New look at CP violation

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The CP-violating (CPV) kaonic matrix elements ϵ', $\mathrm{Im}A_0$, and $\mathrm{Im}A_2$ are used to compute CPV observables in baryonic systems. The numerical results for the electric dipole moment and the difference between Λ and $\bar{\Lambda}$ decay parameters are similar to other approaches that use quark operators. Relations between the neutron electric dipole moment, CPV pion-nucleon coupling constants, and Λ-decay parameters are derived.

I. INTRODUCTION

Efforts to understand the nature of CP violation (CPV) have gained new impetus from recent experiments on the K^0-$\bar{K}^0$ (Ref. 1) and B^0-$\bar{B}^0$ systems.[2] Several different experiments are planned[3] to further elucidate the CP-violating aspects of kaons. It is very clear that any discovery of CP violation or time-reversal violation (TRV) in other systems would by very important. Indeed, there are systematic significant efforts to study TRV in nuclei and atoms.[4]

The experimental efforts have stimulated theorists to search for and predict the size of new CPV effects. However, the very small value of the upper limit on the electric dipole moment (EDM) of the neutron [$D_n \le 5 \times 10^{-26}$ e cm (Ref. 5), $-(1.4\pm0.6)\times10^{-25}$ e cm (Ref. 6)] places very strong constraints on all of the theories. Standard-model estimates[7] of the neutron EDM employ the Cabibbo-Kobayashi-Maskawa (CKM) matrix.[8] In those treatments the CPV occurs only at the three-loop order. Typical results[7] are $D_n = 10^{-31}-10^{-33}$ e cm. The spread in the results arises mainly from the uncertainties in relating the hadron CPV properties to matrix elements of quark operators. For example, computing ϵ (the CPV mixing parameter in the K^0-$\bar{K}^0$ system) from the CKM matrix requires detailed knowledge of the kaon wave functions. Relating the CKM matrix to the neutron EDM is even more difficult.

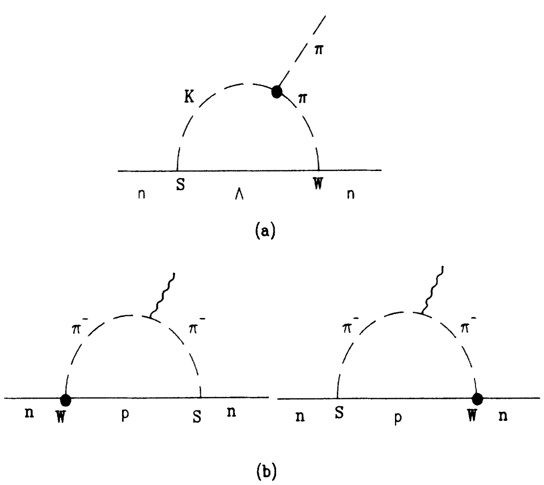

FIG. 1. (a) A sample CPV graph. The CPV occurs in the blob. (b) EDM in terms of the effective CPV pion-nucleon interaction (blob). The S, W stand for strong, weak vertices.

The aim of the present work is to provide alternative estimates of CPV observables such as the neutron EDM and the differences between Λ and $\bar{\Lambda}$ decay.[9,10] In our approach measured hadronic matrix elements (including the CPV kaon observables) are used to compute Feynman graphs at the hadronic level. Examples are shown in Fig. 1. By employing the hadronic basis one avoids the difficult steps of obtaining the quark CPV parameters from the kaonic system and inserting these quark parameters into the matrix elements involving the baryons. However, there are other problems, see Sec. V.

Here is an outline of our approach. Since measured CPV matrix elements (generated only in the kaon system) are to be used, it is necessary to review the standard notation. This is done in Sec. II. Our hadronic approach is applied to the computation of the neutron EDM in Sec. III. The differences between Λ and $\bar{\Lambda}$ decay observables are predicted in Sec. IV. Experimental interest[11] in comparing Λ and $\bar{\Lambda}$ decay has been stimulated by the work of Refs. 9 and 10. The output of Secs. III and IV can be parametrized in terms of effective CPV meson-nucleon coupling constants. Section V contains a critical discussion of our approach in which various uncertainties are discussed. Section VI contains a summary, including a set of relations between various possible CPV observables.

II. CP VIOLATION IN K^0 DECAY

We use the formalism of Wolfenstein.[12] The central point is that a phenomenology requires three real CPV quantities, while the neutral-kaon data provides only two numbers. Thus, there is a void which can, in principle, be filled with the output of experiments on other systems.

The observations of CPV, in kaonic systems, can be summarized[12] in terms of two complex parameters η_{+-} and η_{00} and the charge asymmetry δ defined by

$$\eta_{+-}=\frac{A(K_L\to\pi^+\pi^-)}{A(K_S\to\pi^+\pi^-)}\ ,$$
$$\eta_{00}=\frac{A(K_L\to\pi^0\pi^0)}{A(K_S\to\pi^0\pi^0)}\ , \qquad (2.1)$$
$$\delta=\frac{\Gamma(K_L\to\pi^-l^+\nu)-\Gamma(K_L\to\pi^+l^-\bar{\nu})}{\Gamma(K_L\to\pi^-l^+\nu)+\Gamma(K_L\to\pi^+l^-\bar{\nu})}\ ,$$

where A stands for amplitude and l is either e or μ. The experimental results are tabulated in Ref. 13.

In general, CPV may occur either in the mass matrix of the K^0-$\bar{K}^0$ system or in the decay amplitude. The CP eigenstates are

$$K_1=(K^0-\bar{K}^0)/\sqrt{2},\quad CP=+\ ,$$
$$K_2=(K^0+\bar{K}^0)/\sqrt{2},\quad CP=-\ , \qquad (2.2)$$

where $\bar{K}^0=-(CP)K^0$. In the K_1-K_2 representation the complex mass matrix takes the form

$$M-i\frac{\Gamma}{2}=\begin{bmatrix} M_1 & im' \\ -im' & M_2\end{bmatrix}-\frac{i}{2}\begin{bmatrix}\gamma_1 & i\gamma' \\ -i\gamma' & \gamma_2\end{bmatrix}, \qquad (2.3)$$

assuming CPT invariance. The off-diagonal terms that mix K_1 and K_2 are the result of CP violation. To a good approximation[12] the diagonal values are equal to the eigenvalues

$$M_1-M_2+i(\gamma_1-\gamma_2)/2=(M_S-M_L)+i(\Gamma_S-\Gamma_L)/2$$
$$=-\Delta M+i\Gamma_S/2\ , \qquad (2.4)$$

where ΔM is the mass difference between K_L and K_S and in the last line we use $\Gamma_L \ll \Gamma_S$. The factor i and the antisymmetry indicate that the term m' violates not only CP but also T as expected from the CPT theorem.

The decay amplitudes of main interest are $K^0\to 2\pi$ written as

$$A[K^0\to\pi\pi(I)]=A_I\exp(i\delta_I)$$
$$A[\bar{K}^0\to\pi\pi(I)]=-A_I^*\exp(i\delta_I)\ , \qquad (2.5)$$

where I is the $\pi\pi$ isospin and δ_I is the corresponding $\pi\pi$ phase shift. From CPT invariance and unitarity one can show that A_I is real if CP is not violated and also that γ' in Eq. (2.3) is given to a good approximation by:[12]

$$\frac{i\gamma'}{\Gamma_S}=i\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\ . \qquad (2.6)$$

Thus the phenomenology contains the CPV quantities m', $\mathrm{Im}A_0$, and $\mathrm{Im}A_2$. The observables can be expressed in terms of these three numbers. As a result of CP violation in the mass matrix (2.3) the mass eigenstates differ from the CP eigenstates:

$$K_S=(K_1+\tilde{\epsilon}K_2)/(1+|\tilde{\epsilon}|^2)^{1/2}\ , \qquad (2.7a)$$

$$K_L=(K_2+\tilde{\epsilon}K_1)/(1+|\tilde{\epsilon}|^2)^{1/2}\ , \qquad (2.7b)$$

$$\tilde{\epsilon}=i\frac{m'-i\gamma'/2}{\Delta M+i\Gamma_S/2}\ . \qquad (2.8)$$

One can also show[12]

$$\eta_{+-}=\epsilon+\epsilon'/(1+w/\sqrt{2})\ , \qquad (2.9a)$$

$$\eta_{00}=\epsilon-2\epsilon'/(1-\sqrt{2}w)\ , \qquad (2.9b)$$

$$\epsilon=\tilde{\epsilon}+i(\mathrm{Im}A_0/\mathrm{Re}A_0)\simeq\frac{1}{\sqrt{2}}e^{i\theta}\left[\frac{m'}{\Delta M}+\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\ , \qquad (2.10)$$

$$\epsilon'=\frac{1}{\sqrt{2}}e^{i\theta'}w\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2}-\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\ , \qquad (2.11)$$

$$w=\mathrm{Re}A_2/\mathrm{Re}A_0=0.045\ , \qquad (2.12a)$$

$$\theta=\arctan(2\Delta M/\Gamma_S)=43.67°\pm 0.14°\ , \qquad (2.12b)$$

$$\theta'=\delta_2-\delta_0+\frac{\pi}{2}=48°\pm 8°\ . \qquad (2.12c)$$

The numerical values are experimental results.[13,14]

One can show[12] that the asymmetry parameter δ can be expressed as

$$\delta=2\,\mathrm{Re}\tilde{\epsilon}=2\,\mathrm{Re}\epsilon\ . \qquad (2.13)$$

Equation (2.13) is well satisfied by the experiments.[12] Thus precise experimental knowledge of η_{+-} and η_{00} determines δ. Thus there are only two experimental observables to be determined from $\Delta m'$, $\mathrm{Im}A_0$, and $\mathrm{Im}A_2$.

