Symmetry breaking and hyperon decays in the Skyrme model

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We compute the vector and axial-vector-current matrix elements needed for the Cabibbo-theory description of the semileptonic hyperon decays using exact eigenstates of the SU(3) Skyrme-model collective Hamiltonian. In a qualitative sense we confirm that the symmetry-breaking corrections are relatively small for the Cabibbo matrix elements while there is a drastic reduction from the SU(3) limit of the matrix element $\langle \text{proton}|\bar{s}\gamma_\mu\gamma_5 s|\text{proton}\rangle$ which figures prominently in analyses of the European Muon Collaboration "proton-spin" experiment.

I. INTRODUCTION

The recent European Muon Collaboration (EMC) result[1] on the polarized proton structure function implies,[2] with some seemingly reasonable assumptions, that the main form factor of the three-flavor U(1) axial-vector-current matrix element between proton states approximately vanishes at zero-momentum transfer. This is in direct contradiction to the time-honored folk picture of the proton as a system of three quarks exhibiting only noninteracting spin and color degrees of freedom at low energies. On the other hand, the EMC result is in good agreement with the SU(3) Skyrme model[3-5] which predicts[6,7] this form factor to vanish. Hence it is clearly desirable to understand this aspect of the SU(3) Skyrme model in more detail.

At the outset it should be remarked that the Skyrme model describes the baryons in a collective manner in which the role of spontaneously broken chiral symmetry is emphasized. It is not in contradiction with the full quark-gluon QCD description,[8] being presumably a reasonable effective model for it at low energies. There are actually many variations on the Skyrme model which typically include field degrees of freedom other than the pseudoscalars. These seem to us better and for some purposes necessary[9] approximations but in the present paper we shall for simplicity and definiteness consider the basic SU(3) Skyrme model of pseudoscalars only.

Here we shall use the SU(3) Skyrme model to calculate the Cabibbo[10] matrix elements of the vector and axial-vector currents. These are of relevance even though they involve flavor-changing currents such as $\bar{u}\gamma_\mu(1+\gamma_5)s$ and $\bar{u}\gamma_\mu(1+\gamma_5)d$ rather than the flavor-conserving currents such as $\bar{u}\gamma_\mu\gamma_5 u$, $\bar{d}\gamma_\mu\gamma_5 d$, and $\bar{s}\gamma_\mu\gamma_5 s$ which enter into discussions of the EMC experiment. The reason is that SU(3) invariance can be assumed to try to relate the currents measured in hyperon decays to the octet parts of the flavor-conserving currents. This is, of course, a very simple calculation. We will be interested instead in the effects of SU(3)-symmetry breaking. A useful feature of the SU(3) Skyrme model is that its collective Hamiltonian can be diagonalized exactly[4] (in a numerical sense) including SU(3)-symmetry breaking. In an earlier paper we found that the SU(3)-symmetry breaking made an important difference in the analysis of the EMC data as presented by Brodsky, Ellis, and Karliner.[6] We did not however investigate the effects of symmetry breaking in the Cabibbo scheme itself.

For the reader's convenience we first mention three conclusions of Ref. 6 and how these are modified by a more precise treatment of the same SU(3) Skyrme model used there.

(i) The basic SU(3) Skyrme model including only pseudoscalars predicts a vanishing U(1) axial-vector-current matrix element of the proton.

(ii) Feature (i) can be slightly modified by including derivative-type symmetry-breaking terms anyway needed to split the pion and kaon decay constants.

(iii) The proton matrix element of the strange axial-vector current $\bar{s}\gamma_\mu\gamma_5 s$ is relatively large, about 30% of the $\bar{d}\gamma_\mu\gamma_5 d$ matrix element. This point is also made in practically all phenomenological interpretations[2] of the EMC result and follows just from exact SU(3) symmetry applied to the axial-vector matrix elements.

Point (i), which is the main one, is correct. However, we have shown[7] that point (ii) is actually not correct. The extra terms mentioned above contribute to the *induced* axial-vector form factor which is not the relevant one for interpretation of the EMC experiment. We also showed[7] that point (iii) does not hold in the SU(3) Skyrme model—the proton matrix element of $\bar{s}\gamma_\mu\gamma_5 s$ is reduced to perhaps as little as 4% of the $\bar{d}\gamma_\mu\gamma_5 d$ matrix element.

Since the matrix element of the U(1) axial-vector current $\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d+\bar{s}\gamma_\mu\gamma_5 s$ vanishes in the SU(3) Skyrme model we may equivalently choose to measure the strange axial-vector current by the matrix element of the eighth component of an octet, namely, $\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d-2\bar{s}\gamma_\mu\gamma_5 s$. This matrix element is evidently related by SU(3) Clebsch-Gordan coefficients to the famous F and D parameters measured in the Cabibbo theory[10] of hyperon decay. Since SU(3) seems to hold to good accuracy in the Cabibbo theory one would expect this to be a reliable relation and hence for point (iii) above to hold. But we have seen that (iii) is not true in the SU(3) Skyrme model. The purpose of this paper is to find out what is happening by investigating the Cabibbo currents in the SU(3) Skyrme model.

