CP and P determination and other applications of τ-lepton and t-quark polarimetry

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At an e^+e^- collider when the decay $X \to f\bar{f}$ occurs, the X rest frame is often known. We assume here that this is the case and work out symmetry tests in this frame. In this paper the P and CP symmetry tests for the decay of integer-spin systems into a pair of vector bosons $V_1 V_2$ are generalized to the $X \to \bar{\tau}\tau$ and $\to t\bar{t}$ decay modes where the τ's and t's decay weakly. Polarization effects are present in τ decays and are expected to be in top-quark decays provided m_t is not $\sim\sqrt{2}m_W$ (113 GeV). They also are expected to occur if other new charged fermions should be discovered. In spite of missing neutrino and antineutrinos in the final state, we find that powerful P and CP symmetry tests can be obtained. At hadron colliders the X rest frame is normally not known. However, these tests may assist or stimulate similar symmetry-test investigations, for example, via Monte Carlo simulations that include the angular momentum properties of the decay process. These τ and/or t polarimetry results for P and CP determination, for determination of the chiral or helicity structure of f or X decay amplitudes, for testing for CP violation, etc., are of current interest. For example, they should be useful in Z and Z' physics and in searches for the top quark and for Higgs bosons.

I. INTRODUCTION

At an e^+e^- collider when the decay $X \to \bar{f}f$ occurs, the X rest frame is often known kinematically. We assume here that this is the case and work out symmetry tests in this frame. In this paper the P and CP symmetry tests for the decay of integer-spin systems into a pair[1,2] of vector bosons $V_1 V_2$ are generalized to the $X \to \bar{\tau}\tau$ and $\to t\bar{t}$ decay modes where the τ's and t's decay weakly. Polarization effects are present in τ decays and are expected to be in top-quark decays provided m_t is not $\sim\sqrt{2}m_W$ (113 GeV). They also are expected to occur if other new charged quarks or leptons are discovered. With the assumption that the X rest frame is known, in spite of missing neutrino and antineutrinos in the final state, we find that powerful P and CP symmetry tests can be obtained.

At hadron colliders the X frame is normally not known. However, these tests may assist or stimulate similar symmetry test investigations via Monte Carlo simulations that properly include spin and other angular momentum properties *in the decay process.* A very important point in this context is that our results for $X \to f\bar{f}$ are independent of the polarization state of X. Consequently these effects are present even if the simulation does not include completely the angular momentum properties of the production of X. Likewise, in the actual experiments these effects do not require the experimentalist to polarize the X system or to know or measure anything about the polarization state of the X. The X can be polarized or unpolarized; it does not matter.

τ and t polarimetry results are of current interest to Z and Z' physics and to searches for the t quark and for Higgs bosons. At hadron colliders, H (or π_{TC})$\to \tau^+\tau^-$ is distinguishable from the γ^*, Z^* background because of their opposite helicity structure. Polarimetry determines the chiral structures of $t \to Wb$ and $X \to t\bar{t}$. If $m_t > 200$ GeV, the dominantly longitudinally polarized W's from $t \to Wb$ will be a striking signature for finding the t quark and Higgs bosons.

The results reported here were derived by the methods[1] explained in Refs. 3 and 4 for $X \to \bar{f}f$ with spin $J=0$. In the Appendix of Ref. 4 is an explanation of the kinematic procedures to obtain the necessary τ (t-quark) rest-frame variables when at most one final momentum, i.e., that of the ν or $\bar{\nu}$, is unknown in each f decay.

In Sec. II we define the α, δ, and ζ parameters which very conveniently characterize all the sequential decay correlation functions considered later in the paper. The first two tables display the relations among the helicity amplitudes for $X \to \bar{f}f$ which follow, respectively, from invariance with only CP and under both P and C separately.

Formulas for the various sequential decay correlation functions are listed in Sec. III for the $\bar{\tau}\tau$ mode and in Sec. IV for the $t\bar{t}$ mode. In Table III an ordered procedure for determination of the CP quantum number of X is given. The CP quantum number of X can always be determined by measurement of α, δ, and ζ.

A signature that P and C are not separately conserved is $\alpha \neq 0$. When, on the other hand, $X \to \bar{f}f$ is invariant under P and C separately, we find that if $\delta \neq 0$, measurement will always relate the P quantum number and the signature, $\eta_P = \zeta(-)^J$. An ordered procedure to do so is in Table IV.

When $\delta \neq 0$, helicity conservation is violated along the f and $\bar{f}$ line.

For $X \to \bar{\tau}\tau$ with an X mass $M > 10$ GeV, we find that the hadron-hadron correlation function is comparable or superior to the lepton-lepton correlation function for determining γ_{CP} or ζ in $\eta_P = \zeta(-)^J$. This conclusion is important for exploratory searches at e^+e^- colliders by

$e^+e^- \to Hl^+l^-$, $e^+e^- \to (Z \text{ or } Z')H$, or in the decays $(Z$ or $Z') \to l^+l^-H$. H (or π_{TC}) might also be produced by the decays of a heavy vector boson V at an e^+e^- collider. For a single Higgs doublet, in Ref. 4 there is a short discussion of the important issues of the rates and backgrounds involved in using $V \to H\gamma$ and the $\{\mu^+e^-\}$ spectrum of H to measure γ_{CP}. Though often model dependent, these two issues must always be considered in specific applications of the tests in the present paper.

In Sec. V tests for the violation of both CP and of P are given. Two additional parameters ϵ and ζ_X are then necessary to describe the decay correlation function. If $\delta \neq 0$, there is the conclusive signature that always $|\zeta| \neq 1$ when both CP and P are violated.

We stress that this leaves one prominent exception to both the test for P determination and the three tests for unexpected violation of both CP and P. This is an X with $J \geq 0$ when $\delta = 0$, i.e., when helicity is conserved along the f and $\bar{f}$ line. Note that since $\delta \simeq (m_\tau/M)^2 \simeq 3.8 \times 10^{-4}$, this is approximately the case predicted for Z decay in the standard model.

The tests reported here for $X \to \bar{f}f$ could also be used if heavier spin-$\frac{1}{2}$ fermions f are discovered. For example, for a heavy down-type quark the tests are obvious since the semileptonic decay correlations for $t \to e^+$ and $b' \to \bar{\nu}$ are identical, and for $t \to \nu$ and $b' \to e^-$ are identical, in the approximation of neglecting corrections involving the masses of all the final particles in the semileptonic decay.

In tree-level formulas in this paper we have explicitly included the chiral polarization parameter

$$\xi_A = \frac{|g_L|^2 - |g_R|}{|g_L|^2 + |g_R|} = \frac{2\,\mathrm{Re}(v_A a_A^*)}{|v_A|^2 + |a_A|^2}$$

for $\tau^- \to A^-\nu$. We have also explicitly included the Michel polarization parameter ξ for $\tau^- \to l^-\nu\bar{\nu}$. For a pure $V-A$ coupling these are one. Consequently, these results can also be used for studying the chiral properties of $t \to Wb$ as well as for $X \to t\bar{t}$.

In Sec. VI, we discuss how the results can be used at hadron colliders to help distinguish H $(\pi_{TC}) \to \tau^+\tau^-$ from the γ^*, Z^* background because of their opposite helicity structure. This would be particularly important to searches for an intermediate-mass Higgs boson if the top quark is very heavy, $m_t > 200$ GeV. The tests would also be useful in studying H $(\pi_{TC}) \to t\bar{t}$ around the $t\bar{t}$ threshold where a dip is expeced in the H $(\pi_{TC}) \to WW, ZZ$ branching ratios. *If the t quark is very heavy, the W bosons in t-quark decays would be predominantly longitudinally polarized* ($\Gamma_L/\Gamma_T = 3$ *if* $m_t = 200$ *GeV), which would be a very striking signature in t-quark searches and in* $X \to t\bar{t}$ *physics.*

II. CONSTRAINTS ON $X \to \bar{f}f$ AMPLITUDES FROM P AND C, OR ONLY CP, INVARIANCE

In the decay of a spin-J system into a pair of spin-$\frac{1}{2}$ particles, the $X \to \bar{f}f$ decay helicity amplitude $t_{\lambda_1\lambda_2}$ is defined by

$$\langle \Theta, \Phi, \lambda_1, \lambda_2, |JM\rangle = D^{J*}_{M\lambda}(\Phi, \Theta, -\Phi) t_{\lambda_1\lambda_2} \tag{1}$$

where λ_1, λ_2 denote, respectively, the helicities of $\bar{f}, f$, and $\lambda = \lambda_1 - \lambda_2$. So for $J=0$ there can be two nonvanishing amplitudes and for $J \geq 1$ there can be four such amplitudes. In the X rest frame, Θ and Φ are the polar and azimuthal angles of $\bar{f}$ relative to the $|JM\rangle$ quantization axis for the X system.