It is worthwhile to present a physical interpretation of the parameters ϵ and ϵ'. The relevant graphs are those of Figs. 2(a) and 2(b). One source of CPV ($\tilde{\epsilon}$) is that due to mixing between bare K_L and K_S systems, Fig. 2(a). (The bare K_L is simply the K_2.) The graph of Fig. 2(a) is proportional to $X=2mM_{12}/(q^2-M_S^2)$, where M_{12} is the matrix element for mixing between the bare K^0 and $\bar{K}^0$ mesons and m is the K_0 mass. For an on-shell K_L, $q^2=M_L^2$ and X is simply $\tilde{\epsilon}=(1.6+1.6i)\times 10^{-3}$. But in evaluating graphs such as Fig. 1(a), the kaon is off shell and its propagator is large compared with $M_L^2-M_S^2$. Thus the effects of M_{12} and $\tilde{\epsilon}$ turn out to be negligible when off-shell kaons are involved. (We shall show below that $\tilde{\epsilon}\approx\epsilon$.)

Figure 2(a) shows mixing between the bare K_L and K_S states. However, there can be another effect, CPV in the $K_L\to\pi\pi$ decay matrix, Fig. 2(b). For free kaons this is proportional to ϵ' which is limited to about $10^{-3}\epsilon$. At first glance, the effect of ϵ' may be thought to be negligible. But with this direct CPV the factor X does not ap-

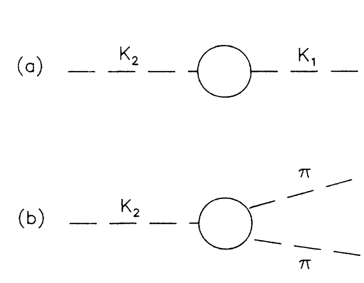

FIG. 2. Sources of CPV. (a) Mixing between bare K_L and K_S states proportional to $\tilde{\epsilon}$. (b) Direct term proportional to ϵ'.

pear. In our calculations, the influence of CPV in the decay operators dominates. The value of $|\epsilon|$ is given by[13]

$$|\epsilon| = (2.259 \pm 0.018) \times 10^{-3} . \qquad (2.14)$$

The recent CERN experiment (Burkhardt *et al.*[1]) finds

$$\mathrm{Re}\frac{\epsilon'}{\epsilon} = (3.2 \pm 1.0) \times 10^{-3} , \qquad (2.15a)$$

whereas a Fermilab experiment (Woods *et al.*[1]) publishes

$$\mathrm{Re}\frac{\epsilon'}{\epsilon} = (3.2 \pm 2.8 \pm 1.2) \times 10^{-3} . \qquad (2.15b)$$

A newer value announced at the 1989 Lepton-Photon Conference is[15]

$$\mathrm{Re}\frac{\epsilon'}{\epsilon} = (-0.5 \pm 0.9) \times 10^{-3} . \qquad (2.15c)$$

More precise values from both experiments are anticipated in the next year. Note that a precise knowledge of $|\epsilon'|$ and $|\epsilon|$ and the determination of the phases with Eqs. (2.10) and (2.11) are not sufficient to determine m', $\mathrm{Im}A_0$, and $\mathrm{Im}A_2$.

It is convenient to define the phases of A_0 and A_2 as

$$\xi \equiv \mathrm{Im}A_0/\mathrm{Re}A_0 , \qquad (2.15d)$$

$$\xi' \equiv \mathrm{Im}A_2/\mathrm{Re}A_2 . \qquad (2.15e)$$

The phases of ϵ and ϵ' are approximately equal (because $\theta \approx \theta'$), so that Eqs. (2.11) and (2.10) can be combined to yield

$$\mathrm{Re}\frac{\epsilon'}{\epsilon} = \frac{w(\xi' - \xi)}{m'/\Delta M + \xi} . \qquad (2.15f)$$

If $\xi' \ll \xi$, use Eqs. (2.10) and (2.11) to obtain

$$\xi \approx \frac{-2}{w}\mathrm{Re}\left[\frac{\epsilon'}{\epsilon}\right]\mathrm{Re}\epsilon . \qquad (2.15g)$$

Using the recent experimental values leads to

$$\xi \approx \begin{cases} -(2.2 \pm 1.0) \times 10^{-4} \text{ (CERN)}, \\ (3.3 \pm 6.0) \times 10^{-5} \text{ (Fermilab)}. \end{cases} \qquad (2.15h)$$

It is clear that measuring ϵ' is a difficult task. One reason is the appearance of the factor $w = 0.045$ in Eq. (2.11). Another possibility is that the value of ϵ' is artificially decreased by a cancellation between the $\Delta I = \frac{1}{2}$ and $\frac{3}{2}$ amplitudes ξ and ξ'. Flynn and Randall[16] study the effects of the electromagnetic penguin graphs and find that the value of ξ' can be substantial. For the extreme case of $\xi' \approx \xi$, the measurements of $\mathrm{Re}(\epsilon'/\epsilon)$ provide no constraint on either ξ or ξ'.

The difference between the values of $\tilde{\epsilon}$ and ϵ is of conceptual interest. This is because graphs, such as Fig. 2(a), that depend on mixing between *CP* eigenstates, are proportional to $\tilde{\epsilon}$ rather than the measurable ϵ. However, the measured (or implied) values of ϵ' are small enough (implying ξ is small) so that $\tilde{\epsilon}$ and ϵ are essentially the same via Eq. (2.10).

Wu and Yang[17] emphasized that the parameters m', $\mathrm{Im}A_0$, and $\mathrm{Im}A_2$ cannot be determined because it is possible to transform the phase of the strange (s) quark:

$$s \to s e^{-i\alpha} \approx s(1 - i\alpha), \quad \bar{s} \to \bar{s} e^{i\alpha} \approx \bar{s}(1 + i\alpha) . \qquad (2.16)$$

Under (2.16) one finds that any one of the three parameters can be set of zero. Wu and Yang set $\mathrm{Im}A_0$ to zero. However, theoretical formulations such as the standard model lead to nonzero values for all three parameters. We do not take any of the parameters to zero in our calculations. (We checked that the results presented below are unchanged if we adopt the Wu-Yang phase convention.) The key point is to use Eq. (2.16) for *all* of the s and $\bar{s}$ quarks.

Note again that ϵ' is suppressed by a factor of $w = 0.045$. Thus processes (other than kaon decays) which depend on $\mathrm{Im}A_0$ and $\mathrm{Im}A_2$ but without the w factor may be a fruitful source of information about CPV.

III. CPV IN πNN AND THE NEUTRON ELECTRIC DIPOLE MOMENT

As a first step, we compute the CPV coupling constants for $N \rightleftharpoons N\pi$. These can be used to compute the neutron electric dipole moment (EDM) and to estimate the CPV and TRV of the nucleon-nucleon and nuclear systems.

If one assumes that the CPV occurs among the kaons the relevant graphs are those of Figs. 3(a) and 3(b). The heavy blob contains the CPV which may be related to the parameters m' and A_I of Sec. II. The strong emission of a kaon occurs via a transition of the nucleon into a hyperon, here the Λ. However, the reconversion of the Λ into a nucleon requires a weak interaction. Hence one can already anticipate the neutron EDM (D_n) calculation. The result will be of order

$$D_n \approx \frac{e}{M_n} g_S g_W^2 \epsilon / 4\pi$$
$$\approx e(0.2 \times 10^{-13}\ \mathrm{cm}) 10 (10^{-14} \times 10^{-3})/4\pi$$
$$\approx 10^{-31}\ e\ \mathrm{cm} ,$$

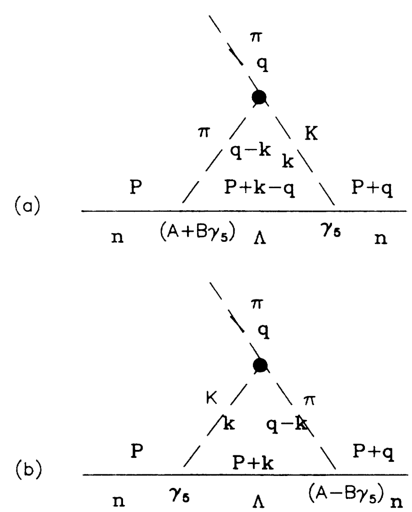

FIG. 3. Graphs for the CPV π^0-N and coupling constants.

