E_{1+}/M_{1+} and S_{1+}/M_{1+} and their Q^2 dependence in $\gamma_v N \to \Delta$ with relativized quark-model wave functions

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We calculate the multipole ratios E_{1+}/M_{1+} (electric quadrupole to magnetic dipole) and S_{1+}/M_{1+} (scalar quadrupole to magnetic dipole) for the reaction $\gamma_v N \to \Delta$. We use the nonrelativistic transition operators but wave functions expanded in a large harmonic-oscillator basis which are the solutions of a "relativized" Hamiltonian (relativized wave functions), in order to test the dependence of the results on truncation of the wave-function basis. Our results show an M_{1+} multipole closer to the experimental data at the photon point, $E_{1+}(0)/M_{1+}(0) \simeq -0.2$, and smaller values of these ratios than previous calculations at all Q^2. The current-conservation condition between S_{1+} and L_{1+} (the longitudinal quadrupole multipole) is also satisfied to a better approximation with the relativized wave functions at values of Q^2 where the model is believable.

I. INTRODUCTION

It has been acknowledged for some time[1–5] that in the quark model the presence of a nonzero electric quadrupole multipole in the photodecay amplitude for the Δ (i.e., $\Delta \to N\gamma$) is a signal for the presence of D-wave components in the wave functions of the nucleon or the Δ. Since these are both spatial ground states in the quark model, these D-wave components are mixed in by the tensor part of the color-magnetic spin-spin interaction. Taking the ratio of the strength E_{1+} of this multipole to that of the dominant magnetic dipole multipole M_{1+} removes some of the model dependence, and is a sensitive test of the presence of color-magnetic interactions in these states; it has been investigated by many authors[6] in various models. We will focus in this paper on nonrelativistic-quark-model calculations,[1–3,5,7,8] which predict a ratio at the photon point of roughly -0.3% to -0.8% (see Table I), which agrees very roughly with the present, rather uncertain experimental situation.[9]

If the inverse process is considered, that of production of the Δ by a nucleon absorbing a virtual photon from a scattered electron (electroproduction of the Δ), then as well as these amplitudes (due to photons with helicity ± 1) there is an additional helicity-zero amplitude allowed. This can be calculated using the zero component of the four-vector electromagnetic current, or the three-vector part, and these should be related by current conservation. Bourdeau and Mukhopadhyay[7] tested this relation using two different sets[10] of nonrelativistic-quark-model (NRQM) wave functions, and made conclusions about the validity of these wave functions and of the NRQM based on their results. In particular they conclude (as pointed out earlier by Drechsel and Giannini[4]) that truncation at the first excited level of the oscillator basis, or perhaps the nonrelativistic approximation,[11] is responsible for the failure of this relation. They use an extension of the Siegert theorem[12] to approximately equate the scalar multipole S_{1+} with E_{1+} calculated at the photon point, since their results suggest that the S_{1+} multipole depends weakly on the details of the wave functions. In this way they make a prediction for E_{1+}/M_{1+}.

We feel it is interesting to test these assertions by enlarging the oscillator basis. Is the lack of current conservation only due to this truncation, and is it justified[13] to use the Siegert theorem? How does the extension of the basis (and the difference in the source of the wave functions) affect the multipole ratios at the photon point and their behavior as a function of $-K^2$?

II. THE MODEL

A. The helicity-1 nonrelativistic transition operator

Our starting point is the nonrelativistic interaction Hamiltonian

$$H_{\text{int}} = -\sum_i \left\{ \frac{e_i}{2m_i}[\mathbf{p}_i \cdot \mathbf{A}(\mathbf{r}_i) + \mathbf{A}(\mathbf{r}_i) \cdot \mathbf{p}_i] + \boldsymbol{\mu}_i \cdot \nabla \times \mathbf{A}(\mathbf{r}_i) \right\}, \quad (1)$$

where $\boldsymbol{\mu}_i = e_i \boldsymbol{\sigma}_i / 2m_i$ is the magnetic moment of the ith quark, e_i, m_i, $\boldsymbol{\sigma}_i/2$, and $\mathbf{p}_i$ are its charge, (constituent) mass, spin, and momentum, and $\mathbf{A}(\mathbf{r}_i)$ is the electromagnetic potential at position $\mathbf{r}_i$. For definiteness we will address the process of (for now real) photoproduction of the Δ in its center-of-mass frame. Then the amplitudes $A_{1/2}$ and $A_{3/2}$ are defined in terms of the helicity states of the nucleon and Δ by

$$A_\lambda = \langle \Delta(J=\tfrac{3}{2}, M=\lambda) | H_{\text{int}} | \gamma(\lambda_\gamma = 1) N(J=\tfrac{1}{2}, M=-\lambda_N) \rangle , \quad (2)$$

TABLE I. Values of the ratio E_{1+}/M_{1+} at the photon point ($K^2=0$) in various quark-model calculations.