α, δ, ζ formalism. The sequential-decay correlation functions considered later on in this paper are found to depend on a parameter ζ defined by

$$\zeta = \frac{2\,\mathrm{Re}(t_{++} t_{--}^*)}{|t_{++}|^2 + |t_{--}|^2}, \tag{2}$$

which simplifies to

$$t_{++} = \zeta t_{--}$$

when $X \to \bar{f}f$ is CP invariant, and on the following three quadratic functions of the above $t_{\lambda_1\lambda_2}$ amplitudes:

$$\begin{aligned} \sigma &\equiv \tfrac{1}{2}(|t_{++}|^2 + |t_{--}|^2), \\ \tau &\equiv \tfrac{1}{2}(|t_{-+}|^2 + |t_{+-}|^2), \\ v &\equiv \tfrac{1}{2}(|t_{-+}|^2 - |t_{+-}|^2). \end{aligned} \tag{3}$$

It is useful for discrete symmetry tests to renormalize σ, τ, and v by defining

$$\mathcal{N} \equiv \sum_{\lambda_1\lambda_2} |t_{\lambda_1\lambda_2}|^2 \tag{4}$$

so the important parameters α and δ are defined by

$$\alpha \equiv \frac{|t_{-+}|^2 - |t_{+-}|^2}{\mathcal{N}} = \frac{v}{\sigma + \tau}, \tag{5}$$

$$\delta \equiv \frac{|t_{++}|^2 + |t_{--}|^2}{\mathcal{N}} = \frac{\sigma}{\sigma + \tau}. \tag{6}$$

By Eqs. (4)–(6), a sequential-decay correlation function to be considered later in this paper is proportional to $\mathcal{N}$, which serves as its overall normalization factor. Measurement of the relative proportions of the different terms in a sequential-decay correlation function then determines the α, δ, and ζ parameters.

Case: CP good. When $\bar{f}$ and f are an antiparticle-particle pair, then by CP invariance the nonvanishing amplitudes in Eq. (1) are related by

$$t_{\lambda_1\lambda_2} = \gamma_{CP} t_{-\lambda_2, -\lambda_1}, \tag{7}$$

where γ_{CP} is the CP quantum number of X. For a CP-even (-odd) system, $\gamma_{CP} = +1$ (-1), respectively. In Table I are listed explicitly the relations among the $t_{\lambda_1\lambda_2}$ decay helicity amplitudes and the associated allowed α, δ, and ζ parameter ranges.

Case: P and C separately good. For $\bar{f}$ and f an antiparticle-particle pair, by P invariance the nonvanishing amplitudes in Eq. (1) are related by

TABLE I. Relations among the helicity amplitudes describing the decay $X \to \bar{f} f$ which follow from invariance under CP, $\gamma_{CP} = CP$ eigenvalue of X, where $f\bar{f}$ are a spin-$\frac{1}{2}$ particle-antiparticle pair. The associated α, δ, ζ ranges are tabulated and show that γ_{CP} can always be determined. For $\delta \neq 0$, ζ is measurable and $\gamma_{CP} = \zeta$. If $\delta \neq 1$ and/or $\alpha \neq 0$, then $\gamma_{CP} = +1$ and $J \geq 1$.

Nonvanishing amplitudes	Parameter ranges and allowed γ_{CP} ($0 \leq \delta \leq 1$, $-1 \leq \alpha \leq +1$, $\zeta = \pm 1$)
	$J=0$
$t_{++} = \gamma_{CP} t_{--}$	$\delta = 1$, $\alpha = 0$, $\zeta = \gamma_{CP}$
	$J \geq 1$
Case $\gamma_{CP} = -1$	
$t_{++} = -t_{--}$	$\delta = 1$, $\alpha = 0$, $\zeta = \gamma_{CP} = -1$
Case $\gamma_{CP} = +1$	
$t_{++} = t_{--}$	$(1-\delta) \geq \lvert\alpha\rvert$, $\zeta = \gamma_{CP} = +1$
t_{-+}, t_{+-}	

$$t_{\lambda_1 \lambda_2} = \eta_P (-)^J t_{-\lambda_1, -\lambda_2}, \tag{8a}$$

where η_P is the parity quantum number of X with spin J. And by C invariance,

$$t_{\lambda_1 \lambda_2} = C_n (-)^J t_{\lambda_2 \lambda_1}, \tag{8b}$$

where C_n is the charge-conjugation eigenvalue of X. Table II lists the relations among the $t_{\lambda_1 \lambda_2}$ decay helicity amplitudes, the associated α, δ, ζ ranges, and also the allowed states. Note that when P and C are separately good some $X \to \bar{f} f$ decays are not allowed. These are $J=0$ if $C_n = -1$; $J_{\rm odd} \geq 1$ if $\eta_P = -1$ and $C_n = +1$; and $J_{\rm even} \geq 1$ if $\eta_P = +1$ and $C_n = -1$.

In assessing what can be measured and how, it is instructive to compare Table II with Table I. In Table II, the α parameter always vanishes. So if measurement should show $\alpha \neq 0$, then from Eqs. (8) and (9) one knows that P and C invariance for $X \to \bar{f} f$ are both separately violated (and of course that $J \neq 0$). When P and C are separately good, besides the γ_{CP} results analogous to those of Table I, we find that

$$\eta_P = \zeta (-)^J \quad (\textit{if } \delta \neq 0) \tag{9}$$

but that η_p is undetermined by this method if $\delta = 0$.

Signature for violation of helicity conservation along f and $\bar{f}$ line. When helicity is conserved along the f and $\bar{f}$ lines, then $\delta = 0$. The converse is also true, but to show that δ vanishes, instead of just being in some sense "small" would usually be unrealistic experimentally. So we note that $\delta \neq 0$ is a signature for the violation of helicity conservation along the f and $\bar{f}$ line. Also note that $\delta \neq 1$ is the signature for the presence of some amplitudes that do conserve helicity along the f and $\bar{f}$ line.

Formulation in terms of CP and P-even and -odd amplitudes, and of C- and P-even and -odd amplitudes. In using sequential-decay correlation functions to analyze for possible discrete symmetry violations, it is often useful to define amplitudes that are even and odd under the discrete symmetry operation of interest. Sometimes this also better displays to what extent measurement gives a complete determination of the elementary amplitudes.

We define

$$f^{(\pm)} \equiv t_{++} \pm t_{-}, \quad f^{(\pm)} \equiv |f^{(\pm)}| e^{i\theta}_{\pm} \tag{10}$$

so by Eq. (7), under CP,

TABLE II. Relations among the helicity amplitudes describing the decay $X \to \bar{f} f$ which follow from invariance under P and separately under C, η_P = parity eigenvalue and C_n the charge-conjugation eigenvalue of X. The associated α, δ, ζ ranges and allowed states are tabulated. Note that for J odd the upper signs are to be taken in the lower part of the table, and for J even, the lower signs. For $J=0$, η_P can always be determined. As in Table I, γ_{CP} can always be determined. For $\delta \neq 0$, $\gamma_{CP} \equiv \eta_P C_n = \zeta$ and $\eta_P = \zeta (-)^J$. If $\delta \neq 1$, then $\gamma_{CP} = 1$ and $J \geq 1$.

