^0} < 2m_\pi$ are ruled out by the TASSO limit. This conclusion requires only a 5% efficiency for a Higgs-boson decay event, relative to the $\mu^+\mu^-$ acceptance assumed in their analysis.

(3) From TASSO and CLEO data it may be possible to rule out Higgs-boson masses between 500 MeV and about 900 MeV if the $\phi^0\to\pi\pi$ enhancement does not unduly suppress the $\phi^0\to\mu^+\mu^-$ branching ratio. Higgs-boson masses in the vicinity of the $f_0(975)$ scalar-meson resonance cannot be ruled out due to the strong final-state $\pi\pi$ interaction.

(4) Higgs-boson masses between about 1.2 GeV and 3.0 GeV are ruled out based on experimental results from CLEO and ARGUS.

(5) For $3.0 < m_{\phi^0} < 3.2$ GeV the decay $B\to\psi X$ is an important standard-model background for Higgs detection. Nevertheless, the recent CLEO data is close to ruling out a Higgs boson in this mass range.

(6) Higgs-boson masses between 3.2 GeV and $2m_\tau$ are ruled out by the ARGUS and CLEO data.

(7) There are no constraints for $m_{\phi^0} > 2m_\tau$.

Finally, we remind the reader that all the above conclusions apply only for three generations as implicitly assumed in obtaining Eq. (4.2), and for $m_t \approx m_W$ as required to have a very light standard-model Higgs boson.

Let us now consider the exclusive channel $B\to\phi^0K$, which is quite analogous to $K\to\pi\phi^0$ with the exception that in the B decay case the contribution from the two-quark transition operator should be much larger than that from the four-quark operator. The width for the B decay case[54,55,18] is best computed using the ratio

$$r_K \equiv \frac{B(B\to\phi^0 K)}{B(B\to\phi^0 X)} \tag{4.5}$$

and the result for the inclusive branching ratio of Eq. (4.2). The ratio r_K has been computed in a nonrelativistic approximation in Ref. 55, but, as pointed out in Ref. 18, this approximation cannot really be justified since there is considerable sensitivity to the component of the K-meson wave function with large relative quark momentum. For small m_{ϕ^0} values, Ref. 55 estimates that $r_K \sim 0.08$, rising to roughly $r_K \sim 0.2$ by $m_{\phi^0}\sim 3$ GeV. Reference 18 obtains similar results at small m_{ϕ^0} for one choice of wave function, but also finds that r_K could be as much as a factor of 7 to 10 smaller for reasonable alternative choices for the wave-function parameters. In the following discussion we shall adopt $r_K = 0.01$ as a conservative lower bound.

As we have seen, in the mass range of interest (below $m_B - m_K$), the ϕ^0 decays primarily to $\mu^+\mu^-$, $\pi\pi$, and to KK and multimeson final states for larger m_{ϕ^0}. Published experimental upper limits are available from CLEO:[56] $B(B^0\to K^0\mu^+\mu^-) < 4.5\times 10^{-4}$ and $B(B^+\to K^+\mu^+\mu^-) < 3.2\times 10^{-4}$. Given the uncertainties due to experimental acceptance, and conservative choices for r_K and $B(\phi^0\to\mu^+\mu^-)$, these experimental limits are not strong enough to rule out any range of m_{ϕ^0}.

However, more stringent limits from CLEO have recently appeared.[52] For charged B decays (summed over charge-conjugate final states) they find

(i) $B(B\to\mu^+\mu^- K) \lesssim 4-6\times 10^{-5}$ for $2m_\mu < M_{\mu^+\mu^-} < 3.6$ GeV ;

(ii) $B(B\to\pi^+\pi^- K) \lesssim 2.5-20\times 10^{-5}$ for $2m_\pi < M_{\pi^+\pi^-} < 3.6$ GeV ;

and

(iii) $B(B\to K^+K^- K) \lesssim 2.5-20\times 10^{-5}$ for $2m_K < M_{K^+K^-} < 3.6$ GeV .