It turns out that the proton matrix element of the diagonal current $\bar{u}\gamma_\mu\gamma_5 u+\bar{d}\gamma_\mu\gamma_5 d-2\bar{s}\gamma_\mu\gamma_5 s$ is very much more drastically suppressed (to less than one-quarter of its original value) by SU(3)-symmetry breaking than are the matrix elements of the off-diagonal currents needed in the Cabibbo theory. The latter typically suffer corrections on the order of 30%. Thus it seems possible to make the *qualitative* statement that SU(3) invariance can be reasonably good for the axial-vector Cabibbo matrix elements but quite bad for relating to them the diagonal current matrix element relevant for understanding the EMC experiment. The reason this is a *qualitative* statement is that the Skyrme model does not do too good a job of predicting the absolute magnitudes of the axial-vector matrix elements (e.g., g_A is predicted in the range 0.6–0.8 rather than 1.25). Most conservatively, therefore, our calculation sheds some doubt on the applicability of SU(3) invariance in the present context.

Another possible interesting aspect of this calculation (apart from the fact that it has not been discussed before) is that it would hold, modulo an overall factor, for any *generalized* Skyrme-model (e.g., one including vector mesons) which yields a collective Hamiltonian [see Eq. (4.2)] of the same form. If the generalized model were to produce an overall factor such that g_A comes out correctly, we could start to take its predictions very seriously rather than just in a qualitative sense. In such an event the symmetry-breaking corrections to the Cabibbo scheme computed here would provide a very severe test of the model which would constrain the effective symmetry-breaking parameter $\frac{3}{2}\gamma\beta^2 \equiv \omega^2$ [see Eqs. (3.1) and (3.4)].

To put our work in perspective we remark that the first papers[3] on the SU(3) Skyrme model treated the mass splittings to first order in the SU(3)-symmetry breaking (essentially m_K^2 which measures the strange-quark mass m_s) and neglected any symmetry-breaking effect on the baryon wave functions. Reference 6 also followed this simplified approach. More recently, by carrying out an exact numerical diagonalization of the collective Hamiltonian, Yabu and Ando[4] showed that this "first-order" approach is not correct. In some previous papers[5] we have given additional interpretation and suggested improvement of the Yabu-Ando scheme. We interpret the situation in the following way: when the original "first-order" treatment is adopted the baryon wave function knows nothing about the mass of the strange quark. It is "deluded" into treating it as very light, thereby making it easy to produce $s\bar{s}$ pairs in the proton. Indeed the proton was observed[11] to have a large "strange content" at this level of approximation. On the other hand, when an exact diagonalization of the Hamiltonian (or higher-order perturbation theory) is employed[4,5] the baryon "knows" that the s quark is relatively heavy and therefore contains fewer of them. As the s-quark mass is increased the "strange content" of the proton goes down. However the *deviations* from the SU(3)-symmetry relations increase.

In Sec. II the underlying chiral Lagrangian is written down and the relevant currents are computed at the microscopic level. The Cabibbo matrix elements are tabulated there for convenience. Section III contains a very brief discussion of the collective Hamiltonian. In Secs. IV and V the zero-momentum-transfer matrix elements of the vector and axial-vector currents are computed for an arbitrary symmetry-breaking parameter. Finally, Sec. VI contains some discussion and a comparison with the experimental g_A/g_V ratios for strangeness-changing hyperon decays on the assumption that all axial-vector matrix elements can be uniformly rescaled.

II. VECTOR AND AXIAL-VECTOR CURRENTS

The underlying SU(3) chiral action is

$$\Gamma = \int d^4x(\mathcal{L}_0 + \mathcal{L}_{\rm SB}) + \Gamma_{\rm WZ} \ . \tag{2.1}$$

Here

$$\mathcal{L}_0 = \frac{F_\pi^2}{8}\mathrm{Tr}(\alpha_\mu\alpha_\mu) + \frac{1}{32e^2}\mathrm{Tr}([\alpha_\mu,\alpha_\nu]^2) \ , \tag{2.2}$$

with $\alpha_\mu \equiv \partial_\mu U\, U^\dagger$, while the Wess-Zumino action[12] in terms of the one-form $\alpha = \alpha_\mu dx^\mu$ is

$$\Gamma_{\rm WZ} = \frac{iN_c}{240\pi^2}\int \mathrm{Tr}(\alpha^5), \tag{2.3}$$

the integral being over a five-dimensional manifold whose boundary is Minkowski space. Finally the symmetry-breaking terms are (see Ref. 9 for notation and the relevant analysis)

$$\mathcal{L}_{\rm SB} = \mathrm{Tr}[(\beta' T + \beta'' S)(\partial_\mu U\, \partial_\mu U^\dagger\, U + U^\dagger \partial_\mu U\, \partial_\mu U^\dagger) + (\delta' T + \delta'' S)(U + U^\dagger - 2)] \ , \tag{2.4}$$

wherein $T = \mathrm{diag}(1,1,0)$ and $S = \mathrm{diag}(0,0,1)$. The parameters δ' and δ'' are analogous to "quark mass" coefficients and are determined from the meson sector of the theory. Similarly β' and β'' adjust[13] $F_k \neq F_\pi$ in the meson sector.