Nonvanishing amplitudes	Parameter ranges ($\alpha = 0$, $0 \leq \delta \leq 1$, $\zeta = \pm 1$)	Allowed states
	$J=0$	
$t_{++} = \eta_P t_{--}$	$\delta = 1$	$C_n = +1$, $\eta_P = \zeta$
	$J \geq 1$: $J_{\rm odd/even}$	
Case $\gamma_{CP} = -1$:		
When $\eta_P = \pm 1$, $C_n = \mp 1$		
$t_{++} = -t_{--}$	$\delta = 1$, $\zeta = -1$	$\eta_P = -(-)^J = -C_n$
Case $\gamma_{CP} = +1$:		
When $\eta_P = \mp 1$, $C_n = \mp 1$		
$t_{++} = t_{--}$	$0 \leq \delta \leq 1$, $\zeta = +1$	$\eta_P = (-)^J = C_n$
$t_{+-} = t_{-+}$		
When $\eta_P = \pm 1$, $C_n = \pm 1$		
$t_{+-} = -t_{-+}$	$\delta = 0$	$\eta_P = -(-)^J = C_n$

$$f^{(\pm)}=\pm\gamma_{CP}f^{(\pm)} \tag{11}$$

and by Eq. (8), under P,

$$f^{(\pm)}=\pm\eta_P(-)^J f^{(\pm)} . \tag{12}$$

Similarly, for

$$g^{(\pm)}\equiv t_{+-}\pm t_{-+}, \quad g^{(\pm)}\equiv|g^{(\pm)}|e^{i\theta}_{\pm} , \tag{13}$$

under P,

$$g^{(\pm)}=\pm\eta_P(-)^J g^{(\pm)} \tag{14}$$

and by Eq. (9), under C,

$$g^{(\pm)}=\pm C_n(-)^J g^{(\pm)} . \tag{15}$$

In terms of these amplitudes, the α, δ, and ζ parameters are

$$\zeta=\frac{|f^{(+)}|^2-|f^{(-)}|^2}{|f^{(+)}|^2+|f^{(-)}|^2} , \tag{16}$$

$$\delta=\frac{|f^{(+)}|^2+|f^{(-)}|^2}{2\mathcal{N}} , \tag{17}$$

$$\alpha=\frac{|g^{(+)}||g^{(-)}|\cos(\phi_+-\phi_-)}{2\mathcal{N}} , \tag{18}$$

where $\mathcal{N}=(|f^{(+)}|^2+|f^{(-)}|^2+|g^{(+)}|^2+|g^{(-)}|^2)/2$. For both $|f^{(+)}|\neq 0$ and $|f^{(-)}|\neq 0$ simultaneously, it is of course necessary that both CP invariance and P invariance be violated in $X\to\bar{f}f$.

III. τ-LEPTON POLARIMETRY: CP OR P DETERMINATION BY $\bar{\tau}\tau$ DECAY MODE

Using the above α, δ, ζ formalism for characterizing the $X\to\bar{f}f$ amplitudes, we now consider the sequential decays

$$X\to\tau_1^+\tau_2^- ,\quad \tau_2^-\to B^- + \text{neutrinos},\quad \tau_1^+\to A^+ + \text{neutrinos} \tag{19}$$

for X of arbitrary spin J.

For specific cases of observed final particles A^+ and B^- in Eq. (19), we will discuss in turn the associated decay correlation functions

$$I(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau;\phi) .$$

In part D of this section and in the associated tables we explain how to systematically use these results for discrete symmetry tests.

In the τ_1^+ rest frame θ_1^τ and ϕ_1^τ are the polar and azimuthal angles of A^+ in the usual helicity coordinate system and E_1^τ is the A's energy. Correspondingly θ_2^τ, ϕ_2^τ, and E_2^τ specify B^- in the τ_2^- rest frame. The important azimuthal angle ϕ between the τ^+A^+ momenta-γ^-B^- momenta planes in the X rest frame is defined by

$$\phi=\phi_1+\phi_2 . \tag{20}$$

For a purely leptonic τ decay such as $\tau^\pm\to l^\pm\nu\bar{\nu}$ the above sequential-decay correlation function depends on the usual decay density matrix

$$\rho_{(1/2)(1/2)}(\theta_i^\tau,E_i^\tau)=R(x_i^\tau)\pm\xi_i\cos\theta_i^\tau S(x_i^\tau), \quad i=1,2 \tag{21}$$

for the decay of a fully polarized τ^+ (τ^-) at rest with $x^\tau=E^\tau/\mathcal{E}$ where $\mathcal{E}=(m_\tau^2+\mu^2)/(2m_\tau)$ is the maximum l^+ (l^-) energy in the τ rest frame. In the massless l^+ (l^-) limit, i.e., $\mu^2\to 0$,

$$R\to(E^\tau)^2(3-2x^\tau), \quad S\to(E^\tau)^2(-1+2x^\tau) . \tag{22}$$

For a two-body τ decay such as $\tau^+\to h_A^+\bar{\nu}$ the helicity polar angle θ_1^τ between the h_A^+ hadron's momentum and its associated τ's z axis in the τ rest frame is determined by measurement of the h_A^+'s energy $E_A=E_1^\tau$ in the X rest frame. So E_A and θ_1^τ are equivalent sequential decay variables since $0\le\theta_1^\tau\le\pi$; with M the mass of the X system,

$$\cos\theta_1^\tau=\frac{4E_A m_\tau^2-M(m_\tau^2-m_A^2)}{(m_\tau^2-m_A^2)\sqrt{M^2-4m_\tau^2}}$$
$$=[2(E_A/E_A^{\max})-1][1+O((m_\tau/M)^2)] . \tag{23}$$

The decay density matrix for the two-body mode $\tau^+\to h_A^+\nu$ also depends on the normalized parameter $\mathcal{S}_A$ with

$$\mathcal{S}_A=\begin{cases}1 & \text{for } A=\pi,K ,\\ \dfrac{m_\tau^2-2m_A^2}{m_\tau^2+2m_A^2} & \text{for } A=\rho,K^*,a_1\end{cases} \tag{24}$$

(numerically $\mathcal{S}_\rho=0.457$, $\mathcal{S}_{K^*}=0.333$, and $\mathcal{S}_{a_1}=-0.11$).

A. μ^+,e^- correlation functions: $\{\mu^+e^-\}$ spectrum

For the sequential decay

$$X\to\tau_1^+\tau_2^- ,\quad \tau_2^-\to e_2^-\bar{\nu}_2\nu_2 ,\quad \tau_1^+\to\mu_2^+\nu_1\bar{\nu}_1 \tag{25}$$

it follows by Lorentz invariance that the "standard decay correlation function" is (see Sec. 2 of the first paper in Ref. 3)

$$I(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau;\phi)=C(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau)+A_0(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau)\cos\phi , \tag{26}$$

where

$$A_0=\tfrac{1}{2}\mathcal{N}\zeta\delta D(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau)$$
$$=-\tfrac{1}{2}\mathcal{N}\zeta\delta\xi_1 S(E_1^\tau)\xi_2 S(E_2^\tau)\sin\theta_1^\tau\sin\theta_2^\tau \tag{27}$$

and

$$C=\tfrac{1}{2}\mathcal{N}[\delta S(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau)+(1-\delta)T(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau)$$
$$-\alpha U(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau)] \tag{28}$$

with

$$S(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau)=R(E_1^\tau)R(E_2^\tau)-\xi_1 S(E_1^\tau)\xi_2 S(E_2^\tau)\cos\theta_1^\tau\cos\theta_2^\tau\ ,$$

$$T(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau)=R(E_1^\tau)R(E_2^\tau)+\xi_1 S(E_1^\tau)\xi_2 S(E_2^\tau)\cos\theta_1^\tau\cos\theta_2^\tau\ , \quad (29)$$

$$U(E_1^\tau,\theta_1^\tau;E_2^\tau,\theta_2^\tau)=\xi_1 S(E_1^\tau)R(E_2^\tau)\cos\theta_1^\tau+R(E_1^\tau)\xi_2 S(E_2^\tau)\cos\theta_2^\tau\ ,$$