TABLE I. Parameters used in the calculations.

Parameter	Value	Origin
$\lvert\langle K_S \lvert H \rvert \pi^+\pi^- \rangle\rvert$	3.91×10^{-4} MeV	
$\lvert\langle K_S \lvert H \rvert \pi^0\pi^0 \rangle\rvert$	2.64×10^{-4} MeV	Ref. 10
ϵ'/ϵ	See text	Refs. 1 and 15
ϵ	$(1.64+1.57i)\times10^{-3}$	Ref. 13
$G^2_{N\Lambda K}/4\pi$	13.6	Ref. 19
$G^2/4\pi$	14.3	Ref. 19
B	$10.76/\sqrt{3}$	Refs. 13 and 18
B_c	$-10.76\sqrt{2/3}$	
$\mathrm{Re}A_0$	3.4×10^{-4} MeV	
$w=\mathrm{Re}A_2/\mathrm{Re}A_0$	0.045	

in rough agreement with typical standard-model evaluations.

A. Evaluation of CPV πN couplings

The evaluation of Figs. 3(a) and 3(b) requires information about the strong, weak, and CPV meson-baryon interactions. The parameters of CPV are described first. The CPV occurs in the matrix element via a physical kaon (K) decaying into two pions, recall Eq. (2.5). Here the pions are virtual (the center-of-mass energy is less than $2m_\pi$) so that one may set the $\pi\pi$ phase shift δ_I to zero. The K^0 decay amplitude enters in Fig. 3(a) and the complex conjugate of that amplitude enters in Fig. 3(b). The strong meson (ϕ)-baryon (B) Lagrangian densities (L) that we use are

$$L_{N\Lambda K}=G_{N\Lambda K}i\bar{B}_N\gamma_5 B_\Lambda\phi_K\ ,$$
$$L_{NN\pi}=Gi\bar{B}_N\gamma_5\boldsymbol{\tau}\cdot B_N\boldsymbol{\phi}_\pi\ . \tag{3.1}$$

The weak $\Lambda\rightleftharpoons N\pi$ vertex functions are

$$L_{\Lambda N\pi}=G_F m_\pi^2\bar{B}_N(A-B\gamma_5)B_\Lambda\phi_{\pi^0}\ ,$$
$$L_{N\pi^0\Lambda}=G_F m_\pi^2\bar{B}_\Lambda(A+B\gamma_5)B_N\phi_{\pi^0}\ , \tag{3.2}$$

where G_F, the universal Fermi coupling constant is $G_F=1.03\times10^{-5}/M_p^2$. The $\Lambda\leftrightarrow p\pi^-$ interactions have the form of Eq. (3.2) but with constants A_c and B_c. The measured ratios A_c/A and B_c/B are consistent with the $(-\sqrt{2})$ of the $\Delta I=\frac{1}{2}$ rule. We use the numerical parameters given in Table I. Each of these is determined from experimental data and all the input necessary to do the calculations are now specified.

Computation of the sum M of the graphs of Figs. 3(a) and 3(b) yields

$$M=\frac{G_{nK\Lambda}}{(4\pi)^2}2(G_F m_\pi^2)\mathrm{Im}(\langle K^0|H|\pi\pi\rangle)BI\bar{u}(p+q)u(p)\ , \tag{3.3a}$$

where

$$I=\int_0^1 dx_1\int_0^{1-x_1}dx_2\frac{m_\Lambda-m_n+m_n(1-x_1-x_2)}{\{[qx_2+(p+q)(1-x_1-x_2)]^2+m_k^2x_1+(m_\pi^2-q^2)x_2+(m_\Lambda^2-m_n^2)(1-x_1-x_2)]\}}\ . \tag{3.3b}$$

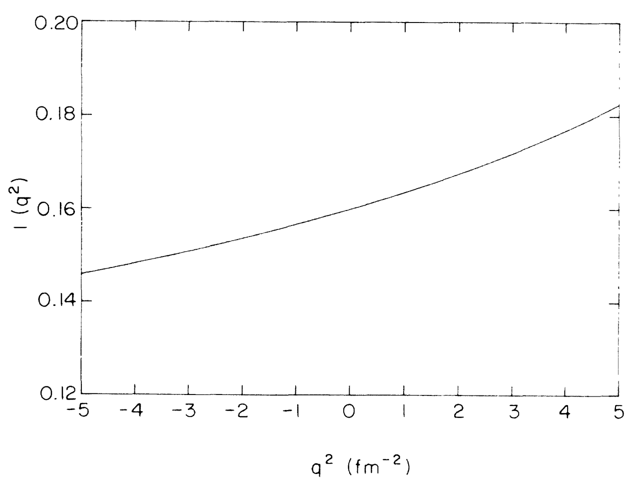

FIG. 4. Variation of $I(q^2)$ [Eq. (3.3)] with q^2.

The integrals are convergent, since the only potential logarithmic divergence goes as $\int d^4k\, k_\mu F(k^2)=0$. Numerical evaluation yields $I=0.16$ fm at $q^2=0.0$. There is only a slow variation with q^2, see Fig. 4. (Note that the term of Fig. 3 proportional to A goes as $i\gamma_5$ and is merely a second-order weak correction to the usual strong interaction.)

To proceed we must relate $\mathrm{Im}\langle K^0|H|\pi\pi\rangle$ to $\mathrm{Im}A_I$. One may use (2.1) and (2.5) with $\delta_I=0$ to show that

$$\langle K^0|H|\pi^+\pi^-\rangle=\sqrt{2/3}(A_0+A_2/\sqrt{2})\ , \tag{3.4a}$$
$$\langle K^0|H|\pi^0\pi^0\rangle=-\sqrt{1/3}(A_0-\sqrt{2}A_2)\ . \tag{3.4b}$$

The use of (3.4a), Table I and the value $\mathrm{Re}A_2/\mathrm{Re}A_0=0.045$ gives $\mathrm{Re}A_0\approx3.38\times10^{-4}$ MeV. A similar use of Eq. (3.4b) yields $\mathrm{Re}A_0\approx3.25\times10^{-4}$ MeV. Using Eqs. (3.4) and measured rates for $K_S\to\pi\pi$ leads to $\mathrm{Re}A_2/\mathrm{Re}A_0=0.032$ and $\mathrm{Re}A_0\approx3.4\times10^{-4}$ MeV. To be definite we use the values of Table I.

We may now put everything together:

$$M(\pi^+ n \to p) = \frac{G_{nK\Lambda}}{(4\pi)^2} 2(\mathrm{Re}\,A_0) G_F m_\pi^2 \left[\frac{2}{3}\right]^{1/2} \left[\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0} + \left[\frac{1}{2}\right]^{1/2} \frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} w\right] B_c I \bar{u}(p+q) u(p) \ , \tag{3.5a}$$

$$M(\pi^0 n \to n) = -\frac{G_{nK\Lambda}}{(4\pi)^2} 2(\mathrm{Re}\,A_0) \frac{G_F m_\pi^2}{\sqrt{3}} \left[\frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0} - \sqrt{2} \frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} w\right] B I \bar{u}(p+q) u(p) \ . \tag{3.5b}$$

Thus,

$$M(\pi^+ n \to p) = -6.9 \times 10^{-14} \left[\xi + \frac{\xi'}{\sqrt{2}} w\right] \ , \tag{3.6a}$$

$$M(\pi^0 n \to n) = 3.5 \times 10^{-14} (\xi - \sqrt{2} \xi' w) \ . \tag{3.6b}$$

To obtain the πN CPV coupling constants one needs values of ξ and ξ'. However, no data exist. The equation for ϵ',

$$\epsilon' = \frac{1}{\sqrt{2}} w e^{i\theta'} (\xi' - \xi) \ , \tag{3.7}$$

or $|\epsilon'| \approx w/\sqrt{2} |\xi' - \xi|$, depends on a different linear combination of ξ and ξ' than appearing in Eq. (3.6). Thus the neutral K data do not determine g_{CPV}. However, one can make a series of estimates.