In specific mass regions, the limits are often near the minimum of the stated ranges. However, in numerous mass bins, the limits peak sharply at values near the maximum limits indicate above. These mass bins correspond to bins where candidate events have been observed. These results can be converted to limits on $B(B\to\phi^0 K)$ as follows. For $2m_\mu < m_{\phi^0} < 1$ GeV we know that $B(\phi^0\to\mu^+\mu^-)+B(\phi^0\to\pi^+\pi^-)+B(\phi^0\to\pi^0\pi^0)=1$; using (i) and (ii) (and isospin conservation) gives

$$B(B^-\to\phi^0 K^-) \lesssim 10^{-4} . \tag{4.6}$$

Using Eq. (4.2) and the conservative $r_K = 0.01$ choice yields a result which is about 36 times larger, and ϕ^0 masses in the above range would be ruled out. For m_{ϕ^0} above $2m_K$ Ref. 52 uses the experimentally measured ratio of $\sigma(e^+e^-\to K^+K^-)/\sigma(e^+e^-\to \text{all})$ to estimate $B(\phi^0\to K^+K^-)$ in order to convert their limit (iii) above to a limit on $B\to\phi^0 K$. Using these estimates, and assuming that $r_K \gtrsim 0.01$, we find that the above limit translates to $B(B\to\phi^0 X) \lesssim 0.06$ for $2m_K < m_{\phi^0} < 2$ GeV, $\lesssim 0.3$ for 2 GeV $< m_{\phi^0} < 2.5$ GeV—thereby eliminating $2m_K < m_{\phi^0} < 2.5$ GeV. The less conservative exclusive-to-inclusive ratio assumed in Ref. 52, based on Refs. 55 and 54, allows exclusion of Higgs-boson masses in the full range $2m_K < m_{\phi^0} < 2m_\tau$. A slightly different approach is to use Eq. (4.2) to convert the exclusive limits (i)–(iii) into

$$\begin{aligned} &B(\phi^0\to\mu^+\mu^-) < (1-3)\times 10^{-4}/r_K ,\\ &B(\phi^0\to\pi^+\pi^-) < (1-6)\times 10^{-4}/r_K ,\\ &B(\phi^0\to K^+K^-) < (1-6)\times 10^{-4}/r_K , \end{aligned} \tag{4.7}$$

in the relevant mass ranges. Clearly multibody final states would have to be the dominant component of ϕ^0 decays in order that *all* of these two-body modes have branching ratio below the $\sim 1-6\%$ upper limit obtained for $r_K = 0.01$. These numbers can be compared to the estimates from Ref. 52 for the K^+K^- component of ϕ^0 decays which, for example, falls below 3% only for $m_{\phi^0} \gtrsim 2$ GeV. As already noted, a close examination of the limits on the branching ratios of Eq. (4.7) as a function of KK mass, in combination with $r_K \gtrsim 0.01$ and the estimates of Ref. 52 for the two-body K^+K^- fraction, allows one to conclude that $2m_K \lesssim m_{\phi^0} \lesssim 2.5$ GeV is excluded.

Let us now summarize the existing limits on the ϕ^0 from rare B decays. Altogether we see that B decays come close to ruling out m_{ϕ^0} in the entire range 50 MeV $< m_{\phi^0} < 2m_\tau$. Mark II data for $B\to Ke^+e^-$ rules out 50 MeV $< m_{\phi^0} < 2m_\mu$. Data for $B\to\mu^+\mu^- X$ can reliably rule out $2m_\mu < m_{\phi^0} < 2m_\pi$ and 1.5 GeV $\lesssim m_{\phi^0} < 2m_\tau$, excluding a small window around $m_{\phi^0}\approx m_\psi$. Exclusive charmless B decays, discussed above, can rule out $2m_\mu < m_{\phi^0} \lesssim 2.5$ GeV, if the model-dependent parameter r_K is larger than 0.01. For $m_{\phi^0} > 2m_\tau$ we must turn to other techniques.