Because of the missing neutrals in Eq. (25), the $\tau^\pm$ rest frames are not directly accessible[3,4] so we reexpress $I(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau;\phi)$ in terms of three independent observables in the X rest frame:

$$E_{\bar\mu}\equiv E_1,\ \text{the } \mu_1^+ \text{ energy}\ ,$$

$$E_e\equiv E_2,\ \text{the } e_2^- \text{ energy}\ ,$$

$$z\equiv\cos\psi\ ,$$

where ψ is the opening angle between the μ^+ and e^- momenta, and then we integrate out the other two independent unobservable variables. We obtain

$$I(E_{\bar\mu},E_e,\cos\psi)=\delta S(E_{\bar\mu},E_e,\cos\psi)+(1-\delta)T(E_{\bar\mu},E_e,\cos\psi)-\alpha U(E_{\bar\mu},E_e,\cos\psi)+\zeta\delta D(E_{\bar\mu},E_e,\cos\psi)\ , \quad (30)$$

where the explicit analytic expressions for S and D are listed in Appendix A of the first paper of Ref. 3 and those for T and U are listed in Appendix B of Ref. 5 in the approximation that the muon and electron masses can be neglected. The reader is referred to Refs. 4 and 5 for representative plots of these functions for an X mass $M \gg m_\tau$ where m_τ is the tau mass, i.e., $M\sim 70-90$ GeV.

It is convenient to introduce the scaled lepton energies

$$x\equiv E_{\bar\mu}/E_{\max},\quad y\equiv E_e/E_{\max}\ , \quad (31)$$

where $E_{\max}=m_\tau/[2\gamma(1-\beta)]$ is the maximum available charged-lepton energy in the X rest frame (γ and β are the usual relativistic boost variables connecting the X and τ rest frames, $\gamma=M/2m_\tau$). Corresponding to Eq. (30), for $\tau^+\to\mu^+\nu\bar\nu$ the muon's energy distribution in the X rest frame is

$$I(E_{\bar\mu})=f(x)-\xi\alpha g(x)\ , \quad (32)$$

where

$$f(x)=\tfrac{5}{6}-\tfrac{3}{2}x^2+\tfrac{2}{3}x^3$$

$$g(x)=1/\beta[-\tfrac{1}{6}+\tfrac{3}{2}x^2-\tfrac{4}{3}x^3+(1-\beta)2x^2(x-1)]\ . \quad (33)$$

The muon-energy–electron-energy correlation function is

$$I(E_{\bar\mu},E_e)=\delta S(E_{\bar\mu},E_e)+(1-\delta)T(E_{\bar\mu},E_e)-\alpha U(E_{\bar\mu},E_e)\ , \quad (34)$$

where

$$S(E_{\bar\mu},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[f(x)f(y)-\xi_1\xi_2 g(x)g(y)]\ ,$$

$$T(E_{\bar\mu},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[f(x)f(y)+\xi_1\xi_2 g(x)g(y)]\ , \quad (35)$$

$$U(E_{\bar\mu},E_e)=\frac{\pi}{64}\gamma^{-2}\beta^{-2}m_\tau^6[\xi_1 g(x)f(y)+\xi_2 f(x)g(y)]\ .$$

Note, when $\alpha=0$, Eq. (35) simplifies to

$$I(E_\mu,E_e)|=[f(x)f(y)+(1-2\delta)\xi_1\xi_2 g(x)g(y)]\ . \quad (36)$$

B. Hadron-hadron correlation functions: $\{h_A^+h_B^-\}$ spectrum

For the decay sequence

$$X\to\tau_1^+\tau_2^-,\quad \tau_2^-\to h_B^-\nu_2,\quad \tau_1^+\to h_A^+\nu_1 \quad (37)$$

the standard decay correlation function assuming CP invariance is

$$I(\theta_1^\tau;\theta_2^\tau;\phi)=C(\theta_1^\tau;\theta_2^\tau)+A_0(\theta_1^\tau;\theta_2^\tau)\cos\phi \quad (38)$$

where the hadronic helicity polar angles θ_1^τ and θ_2^τ for h_A^+ and h_B^- in their τ's rest frame are, respectively, determined by the observed hadronic energies E_A and E_B in the X rest frame as in Eq. (23). We find it simpler to use θ_1^τ and θ_2^τ, instead of E_A and E_B, in our analysis. Note also that $\cos\phi$ in Eq. (38) is determined by E_A, E_B, and $\cos\psi_{AB}$, where ψ_{AB} is the opening angle between h_A^+ and h_B^- moment in the X rest frame (see Appendix A of Ref. 4). Therefore, Eq. (38) is an observable distribution in the X rest frame, assuming that the detected h_A^+ and τ^+ directions in $\tau^+\to h_A^+\nu$ are not forced to be measurably the same due to a very large τ momentum. Likewise for h_B^-. The associated differential counting rate is

$$dN=I(\theta_1^\tau;\theta_2^\tau;\phi)d(\cos\theta_1^\tau)d(\cos\theta_2^\tau)d\phi\ . \quad (39)$$

In Eq. (38),

$$A_0=\tfrac{1}{2}\mathcal{N}\zeta\delta D(\theta_1^\tau;\theta_2^\tau)=-\tfrac{1}{2}\mathcal{N}\zeta\delta\xi_A\mathcal{S}_A\xi_B\mathcal{S}_B\sin\theta_1^\tau\sin\theta_2^\tau\ ; \quad (40)$$

and

$$C=\tfrac{1}{2}\mathcal{N}[\delta S(\theta_1^\tau;\theta_2^\tau)+(1-\delta)T(\theta_1^\tau;\theta_2^\tau)-\alpha U(\theta_1^\tau;\theta_2^\tau)] \quad (41)$$

with

$$S(\theta_1^\tau;\theta_2^\tau)=1-\xi_A\mathcal{S}_A\xi_B\mathcal{S}_B\cos\theta_1^\tau\cos\theta_2^\tau\ ,$$

$$T(\theta_1^\tau;\theta_2^\tau)=1+\xi_A\mathcal{S}_A\xi_B\mathcal{S}_B\cos\theta_1^\tau\cos\theta_2^\tau\ , \quad (42)$$

$$U(\theta_1^\tau;\theta_2^\tau)=-\xi_A\mathcal{S}_A\cos\theta_1^\tau-\xi_B\mathcal{S}_B\cos\theta_2^\tau\ ,$$

where $\mathcal{S}_A$ and $\mathcal{S}_B$ are given by Eq. (24).

The α, δ, and ζ parameters can be determined by measurement of $I(\theta_1^\tau;\theta_2^\tau;\phi)$ *or equivalently* by measurement of the following associated integrated distributions.

(i) If the entire θ_1^τ, θ_2^τ acceptance is integrated over

$$F(\phi)\equiv\int d(\cos\theta_1^\tau)\int d(\cos\theta_2^\tau)I(\theta_1^\tau;\theta_2^\tau;\phi) \quad (43)$$

$$=2\mathcal{N}[1-\zeta\delta(\pi/4)^2\xi_A\mathcal{S}_A\xi_B\mathcal{S}_B\cos\phi] \quad (44)$$

For $A,B=\pi$ or K, the size of the $\zeta\delta$ signature is comparable to that in the $\phi\phi$ symmetry test for the η_c where about 20 events were initially used[6] to determine the η_c's parity. From Eq. (24), one sees that the signature is reduced by about a factor of 2 for a $\tau\to\rho\nu$ mode, and by a factor of 3 for a $\tau\to K^*\nu$ mode. Since the $\tau^+\to\rho^+\nu$ branching ratio is about twice that for $\tau^+\to\pi^+\nu$, the $\tau^+\to\rho^+\nu$ and $\tau^+\to\pi^+\nu$ modes should be of comparable significance for measurement of $\zeta\delta$. Clearly, ignoring the possibly important issues of backgrounds and detection efficiencies which will depend on what X is and how it is produced, *for $M>10$ GeV the hadron-hadron correlation function is comparable or superior for determining $\zeta\delta$ to the lepton-lepton correlation function discussed above in Sec. III A.*

(ii) If the entire range of the azimuthal angle ϕ is integrated over, the hadron-energy–hadron-energy correlation function is obtained, which equals $C(\theta_1^\tau,\theta_2^\tau)$ of Eq. (41):

$$I(E_A,E_B)\equiv\frac{1}{2\pi}\int_0^{2\pi}d\phi\, I(\theta_1^\tau,\theta_2^\tau,\phi)$$

$$=C(\theta_1^\tau,\theta_2^\tau)\ . \quad (45)$$

Absorbing the $\frac{1}{2}\mathcal{N}$ factor, we have

$$I(E_A,E_B)=1+(1-2\delta)\xi_A\mathcal{S}_A\xi_B\mathcal{S}_A\cos\theta_1^\tau\cos\theta_2^\tau + \alpha(\xi_A\mathcal{S}_A\cos\theta_1^\tau+\xi_B\mathcal{S}_B\cos\theta_2^\tau)\ , \quad (46)$$

which depends on both δ and α.