In the standard model (SM) with penguin dominance $|\xi| \gg |\xi'|$ and $|\epsilon'| \neq 0$. Then $\xi = \xi^{\mathrm{SM}}$ and is given by Eq. (2.15h). This gives a CPV pion-nucleon coupling constant defined as $g_{\mathrm{CPV}}^{\mathrm{SM}}$:

$$\begin{aligned} g_{\mathrm{CPV}}^{\mathrm{SM}}(\pi^+ n, p) &= -1.5 \times 10^{-17} \quad (\mathrm{CERN}) \ , \\ g_{\mathrm{CPV}}^{\mathrm{SM}}(\pi^+ n, p) &= (2.3 \pm 4.2) \times 10^{-18} \quad (\mathrm{Fermilab}) \ , \end{aligned} \tag{3.8}$$

if the CERN or the Fermilab value of Eq. (2.15h) is used. It may turn out that $|\epsilon'|/|\epsilon| \approx 0$, but with ξ and ξ' each not zero. Indeed if $\xi = \xi'$ then there are no experimental limits on either. Then how large can ξ possibly be? The relevant standard-model phase is $\mathrm{Im} C_5 \le 2 \times 10^{-4}$ in the notation of Ref. 9. Then the upper limit is essentially the same as that of the CERN value of Eq. (2.15h). One can make a more optimistic hypothesis. Suppose the CPV $K \to 2\pi$ decay is enhanced due to the presence of the nucleon. This is possible, see Fig. 5, if the nucleon has a significant $s\bar{s}$ content.[20] Obtaining a reasonable estimate of such an effect is beyond the scope of this work. But one can make a wild guess (WG) that there is an enhancement by 10 over the standard-model phase. Then one obtains $\xi = \xi^{\mathrm{WG}}$:

$$\begin{aligned} \xi^{\mathrm{WG}} &= 2 \times 10^{-3} \\ &\text{and} \\ g_{\mathrm{CPV}}^{\mathrm{WG}}(\pi^+ n, p) &= 1.5 \times 10^{-16} \ . \end{aligned} \tag{3.9}$$

The wild-guess value would also be obtained if $\xi' - \xi \approx \pm 0.1 \xi$. Equations (3.8) and (3.9) are chosen to yield relatively large values of ξ and g_{CPV}. However, the possibility

$$\xi + \frac{\xi'}{\sqrt{2}} w = 0 \tag{3.10a}$$

is not ruled out. In that case one obtains at lower limit (LL), the pessimistic value

$$g_{\mathrm{CPV}}^{\mathrm{LL}}(\pi^+ n, p) = 0 \ . \tag{3.10b}$$

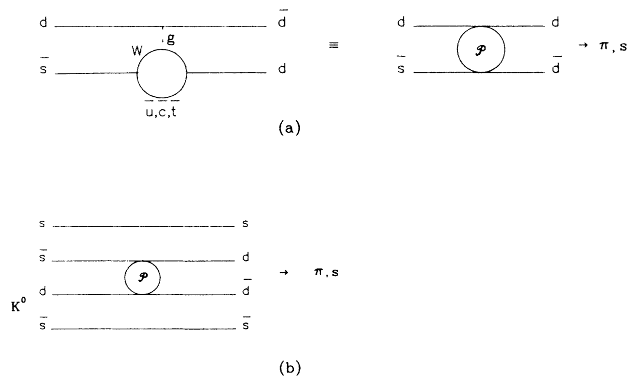

FIG. 5. Enhancement of $K \to 2\pi$ due to nucleonic $s\bar{s}$ pairs. (a) Standard penguin diagram. (b) Possible nucleonic enhancement.

B. Evaluating the neutron electric dipole moment

The next step is to compute the neutron electric dipole moment D_n. We proceed by using the values of g_{CPV} of Eqs. (3.7)–(3.10) to compute the graph of Fig. 1(b). The estimates of Crewther *et al.*[21] and Morgan and Miller[22] are relevant. These works are motivated by term $\bar{\theta}$ of QCD. The value of the CPV πNN vertex function (g_{CPV}) is obtained in terms of $\bar{\theta}$. Then D_n is computed in terms of $g_{\mathrm{CPV}}(\pi^+ n, p)$. Hence we can use our values of $g_{\mathrm{CPV}}(\pi^+ n, p)$ in the expressions of Refs. 21 and 22. Crewther *et al.*[21] use a current-algebra estimate. Then in our notation their result is

$$\begin{aligned} D_n^C &= \frac{G}{4\pi^2} \frac{g^{\mathrm{CPV}}(\pi^+ n, p)}{\sqrt{2}} M_n \ln \left[\frac{M_n}{M_\pi}\right] \\ &= 9.6 \times 10^{-15} g^{\mathrm{CPV}}(\pi^+ n, p) e \ \mathrm{cm} \ . \end{aligned} \tag{3.11a}$$

Morgan and Miller[22] make use of the cloudy bag model[23] which allows one to relate the pionic properties of the nucleon to observables. In Ref. 22 both quark and pion terms yield contributions to D_n, but only the pionic term (D_n^π) is relevant here. D_n^π of Ref. 21 differs from that of Ref. 20 in two ways: terms of all order in m_π are kept and, the radius R of the three-quark nucleon bag provides a cutoff for the integral. These effects reduce the magnitude of the computed D_n^π. The result is about half as much as that of Crewther *et al.* For $R = 0.6$ fm,

$$D_n^{\pi} \approx 4.8 \times 10^{-15} g^{\rm CPV}(\pi^+ n,p) e \text{ cm} . \quad (3.11b)$$

We use both values in (3.8b), (3.9), and (3.10b) to provide a range of estimates of D_n:

$$\begin{aligned} D_n &\approx (5-10) \times 10^{-15} g^{\rm CPV}(\pi N,N) \\ &\approx (3.5-7) \times 10^{-28} (\xi + \xi' \omega \sqrt{2}) \\ &\approx (3.5-7) \times 10^{-28} \xi . \end{aligned} \quad (3.12)$$

The values of ξ from Eqs. (2.15h), (3.9), and (3.10a) yield

$$\begin{aligned} D^{\rm SM} &= -(7-14) \times 10^{-32} \ e \text{ cm} \quad (\text{CERN}) , \\ D^{\rm SM} &= (1 \pm 2) \times 10^{-32} \ e \text{ cm} \quad (\text{CERN}) , \\ D^{\rm WG} &= (7-14) \times 10^{-31} \ e \text{ cm} , \\ D^{\rm LL} &= 0 . \end{aligned} \quad (3.13)$$

Using the current experimental upper limit (UL) of $D = D^{\rm UL} = (3 \times 10^{-25}\ e \text{ cm})$ in the third line of Eq. (3.12), an upper limit on $\xi = \xi^{\rm UL}$ can be obtained. The value of $\xi^{\rm UL} \approx 1000 \gg 2\pi$. But the measurements of ϵ' already constrain ξ to be very small. Thus the current upper limits on D_n do not place a useful constraint on ξ.

The value of $D^{\rm SM}$ is consistent with the standard-model estimates using quarks. The large value in Eq. (3.12) is at the upper limit value of Ref. 7. This is as it should be, since Eqs. (3.7) and the larger value of (3.13) are based on the assumption of a relatively large value of $|\epsilon'|$. Thus, we provide a confirmation of the quark-based standard-model estimates of D_n.

IV. CPV IN Λ AND $\bar{\Lambda}$ DECAY

The basic reason for investigation of these effects can be seen by comparing Fig. 6(a) with Fig. 1(a). One immediately sees $G^{\rm CPV}(\Lambda, N\pi)/G^{\rm CPV}(N,N\pi) \sim \frac{1}{2}(G/G_F m_\pi^2) \approx 10^7$, since two strong interactions appear in Λ decay. An immediate consequence of such a CPV coupling constant is the existence of a time-reversal-violating TRV signal in the weak decays $\Lambda \to N\pi$. However, such effects are masked by the influence of $N\pi$ final-state-interaction effects. A better way is to compare Λ and $\bar{\Lambda}$ decays. In principle, this can be done using $p\bar{p}$ collisions.[9,11] Plans to carry out a dedicated experiment are in progress.[11]

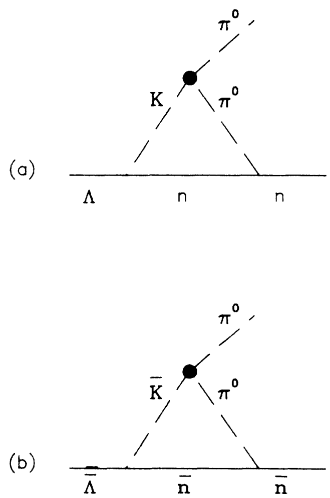

FIG. 6. S-wave terms in $\Lambda \to N\pi$, $\bar{\Lambda} \to \bar{N}\pi$ decays. CPV occurs in the blob.

A. General formalism

The formulas necessary to compute CPV in nonleptonic hyperon decays are given by Donoghue, He, and Pakvasa[9] (DHP). We use their formulas and notation and begin by presenting a brief summary.