Reference	$E_{1+}/M_{1+}(\%)$
Gershtein and Dzhikiya (Ref. 1)	-0.32
Isgur, Karl, and Koniuk (Ref. 2)	-0.41
Weyrauch and Weber (Ref. 5)	$-0.69, -0.81$[a]
Bourdeau and Mukhopadhyay (Ref. 7)	$-0.6, -0.3$[b]
Gogilidze, Surovtsev, and Tkebuchava (Ref. 8)	-0.65
This work	-0.21

[a]This first value is with the color-magnetic hyperfine interaction only; the second has a one-pion-exchange contribution without the zero range part. Both are calculated with the electromagnetic-charge operator, and not the current.

[b]The two values quoted for Ref. 7 are those calculated using S_{1+} and the Siegert theorem to estimate E_{1+}, and the usual approach.

where to conserve helicity we have (see Fig. 1) $\lambda_N=-\frac{1}{2}$ for $A_{1/2}$ and $\lambda_N=\frac{1}{2}$ for $A_{1/2}$, where the (incoming) photon's three-momentum $\mathbf{k}$ is in the z direction. If we insert, as in Ref. 1, the field

$$\mathbf{A}(\mathbf{r}_i)=\sum_{k,\lambda}\left[\frac{\pi}{k}\right]^{1/2}(\epsilon_{k,\lambda}a_{k,\lambda}e^{i\mathbf{k}\cdot\mathbf{r}_i}+\epsilon^*_{k,\lambda}a^\dagger_{k,\lambda}e^{-i\mathbf{k}\cdot\mathbf{r}_i}) \quad (3)$$

into Eqs. (1) and (2), then we can write the amplitudes as

$$A_\lambda=\sum_i\langle\Delta(J=\tfrac{3}{2},M=\lambda)|H_i^t|N(J=\tfrac{1}{2},M=\lambda-1)\rangle\ . \quad (4)$$

Here the expectation value is taken to mean integration over the coordinates $\boldsymbol{\rho}=(\mathbf{r}_1-\mathbf{r}_2)/\sqrt{2}$, $\boldsymbol{\lambda}=(\mathbf{r}_1+\mathbf{r}_2-2\mathbf{r}_3)/\sqrt{6}$, and $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_3)/3$, and expectation value of spin- and flavor-dependent quantities between the spin wave functions $\chi_\Delta^\dagger$ and χ_N and the flavor wave functions ϕ_Δ and ϕ_N. We have already specialized to the equal-mass case of interest, and we will show that it is sufficient to know the form of the operator for the third quark in the sum in Eq. (1). H_3^t in Eq. (4) can be written as

$$H_3^t=\frac{e_3}{m}\left[\frac{\pi}{k}\right]^{1/2}e^{ikr_{3z}}\left[p_{3+}+k\frac{\sigma_{3+}}{2}\right]\ , \quad (5)$$

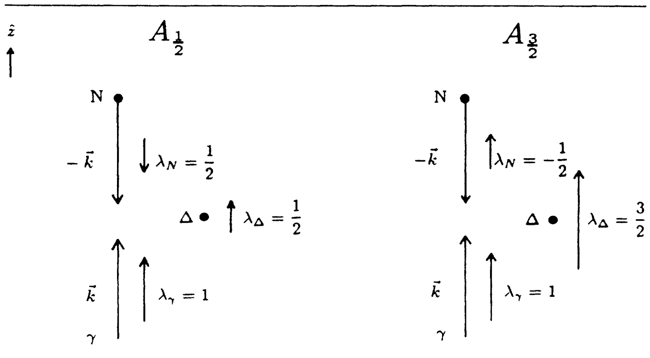

FIG. 1. Momentum and helicity assignments in the Δ center-of-momentum frame for the (photon) helicity-1 transitions.