B. Higgs bosons in upsilon decays

As first suggested by Wilczek,[57] the Higgs boson can be searched for in the radiative decays of heavy vector mesons, V, such as the ψ and Υ. The QCD radiative corrections to the rate for $V\to\phi^0\gamma$ have been calculated in Refs. 58 and 59, and are large: for $m_{\phi^0} \ll M$, the radiative corrections result in an 84% decrease in the absolute rate. The large size of these corrections indicates that higher orders in perturbation theory may be significant.

We can also determine the effects of one-loop QCD corrections upon the ratio $R \equiv \Gamma(V\to\phi^0\gamma)/\Gamma(V\to\mu^+\mu^-)$ by including corrections to the denominator. One finds[60]

$$\Gamma(V\to\mu^+\mu^-) = \Gamma_0(V\to\mu^+\mu^-)\left[1-\frac{4\alpha_s C_F}{\pi}\right] , \tag{4.8}$$

which is a 34% reduction for $\alpha_s = 0.2$. Therefore, the one-loop QCD corrections to the ratio R will be somewhat less than the large radiative corrections to the numerator alone.

In addition to the QCD radiative corrections, there may also be large bound-state and relativistic corrections. These are reviewed in some detail in Ref. 2, and we only provide a brief summary here. First, one must consider the bound-state corrections. These were originally examined in Ref. 61, where it was stated that the computations are reliable for Higgs-boson masses in the range 7.5 GeV $\lesssim m_{\phi^0} \lesssim M_\Upsilon$. The result is to reduce R_Υ in the above m_{ϕ^0} range by a factor which is always larger than 2. Second, there are relativistic corrections to the decay rate for $\Upsilon \to \phi^0\gamma$. A formalism for computing relativistic corrections to quarkonium decay has been developed by Aznauryan *et al.*[62] These authors state that their results are reliable for small m_{ϕ^0} and find, for $m_{\phi^0} \lesssim 4$ GeV, that R_Υ is reduced by a factor of 2.

It is not obvious how to combine the QCD and the relativistic corrections since they are both of $O(v^2/c^2)$. However, the QCD radiative corrections reflect hard-gluon effects, whereas the relativistic corrections are due to soft-gluon effects. This suggests that it is correct to include the two corrections independently. In particular, the authors of Ref. 62 specifically state that their calculation omits the diagrams associated with the hard QCD corrections computed in Refs. 58 and 59. Thus, it seems that one must include the suppression from relativistic effects on top of the QCD-corrected prediction. Clearly, in light of these remarks, additional theoretical work is needed before a firm limit on the Higgs-boson mass can be extracted from the decay $\Upsilon \to \phi^0\gamma$. In particular, the relativistic corrections of Aznauryan *et al.*[62] need to be confirmed by an independent calculation.

CUSB has performed a search at the Cornell Electron Storage Ring for Higgs bosons in $\sim 8\times 10^5 \Upsilon(1S)$ and $\sim 6\times 10^5 \Upsilon(3S)$ radiative decays.[63] When they include QCD radiative corrections (but not relativistic corrections) in the prediction, they obtain a limit of $m_{\phi^0} > 5.4$ GeV at the 90% confidence level and $m_{\phi^0} > 4.8$ GeV at the 95% level. They currently claim sensitivity only for $m_{\phi^0} \gtrsim 600$ MeV.[64] If the relativistic corrections are also included, it seems that no limit on m_{ϕ^0} can be found from the CUSB data.