The amplitude M for the hyperonic nonleptonic decay $B^a \to B^b \pi^c$ is a sum of a parity-violating S-wave term $S(B_c^a)$ and a parity-conserving P-wave term $P(B_c^a)$:

$$M(B^a \to B^b \pi^c) = S(B_c^a) + P(B_c^a) \sigma \cdot \hat{\mathbf{q}} . \quad (4.1)$$

Here a,b,c refer to charges and $\mathbf{q}$ is the momentum of the final baryon in the B^a rest frame. The experimental observables are the total decay rate Γ and α,β which determine the decay angular distributions and polarization of the final baryon. These are given in terms of S and P by

$$\begin{aligned} \alpha &= 2\,\mathrm{Re} S^* P / (|S|^2 + |P|^2) , \\ \beta &= 2\,\mathrm{Im} S^* P / (|S|^2 + |P|^2) . \end{aligned} \quad (4.2)$$

For antihyperon decays one uses the parameters $\bar{S}$, $\bar{P}$, $\bar{\alpha}$, and $\bar{\beta}$.

The following quantities vanish if CP is conserved:

$$\begin{aligned} \Delta &= \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}, \quad A = \frac{\Gamma\alpha + \bar{\Gamma}\bar{\alpha}}{\Gamma\alpha - \bar{\Gamma}\bar{\alpha}} \approx \frac{\alpha + \bar{\alpha}}{\alpha - \bar{\alpha}} + \Delta , \\ B &= \frac{\Gamma\beta + \bar{\Gamma}\bar{\beta}}{\Gamma\beta - \bar{\Gamma}\bar{\beta}} = \frac{\beta + \bar{\beta}}{\beta - \bar{\beta}} + \Delta . \end{aligned} \quad (4.3)$$

DHP parametrize the decay amplitudes as

$$S = \sum_i S_i e^{i(\delta_i + \phi_i^S)}, \quad P = \sum_i P_i e^{i(\delta_i + \phi_i^P)} , \quad (4.4)$$

in which i denotes the possible final isospin state, S_i and P_i are real, δ_i is the strong final-state-interaction phase. The phases $\phi_i^{S,P}$ contain the information about CPV. If these are zero $\bar{S} = -S$ and $\bar{P} = P$, and the observables of Eq. (4.3) would vanish. Under the CP transformation $S_i \phi_i^S \to +S_i \phi_i^S$ and $P_i \phi_i^P \to -P_i \phi_i^P$ so that

$$\bar{S} = -\sum_i S_i e^{i(\delta_i - \phi_i^S)}, \quad \bar{P} = \sum_i P_i e^{i(\delta_i - \phi_i^P)} . \quad (4.5)$$

DHP give expressions for all of the nonleptonic hyperon decays. We consider only the Λ decays: $\Lambda \to p\pi^-$ (Λ_-^0) and $\Lambda \to n\pi^0$ (Λ_0^0) here. Then[9]

$$S(\Lambda_-^0) = -\sqrt{2/3} S_{11} e^{i(\delta_1 + \phi_1^S)} + \sqrt{1/3} S_{33} e^{i(\delta_3 + \phi_3^S)} , \quad (4.6)$$

$$P(\Lambda_-^0) = -\sqrt{2/3} P_{11} e^{i(\delta_{11} + \phi_1^P)} + \sqrt{1/3} P_{33} e^{i(\delta_{31} + \phi_3^P)} ,$$

where $S_{ij}\equiv S_{2\Delta I,2I}$, $P_{ij}\equiv P_{2\Delta I,2I}$, $\phi_i^{S,P}=\phi_{2I}^{S,P}$, $\delta_I=\delta_{2I}^S$, $\delta_{I1}=\delta_{2I}^P$, ΔI is the magnitude of the change in baryon isospin, and I is the final π-baryon isospin. The values of the parameters are summarized by[9]

$$\delta_1=6.0^\circ,\quad \delta_{11}=-1.1^\circ\ ,$$
$$\delta=-3.8^\circ,\quad \delta_{31}=-0.7^\circ\ , \tag{4.7}$$
$$\frac{S_{33}}{S_{11}}=\frac{-0.3}{32.8},\quad \frac{P_{33}}{P_{11}}=-\frac{0.06}{12.4}\ .$$

It is therefore an excellent approximation to take

$$S(\Lambda_-^0)=-\sqrt{2/3}S_{11}e^{i(\delta_1+\phi_1^S)}=-\sqrt{2}S(\Lambda_0^0)\ ,$$
$$P(\Lambda_-^0)=-\sqrt{2/3}P_{11}e^{i(\delta_{11}+\phi_1^P)}=-\sqrt{2}P(\Lambda_0^0)\ . \tag{4.8}$$

Then one finds

$$\Delta(\Lambda_-^0)=\sqrt{2}(S_{33}/S_{11})\sin(\delta_3-\delta_1)\sin(\phi_3^S-\phi_1^S)\ ,$$
$$A(\Lambda_-^0)=-\tan(\delta_{11}-\delta_1)\sin(\phi_1^P-\phi_1^S)=A(\Lambda_0^0)\ , \tag{4.9}$$
$$B(\Lambda_-^0)=\cot(\delta_{11}-\delta_1)\sin(\phi_1^P-\phi_1^S)=B(\Lambda_0^0)\ .$$

The term Δ has three very small factors, so we concentrate on A and B. Note that $B=-64.5A$.

The problem of computing CPV reduces to that of computing ϕ_1^S and ϕ_1^P. We have

$$S=-\sqrt{2/3}S_{11}e^{i\delta_1}(1+i\phi_1^S)\ ,$$
$$\bar{S}=+\sqrt{2/3}S_{11}e^{i\delta_1}(1-i\phi_1^S)\ , \tag{4.10}$$

so that

$$i\phi_1^S=\frac{S+\bar{S}}{S-\bar{S}}\ . \tag{4.11}$$

Similarly,

$$P=-\sqrt{2/3}P_{11}e^{i\delta_{11}}(1+i\phi_1^P)\ ,$$
$$\bar{P}=\sqrt{2/3}P_{11}e^{i\delta_{11}}(1-i\phi_1^P)\ , \tag{4.12}$$
$$i\phi_1^P=\frac{P-\bar{P}}{P+\bar{P}}\ .$$

The parity-violating (ϕ_1^S) and parity-conserving (ϕ_1^P) CPV phases have different physical origins, and are treated separately. Indeed since these are separately small, we may write [using (4.7) and (4.9)]

$$B=B_S+B_p,\quad B_S=-8.0\phi_1^S,\quad B_P=8.0\phi_1^P\ . \tag{4.13}$$

B. Computing ϕ_1^S

The S-wave terms arise as in Figs. 6(a) and 6(b). The effects of the two strong $i\gamma_5$ couplings lead to an S-wave amplitude since $\gamma_5^2=1$.

In our approach the CPV difference between Λ and $\bar{\Lambda}$ decays arise from CPV in the $K\to\pi\pi$ and $\bar{K}\to\pi\pi$ amplitudes. Recall that

$$\langle K|H|\pi\pi(I)\rangle=A_Ie^{i\delta_I}\ ,$$
$$\langle \bar{K}|H|\pi\pi(I)\rangle=-A_I^*e^{i\delta_I}\ . \tag{4.14}$$

Using CP conservation of the strong amplitudes and setting $\delta_I=0$ for the neutral $\pi\pi$ system at subthreshold energies one finds that the amplitudes for $\Lambda\to n\pi^0$ and $\bar{\Lambda}\to\bar{n}\pi^0$ can be written as

$$S=FA_0+S_{\rm CPC}\ ,$$
$$\bar{S}=-FA_0^*-S_{\rm CPC}\ . \tag{4.15}$$

Here F stands for the magnitude of the Feynman diagram of Fig. 6(a) or Fig. 6(b), not including the A_0 factor. The $\Delta I=\frac{1}{2}$ rule is used in obtaining Eq. (4.15). The effects of other CP-conserving (CPC) diagrams are denoted as $S_{\rm CPC}$. The use of Eqs. (4.12) and (4.15) to compute ϕ_1^S yields

$$\phi_1^S=\frac{F\,{\rm Im}\,A_0}{S_{11}^{\rm expt}}=\frac{F\,{\rm Re}\,A_0}{S_{11}^{\rm expt}}\xi\ . \tag{4.16a}$$

The experimentally measured value of $S_{\rm CPC}$ ($S_{11}^{\rm expt}$) appears in the denominator. One knows[9]

$$S_{11}^{\rm expt}=-40\times10^{-8}\ . \tag{4.16b}$$

The next step is to compute the Feynman-diagram term $(F){\rm Re}(A_0)$. The possible values of ξ are given in Eqs. (2.15), (3.9), and (3.10). A straightforward evaluation yields

$$F=-\frac{1}{\sqrt{3}}\frac{1}{(4\pi)^2}g_{\Lambda KN}g_{n\pi\pi}\bar{u}_n(p+q)u_\Lambda(p)J\ , \tag{4.17a}$$

with

$$J=\int_0^1dx_1\int_0^{1-x_1}dx_2\frac{m_\Lambda x_1-m_n(1-x_2)}{m_n^2(1-x_1-x_2)(1-x_2)-m_\Lambda^2x_1(1-x_1-x_2)+m_K^2x_1+m_\pi^2x_2-q^2x_1x_1}\ . \tag{4.17b}$$