where we have used the polarization vector $\boldsymbol{\epsilon}=(-1/\sqrt{2})(1,i,0)$, and written a single term for the derivative operator by noting that p_{3+} commutes with $e^{ikr_{3z}}$. To write (4) as a simple integral over the internal coordinates we must perform the integration over the center-of-mass coordinate $\mathbf{R}$. To do so we insert plane waves for the center-of-mass motion of both baryons (one with momentum zero) and note that $\mathbf{r}_3=\mathbf{R}-\sqrt{2/3}\boldsymbol{\lambda}$, with the result that

$$H_3^t\rightarrow-\frac{e_3}{m}\left[\frac{\pi}{k}\right]^{1/2}e^{-ik\sqrt{2/3}\lambda_z}\left[\sqrt{2/3}p_{\lambda+}-k\frac{\sigma_{3+}}{2}\right]\ . \quad (6)$$

Then, finally the multipoles E_{1+} and M_{1+} are given by

$$E_{1+}=\frac{1}{2\sqrt{3}}(A_{3/2}-\sqrt{3}A_{1/2})\ ,$$
$$M_{1+}=-\frac{1}{2\sqrt{3}}(3A_{3/2}+\sqrt{3}A_{1/2})\ . \quad (7)$$

If we wish to present results for the multipoles themselves and not just ratios of multipoles, for the general case away from the photon point $K^2=0$, then we must[14] use an *ansatz* for the "normalization of a virtual photon." We adopt the convention of Bourdeau and Mukhopadhyay,[7] that $\sqrt{\pi/k}$ in Eqs. (5) and (6) is replaced by $\sqrt{\pi/K_0}$, where $K_0=(M_\Delta^2-M_N^2)/2M_\Delta$, the value of k at the photon point. The k in the exponential in (5) and (6) is given by conservation of energy to be

$$k^2=\left[\frac{M_\Delta^2+M_N^2-K^2}{2M_\Delta}\right]^2-M_\Delta^2\ , \quad (8)$$

where K^2 is the square of the photon four-momentum (0 for real photons).

B. The helicity-0 nonrelativistic transition operators

The helicity-0 operators are found in much the same way as the helicity-1 operators. We again concentrate on photoproduction of the Δ in its center-of-mass frame, now with the photon polarization vector $\boldsymbol{\epsilon}=(0,0,1)$. The equivalent of (4) for the longitudinal multipole is

$$L_{1+}=\sum_i\frac{1}{\sqrt{2}}\langle\Delta(J=\tfrac{3}{2},M=\tfrac{1}{2})|H_i^l|N(J=\tfrac{1}{2},M=\tfrac{1}{2})\rangle\ , \quad (9)$$

where H_3^l has the form

$$H_3^l=-\frac{e_3}{m}\left[\frac{2\pi}{k}\right]^{1/2}\frac{1}{2}(p_{3z}e^{ikr_{3z}}+e^{ikr_{3z}}p_{3z})\ . \quad (10)$$

Performing the integration over the center-of-mass coordinate in (9) we find that

$$H_3^l\rightarrow\frac{e_3}{m}\left[\frac{2\pi}{k}\right]^{1/2}\left[\sqrt{2/3}p_{\lambda z}+\frac{k}{2}\right]e^{-ik\sqrt{2/3}\lambda_z}\ . \quad (11)$$

The scalar multipole is defined by

$$S_{1+}=\frac{1}{\sqrt{2}}\sum_i \langle \Delta(J=\tfrac{3}{2},M=\tfrac{1}{2})|H_i^s|N(J=\tfrac{1}{2},M=\tfrac{1}{2})\rangle \ , \tag{12}$$

where the operator H_i^s is the zero component of the electromagnetic-current four-vector that has H_i^l as its z component. The correct operator may be found by examining the electromagnetic current for the quark-photon vertex, putting in the explicit form of the four-spinors for the quarks in terms of Pauli spinors and Pauli matrices, and the result is that in the nonrelativistic limit the current is proportional to $(1,(\mathbf{p}_i+\mathbf{p}_i')/2m_i)$, where $\mathbf{p}_i$ and $\mathbf{p}_i'$ are the initial and final momenta of the struck quark, as in Eq. (10). This gives the correct form of H_3^s to be

$$H_3^s=-\frac{e_3}{m}\left[\frac{2\pi}{k}\right]^{1/2} e^{ikr_{3z}} \ , \tag{13}$$

which when we integrate over the center-of-mass coordinate gives

$$H_3^s\to-\frac{e_s}{m}\left[\frac{2\pi}{k}\right]^{1/2} e^{-ik\sqrt{2/3}\lambda_z} \ . \tag{14}$$