Preliminary results from a second experiment on radiative Υ decays have also appeared.[65] This experiment places limits on $\Upsilon \to \gamma X$, where $X \to \pi^+\pi^-$, K^+K^-, or $p\bar{p}$. The only useful limit is for the $\gamma\pi^+\pi^-$ final state, for which they obtain $B(\Upsilon \to \gamma\pi^+\pi^-) < (3\text{–}4.5)\times 10^{-5}$ for 270 MeV $< M(\pi^+\pi^-) \lesssim 670$ MeV and $< (2.5\text{–}3)\times 10^{-5}$ for 1.1 GeV $< M(\pi^+\pi^-) < 3.5$ GeV. Using the $B(\phi^0 \to \pi^+\pi^-)$ found in the absence of $\pi\pi$ enhancements and $B(\Upsilon \to \gamma\phi^0)$ as computed by including QCD corrections but *not* relativistic corrections, $B(\Upsilon \to \gamma\phi^0)\times B(\phi^0 \to \pi^+\pi^-)$ exceeds the limits quoted above in the range 290 MeV $< m_{\phi^0} < 570$ MeV; large enhancements for the $\phi^0 \to \pi\pi$ mode would allow one to exclude an even larger range -270 MeV $< m_{\phi^0} < 670$ MeV. But, if relativistic corrections are included and $\pi\pi$ enhancements are not large, no m_{ϕ^0} values can be excluded using the above $\gamma\pi^+\pi^-$ final-state branching-ratio limit. Above $m_{\phi^0} = 2m_K$, the $\pi\pi$ modes will be rapidly overshadowed by the $K\bar{K}$ modes. The experimental limits from Ref. 65 are somewhat weaker for these modes, and precise estimates for ϕ^0 decays to $K\bar{K}$ in the region $1 < M(K^+K^-) < 3.5$ GeV (where the experimental data exists) are not currently available. Thus, restrictions on m_{ϕ^0} from the data of Ref. 65 in the region $m_{\phi^0} > 1$ GeV are not currently possible. In the region from ~ 670 MeV to ~ 1 GeV there is a large-ρ resonance contribution to the $\pi\pi$ final state that yields branching ratios far above those expected from ϕ^0 contributions.

To summarize, limits obtained from Υ decays are sensitive to theoretical uncertainties, including those associated with higher-order QCD corrections and relativistic corrections. With our present understanding of the nature of these corrections, no limit can be ascertained from current data.

V. CONCLUSIONS

From our analysis of very light Higgs bosons, we conclude that $m_{\phi^0} < 11.5$ MeV is ruled out by nuclear-physics experiments. For larger Higgs-boson masses we are potentially sensitive to a variety of theoretical uncertainties. Those we have focused on are (a) the uncertainty in $B(K \to \phi^0\pi)$ due to imprecise knowledge of the non-spectator contributions; (b) the uncertainty in $B(\phi^0 \to \pi\pi)$ [relevant for determining $B(\phi^0 \to \mu\mu)$ for $m_{\phi^0} > 2m_\pi$] coming from the possibility of resonant enhancements in this channel; (c) the uncertainty in the wave-function-dependent parameter r_K [see Eq. (4.5)] specifying the rate for the exclusive $B \to K\phi^0$ decays; and (d) the uncertainties in $\Upsilon \to \gamma\phi^0$ from higher-order QCD corrections and relativistic wave-function corrections. Below we summarize the Higgs-boson-mass limits examined in this paper. In dealing with the uncertainties described above, we shall proceed as follows. We assume that the imprecisely known parameters (such as $m_t^2 V_{td}^* V_{ts}$ and the A parameter) are not fine-tuned in such a way as to make the $K \to \pi\phi^0$ amplitude artificially small. We adopt a conservatively small choice for r_K ($r_K = 0.01$). Sensitivity to the size of $B(\phi^0 \to \mu^+\mu^-)$ is indicated; the limits given below are all valid if the $\pi\pi$ decay mode enhancement is of the moderate magnitude found in Ref. 46. We then have the following results.

(1) 10 MeV $< m_{\phi^0} \lesssim 100$ MeV is ruled out by SINDRUM data on $\pi^+ \to e^+\nu e^+e^-$.

(2) 50 MeV $< m_{\phi^0} \lesssim 2m_\mu$ is excluded by NA-31 results on $K_L^0 \to \pi^0 e^+e^-$, with little sensitivity to theoretical uncertainties.