(iii) Finally, for determination of α, it is simplest to integrate Eq. (38) over E_B. This gives the single τ^+ decay's energy distribution[3]

$$I(E_A)=\tfrac{1}{2}\int_{-1}^{1}d(\cos\theta_2^\tau)I(E_A,E_B)$$

$$=1+\alpha\xi_A\mathcal{S}_A\cos\theta_1^\tau\ , \quad (47)$$

which depends only on α.

C. Lepton-hadron correlation functions: $\{\mu^+h_B^-\}$ spectrum

In terms of the notation of Eqs. (23) and (24) for describing the final observed hadron h_B^-, for the decay sequence

$$X\to\tau_1^+\tau_2^-,\quad \tau_2^-\to h_B^-\nu_1,\quad \tau_1^+\to\mu_1^+\nu_1\nu_1 \quad (48)$$

the standard decay correlation function is

$$I(\theta_1^\tau,E_2^\tau;\theta_B^\tau;\phi)=C(\theta_1^\tau,E_2^\tau;\theta_B^\tau)+A_0(\theta_1^\tau,E_2^\tau;\theta_B^\tau)\cos\phi\ , \quad (49)$$

where

$$A_0=\tfrac{1}{2}\mathcal{N}\zeta\delta D(E_1^\tau,\theta_1^\tau;\theta_1^\tau)$$

$$=\tfrac{1}{2}\mathcal{N}\zeta\delta\xi S(E_1^\tau)\xi_B\mathcal{S}_B\sin\theta_1^\tau\sin\theta_B^\tau \quad (50)$$

and

$$C=\tfrac{1}{2}\mathcal{N}[\delta S(E_1^\tau,\theta_1^\tau;\theta_B^\tau)+(1-\delta)T(E_1^\tau,\theta_1^\tau;\theta_B^\tau) - \alpha U(E_1^\tau,\theta_1^\tau;\theta_B^\tau)] \quad (51)$$

with

$$S(E_1^\tau,\theta_1^\tau;\theta_B^\tau)=R(E_1^\tau)+\xi S(E_1^\tau)\xi_B\mathcal{S}_B\cos\theta_1^\tau\cos\theta_B^\tau\ ,$$

$$T(E_1^\tau,\theta_1^\tau;\theta_B^\tau)=R(E_1^\tau)-\xi S(E_1^\tau)\xi_B\mathcal{S}_B\cos\theta_1^\tau\cos\theta_B^\tau\ , \quad (52)$$

$$U(E_1^\tau,\theta_1^\tau;\theta_B^\tau)=\xi S(E_1^\tau)\cos\theta_1^\tau-R(E_1^\tau)\xi_B\mathcal{S}_B\cos\theta_B^\tau\ .$$

For $M\gg m_\tau$, the associated lepton-energy–hadron-energy correlation function is

$$I(E_{\bar\mu},\cos\theta_B^\tau)=\delta S(E_{\bar\mu},\cos\theta_B^\tau)+(1-\delta)T(E_{\bar\mu},\cos\theta_B^\tau) - \alpha U(E_{\bar\mu},\cos\theta_B^\tau)\ , \quad (53)$$

where

$$S(E_{\bar\mu},\cos\theta_B^\tau)=\frac{\pi}{8}\gamma^{-1}\beta^{-1}m_\tau^3[f(x)+\xi g(x)\xi_B\mathcal{S}_B\cos\theta_B^\tau]\ ,$$

$$T(E_{\bar\mu},\cos\theta_B^\tau)=\frac{\pi}{8}\gamma^{-1}\beta^{-1}m_\tau^3[f(x)-\xi g(x)\xi_B\mathcal{S}_B\cos\theta_B^\tau]\ , \quad (54)$$

$$U(E_{\bar\mu},\cos\theta_B^\tau)=\frac{\pi}{8}\gamma^{-1}\beta^{-1}m_\tau^3[\xi g(x)-f(x)\xi_B\mathcal{S}_B\cos\theta_B^\tau]\ .$$

As in Sec. III A above, Eq. (53) can be used to determine α and δ. However, in general the ζ parameter will remain unknown. In order to obtain the formula for determining ζ when $\delta\neq0$, it is necessary to reexpress $I(\theta_1^\tau,E_1^\tau;\theta_B^\tau;\phi)$ of Eq. (49) in terms of the X rest frame observables $E_{\bar\mu},\cos\theta_B^\tau$ and $z=\cos\psi_{\bar\mu B}$ and then integrate out the other independent unobservable variable, in analogy to what was done in obtaining Eq. (30) from Eq. (26) in Refs. 3 and 5.

D. Determination of γ_{CP}, and possibly η_P, for X

In Tables III and IV are listed, respectively, an ordered procedure for determining the γ_{CP} quantum number of X and a similar procedure for possibly relating η_P and $(-)^J$ of X. Important remarks are in the table captions. Note that when $I(E_{\bar\mu},E_e,\cos\psi)$ is to be measured[3,4] to obtain ζ for an X mass $M\gtrsim10$ GeV a 10–20 % accuracy measurement is required; for $M\lesssim10$ GeV but not near the nonrelativistic limit about 10% accuracy is needed; but for $M\simeq2m_\tau$ (i.e., near the nonrelativistic limit) the signature is very large since then $|D/S|$ of Eq. (30) is roughly $\sim50\%$.

IV. t-QUARK POLARIMETRY: CP OR P DETERMINATION OF $\bar{t}t$ DECAY MODES

We consider

$$X\to t\bar{t},\quad \to W^-b\ ,\quad \to W^+b \quad (55)$$

From[7,8] present experiments $76\le m_t\lesssim260$ GeV, and we

TABLE III. An ordered procedure for determination of the CP quantum number γ_{CP} of X when $X \to \bar{f} f$ is CP invariant. γ_{CP} can always be determined from measurement of the α, δ, and ζ parameters. In ordering (i) before (ii) it has been assumed that $I(E_A)$ can be better measured than $I(E_A, E_B)$. Note about (iii) that while measurement of $I(E_A, E_B)$ determines α and δ, to obtain ζ measurement of $F(\phi)$ or of $I(E_{\bar{\mu}}, E_e, \cos\psi)$ is required.

Result		Measure	$\tau^+\tau^-$ Equation Nos.	$t\bar{t}$ Equation Nos.
(i) $\alpha \neq 0$	$\to \gamma_{CP} = +1$	$I(E_A)$	(32), (47)	(32), (47)
(ii) $\alpha = 0, \delta \neq 1$	$\to \gamma_{CP} = +1$	$I(E_A, E_B)$	(34), (46), (53)	(34), (46)
(iii) $\alpha = 0, \delta = 1$	$\to \gamma_{CP} = \zeta$	$F(\phi)$ or $I(E_{\bar{\mu}}, E_e, \cos\psi)$ to obtain ζ	(44), (30), (38)	(57), (44), (38)

will treat separately the case of $m_t \lesssim 85$ GeV, or "virtual" W production and the case of $m_t \gtrsim 100$ GeV, or "real" W production. If m_t lies in the transition region $m_t \simeq m_W + m_b$, then corrections[9] due to the finite W width would need to be included.