The vector and axial-vector V_μ and A_μ currents, may be found[14] by "gauging" (2.1) with external gauge fields $v_\mu = v_\mu^a Q_a$ and $a_\mu = a_\mu^a Q_a$ and evaluating the functional derivatives:

$$V_\mu^a = \left.\frac{\delta\Gamma(U, v_\mu, a_\mu)}{\delta v_\mu^a}\right|_{v_\mu = a_\mu = 0} , \tag{2.5}$$

$$A_\mu^a = \left.\frac{\delta\Gamma(U, v_\mu, a_\mu)}{\delta a_\mu^a}\right|_{v_\mu = a_\mu = 0} . \tag{2.6}$$

Note that the Q_a are 3×3 Hermitian U(3) generators. This yields, for the currents,

$$\begin{aligned} &-\frac{iF_\pi^2}{4}\mathrm{Tr}[Q^a(\alpha_\mu \mp \beta_\mu)] \ , \\ &-\frac{i}{8e^2}\mathrm{Tr}\{Q^a([\alpha_\nu,[\alpha_\mu,\alpha_\nu]] \mp [\beta_\nu,[\beta_\mu,\beta_\nu]])\} \ , \\ &-\frac{N_c}{48\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{Tr}[Q^a(\alpha_\nu\alpha_\rho\alpha_\sigma \pm \beta_\nu\beta_\rho\beta_\sigma)] + \cdots \ , \end{aligned} \tag{2.7}$$

wherein the upper (lower) sign corresponds to V_μ^a (A_μ^a). Furthermore $\beta_\mu = U^\dagger \partial_\mu U$. The center dots indicate the contributions to the currents of the derivative-type symmetry-breaking terms. This type of contribution was shown in the second paper of Ref. 5 to be not very important for *baryonic* matrix elements.

For our present purposes we are only interested in the zero-momentum-transfer matrix elements of the above currents. Equivalently, we need the space integrals $\int V_0^a d^3x$ and $\int A_i^a d^3x$.

In order to compare our results with the SU(3)-symmetry Cabibbo model we need the matrix elements of $\int V_0^{\pi^-} d^3x$, $\int V_0^{k^-} d^3x$, $\int A_i^{\pi^-} d^3x$, and $\int A_i^{k^-} d^3x$ appropriate to various semileptonic hyperon decays. For ordinary neutron $\rightarrow$ proton β decay we have

$$\int d^3x \langle p | V_0^{\pi^-} | n \rangle = 1 \, , \quad \int d^3x \langle p | A_i^{\pi^-} | n \rangle = -g_A \langle \sigma_i \rangle \, . \tag{2.8}$$

The matrix element coefficients of interest are given in Table I. The axial-vector coefficients are given in terms of the usual dimensionless parameters F and D which satisfy

$$F + D = g_A \approx 1.25 \, . \tag{2.9}$$

Notice that the parametrization of the matrix elements in Table I is only meaningful in the exact SU(3)-symmetry limit. In particular it is meaningless to speak of the parameters F and D when SU(3) symmetry is broken.

III. COLLECTIVE HAMILTONIAN

The collective Hamiltonian appropriate to baryonic excitations of (2.1) has been widely discussed in the literature.[3-5] It is given by

$$H = M_{\rm cl} + \frac{J(J+1)}{2\alpha^2} + \frac{1}{2\beta^2}[C_2(\mathrm{SU}(3)) - J(J+1) - \tfrac{3}{4}] + \frac{\gamma}{2}[1 - D_{88}(A)] \, , \tag{3.1}$$

where the dynamical variables A comprise a 3×3 unitary matrix and whose canonical momenta appear implicitly in the spin Casimir operator $J(J+1)$ and the flavor Casimir operator $C_2(\mathrm{SU}(3))$. $M_{\rm cl}$ is the classical soliton mass while α^2 is the ordinary moment of inertia and β^2 is the moment of inertia for flavor-space rotations. γ is the symmetry-breaking parameter corresponding to (2.4) and $D_{ij}(A)$ is the octet representation matrix

$$D_{ij}(A) = \tfrac{1}{2}\mathrm{Tr}(\lambda_i A \lambda_j A^\dagger) \, . \tag{3.2}$$

The above parameters are all calculable in terms of the Lagrangian (2.1). But we would like to stress that the collective Hamiltonian H may arise from a more general SU(3) chiral Lagrangian than (2.1). In this sense the present calculation may apply to models beyond the Skyrme model of pseudoscalars only. It is also important to remark, as first pointed out by Yabu and Ando[4] that the last, symmetry-breaking term must be treated beyond first-order perturbation theory. Those authors wrote down a differential equation for the exact diagonalization of (3.1). Their basic results can be recaptured by perturbation theory beyond leading order. Then it is seen that the spin-$\frac{1}{2}$ baryons, for example, are no longer pure octets but contain sizable admixtures of the $\overline{10}$ and 27 representations. For example, the proton looks like[5]

$$|p\rangle = |p,8\rangle + \frac{\gamma\beta^2}{6\sqrt{5}}|p,\overline{10}\rangle + \frac{\sqrt{6}\gamma\beta^2}{50}|p,27\rangle + \cdots \, , \tag{3.3}$$