At $q^2=0$, $J=0.12$ fm. Then using the numbers in Table I we find

$$\phi_1^S=0.33\xi \tag{4.18}$$

and [from (4.13)] $B^S=2.6\xi$. If $\xi=\xi^{\rm SM}$ [Eq. (2.15h)] $B^S=B_{\rm SM}^S$,

$$B_{\rm SM}^S=-5.8\times10^{-4}\ \ ({\rm CERN})\ ,$$
$$B_{\rm SM}^S=(9.0\pm16)\times10^{-5}\ \ ({\rm Fermilab})\ . \tag{4.19}$$

If $\xi=\xi^{\rm WG}$ [Eq. (3.9)] $B^S=B_{\rm WG}^S$

$$B_{\rm WG}^S=5.8\times10^{-3}\ . \tag{4.20}$$

Of course, the value $\xi=0$ is not ruled out so a lower limit (LL) is

$$|B_{\rm LL}^S|=0.0\ . \tag{4.21}$$

Note that the "standard-model" value $B_{\rm SM}^S$ (with CERN data) is about five times larger than the DHP value.

If the small measured value of ϵ'/ϵ is due to a cancellation between ξ' and ξ, the value of B could approach the magnitude of the wild-guess values.

C. Parity-conserving CPV

In our approach the P-wave (CPV) phase ϕ_1^P arises from $K\rightarrow 3\pi$ amplitude as in Fig. 7. Three γ_5 operators act on the baryons, so the net result is the single γ_5 needed for a parity-conserving vertex. We may then write the P-wave amplitudes for $\Lambda\rightarrow\Lambda\pi^0$, $\bar\Lambda\rightarrow\bar\Lambda\pi^0$ as

$$\begin{aligned} P&=F_a\langle K|H|\pi\pi\pi\rangle+F_b\langle K|H|\pi\pi\pi\rangle^*+P_{\rm CPC}\ ,\\ \bar P&=F_a\langle \bar K|H|\pi\pi\pi\rangle+F_b\langle \bar K|H|\pi\pi\pi\rangle^*+P_{\rm CPC}\ , \end{aligned} \tag{4.22}$$

in which the CP-conserving terms are $P_{\rm CPC}$. The diagram of Fig. 7(a) is written as $F_a\langle K|H|\pi\pi\pi\rangle$, and that for Fig. 7(b) is $F_b\langle \bar K|H|\pi\pi\pi\rangle^*$. Another graph, that of Fig. 7(c) does not contribute to CPV because of a cancellation. Then we find, with Eq. (4.12),

$$i\phi_1^P=\frac{F_a\langle K_1|H|\pi\pi\pi\rangle+F_b\langle K_1|H|\pi\pi\pi\rangle^*}{\sqrt{2}P_{\rm CPC}}\ . \tag{4.23}$$

Recall that

$$|K_1\rangle=\frac{1}{\sqrt{2}}(|K\rangle-|\bar K\rangle)=|K_S\rangle-\bar\epsilon|K_L\rangle\ , \tag{4.24}$$

so

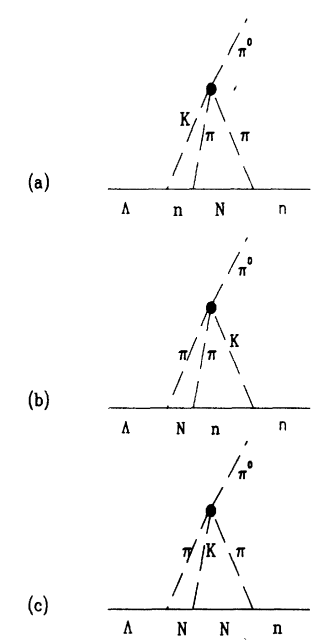

FIG. 7. P-wave terms in $\Lambda\rightarrow N\pi$, $\bar\Lambda\rightarrow\bar N\pi$ decays. CPV occurs in the blob.

$$\begin{aligned} i\phi_1^P=&-\frac{\sqrt{2}}{P_{\rm CPC}}i\,{\rm Im}(\bar\epsilon)\langle K_L|H|\pi\pi\pi\rangle\\ &+\frac{F_a\langle K_S|H|\pi\pi\pi\rangle+F_b\langle K_S|H|\pi\pi\pi\rangle^*}{\sqrt{2}P_{\rm CPC}}\ . \end{aligned} \tag{4.25}$$

Since $\bar\epsilon$ and $\langle K_L|H|\pi\pi\pi\rangle$ are measured, it is tempting to keep only the first term of Eq. (4.16). This yields a negligible result, if the effects of the virtuality of the K_L are included: In free space, the mixing between K_L and K_S can be expressed as the conversion of a bare $K_L(K_2)$ into a bare $K_S(K_1)$ as in Fig. 2(a). The graph is

$$\begin{aligned} &\frac{2mM_{12}}{q^2-m_s^2+i(\Gamma_S m/2)}\langle K_S|H|\pi\pi\rangle\\ &\quad=\bar\epsilon\frac{m_L^2-m_S^2+i(\Gamma_S m/2)}{q^2-m_S^2+i(\Gamma_S m/2)}\langle K_S|H|\pi\pi\rangle\ . \end{aligned} \tag{4.26}$$

For on-shell kaons $q^2=m_L^2$ and the term is proportional to $\bar\epsilon$ as in Sec. II. However, for off-shell kaons the factor

$$\frac{m_L^2-m_S^2+i\Gamma_S m/2}{q^2-m_S^2+i\Gamma_S m/2}\equiv F \tag{4.27}$$

essentially vanishes because the numerator is so small. The rare opportunity of seeing CPV in K_L-K_S mixing in free space arises from the small energy denominator. This denominator is absent when kaons are virtual. Numerical calculation shows that including the effects of the factor F of Eq. (4.27) causes a suppression by a factor of 1.5×10^{-17}.

Neglecting the effects of K^0-$\bar K^0$ mixing in Eq. (4.25) leads to

$$i\phi_1^P=-\frac{F_a\langle K_1|H|\pi\pi\pi\rangle+F_b\langle K_1|H|\pi\pi\pi\rangle^*}{\sqrt{2}P_{\rm CPC}}\ . \tag{4.28}$$

We need the value of $P_{\rm CPC}(P_{11}^{\rm expt})$ which appears in the denominator. One knows[9]

$$P_{11}^{\rm expt}=-15\times10^{-8}\ . \tag{4.29}$$

The next step is to estimate $\langle K_1|H|\pi\pi\pi\rangle$. Since K_1 is a pure $CP=1$ eigenstate, $\langle K_1|H|\pi\pi\pi\rangle=0$ for three $l=0$ pions. The CPV occurs in $H|\pi\pi\pi\rangle$, so that $\langle K_1|H|\pi\pi\pi\rangle$ is analogous to ϵ' of the two-pion decay. The quantity is accessible in the observation of the time dependence of the decay rate of an initially pure K^0 and/or $\bar K^0$ beams. At present no signature for $K^0\rightarrow3\pi$ events have been found.[13]

We repeat the Particle Data Group[13] (PDG) analysis in order to obtain reasonable estimates. For $K_S\rightarrow3\pi$, the ratios of amplitude

$$\begin{aligned} \eta_{+-0}&=\frac{A(K_S\rightarrow\pi^+\pi^-\pi^0)}{A(K_L\rightarrow\pi^+\pi^-\pi^0)}\ ,\\ \eta_{000}&=\frac{A(K_S\rightarrow\pi^0\pi^0\pi^0)}{A(K_L\rightarrow\pi^0\pi^0\pi^0)} \end{aligned} \tag{4.30}$$

are the quantities that measure CPV. If CPT invariance holds and there are no $\Delta I=\frac{5}{2}$ or $\frac{7}{2}$ transitions, then

$\mathrm{Re}\eta_{+-0}$ can be neglected. Furthermore $\mathrm{Im}\eta_{+-0} = \mathrm{Im}\eta_{000} \equiv \mathrm{Im}\eta$. Then[24]

$$\mathrm{Im}\eta_{+-0} = \frac{\sqrt{\Gamma(K_S \to \pi^+\pi^-\pi^0)}}{\sqrt{\Gamma(K_L \to \pi^+\pi^-\pi^0)}} < 0.12 \quad \text{(Ref. 25)}$$
$$\mathrm{Im}\eta_{000} = \frac{\sqrt{\Gamma(K_S \to \pi^0\pi^0\pi^0)}}{\sqrt{\Gamma(K_L \to \pi^0\pi^0\pi^0)}} < 0.23 \quad \text{(Ref. 23)} . \qquad (4.31)$$