From the present lower bound on m_t, in the standard model the weak lifetime of the top quark is so short that the top quark will decay before QCD bound state or other hadronization effects depolarize the top quark, unlike for b and c quarks. In the present analysis we therefore study t-quark polarimetry and its applications assuming that this is true in nature. If the EW symmetry breaking is due to a $t\bar{t}$ condensate as some have proposed, then the "new strong interactions" for this phenomena could, indeed, depolarize the t quarks before the weak decay occurs. If the idea of a $t\bar{t}$ condensate remains viable, then when the associated dynamics is further developed, what happens to the t quark's polarization will require careful investigation to see what is and is not model dependent.

Tables III and IV list how the results can be used to determine γ_{CP} and possibly η_P in terms of $(-)^J$ for the X system.

A. Case $m_t \lesssim 85$ GeV

$\{\mu^+ e^-\}$ spectrum. We consider $t \to \mu^+ b \nu$ and $\bar{t} \to e^- b \bar{\nu}$ so the formulas of Sec. III A apply if R and S of Eq. (22) for τ^+ decay are simply replaced by ($R_\tau \to R_t$, $S_\tau \to -S_t$) those[4] for t decay [the explicit analytic expressions for the quantities in Eq. (30) are, however, not in the literature]. In the μ^+ energy distribution and in the $I(E_{\bar{\mu}}, E_e)$ for the X rest frame, f and g are replaced by $(f \to f^t, g \to -g^t)$

$$f^t(x) = 1 - 3x^2 + 2x^3 ,$$

$$g^t(x) = \frac{1}{3\beta}\left[-\tfrac{1}{3} + 2x - 3x^2 + \tfrac{4}{3}x^3 - (1-\beta)x(1-x)^2\right] \tag{56}$$

when $M \gg 2m_t$. When $M \simeq 2m_t$, Eqs. (B5) and (B6) of Ref. 5 with respectively $\rho = 0$ and $\delta = 0$ are to be used.

$\{\nu\bar{\nu}\}$ spectrum. This "spectrum" is of interest here because if the b and $\bar{b}$ jets' momenta are measured, then most of the necessary ν and $\bar{\nu}$ variables in the t and t rest frame can be determined[4] in spite of the twofold kinematic ambiguity in the t and $\bar{t}$ momenta directions. Now R_t and $-S_t$, and f^t and $-g^t$, have the same functional form as in Sec. III A. If the entire θ_1^ν, θ_2^ν acceptance is integrated over, then, in terms of t and $\bar{t}$ rest-frame variables,

TABLE IV. An ordered procedure for possibly relating the P quantum number η_P of X and $(-)^J$, the signature of X. This is of interest when $X \to \bar{f} f$ is both P invariant and C invariant (this implies $\alpha = 0$).

Result		Measure	$\tau^+\tau^-$ Equation Nos.	$t\bar{t}$ Equation Nos.
(i) $\delta \neq 0$ or 1	$\to \eta_P = (-)^J = C_n$	$I(E_A, E_B)$	(34), (46), (53)	(34), (46)
(ii) $\delta = 1$	$\to \eta_P = \zeta(-)^J$	$F(\phi)$ or $I(E_{\bar{\mu}}, E_e, \cos\psi)$ to obtain ζ	(44), (30), (38)	(57), (44), (38)
(iii) $\delta = 0$	Technique fails. Cannot fail if $J = 0$.			

$$F(E_1^\nu,E_2^\nu,\phi_{\nu\bar\nu}) \equiv \int_{-1}^{1} d(\cos\theta_1^\nu)\int_{-1}^{1} d(\cos\theta_2^{\bar\nu}) I(\theta_1^\nu,E_1^\nu;\theta_2^{\bar\nu},E_2^{\bar\nu};\phi_{\nu\bar\nu})$$
$$\equiv 2\mathcal{N}[R(E_1^\nu)R(E_2^{\bar\nu}) - \zeta\delta(\pi/4)^2\xi_1 S(E_1^\nu)\xi_2 S(E_2^{\bar\nu})\cos\phi_{\nu\bar\nu}]$$
$$\underset{E_1^\nu,E_2^\nu \text{ maximum}}{\longrightarrow} 2\mathcal{N}[1-\zeta\delta(\pi/4)^2\xi_1\xi_2\cos\phi_{\nu\bar\nu}] \quad (57)$$

so the size of the signature for $\zeta\delta$ is comparable to the $\phi\phi$ parity test in η_c decay.

B. Case $m_t \gtrsim 100$ GeV

$\{W^+W^-\}$ *spectrum.* If we neglected the b-quark mass, then the expressions in Sec. III B now apply with θ_1^τ and θ_2^τ (E_A and E_B) the helicity polar angles (energies) of the $W^\pm$, respectively, in the $t,\bar t$ rest frames. Replace $\xi_A \to -\xi_A$, $\xi_B \to -\xi_B$, and the factors

$$\mathcal{S}_t = \frac{m_t^2-2m_W^2}{m_t^2+2m_W^2} = \frac{\Gamma_L-\Gamma_T}{\Gamma_L+\Gamma_T} \quad (58)$$

appear in place of $\mathcal{S}_A$ and $\mathcal{S}_B$. This factor can suppress the signatures needed to determine the α, δ, and ζ parameters because $\mathcal{S}_t$ vanishes at $m_t=\sqrt{2}m_W \cong 113$ GeV. It increases to $\mathcal{S}_t = \mp\frac{1}{3}$, respectively at threshold $m_t \simeq m_W$, and at $m_t = 2m_W \simeq 162$ GeV. For $m_t \gtrsim 200$ GeV, however, the W bosons will be predominantly longitudinally polarized as we discuss in Sec. VI.

We also stress that if m_t is heavy, then the results of Sec. III B can be used to study the chiral properties of $t \to Wb$ and properties of the $X \to t\bar t$ coupling.

C. Case $t \to H^+ b$

If charged Higgs bosons $H^\pm$ are lighter than about $(m_t - m_b)$, the $t \to H^+ b$ decay mode may need to be considered as well as $t \to W^+ b$. In this case the remarks above for the W^+W^- spectrum now also apply to the H^+H^- and HW spectra *except* that, since now $H^\pm$ has spin zero, $\mathcal{S}_t = 1$. *Thus, if kinematically allowed, the H^+ can be used for t polarimetry analogously to how π^+ is used for $\tau^+ \to \pi^+\bar\nu$.*

The $H^\pm$ is expected[12] to be very difficult to observe at a hadron collider but it should be observable at e^+e^- colliders. H^+ is expected to decay into $c\bar s$, $c\bar b$, $\tau^+\nu$, and, if kinematically allowed, W^+H^0 where H^0 is a neutral Higgs boson.

V. TESTS FOR "CP AND P" VIOLATION

When CP and P are both violated in $X \to \bar f f$, two additional distribution parameters must be introduced to describe I. They are

$$\epsilon \equiv \frac{|t_{++}|^2-|t_{--}|^2}{\mathcal{N}} = \frac{|f^{(+)}||f^{(-)}|\cos(\theta_- - \theta_+)}{\mathcal{N}} \quad (59)$$

and

$$\zeta_X \equiv \frac{2\,\mathrm{Im}(t_{++}t_{--})}{|t_{++}|^2+|t_{--}|^2} = \frac{2|f^{(+)}||f^{(-)}|\sin(\theta_- - \theta_+)}{|f^{(+)}|^2+|f^{(-)}|^2} . \quad (60)$$

Note the amplitudes $f^{(\pm)}$ were defined in Eq. (10). The experimental signatures for the violation of both CP invariance and P invariance are listed in Table V along with some important remarks.

Here in the text for the decay modes considered earlier, we give the generalizations of their sequential decay correlation functions. When the analysis of Sec. II is generalized to the case where CP invariance is *not* assumed for $X \to \bar f f$, there is an extra contribution I_x; that is, now

$$I_{\text{total}} = I_0 + I_X , \quad (61)$$

where I_X depends on ζ_X and ϵ, and I_0 denotes the corresponding distribution treated earlier in this paper. In this section we will suppress the "total" subscript when referring to the entire distribution function.