which illustrates that the effective symmetry breaker is the product

$$\omega^2 \equiv \tfrac{3}{2}\gamma\beta^2 \, . \tag{3.4}$$

IV. VECTOR MATRIX ELEMENTS

Here we shall calculate the integrated vector currents $\int d^3x \, V_0^a$ as obtained from (2.7). We shall neglect the relatively small SU(3)-symmetry breaking due to the derivative-type terms in (2.4). The quantities of interest are simply the flavor-SU(3) generators. Because the baryons are not pure octets, this computation is not trivial. However the vector charges for the strangeness-conserving decays are not affected by SU(3)-symmetry breaking and continue to be given by their values in Table I. It is well known that the strangeness-changing vector charges are not renormalized to first order in perturbation theory (Ademollo-Gatto theorem[15]). However since first-order perturbation theory is inadequate for the SU(3) Skyrme model there is no guarantee that there will not be important second-order contributions. To see what is happening we may compute a typical matrix element by perturbation theory. Let us denote the generators by G_{YII_3} (normalized so $I_3 = G_{010}/\sqrt{2}$ and $Y = \sqrt{2/3}G_{000}$) and compute the strangeness-changing matrix element $\langle p | G_{1\,1/2\,1/2} | \Lambda \rangle$. Notice that the $|p\rangle$ state is given in (3.3) and the $|\Lambda\rangle$ is

$$|\Lambda\rangle = |\Lambda,8\rangle + \tfrac{3}{50}\gamma\beta^2|\Lambda,27\rangle + \cdots \, . \tag{4.1}$$

Then, remembering that the group generators do not take us out of a given irreducible representation we have, to order $(\gamma\beta^2)^2$,

TABLE I. (a) Vector and axial-vector matrix elements for the strangeness-conserving baryon decays in the limit of exact SU(3) symmetry. (b) The same as (a) for the strangeness-changing baryon decays. Note that $D/F = 1.8$ while $D + F = g_A$.

(a)

	$n \rightarrow p$	$\Sigma^- \rightarrow \Lambda$	$\Sigma^- \rightarrow \Sigma^0$	$\Xi^- \rightarrow \Xi^0$
V^{π^-}	1	0	$\sqrt{2}$	-1
A^{π^-}	g_A	$\frac{2D}{\sqrt{6}}$	$\sqrt{2}F$	$D-F$

(b)

	$\Lambda \rightarrow p$	$\Sigma^- \rightarrow n$	$\Xi^- \rightarrow \Lambda$	$\Xi^- \rightarrow \Sigma^0$
V^{k^-}	$-3/\sqrt{6}$	-1	$3/\sqrt{6}$	$1/\sqrt{2}$
A^{k^-}	$-\frac{(3F+D)}{\sqrt{6}}$	$D-F$	$\frac{3F-D}{\sqrt{6}}$	$\frac{F+D}{\sqrt{2}}$

$$\langle p|G_{1\,1/2\,1/2}|\Lambda\rangle = N[\langle p,8|G_{1\,1/2\,1/2}|\Lambda,8\rangle + \frac{3\sqrt{6}}{2500}(\gamma\beta^2)^2\langle p,27|G_{1\,1/2\,1/2}|\Lambda,27\rangle] \,,$$
$$N \simeq 1 - \tfrac{1}{2}(\gamma\beta^2)^2(\tfrac{3}{500}+\tfrac{1}{180}) \,. \tag{4.2}$$

It is obvious from this derivation why the Ademollo-Gatto theorem holds. Evaluating the matrix elements in (4.2) in terms of SU(3) Clebsch-Gordan coefficients finally gives

$$\langle p|G_{1\,1/2\,1/2}|\Lambda\rangle = -\sqrt{3/2} + (\gamma\beta^2)^2\left[\frac{-3\sqrt{6}}{1250} + \sqrt{3/2}(\tfrac{3}{1000}+\tfrac{1}{360})\right] \,. \tag{4.3}$$

Actually we shall evaluate the matrix elements as a function of $\omega^2 = \frac{3}{2}\gamma\beta^2$ by an exact diagonalization of (3.1). To simplify the calculation we employ the identity

$$\langle\psi'|[H,G_i]|\psi\rangle = (E'-E)\langle\psi'|G_i|\psi\rangle \tag{4.4}$$

for any operator G_i. Taking $G_i = G_{1\,1/2\,1/2}$ and using (3.1),

$$\langle\psi'|G_{1\,1/2\,1/2}|\psi\rangle = \frac{\sqrt{3}}{4}\frac{\gamma}{E'-E}\langle\psi'|D_{1\,1/2\,1/2,000}(A)|\psi\rangle \,, \tag{4.5}$$

where an evident "spherical basis" has been used for $D_{ij}(A)$. The right-hand side (RHS) of (4.5) is now straightforward to evaluate[16] with the known exact eigenfunctions of (3.1). The results for the experimentally relevant strangeness-changing vector matrix elements are shown in Fig. 1 as a function of the symmetry-breaking parameter ω^2. Note that these quantities should be multiplied by the appropriate Clebsch-Gordan coefficients given in Table I. As expected, the deviation from unity is at least quadratic in ω^2. For $\omega^2 \approx 10$, which is suggested by a recent treatment of the Skyrme model of pseudoscalars only (see Weigel, *et al.*[5]), a deviation of about 20% from the symmetry limit is predicted for $\langle n|V_0^{k^-}|\Sigma^-\rangle$.