(3) $m_{\phi^0} \lesssim 55$ MeV is excluded by $K^+ \to \pi^+ +$nothing and $K^+ \to \pi^+ + \gamma\gamma$ limits from Refs. 40 and 39, respectively.

(4) ~ 60 MeV $< m_{\phi^0} \lesssim 100$ MeV is excluded by the $K^+ \to \pi^+e^+e^-$ decay experiment of Ref. 13.

(5) $2m_\mu < m_{\phi^0} < 320$ MeV is excluded by measurements of $K \to \pi\mu^+\mu^-$, in particular by the analysis of BNL-787

in Ref. 15, unless $B(\phi^0 \to \mu^+\mu^-) \lesssim 0.1$ due to a large enhancement of the $\phi^0 \to \pi\pi$ decay mode.

(6) 10 MeV $< m_{\phi^0} \lesssim 100$ MeV is excluded by limits on $K^+ \to \pi^+ + X$.

(7) 50 MeV $< m_{\phi^0} < 2m_\mu$ is excluded by the Mark II data on $B \to Ke^+e^-$.

(8) $2m_\mu < m_{\phi^0} < 2m_\pi$ is ruled out by TASSO data on $B \to \mu^+\mu^- X$.

(9) $2m_\pi < m_{\phi^0} < 800$ MeV is ruled out by TASSO data if $B(\phi^0 \to \mu^+\mu^-) \gtrsim 0.1$, assuming that the acceptance for Higgs-boson decay events is greater than 60%.

(10) 500 MeV $\lesssim m_{\phi^0} \lesssim 900$ MeV is ruled out by CLEO data on $B \to \mu^+\mu^- X$ unless $B(\phi^0 \to \mu^+\mu^-)$ is more strongly suppressed than a factor of 10.

(11) 1.2 GeV $\lesssim m_{\phi^0} \lesssim 3$ GeV and 3.2 GeV $< m_{\phi^0} < 2m_\tau$ are ruled out by the same CLEO data.

(12) 1.5 GeV $< m_{\phi^0} < 2m_\tau$, with a hole near $m_{\phi^0} \sim m_\psi$, is ruled out by ARGUS data as a by-product of their $B \to \psi X$ measurement, for any m_{ϕ^0} such that $B(\phi^0 \to \mu^+\mu^-) > 10^{-3}$.

(13) $2m_\mu < m_{\phi^0} < 2.5$ GeV is ruled out by $B \to K\mu^+\mu^-, \pi^+\pi^-, K^+K^-$ CLEO results.

(14) No limit from $\Upsilon \to \phi^0\gamma$ can be deduced at this time, due to substantial theoretical uncertainties in the QCD and relativistic corrections to the nonrelativistic prediction.

Modifications to the above statements for different theoretical assumptions have been given in the preceding sections. However, our general conclusion is that there is no mass region below about $2m_\tau$ where a light standard-model Higgs boson might still be allowed. This conclusion should be progressively sharpened as the Brookhaven experiments on rare K decays continue to accumulate data, provided they analyze their data with a possible Higgs boson signal and its possible backgrounds in mind. Also, analysis of the E-731 experimental results[33] as a direct Higgs-boson search may provide results complementary to those available from NA-31. Extension of limits on m_{ϕ^0} to the mass region above $2m_\tau$ may possibly come from Υ decays, but substantial theoretical uncertainties prohibit such an extension at this time.