A. $X \to \bar\tau\tau$ decay mode

$\{\mu^+e^-\}$ *spectrum.* Now to Eq. (26) is added

$$I_X(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau;\phi) = C_X(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau) + A_X(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau)\sin\phi , \quad (62)$$

where

$$A_X = \tfrac{1}{2}\mathcal{N}\zeta_X\delta\xi_1 S_1(E_1^\tau)\xi_2 S(E_2^\tau)\sin\theta_1^\tau\sin\theta_2^\tau ,$$
$$C_X = \tfrac{1}{2}\mathcal{N}\epsilon V(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau) \quad (63)$$

with

$$V(\theta_1^\tau,E_1^\tau;\theta_2^\tau,E_2^\tau) = \xi_1 S(E_1^\tau)R(E_2^\tau)\cos\theta_1^\tau - R(E_1^\tau)\xi_2 S(E_2^\tau)\cos\theta_2^\tau . \quad (64)$$

TABLE V. Signatures for the violation of both CP invariance and P invariance in $X \to \bar f f$ decay where $f\bar f$ are a spin-$\frac{1}{2}$ particle-antiparticle pair.

Signature	Remarks
$\lvert\zeta\rvert \neq 1$	Never fails if $\delta \neq 0$. Fewer events are needed when the $\bar f$ and f rest-frame variables are accessible.
$\epsilon \neq 0$	Can vanish "accidentally" if $(\theta_+ - \theta_-) = \pi/2$ or $3\pi/2$ even though $\lvert f^{(\pm)}\rvert \neq 0$. Measurement of ζ or ζ_X is needed to exclude this possibility. Also if $\delta = 0$, then $\epsilon = 0$.
$\zeta_X \neq 0$	Requires measurement of the coefficient of $\sin\phi$. This necessitates a unique determination of the $\bar f$, or f, momentum direction so, e.g., in $t \to W^+ b$ the W^+ must decay hadronically. For $\tau^+\tau^-$ decay mode the twofold kinematic ambiguity in τ^+ and τ^- momenta directions means ζ_X is not measurable by this technique.

While the ϵ parameter in the case $\tau^+ \to \mu^+ \nu\bar{\nu}$ does appear added to $-\alpha$ in the muon's energy distribution in the X rest frame, i.e., now in place of Eq. (32),

$$I(E_{\bar{\mu}}) = f(x) + \xi(-\alpha+\epsilon) g(x) \ , \tag{65}$$

the ϵ parameter is alone responsible for an antisymmetric term under $x \leftrightarrow y$ in the energy correlation function $I(E_{\bar{\mu}}, E_e)$. That is, we define

$$\Delta(x,y) = \tfrac{1}{2}[I(x,y) - I(y,x)] \tag{66}$$

which corresponds to folding $\{\mu^+ e^-\}$ events with a "minus" about the diagonal $E_{\bar{\mu}} = E_e$. The resulting distribution at the tree level is

$$\Delta(x,y) = \epsilon[\xi_1 g(x) f(y) - \xi_2 g(y) f(x)] \tag{67}$$

with $g(x)$ and $f(x)$ as in Eq. (33). If after this "negative folding," the energy of the softer lepton is integrated out, we find a very simple single-variable distribution $(\xi_1 = \xi_2 = 1)$

$$\Delta(x) \equiv \int_0^x dy\, \Delta(x,y) = \epsilon \tfrac{1}{12} x^3 (1-x)(8 - x - x^2) \ , \tag{68}$$

which has a maximum at $x_{\max} \cong 0.732$ (note that $\int_0^1 dx\, \Delta(x) = \epsilon/35$).

$\{h_A^+ h_B^-\}$ *spectrum.* The extra contribution to Eq. (38) is

$$I_X(\theta_1^\tau, \theta_2^\tau; \phi) = C_X(\theta_1^\tau, \theta_2^\tau) + A_X(\theta_1^\tau, \theta_2^\tau) \sin\phi \tag{69}$$

with

$$A_X = \tfrac{1}{2} \mathcal{N} \zeta_X \delta \xi_A \mathcal{S}_A \xi_B \mathcal{S}_B \sin\theta_1^\tau \sin\theta_2^\tau \ , \tag{70}$$

$$C_X = -\tfrac{1}{2} \mathcal{N} \epsilon (\xi_A \mathcal{S}_A \cos\theta_1^\tau - \xi_B \mathcal{S}_B \cos\theta_2^\tau) \ . \tag{71}$$

The h_A^+ energy distribution now is

$$I(E_A) = 1 + (\alpha - \epsilon) \xi_A \mathcal{S}_A \cos\theta_1^\tau \tag{72}$$

and

$$\Delta(E_A, E_B) = -\epsilon(\xi_A \mathcal{S}_A \cos\theta_1^\tau - \xi_B \mathcal{S}_B \cos\theta_2^\tau) \ . \tag{73}$$

$\{\mu^+ h_B^-\}$ *spectrum.* To Eq. (49) is added

$$I_X(\theta_1^\tau, E_1^\tau; \theta_B^\tau; \phi) = C_X(\theta_1^\tau, E_1^\tau; \theta_B^\tau) + A_X(\theta_1^\tau, E_1^\tau; \theta_B^\tau) \sin\phi \tag{74}$$

with

$$A_X = -\tfrac{1}{2} \mathcal{N} \zeta_X \delta \xi S(E_1^\tau) \xi_B \mathcal{S}_B \sin\theta_1^\tau \sin\theta_B^\tau \ , \tag{75}$$

$$C_X = \tfrac{1}{2} \mathcal{N} \epsilon [\xi S(E_1^\tau) \cos\theta_1^B - R(E_1^\tau) \xi_B \mathcal{S}_B \cos\theta_B^\tau] \ . \tag{76}$$

The negatively folded energy correlation function is

$$\Delta(E_{\bar{\mu}}, \cos\theta_B^\tau) = \epsilon[\xi g(x) + f(x) \xi_B \mathcal{S}_B \cos\theta_B^\tau] \ . \tag{77}$$

B. $X \to t\bar{t}$ decay mode

Using Eqs. (56) for $M \gg 2m_t$ and their analogs[5] when $M \simeq 2m_t$, the results above for the $X \to \bar{\tau}\tau$ mode can be transcribed for the $X \to t\bar{t}$ mode as was done earlier in Sec. IV. For $\{WW\}$, $\{HW\}$, $\{HH\}$ in Eqs. (69)–(73) let $\xi_A \to -\xi_A$, $\xi_B \to -\xi_B$, $\mathcal{S}_{A,B} \to \mathcal{S}_t$, as in Sec. IV for $m_t \gtrsim 100$ GeV.

VI. IMPLICATIONS FOR HIGGS-BOSON AND t-QUARK SEARCHES AT HADRON COLLIDERS

If[7,8] the top quark lies in, or perhaps above, the 180–200 GeV region then in Higgs-boson–technipion searches[10,11] both the $\bar{\tau}\tau$ and the $t\bar{t}$ decay modes[12–15] are of greater experimental interest. (i) As m_t increases the $H/\pi_{\rm TC} \to \tau^+\tau^-$ branching ratio remains large for a greater range of $M_{H/\pi_{\rm TC}}$ values and the $t\bar{t}$ background is also less important. It is also important that both the UA1 Collaboration and the Collider Detector at Fermilab (CDF) Collaboration have shown that it is easy to recognize taus. The same may be true for detectors at the Superconducting Super Collider (SSC) or CERN Large Hadron Collider (LHC). (ii) Since the Higgs boson couples most strongly to the heaviest fermion, if m_t is very large, then the $t\bar{t}$ channel should be prominent above the $t\bar{t}$ threshold assuming $m_H > 2m_t$. If on the other hand $m_H < 2m_t$ occurs due to dynamical symmetry breaking,[16,17] then the $X \to t\bar{t}$ decay mode may turn out to be a colorful new world of X strong-interaction resonancelike physics.