V. AXIAL-VECTOR MATRIX ELEMENTS

Here we shall consider the relevant integrated axial-vector currents $\int d^3x\, A_i^a$ as obtained from (2.7). With the introduction[17] of collective variables $A(t)$ $(A^\dagger = A^{-1})$ according to $U(\mathbf{x},t) = A(t)U_0(\mathbf{x})A^\dagger(t)$, $U_0(\mathbf{x})$ being the classical solution, we have

$$\int d^3x\, A_i^a = cD_{ai}(A) \,, \tag{5.1}$$

where the octet representation matrix $D_{ai}(A)$ is defined in (3.2) and

$$c = \frac{2\sqrt{2}\pi}{3}\int dr\, r^2\left\{F'\left[F_\pi^2 + \frac{4}{e^2}\frac{\sin^2 F}{r^2}\right] + \frac{\sin 2F}{r}\left[F_\pi^2 + \frac{2}{e^2}\left[F'^2 + \frac{\sin^2 F}{r^2}\right]\right]\right\} \,. \tag{5.2}$$

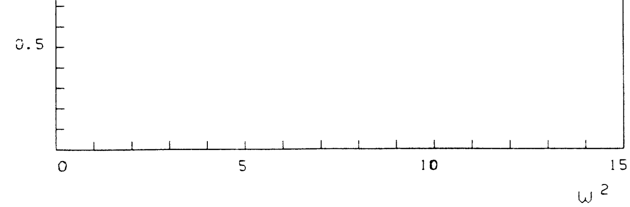

FIG. 1. Dependences of the strangeness-changing vector matrix elements on the SU(3)-symmetry-breaking parameter ω^2. 1:$\Lambda \to p$, 2:$\Sigma^- \to n$, 3:$\Xi^- \to \Lambda$, 4:$\Xi^- \to \Sigma^0$. Note that these values should be multiplied by the appropriate Clebsch-Gordan coefficient for V^{k^-} in Table I(b).

Here $F(r)$ is the usual Skyrme "profile" and $F' = dF/dr$. In (5.1) we have neglected contributions proportional to the angular velocity $A^\dagger\dot{A} = (i/2)\lambda_\alpha\Omega_\alpha$. This point is further discussed in Appendix A. A numerical evaluation yields $c = 3.60$, 2.13, and 1.40 for the choice of the Skyrme constant $e = 3$, 4, and 5.

The diagonal (flavor-conserving) matrix elements of (5.1) for the nucleon were discussed in Ref. 7. The matrix element of $D_{33}(A)$ is proportional to g_A while the matrix element of $D_{83}(A)$ is, because the singlet axial-vector matrix element vanishes in this model, a direct measure of the strange axial-vector current in the proton $\langle p|\bar{s}\gamma_\mu\gamma_5 s|p\rangle$. It was observed that the strange axial-vector matrix element was very substantially suppressed from the value obtained using SU(3) symmetry and a conventional D/F ratio. For the flavor-changing axial-vector currents of present interest we require the matrix elements of the operators

$$D_{\pi^+,3} = \frac{1}{\sqrt{2}}(D_{13} - iD_{23}) = D_{011,010} \,,$$
$$D_{k^+,3} = \frac{1}{\sqrt{2}}(D_{43} - iD_{53}) = D_{1\,1/2\,1/2,010} \,, \tag{5.3}$$

where the "spherical" labels YII_3 are used on the RHS.

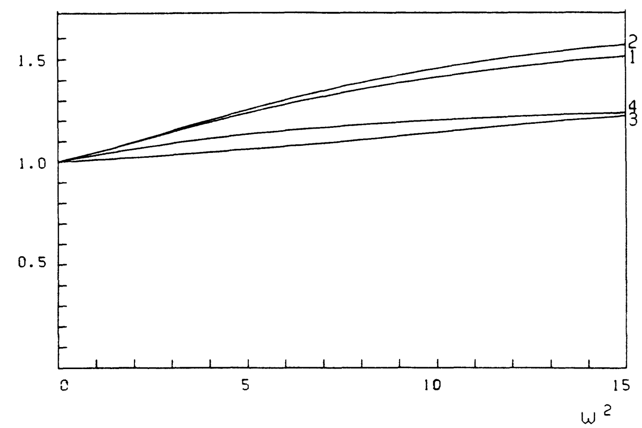

FIG. 2. Dependences of the strangeness-conserving axial-vector matrix elements on the SU(3)-symmetry-breaking parameter ω^2. 1:$\Sigma^- \to \Lambda$,2:$\Sigma^- \to \Sigma^0$,3:$\Xi^- \to \Xi^0$,4:$n \to p$. Note that these values should be multiplied by the Clebsch-Gordan coefficients for A^{π^-} in Table I(a).

RHS.

These corresponding matrix elements are straightforward to evaluate. For orientation let us first discuss the example $\Lambda \to p$ by perturbation theory. Using the approximate wave functions (3.3) and (4.1) as well as a standard formula[18] for the matrix element of a D function between two *irreducible* representations we find

$$\langle p\uparrow|D_{k^+,3}|\Lambda\uparrow\rangle = \frac{2}{5\sqrt{3}} - \frac{7\sqrt{3}}{1125}\gamma\beta^2 + \cdots \ . \tag{5.4}$$

It may be noted that the effect of symmetry breaking is much less drastic than for the flavor-conserving matrix element[7] $\langle p|D_{83}|p\rangle$.