C. Calculational methods

The operators in Eqs. (6), (11), and (14) need to be evaluated between the relativized quark-model wave functions. Details of the spectral calculation used to generate these wave functions may be found in Capstick and Isgur[15] (CI), along with a complete description of the model itself, and the conventions used in the CI wave functions. The main differences between the CI model wave functions, apart from the obvious differences in the baryon Hamiltonian used to generate them, are (1) the wave functions are found by performing a variational calculation in a large harmonic-oscillator basis, up to $N=6$ (the third excited level in the harmonic oscillator) for the Δ and nucleon, yielding a large number of substates in each wave function, (2) the CI calculation did not explicitly symmetrize the spatial-spin-flavor wave functions under exchange of u and d quarks, but rather allowed the Hamiltonian, with $m_u=m_d$, to sort the basis into states with total symmetry in their spatial-spin wave functions (Δ's) and those with mixed (λ-type) symmetry (nucleons). The flavor wave functions for the nucleon and Δ states of the same charge are the same, uud for p and Δ^+ and ddu for n and Δ^0.

Because of (1) it is obvious that the matrix elements of the transition operators should be *calculated* analytically with invariance methods and then *evaluated* with a computer. Once the matrix of individual oscillator substate transition amplitudes is known, then we need only calculate the matrix sum, e.g., for $A_{3/2}$,

$$A_{3/2}=\sum_{i,a,b}\langle \Delta(\tfrac{3}{2},\tfrac{3}{2})|a(\tfrac{3}{2},\tfrac{3}{2})\rangle\langle a(\tfrac{3}{2},\tfrac{3}{2})|H_i^l|b(\tfrac{1}{2},\tfrac{1}{2})\rangle\langle b(\tfrac{1}{2},\tfrac{1}{2})|N(\tfrac{1}{2},\tfrac{1}{2})\rangle \ , \tag{15}$$

where $|a\rangle$ and $|b\rangle$ are the oscillator substates that make up the Δ and nucleon wave functions. In the Appendix we list the formulas used to find the oscillator substate transition amplitudes $\langle a(\tfrac{3}{2},\lambda)|H_3^t|b(\tfrac{1}{2},\lambda-1)\rangle$, $\langle a(\tfrac{3}{2},\tfrac{1}{2})|H_3^l|b(\tfrac{1}{2},\tfrac{1}{2})\rangle$, and $\langle a(\tfrac{3}{2},\tfrac{1}{2})|H_3^s|b(\tfrac{1}{2},\tfrac{1}{2})\rangle$ needed to find the third quark transition amplitudes above.

At first sight it would appear that because of the lack of total permutational symmetry in the spatial-spin wave functions for the nucleon in (4), (9), and (12) that we should have to evaluate the contributions from transitions involving both quark 3 and quark 2 (the wave functions, excluding color, for the nucleon and Δ are always symmetric under exchange of the first and second quark; see Ref. 15). However one can simply argue that these should be the same for the wave functions used here; we will prove in what follows that for the transition $\gamma N\to\Delta$ the transition matrix elements of the operators e_iO_i are the same for all i. If we write the full wave function for the Δ as

$$\begin{aligned}|\Delta\rangle &= C_A(uud)\sum_a c_a\Psi_a(\rho,\lambda)\chi_a\\ &= C_A(uud)\sum_a c_a|a\rangle\\ &= C_A(uud)|S\rangle \ ,\end{aligned} \tag{16}$$

then if we ignore the color and flavor wave functions the sum over a is totally symmetric (S) under quark exchange. Similarly we can write the nucleon wave function as

$$\begin{aligned}|N\rangle &= C_A(uud)\sum_b c_b\Psi_b(\rho,\lambda)\chi_b\\ &= C_A(uud)\sum_b c_b|b\rangle\\ &= C_A(uud)|M^\lambda\rangle \ ,\end{aligned} \tag{17}$$

with the sum over b having mixed λ-type symmetry (M^λ) under quark exchange. We wish to calculate the transition amplitude