Since K_S contains an ϵ admixture of K_L one expects $\mathrm{Im}\eta \equiv \mathrm{Im}\epsilon = 1.6 \times 10^{-3}$. Eq. (4.33) shows that the currently available experimental upper limits are about 100 times larger than this expectation. As explained above, effects proportional to $\tilde{\epsilon}$ and ϵ are suppressed. Thus we neglect the effects of $\tilde{\epsilon}$. However, effects of CPV in the K^0 decay matrix can contribute and are included. Then[26]

$$\mathrm{Im}\eta = \frac{\mathrm{Im}A_1}{\mathrm{Re}A_1} , \qquad (4.32)$$

where A_1 is the amplitude for the K^0 to decay to a three-pion final state of isospin 1. Since $\mathrm{Im}\eta$ is not measured, we need an additional assumption to proceed. It seems reasonable to assume that the CPV amplitude for $K_L \to 3\pi$ proceeds by a CPV $K \to \pi\pi$ decay followed by a PV $\pi\pi \to \pi\pi\pi$ process. See Fig. 8. This gives

$$\mathrm{Im}\eta = \frac{\mathrm{Im}A_1}{\mathrm{Re}A_1} \approx \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} G_F \Lambda^2 \approx \xi G_F \Lambda^2 , \qquad (4.33a)$$

where we have assumed that $\mathrm{Im}A_1/\mathrm{Re}A_1 \approx \mathrm{Im}A_0/\mathrm{Re}A_0$ and Λ is an hadronic appropriate mass scale. The scale Λ may be expected to lie in the range 500–1000 MeV. Then $2.5 \times 10^{-6} \times G_F \Lambda^2 < 10^{-5}$ and $5 \times 10^{-10} < \mathrm{Im}\eta$ if the CERN value (Eq. 2.15) of ξ is used. This implies that

$$\mathrm{Im}\eta = (12 \pm 8) \times 10^{-10} . \qquad (4.33b)$$

This estimate arises from the notion that the standard model would predict CPV phases in the 3π decay, Fig. 8.

The value above is meant only to serve as a preliminary order-of-magnitude estimate. One can determine the value of $\mathrm{Im}A_1$ using current-algebra techniques.[25] First define the amplitude

$$\mathcal{M} = \langle K^0 | \mathcal{H}_W^{\mathrm{PC}} | \pi^+\pi^-\pi^0 \rangle , \qquad (4.34)$$

so that

$$A_1 = \sqrt{2/3}\mathcal{M} . \qquad (4.35)$$

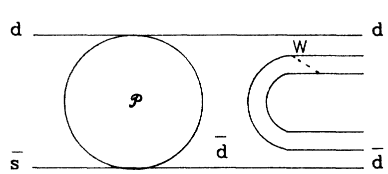

FIG. 8. CPV in $K_1 \to 3\pi$ decay.

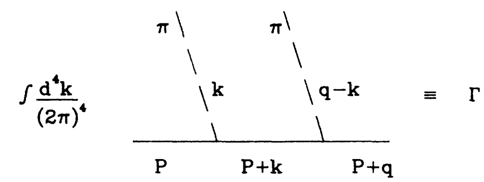

FIG. 9. Two-pion-absorption part of the graphs of Fig. 7.

The charged pions are now projected onto an isospin zero state. The use of PCAC (partial conservation of axial-vector current) to remove the π^0 leads to

$$\mathcal{M} = \langle K^0 | \mathcal{H}_W^{\mathrm{PV}} | \pi^+\pi^- \rangle i/2f_\pi , \qquad (4.36)$$

where f_π is the pion decay constant (≈ 93 MeV). The above result is obtained from the left-handed nature of the weak Hamiltonian using the arguments of Ref. 25. The K^0 decay matrix elements are parametrized by ξ (imaginary part) and in the table. Thus we immediately have

$$\langle K_1 | H | \pi\pi\pi \rangle = 2.8i \times 10^{-6} \xi . \qquad (4.37)$$

To proceed further we evaluate the graphs of Figs. 7(a) and 7(b). A simple approximate evaluation should be sufficient, given the uncertainties inherent in obtaining the unmeasured CPV matrix element for the K_1 to decay into three pions, Eq. (4.37). The first step is to isolate the part of the graph containing two pions, Fig. 9. The quantity Γ is given by

$$\Gamma = \int \frac{d^4k}{(2\pi)^4} \bar{u}(p+q) i g \gamma_5 \frac{1}{\gamma(p+k) - m_n} i g \gamma_5 u(p) \times \frac{i}{k^2 - m_\pi^2} \frac{i}{k^2 - m_\pi^2} . \qquad (4.38)$$

Explicit numerical evaluation shows that using

$$\Gamma = i \left[\frac{g}{4\pi} \right]^2 \frac{m_n}{q^2 - m_n^2} \qquad (4.39)$$

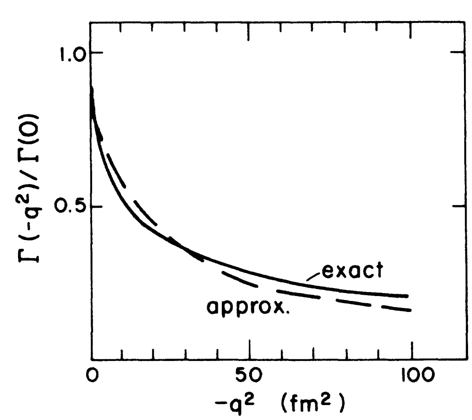

FIG. 10. Comparison of Eq. (4.37) exact and Eq. (4.38) approximate.

is an excellent approximation for $-4m_n^2<q^2<0$. See Fig. 10. For large values of $-q^2$ the integral goes like $1/\ln(Q^2)$. However, the nucleon degrees of freedom should become irrelevant for such large values. At high $-q^2$ Eq. (4.39) represents a mathematically artificial, but physically reasonable cutoff. We therefore use Eq. (4.39) for all negative values of q^2.

The use of Eq. (4.39) allows us to obtain

$$F_a=\frac{g_{nK\Lambda}}{(4\pi)^2}\left[\frac{g}{4\pi}\right]^2 m_n I ,$$
$$I=\int_0^1 dx_1\int_0^{1-x_1}dx_2\frac{m_\Lambda x_1-m_n(1-x_2)}{m_n^2(1-x_1-x_2)(1-x_2)-m_\Lambda^2(1-x_2-x_2)x_1+m_K^2x_1+m_n^2x_2-m_\pi^2x_1x_2} ,$$
$$F_b=\frac{g_{nK\Lambda}}{(4\pi)^2}\left[\frac{g}{4\pi}\right]^2 m_n J , \tag{4.40}$$
$$J=\int_0^1 dx_1\int_0^{1-x_1}dx_2\frac{m_\Lambda x_1-m_n x_1+m_\Lambda x_2}{(m_\Lambda^2-m_K^2x_1-m_\Lambda^2x_2)(1-x_1-x_2)+m_K^2x_1+m_n^2x_2+m_\pi^2x_1x_2} .$$

Numerical evaluation yields

$$I=-0.051\ \text{fm},\quad J=0.19\ \text{fm} , \tag{4.41}$$

so

$$F_a=-0.023,\quad F_b=0.086 . \tag{4.42}$$

At this stage all of the elements required to evaluate ϕ_1^P of Eq. (4.28) are available. Putting the value of the loop integrals (4.41), the coupling constants of the table and the current-algebra value of $\mathrm{Im}A_1$ yields

$$\phi_1^P=2.1\xi . \tag{4.43}$$

This gives

$$B_P=17\xi . \tag{4.44}$$

The evaluation of B_P using the values of ξ of Eq. (2.15) is left for the concluding section.

V. STRONG-INTERACTION UNCERTAINTIES

Our procedure is based on the use of hadronic degrees of freedom. This means that the kaon CPV matrix elements are assumed to have little variation as the kaon goes off its mass shell. But there are no experimental constraints on the on-shell values of ξ and ξ'. Thus the use of on-shell matrix elements is a very significant assumption. Another possible problem is that our graphs seem to represent the first term of a perturbation expansion in the strong coupling constant. For example, terms such as Fig. 11 are ignored. There are phenomenological and theoretical ways of testing the assumptions.

The phenomenological method is to compute ordinary weak decay rates with the same mechanism. Since real pions cannot be emitted from free neutrons, one can look at the β decay via the graph of Fig. 12(a). The β decay of the neutron is already understood in terms of CVC and computed vector couplings. Hence, the check is that term of Fig. 12(a) yields a small contribution to the β decay amplitude. A simple estimate suffices. Compare the rate for Fig. 12(a) to that of a more conventional term Fig. 12(b). One finds

$$\frac{\text{Fig. 12(b)}}{\text{Fig. 12(a)}}\sim\frac{\langle K|H|e\nu\rangle}{\langle\pi|H|e\nu\rangle}\frac{G_F m_\pi^2}{4\pi^2}\approx 10^{-8} , \tag{5.1}$$

where $G_F m_\pi^2$ enters because of the weak πNN vertex. The $1/4\pi^2$ enters because of the loop. In other words, Fig. 12(b) is a second-order weak process, so no difficulty arises.