As before we shall present the results for the relevant decay axial-vector matrix elements obtained by numerically evaluating the operators $D_{\pi^+,3}$ and $D_{k^+,3}$ between the exact eigenfunctions of the Hamiltonian (3.1). In Fig. 2 the dependence of the strangeness-conserving axial-vector matrix elements on the symmetry-breaking parameter ω^2 is shown. These should be multiplied by the appropriate Clebsch-Gordan coefficients given in Table I. [Specifically we must choose $D/F=1.8$ as in (6.1) and $D+F=7c\sqrt{2}/30$ where c is given in (5.2)]. The symmetry-breaking dependence of the strangeness-changing axial-vector matrix elements are similarly displayed in Fig. 3. We can see that the $\Sigma^- \to n$, $\Xi^- \to \Lambda$, and $\Xi^- \to \Xi^0$ transitions are the most stable as symmetry breaking increases while the others may change by as much as 45% for $\omega^2 \approx 10$.

We have checked the relative sign conventions for the vector and axial-vector matrix elements in Appendix B.

VI. DISCUSSION OF RESULTS

Let us first attempt a comparison with the Cabibbo theory, which nicely correlates the decay amplitudes for the modes indicated in Table I with each other. We should recognize that this comparison must inevitably be rather crude. To see this one need only consider neutron β decay. The SU(2) Skyrme model typically predicts[19] g_A in the range 0.6–0.8 depending on the value of the Skyrme constant e. Experiment gives $g_A=1.25$ so a direct comparison to the accuracy of the experiments is not possible. However we feel that a general comparison is useful and results in some interesting qualitative conclusions. Another reason for having some interest in this comparison is that symmetry-breaking corrections to the Cabibbo theory do not seem to have been previously studied in the SU(3) Skyrme model.

For orientation we remark that the original Cabibbo theory assuming SU(3)-symmetry matrix elements works fairly well. Several recent papers[20] which estimate symmetry-breaking corrections in various ways and make global fits to the data have indicated that the SU(3) breaking seems to be rather small.

When SU(3)-symmetry breaking is neglected the various baryon wave functions are SU(3) symmetric. Then the matrix elements must conform to the pattern of Table I. Calculating two particular axial-vector matrix elements and comparing with Table I gives the well-known Skyrme-model prediction

$$D/F = \tfrac{9}{5}\ [\mathrm{SU}(3)\ \text{limit}]\ . \tag{6.1}$$

This is actually not too far from a typical[21] SU(3)-symmetry fit which yields $D/F=1.63$ for the axial-vector matrix elements. Since all the axial-vector matrix elements come out too small it seems reasonable to scale all the axial-vector quantities by the common factor $(g_A)_{\mathrm{expt}}/(g_A)_{\mathrm{pred}}$. With this rescaling we can go on to look at the effect of SU(3)-symmetry breaking on the g_A/g_V ratio for each particular hyperon decay. A possible justification for this procedure in the SU(3) Skyrme

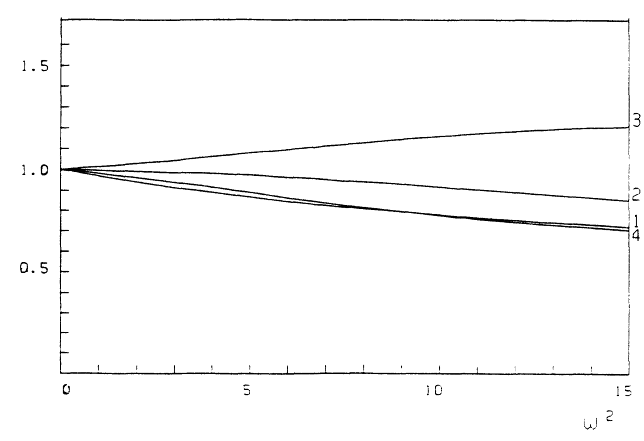

FIG. 3. Dependences of the strangeness-changing axial-vector matrix elements on the SU(3)-symmetry-breaking parameter ω^2. 1:$\Lambda \to p$,2:$\Sigma^- \to n$,3:$\Xi^- \to \Lambda$,4:$\Xi^- \to \Sigma^0$. These values should be multiplied by the values given for A^{k^-} in Table I(b).

model under consideration is that all the axial-vector decay constants roughly scale as $1/e^2$ so physically reasonable values may be obtained by choosing a suitably small value of the Skyrme constant e. Of course, this affects other dynamical predictions too. This procedure is also reasonable in the sense that various different models (e.g., including vector mesons) give the same formula (5.1) but with different values for c.

With a uniform rescaling of all axial-vector quantities so that the calculated g_A is set to 1.25 (for each ω^2) we obtain the predictions shown in Table II for the "g_A/g_V" ratios for the three strangeness-changing hyperon decays $\Lambda\to pe\bar{\nu}_e$, $\Sigma^-\to ne\bar{\nu}_e$, and $\Xi^-\to\Lambda e\bar{\nu}_e$. These are the decays for which a number for g_A/g_V has been listed by the Particle Data Group.[22] The results of Table I(b), Figs. 1 and 3 were used for this determination. It can be seen that the zero symmetry-breaking predictions for $\Lambda\to p$ and $\Sigma^-\to n$ are quite good and the effect of symmetry breaking is to worsen the predictions. For $\Xi^-\to\Lambda$, the predicted value seems a bit lower than the experimental value and the effect of symmetry breaking does not change much. For this last decay it might be remarked that the experimental determination of g_A/g_V *assumed* exact SU(3) symmetry. Also experimental determinations with such assumptions played a role in determining the "world average" value of g_A/g_V for the other modes.