Note added in proof. After this paper was accepted for publication, the first round of experiments at LEP was completed. Soon afterwards, the first Higgs-boson limits were deduced from analysis of Z decays. Assuming a minimal Higgs structure in the context of the standard model, the ALEPH Collaboration has ruled out 32 MeV $\le m_{\phi^0} \le 24$ GeV [D. Decamp *et al.*, Reports Nos. CERN-EP/89-157, 1989 (unpublished); CERN-EP/90-16, 1990 (unpublished)]. [Limits have also been obtained by the OPAL Collaboration in M. Z. Akrawy *et al.*, Report No. CERN-EP/89-174, 1989 (unpublished).] Since the limits obtained in this paper extend from above 32 MeV down to a zero Higgs-boson mass, one can now conclude that a standard-model Higgs boson must have a mass larger than 24 GeV. It would then seem that further searches for a Higgs boson in rare K, B, and Υ decays would be of limited interest. Although this may be so for the minimal Higgs boson of the standard model, it is definitely not the case in more general models with a nonminimal Higgs sector. Specifically, whereas the standard-model Higgs-boson searches in Z decays depend on the existence of a $\phi^0 ZZ$ vertex of strength $gm_Z/\cos\theta_W$, the Higgs-boson searches described in this paper are viable for any Higgs boson with significant coupling to quark pairs. (The Higgs-boson searches in K and B decays depend largely on the $\phi^0 t\bar{t}$ coupling.) In the standard model with a minimal Higgs sector, the $\phi^0 ZZ$ and $\phi^0 q\bar{q}$ couplings are related in a precise way. However, in all nonminimal Higgs models, these two couplings are independent. Moreover, the Higgs coupling to ZZ *must* be suppressed relative to its value in the minimal model, whereas its coupling to quarks can be either enhanced or suppressed. Thus, in nonminimal models, the failure to detect a Higgs boson at LEP does not necessarily rule out the possibility of discovering a light Higgs boson in B or K decay. Since the nonminimal Higgs couplings differ from those of the minimal model, the nature of the Higgs-boson-mass limits which result from the experiments described in this paper are clearly model dependent. Improving the experimental limits can only help to expand the sensitivity to the parameter spaces of more general models.

ACKNOWLEDGMENTS

We appreciate fruitful discussions with P. Nason, S. Raby, L. Randall, S. Rindani, G. West, and R. Willey. We would like to gratefully acknowledge the following experimentalists who assisted us in ascertaining limits on light Higgs bosons: J. Lee-Franzini, Paulo Franzini, L. Littenberg, P. Meyers, H. Nelson, T. Shinkawa, B. Winstein, and M. Zeller. This manuscript has been authored under Contract Nos. DE-AC02-76CH00016, DE-AT03-89ER40492, and DE-AM03-76SF00010 with the U.S. Department of Energy.

---

[1] S. Raby, G. West, and C. M. Hoffman, Phys. Rev. D **39**, 828 (1989).

[2] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Redwood City, CA, 1990).

[3] Some additional experiments are discussed in Ref. 2.

[4] I. Beltrami *et al.*, Nucl. Phys. **A451**, 679 (1986).

[5] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Lett. **78B**, 443 (1978).

[6] D. Kohler, B. Watson, and J. Becker, Phys. Rev. Lett. **33**, 1628 (1974).

[7] S. J. Freedman, J. Napolitano, J. Camp, and M. Kroupa, Phys. Rev. Lett. **52**, 240 (1984).

[8] See, for example, H. Georgi, *Weak Interactions and Modern Particle Theory* (Benjamin-Cummings, New York, 1984).

[9] R. Chivukula and A. Manohar, Phys. Lett. B **207**, 86 (1988); **217**, 568(E) (1989).

[10] S. Dawson, Phys. Lett. B **222**, 143 (1989).