One can make similar checks that mechanisms do not yield unrealistically large *CP*-conserving nonleptonic Λ decays. Consider the *S*-wave term. The *CP*-conserving (CPC) term of Fig. 3(a) is $(S-\bar{S})/2\approx \mathrm{Re}A_0F=13\times10^{-8}<40\times10^{-8}$. Here the last quantity is the measured value. Thus in our phenomology CPV and CPC terms arise from different sources. It is possible to argue that the higher-order effects of the strong interaction increase the CPV and resulting CPC terms. However,

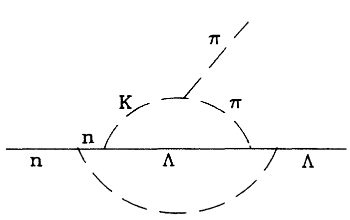

FIG. 11. Strong-interaction correction to Fig. 3.

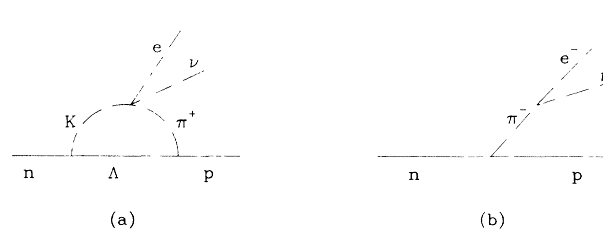

FIG. 12. *CP*-conserving weak decays.

there are other standard CPC terms that approximately account for the standard weak interaction. Hence it seems reasonable to expect that the CPC terms we generate via neutral-kaon exchange are indeed small. In that case our CPV estimates can be of the correct order of magnitude.

The theoretical technique is to reformulate the calculation in terms of dispersion relations. This was the approach of Barton and White[27] (BW). The dominant term of this approach is the meson baryon physical intermediate state. Hence our evaluation of Figs. 2(a) and 2(b) is consistent with the BW approach. The term of Fig. 11 is part of the two-meson baryon intermediate physical state, has a higher threshold and is therefore expected to be relatively small.

Another concern is the possibility we do not use enough intermediate baryon states. We studied the effects of including intermediate Σ baryons, and found that the contributions are about a 20% correction. This is a small effect, so we do not exhibit the Σ calculations explicitly. The inherent errors in our calculation are large enough so that a 20% correction is regarded as small.

A final comment concerns the possible use of chiral perturbation theory. This is an alternate way to compute observables that we have not investigated.

VI. SUMMARY

In this section we give a summary of our results.

A. $\Lambda, \bar{\Lambda}$ decay comparisons

The results of Sec. IV [Eqs. (4.13), (4.19), and (4.33)] are that

$$B = B_S + B_P, \quad B_S = 2.7\xi, \quad B_P = 16.6\xi , \tag{6.1}$$

where $\xi = \mathrm{Im} A_0 / \mathrm{Re} A_0$. Assuming $\xi \gg \xi'$, we get

$$\begin{aligned} B &= (4\pm2)\times10^{-3} \quad \text{(CERN)} \\ &= (-6.2\pm11)\times10^{-4} \quad \text{(Fermilab)} , \end{aligned} \tag{6.2}$$

where the different CERN and Fermilab values of $\mathrm{Re}(\epsilon'/\epsilon)$ are used. The CERN answer of Eq. (6.2) may be viewed as an upper limit. This is because of the recent Fermilab work which shows that the possibility that ϵ' (hence ξ) vanishes is not yet ruled out.

We have stated above that ξ is not determined by ϵ'. Indeed, it is possible that ϵ' is accidentally small because of a cancellation between ξ and ξ', recall Eq. (2.15f) and the subsequent discussion. The size of ξ' depends on the value of electromagnetic penguin terms.[16] In the case $\xi \approx \xi'$ our prediction for B would be enhanced by a factor $\xi/(\xi=\xi')$.

It is worthwhile to compare our CERN results with the standard-model estimates of DHP (Ref. 9). These are based on using the upper limit on the penguin operator $\mathrm{Im} C_5$, as quoted by DHP. This operator leads to a "large" value of $\mathrm{Re}(\epsilon'/\epsilon)$ similar to that observed by the CERN group. Thus if we use the CERN value of Re (ϵ'/ϵ) the input to our and the DHP calculations is similar, so one may compare the methods by comparing the results. DHP find

$$\begin{aligned} &B_S^{\mathrm{DHP}} \approx -6.4\times10^{-4}, \quad B_P^{\mathrm{DHP}} \approx 36\times10^{-4} , \\ &B^{\mathrm{DHP}} \approx 3\times10^{-3} . \end{aligned} \tag{6.3}$$

Our results are rather similar to those of DHP, even though very different techniques are used. This leads to some confidence that one would be able to make accurate predictions of B, once the inputs ϵ' or ξ are known.

B. CPV coupling constants and D_n

In our hadronic approach the idea that CPV originates from the kaonic CPV matrix elements serves as a unifying principle. This allows us to relate the predictions for the CPV pion-nucleon and $\pi\Lambda N$ coupling constants $g^{\mathrm{CPV}}, g^{\mathrm{CPV}}(\Lambda_0^-)$ and the electric dipole moment of the neutron D_n.

The Eqs. (3.6), (3.12), (4.18), and (4.44) determine that

$$\begin{aligned} &g^{\mathrm{CPV}}(\pi_c N, N) \approx 10^{-13}\xi , \\ &g^{\mathrm{CPV}}(\pi_0 n, n) = \tfrac{1}{2} g^{\mathrm{CPV}}(\pi_c N, N) , \\ &g^{\mathrm{CPV}}(\pi_0 p, p) = 0 , \\ &D_n \approx (5.3\pm1.7)\times10^{-28}\xi \; e\,\mathrm{cm} , \\ &g^{\mathrm{CPV}}(\Lambda_0^-) = 10^{-7}\xi (S \text{ wave}) \\ &\qquad\qquad = 7\times10^{-7}\xi (P \text{ wave}) , \\ &g^{\mathrm{CPV}}(\Lambda_0^0) = -\frac{1}{\sqrt{2}} g^{\mathrm{CPV}}(\Lambda_0^-) \end{aligned} \tag{6.4}$$

in which Λ_0^- denotes $\Lambda \to p\pi^-$ and Λ_0^0 denotes $\Lambda \to n\pi^0$. The coupling constants for Λ decay are given by

$$\begin{aligned} &g^{\mathrm{CPV}}(\Lambda_0^-) \equiv -\sqrt{2/3} S_{11} \phi_1^S, \quad S \text{ wave} , \\ &g^{\mathrm{CPV}}(\Lambda_0^-) \equiv -\sqrt{2/3} P_{11} \phi_1^P, \quad P \text{ wave} \end{aligned} \tag{6.5}$$

for the S-wave parity-violating and P-wave parity-conserving CPV coupling constants.

All of our CPV quantities [Eqs. (6.1) and (6.4)] are essentially proportional to ξ. These can be used in connection with any theory that predicts ξ. We may use Eqs. (6.1) and (6.4) to relate observables. Simple division yields

$$\begin{aligned} &B = (3.6\pm1.2)\times10^{28} \frac{D_n}{e\,\mathrm{cm}} , \\ &g^{\mathrm{CPV}}(\Lambda_0^-)(P \text{ wave}) \approx 10^7 g^{\mathrm{CPV}}(\pi_c N, N) \\ &\qquad = (1.3\pm0.4)\times10^{-4} \left[10^{25} \frac{D_n}{e\,\mathrm{cm}} \right] , \\ &g^{\mathrm{CPV}}(\pi_c N, N) = (1.3\pm0.4)\times10^{-11} \left[10^{25} \frac{D_n}{e\,\mathrm{cm}} \right] . \end{aligned} \tag{6.6}$$

A nonzero measurement of any of the CPV quantities $B, g^{\text{CPV}}(\pi N, N)$ or D_n would via Eq. (6.6) lead to a prediction for the others.

We have studied several CPV effects: the pion-nucleon and $\pi\Lambda N$ coupling constants g^{CPV}, $g^{\text{CPV}}(\Lambda_0^-)$, B and the electric dipole moment of the neutron D_n. Of these, the search for a CPV difference between the Λ and $\bar{\Lambda}$ decay parameters (B) is the most likely way to observe a CPV effect.

We conclude that the obvious cannot be overstated. Future progress in this field depends strongly on a new experimental discovery of *CP* violation.

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