A. Searches with $\bar{\tau}\tau$ channel

Three methods have been studied in the literature[13–15] to search for $H/\pi_{\rm TC} \to \bar{\tau}\tau$: (a) associated production[13] of H and Z, (b) production[14] of H with a recoiling, large transverse momentum gluon jet, and (c) inclusively produced[15] H where both τ's are reconstructed using high-multiplicity τ decay modes. Studies of (a) and (b) have found that it is problematic to achieve sufficient mass resolution.

We believe that further analysis, but with hadronic Monte Carlo simulations that include spin correlations in the $X \to \tau^+\tau^-$ decays, is warranted since in $H/\pi_{\rm TC}$ decays both τ^+ and τ^- must have the same helicity whereas in Z decays they have opposite helicities to order $(m_\tau/M)^2 \simeq 3.2 \times 10^{-4}$. This yields distinctively different signatures to the γ^*, Z^* background and to the $H/\pi_{\rm TC}$ in the sequential decay correlation functions $I(E_A, E_B, \psi)$; see Table VI. For the $\{l^+l^-\}$ spectrum, the difference in α and ζ due to very different behavior of the D and U terms in Eq. (30) is a 20% effect in the X rest frame. By Eq. (34), in $I(E_{l^+}, E_{l^-})$ from α and δ it is an $\sim 15\%$ effect. In the case of associated production or of production with a recoiling gluon these effects in $\tau^\pm \to l^\pm \nu\bar{\nu}$ can be enhanced (see remarks on usage of $t\bar{t}$ mode in Ref. 4) due to the greater kinematic information on the $\tau^\pm$ rest frames. For the $\{h_A^+, h_B^-\}$ spectrum, there are larger signal-versus-background effects from all of the α, δ, and ζ parameters based on Table VI because of the tighter two-body decay kinematics and because the substitution rule for changing $\tau^+ \to l^+ \nu\bar{\nu}$ to $\tau^+ \to h_A^+ \nu$ in $I(E_A, E_B)$ is

TABLE VI. Comparison of parameters describing the sequential decay correlation functions for $X\to\tau^+\tau^-, t\bar{t}, \ldots$ in the case X is a Z boson, with the cases when X is a Higgs boson ($\gamma_{CP}=+1$) and when it is a technipion ($\gamma_{CP}=-1$). These differences, due to the opposite $\bar{f}f$ helicity structure, can be used to help separate a H (or π_{TC}) from a γ^*, Z^* background (see text).

Particle	Helicity amplitudes	Parameter values α	δ	ζ	*CP* and *P* violation parameters
Z	$t_{++}\simeq 0$, $t_{--}\simeq 0$	-0.159	0	0	$\epsilon=\zeta_X=0$
H	$t_{+-}=t_{-+}=0$	0	1	1	can measure ζ, ϵ, and ζ_X
π_{TC}		0	1	-1	

$$f(x)\to 1,\quad g(x)\to -\mathcal{S}_A\cos\theta_A^\tau \tag{78}$$

in the case of a pure $V-A$ charge-current coupling in tau decays. The quantities on the right part of Eq. (78) provide a significantly more striking behavior to the energy correlation function $I(E_1,E_2)$ than those in the left half [see discussion in (i) in Sec. III B above].

B. Searches with $t\bar{t}$ channel and for the t quark

The remarks above for the $\{h_A^+ h_B^-\}$ spectrum apply to $t\to W^+ b$ and $\bar{t}\to W\bar{b}$ when $\mathcal{S}_t$ of Eq. (58) is introduced. *Should* $m_t\cong 280$ GeV (200 GeV), then the $\mathcal{S}_t$ suppression factor is not significant because $\mathcal{S}_t\cong 0.72$ (0.51) [equivalently $\Gamma_L/\Gamma_T\cong 7$ (3); i.e., the W's in $t\to Wb$ are predominantly longitudinally polarized]. *The longitudinal polarization of these W's would be a very useful signature in searching for the top quark, as well as in $X\to t\bar{t}$ physics.* Investigations[18,19] have shown that high-momentum longitudinally polarized W's decay predominantly into two jets of approximately equal energy. The background from QCD jet systems (Wjj) and from two jets from WW production will usually have one soft jet and one hard jet, and will be considerably less collimated than in the case of longitudinal W decay.

For lower m_t masses the zero due to the $\mathcal{S}_t$ factor, Eq. (58), at $\sqrt{2}m_W\simeq 113$ GeV means that the W's will be unpolarized. This is consistent with the work in Ref. 20 which for 85 GeV $\leq m_t\leq$ 170 GeV found spin correlation effects to be less than 10% in hadroproduction and decay of t-quark pairs at Fermilab Tevatron energies.

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APPENDIX: HOW TO CHECK SIGNS

The signs in the various correlation functions in this paper can be simply checked by the reader by taking particle A and/or B to maximum or minimum energy and comparing the result to what is allowed and forbidden from the density matrix for the f (or $\bar{f}$) in the f ($\bar{f}$) rest frame. This is equivalent to using simple helicity arguments and angular momentum conservation (see, e.g., Ref. 21).

The density matrix for $\tau^+\to\mu^+\nu\bar{\nu}$ for the final μ^+ with maximum energy at angle θ^τ in the τ^+ rest frame with the τ^+ polarized along the z axis is

$$\rho_{\tau^+}(x^\tau\to 1)=1+\xi\cos\theta^\tau\ , \tag{A1}$$

where $\xi=1$ for a pure $V-A$ coupling. Equation (A1) can be easily checked.[21] Similarly, for $\tau^-\to\mu^-\bar{\nu}\nu$ for a final μ^- with maximum energy at angle θ^τ in the τ^- rest frame with the τ^- polarized along the z axis,

$$\rho_{\tau^-}(x^\tau\to 1)=1-\xi\cos\theta^\tau\ . \tag{A2}$$

For a semihadronic decay mode, $\tau^+\to h_A^+\bar{\nu}$ the density matrix is

$$\rho_{\tau^+}=1-\xi_A\mathcal{S}_A\cos\theta_A^\tau\ , \tag{A3}$$

where θ_A^τ is the direction of h_A^+ in the τ^+ rest frame. And for $\tau^-\to h_B^-\nu$, it is

$$\rho_{\tau^-}=1+\xi_B\mathcal{S}_B\cos\theta_B^\tau\ . \tag{A4}$$

The normalized parameters $\mathcal{S}_A$ and $\mathcal{S}_B$ are defined in Eq. (24). The chiral polarization ξ parameter is defined in the Introduction. Again elementary arguments confirm the signs.

For the t-quark decays, we find for μ^+ at an angle θ^t from $t\to\mu^+\nu b$ when μ^+ has a minimum energy,

$$\rho_t(x^t\to 0)=1+\xi\cos\theta^t \tag{A5}$$

and, for μ^- from $\bar{t}\to\mu^-\bar{\nu}\bar{b}$,

$$\rho_{\bar{t}}(x^t\to 0)=1-\xi\cos\theta^t\ . \tag{A6}$$

When, instead, we consider ν at an angle θ^ν from $t\to\mu^+\nu b$ with ν at maximum energy,

$$\rho_t(x^t\to 1)=1-\xi\cos\theta^\nu \tag{A7}$$

and, for $\bar{\nu}$ from $\bar{t}\to\mu^-\bar{\nu}\bar{b}$,

$$\rho_{\bar{t}}(x^t\to 1)=1+\xi\cos\theta^\nu\ . \tag{A8}$$

Similarly, for h_A^+ from $t\to h_A^+ b$ in the t rest frame with h_A^+ at an angle θ_A,

$$\rho_t=1+\xi_A\mathcal{S}_A\cos\theta_A \tag{A9}$$

and, for h_B^- from $\bar{t}\to h_B^-\bar{b}$ with h_B^- at angle θ_B,

$$\rho_{\bar{t}}=1-\xi_B\mathcal{S}_B\cos\theta_B\ . \tag{A10}$$

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