[11]S. Egli *et al.*, Phys. Lett. B **222**, 533 (1989).
[12]In Ref. 11 they do not include the $(1-2N_H/3b)^2$ factor of Eq. (3.1) in comparing theory to experiment; this prevents one from excluding a Higgs boson in the region above 100 MeV where the theoretical prediction is of the same order as the experimental upper limit.
[13]N. Baker *et al.*, Phys. Rev. Lett. **59**, 2832 (1987).
[14]T. Yamazaki *et al.*, Phys. Rev. Lett. **52**, 1089 (1984).
[15]M. S. Atiya *et al.*, Phys. Rev. Lett. **63**, 2177 (1989).
[16]R. Willey and H. Yu, Phys. Rev. D **26**, 3287 (1982); B. Grzadkowski and P. Krawczyk, Z. Phys. C **18**, 43 (1984); F. Botella and C. Lim, Phys. Rev. Lett. **56**, 1651 (1986); Phys. Rev. D **34**, 301 (1986); R. Godbole, U. Turke, and M. Wirbel, Phys. Lett. B **194**, 302 (1987).
[17]R. Willey, Phys. Lett. **173B**, 480 (1986).
[18]B. Grinstein, L. Hall, and L. Randall, Phys. Lett. B **211**, 363 (1988).
[19]A. Cohen and A. V. Manohar, Phys. Lett. **143B**, 481 (1984).
[20]S. Dawson and H. E. Haber, in *Phenomenology of the Standard Model and Beyond,* proceedings of the Workshop on High Energy Physics Phenomenology, TIFR, Bombay, India, 1989, edited by D. P. Roy and Probir Roy (World Scientific, Singapore, 1989), p. 324.
[21]In principle, A and λ are complex. However, $\mathrm{Im}\lambda \ll \mathrm{Re}\lambda$ and $\mathrm{Im}A \ll \mathrm{Re}A$, so we can neglect these imaginary parts in the subsequent analysis.
[22]H. Y. Cheng and H. L. Yu, Phys. Rev. D **40**, 2980 (1989).
[23]L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1984).
[24]G. Altarelli and P. Franzini, Z. Phys. C **37**, 271 (1988).
[25]J. R. Cudell, F. Halzen, and S. Pakvasa, Phys. Rev. D **40**, 1562 (1989).
[26]A. Linde, Phys. Lett. **63B**, 435 (1976); Pis'ma Zh. Eksp. Teor. Fiz. **19**, 564 (1974) [JETP Lett. **19**, 296 (1974)]; S. Weinberg, Phys. Rev. Lett. **36**, 296 (1976).
[27]Requirements of vacuum stability can be used to derive various limits on the Higgs-boson mass. In particular, Linde and Weinberg (Ref. 26) derived a lower bound on the Higgs-boson mass (denoted by m_{LW}) by requiring that the asymmetric vacuum be the state of lowest energy, i.e., $V(v)-V(0)$ must be less than zero, where V is the one-loop effective potential, and v is the vacuum expectation value of the Higgs field. For $(m_t/m_W)^4 \ll 1$, this bound is about $m_{LW} \simeq 7$ GeV. However, the bound decreases as m_t is raised, with m_{LW} vanishing at $m_t \approx m_W$. For $m_t \gtrsim m_W$, this bound no longer applies. Instead, a different lower bound on m_{ϕ^0} emerges from renormalization-group analyses. It implies that if m_t is significantly larger than m_W then, again, it is not possible to have a light Higgs boson in the minimal version of the standard model.
[28]J. R. Patterson *et al.*, Phys. Rev. Lett. **64**, 1491 (1990). We also thank H. Nelson for helpful conversations on this topic.
[29]$\mathcal{M}(K_L^0 \to \pi^0\phi^0)$ is purely real in the limit where $\epsilon \ll \arg V_{td}^* V_{ts}$. Since $\epsilon \simeq 2\times 10^{-3}$, the numbers quoted above suggest that this is a good approximation. For instance, $\mathrm{Im}\mathcal{M}(K_L^0 \to \pi^0\phi^0) = \pm\mathrm{Re}\epsilon\,\mathrm{Im}\mathcal{M}(K^\pm \to \pi^\pm\phi^0)$, yielding a negligible contribution to the $K_L^0 \to \pi^0\phi^0$ amplitude squared.