While symmetry breaking with $\omega^2=10$ may worsen the g_A/g_V predictions by as much as 30%, it should be emphasized that the matrix element of interest for the proton spin problem $-\langle p|\int d^3x\, A_i^8|p\rangle$ is much more drastically suppressed. As previously shown[7] this matrix element is reduced from 0.42 at $\omega^2=0$ to 0.09 at $\omega^2=10$. Hence qualitatively it seems fair to say that this matrix element is drastically suppressed without causing a major change in the various hyperon decay amplitudes.

Since the above statement claiming a difference between SU(3)-symmetry-breaking effects for the flavor-changing and flavor-conserving axial-vector matrix elements is a *qualitative* one it might be comforting to give a "hand-waving" argument to show how such a situation could be understood in the quark model. Suppose the proton wave function has the schematic form

$$p\sim uud\{1+\eta[u\bar{u}+d\bar{d}+(1-\epsilon)s\bar{s}]\}\ \cdots\ .$$

Here η is a parameter which measures the strength of the quark-antiquark "sea" while ϵ is a (positive) symmetry-breaking parameter which increases as the SU(3)-symmetry breaking increases. Then the flavor-conserving matrix element has the following structure to lowest order:

$$\langle p|(\bar{u}\gamma_\mu\gamma_5u+\bar{d}\gamma_\mu\gamma_5d-2\bar{s}\gamma_\mu\gamma_5s)|p\rangle\sim1-\eta\epsilon\ .$$

On the other hand, if we similarly represent the Λ hyperon as

$$\Lambda\sim u\ ds\{1+\eta'[u\bar{u}+d\bar{d}+(1-\epsilon')s\bar{s}]\}+\cdots\ ,$$

a flavor-changing matrix element would have the lowest-order structure

$$\langle p|\bar{u}\gamma_\mu\gamma_5s|\Lambda\rangle\sim1+\eta(1-\epsilon)\ .$$

It can be seen that for choices such as $\eta=1$ and $\epsilon=0.8$ the symmetry breaking can be greatly magnified for the flavor-conserving matrix element compared to the flavor-changing matrix elements.

It would be tempting to use our results to draw conclusions about the symmetry-breaking parameter ω^2 in the SU(3) Skyrme model. However, given that all axial-vector matrix elements are predicted to be only around half of their true values without symmetry breaking it is clearly premature to take these fine-tuning corrections seriously in other than a general sense. If one were to produce a soliton model with a value of c which gives the correct g_A, then the hyperon decays could be considered to provide strong restrictions on that model. In such a model it would probably be difficult to explain both the pattern of baryon mass splittings (which seems to require $\omega^2\approx10$) as well as the small symmetry-breaking corrections to hyperon decays. This might suggest the necessity of an even more sophisticated method of collective quantization.

Note added in proof. In treating the axial-vector current in (5.1) we mentioned that terms proportional to the "angular velocities" Ω_a were being neglected. These terms are proportional to the coefficient $A_3(r)$ in (A2) of Appendix A and are expected to be small. We have now carried out a rather lengthy calculation (to be described elsewhere) of them. It turns out that they are indeed small (though not completely negligible) for the Yabu-Ando approach to quantizing the SU(3) Skyrme model. However, if one adopts a modified quantization scheme in which an intrinsic kaon field is excited by the collective rotation (see Weigel *et al.*[5]), it turns out that, for the realistic range of the symmetry-breaking parameter ω^2, they make important contributions to the axial-vector charges. This results in only small deviations from the SU(3)-symmetric values of the g_A/g_V ratios in Table II. For the matrix element $\langle p|\bar{s}\gamma_\mu\gamma_5s|p\rangle$ there is still a very substantial reduction from its SU(3)-symmetric value. Because of the improved agreement with the Cabibbo scheme, the overall claim of this paper is strengthened. Furthermore it provides evidence for the importance of

TABLE II. Comparison of experimental and theoretical ratios g_A/g_V for certain hyperon decays as a function of the SU(3)-symmetry-breaking parameter ω^2. The axial-vector matrix elements have been rescaled to set $g_A=1.25$ (for the $n\to p$ mode) at each ω^2. Note that only the magnitude has been experimentally determined for $\Sigma^-\to n$.

	g_A/g_V		
	$\Lambda\to p$	$\Sigma^-\to n$	$\Xi^-\to\Lambda$
$\omega^2=0$	0.71	$-.36$	.18
$\omega^2=5$	0.54	$-.29$	.17
$\omega^2=10$	0.45	$-.23$	.16
Expt.	0.70±0.03	-0.36 ± 0.05	0.25±0.05

exciting the intrinsic kaon field in the SU(3) Skyrme model.

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APPENDIX A: CONTRIBUTIONS TO THE CURRENTS PROPORTIONAL TO THE ANGULAR VELOCITY

In this appendix we comment on the contributions proportional to the angular velocity $A^\dagger \dot{A} = (i/2)\lambda_a \Omega_a$ for the space components of vector and axial-vector currents. We will show that they are important for the space components of the vector currents in the strong-coupling limit in order to recover the SU(2) results. A careful treatment of (2.7) leads to

$$\begin{aligned} V_i^a(\mathbf{r},t) = {} & V_1(r)\epsilon_{ijk}x_j D_{ak} + \frac{\sqrt{3}}{2}B(r)\epsilon_{ijk}x_j\Omega_k D_{a8} \\ & + V_2(r)\epsilon_{ijk}x_j d_{k\alpha\beta}D_{a\alpha}\Omega_\beta \\ & + V_3(r)x_i f_{8\alpha\beta}D_{a\alpha}\Omega_\beta \,, \\ & a=1,\ldots,8,\quad i,j,k=1,\ldots,3,\quad \alpha,\beta=4,\ldots,7 \,. \end{aligned} \tag{A1}$$

Here $V_1(r)$ is the integrand of the moment of inertia α^2 in (3.1) and $B(r)$ is the baryon-number density. The radial functions V_2 and V_3 depend on the cranking of the kaon fields and are irrelevant since only V_0^a is needed to evaluate the corresponding form factor in (2.7) at zero-momentum transfer. Similarly one obtains for the axial-vector current

$$\begin{aligned} A_i^a(\mathbf{r},t) = {} & A_1(r)D_{ai} + A_2(r)x_i x_k D_{ak} \\ & + A_3(r)(x^2\delta_{ij} - x_i x_j)d_{j\alpha\beta}D_{a\alpha}\Omega_\beta \,. \end{aligned} \tag{A2}$$

The integral $\int d^3x(A_1 + \frac{1}{3}x^2 A_2) = c$ in (5.1). As mentioned in Sec. V the term proportional to Ω, i.e., A_3, has been neglected in our discussion of the hyperon decays. A_3 depends again on cranking of the kaon modes but as do V_2 and V_3 it receives a contribution from the WZ term even if a variational ansatz for the kaon field were not imposed. In a somewhat tedious calculation we have evaluated the matrix elements $\langle N|d_{3\alpha\beta}D_{a\alpha}\Omega_\beta|N\rangle$ for $a=3$ and 8. They are found to be small compared to $\langle N|D_{a3}|N\rangle$ at $\omega^2=0$ and to vanish in the strong symmetry-breaking limit. Thus it might appear reasonable to neglect the contributions of A_3 in the discussion of the hyperon decays. The small influence of V_2, V_3, and A_3 on the magnetic moment and axial-vector charge, respectively, at finite symmetry breaking will be considered elsewhere. (See Note added.)

In the limit of large symmetry breaking we obtain, for the terms of the vector currents V_i^3 and V_i^8 which contribute to the nucleon matrix elements,

$$V_i^3 \to V_1(r)\epsilon_{ijk}x_j D_{3k}^{[\mathrm{SU}(2)]} \,, \tag{A3}$$

$$V_i^8 \to \frac{\sqrt{3}}{2}B(r)\epsilon_{ijk}x_j\Omega_k \,. \tag{A4}$$

This gives, for the electromagnetic current $\mathcal{T}_i^{\mathrm{em}}$,

$$\begin{aligned} \mathcal{T}_i^{\mathrm{em}} = V_i^3 + \frac{1}{\sqrt{3}}V_i^8 &= V_1(r)\epsilon_{ijk}x_j D_{3k}^{[\mathrm{SU}(2)]} + \tfrac{1}{2}B(r)\epsilon_{ijk}x_j\Omega_k \\ &= V_i^{3[\mathrm{SU}(2)]} + \tfrac{1}{2}B_i \,, \end{aligned} \tag{A5}$$

where B_i are the space components of the baryon current. Thus we have shown that the SU(2) result will be recovered in the strong symmetry-breaking limit only if the vector current is expanded up to order Ω. This gives evidence that also the effects of $V_2(r)$ and $V_3(r)$ on electromagnetic properties such as magnetic moments have to be taken into account at finite symmetry breaking.

APPENDIX B

Since we are computing both g_A as well as g_V it is amusing to check our (standard) phase conventions to see that the relative sign works out correctly. For simplicity let us restrict attention to the first term in (2.7) and the SU(2) Skyrme model. Making the variations (2.5) and (2.6) on the fundamental QCD Lagrangian gives the vector and axial-vector quark currents corresponding to (2.7): $V_\mu^a = i\bar{q}\gamma_\mu Q^a q$ and $A_\mu^a = i\bar{q}\gamma_\mu\gamma_5 Q^a q$. The normalization is provided by noting that the baryon number is given by $B = -i/3\int d^3x\, V_4^{(0)}$ where $Q^{(0)}=1$. The nonrelativistic reductions of the $n \to p$ matrix elements of these currents are given in (2.8). Reference to Eqs. (3.460) and (3.461) of Sakurai[23] shows that we must choose $g_A \approx +1.25$. Now at the Skyrme-model level we may calculate the isospin charge and the integrated axial-vector current as

$$I^a = -i\int d^3x\, V_4^a = \frac{8\pi i}{3}\int r^2 dr\, \sin^2 F\, \mathrm{Tr}(\dot{A}A^\dagger\tau^a) \cdots \,, \tag{B1}$$

$$\int d^3x\, A_i^a = \frac{2\pi F_\pi^2}{3}\int r^2 dr \left[F' + \frac{\sin 2F}{r}\right] D_{ai}(A) + \cdots \,, \tag{B2}$$

where $D_{ai}(A)$ is defined in (3.2). It is interesting that (B1) is independent of the sign of $F(r)$ while (B2) changes sign if we reverse the sign of $F(r)$. Noting that the matrix element $\langle p\uparrow|D_{a3}|p\uparrow\rangle = -\frac{1}{3}\langle\tau_a\rangle$ and comparing (B2) with (2.8) shows that the integral over r must be positive. This may be accomplished by choosing

$$F(0) = -\pi \,. \tag{B3}$$

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