[30]We quote only those that are potentially useful, given the branching ratios of Eqs. (3.9) and (3.11).
[31]A. S. Carroll *et al.*, Phys. Rev. Lett. **44**, 525 (1980).
[32]BNL Experiment E780, E. Jastrzembski *et al.*, Phys. Rev. Lett. **61**, 2300 (1988).
[33]Fermilab Experiment E731, L. K. Gibbons *et al.*, Phys. Rev. Lett. **61**, 2661 (1988).
[34]P. Bloch *et al.*, Phys. Lett. **56B**, 201 (1975).
[35]R. J. Cence *et al.*, Phys. Rev. D **10**, 776 (1974).
[36]V. Bisi *et al.*, Phys. Lett. **25B**, 572 (1967).
[37]M. Mannelli *et al.*, in *'88 Electroweak Interactions and Unified Theories,* proceedings of the XXIV Rencontre de Moriond, Les Arcs, France, 1989, edited by J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, France, 1988), p. 373.
[38]Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988).
[39]Y. Asano *et al.*, Phys. Lett. **113B**, 195 (1982).
[40]Y. Asano *et al.*, Phys. Lett. **107B**, 159 (1981).
[41]M. S. Atiya *et al.*, Phys. Rev. Lett. **64**, 21 (1990).
[42]In fact the difference in the limits quoted in Refs. 31 and 32 appears to be largely due to a difference in the range of $M(e^+e^-)$ included in quoting the limit.
[43]L. Littenberg (private communication).
[44]T. Shinkawa (private communication).
[45]S. Raby and G. West, Phys. Rev. D **38**, 3488 (1988).
[46]T. Truong and R. Willey, Phys. Rev. D **40**, 3635 (1989).
[47]M. Zeller (private communication).
[48]G. D. Barr *et al.*, Phys. Lett. B **235**, 356 (1990).
[49]A. Snyder *et al.*, Phys. Lett. B **229**, 169 (1989).
[50]M. Althoff *et al.*, Z. Phys. C **22**, 219 (1984).
[51]P. Avery *et al.*, Phys. Rev. Lett. **53**, 1309 (1984).
[52]CLEO Collaboration, M. S. Alam *et al.*, Phys. Rev. D **40**, 712 (1989).
[53]H. Albrecht *et al.*, Phys. Lett. B **199**, 451 (1987).
[54]R. M. Godbole, U. Turke, and M. Wirbel, Phys. Lett. B **194**, 302 (1987).
[55]H. Haber, A. S. Schwartz, and A. E. Snyder, Nucl. Phys. **B294**, 301 (1987).
[56]P. Avery *et al.*, Phys. Lett. B **183**, 429 (1987).
[57]F. Wilczek, Phys. Rev. Lett. **39**, 1304 (1977).
[58]M. Vysotsky, Phys. Lett. **97B**, 159 (1980).
[59]P. Nason, Phys. Lett. B **175**, 223 (1986).
[60]R. Barbieri, R. Gatto, R. Kögerler, and Z. Kunszt, Phys. Lett. **57B**, 455 (1975).
[61]J. Polchinski, S. Sharpe, and T. Barnes, Phys. Lett. **148B**, 493 (1984); J. Pantaleone, M. Peskin, and S.-H. H. Tye, *ibid.* **149B**, 225 (1984).
[62]I. G. Aznauryan, A. S. Bagdasaryan, and N. L. Ter-Isaakyan, Yad. Fiz. **36**, 1278 (1982) [Sov. J. Nucl. Phys. **36**, 743 (1982)]; I. Aznauryan, S. Grigoryan, and S. Matinyan, Pis'ma Zh. Eksp. Teor. Fiz. **43**, 497 (1986) [JETP Lett. **43**, 646 (1986)].
[63]P. Franzini *et al.*, Phys. Rev. D **35**, 2883 (1987); J. Lee-Franzini, in *Proceedings of the XXIV International Conference on High Energy Physics,* Munich, West Germany, 1988, edited by R. Kotthaus and J. H. Kühn (Springer, Berlin, 1989), p. 1432.
[64]R. D. Schamberger (private communication).
[65]ARGUS Collaboration, H. Albrecht *et al.*, Z. Phys. C **42**, 349 (1989).