$$\left\langle\Delta\left|\sum_i e_iO_i\right|N\right\rangle=\langle S|[2(\tfrac{2}{3}O_2)+(-\tfrac{1}{3}O_3)]|M^\lambda\rangle \ , \tag{18}$$

where the O_i are operators which depend only on variables associated with quark i. Now from the properties of the M^λ representation of the permutation group S_3 we have that, under the permutation (23),

$$\langle S|O_2|M^\lambda\rangle\to\langle S|O_3\left[\frac{\sqrt{3}}{2}|M^\rho\rangle-\tfrac{1}{2}|M^\lambda\rangle\right] \ , \tag{19}$$

and since O_3 cannot affect (12) exchange symmetry the first term is zero. Therefore

$$\langle S|O_2|M^\lambda\rangle \to -\tfrac{1}{2}\langle S|O_3|M^\lambda\rangle \tag{20}$$

and so

$$\langle \Delta|\tfrac{2}{3}O_2|N\rangle = \langle \Delta|-\tfrac{1}{3}O_3|N\rangle \ . \tag{21}$$

The eventual result is that the amplitude for all three quarks is the same, for this transition, even though the wave functions themselves are not totally symmetric under quark exchange. This allows us to retain the simplification of calculating only for the third quark.

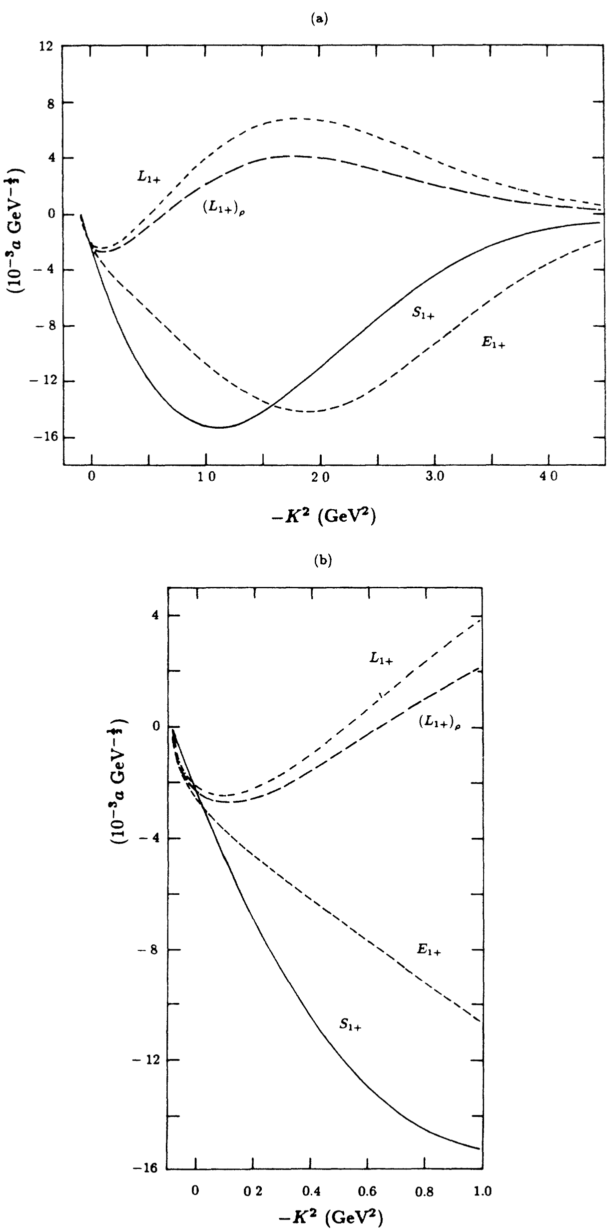

FIG. 2. The multipoles E_{1+}, L_{1+}, S_{1+}, and $(L_{1+})_\rho$ (calculated from S_{1+} using current conservation) with the wave functions of IKK. We have used $\alpha=0.41$ GeV and $m_{u,d}=0.336$ GeV. The multipoles are given in terms of the πN scattering phase factor a; see Ref. 13.

III. RESULTS

A failing of the nonrelativistic calculation which is simply corrected here is that of the size of the dominant multipole M_{1+} at the photon point. If we follow the calculation of Rondon-Aramayo[13] for the πN scattering phase factor a, then the model of Isgur, Karl, and Koniuk[2] (recalculated here) and that of Gershtein and Dzhikiya[1] give 15.4 and 15.6, respectively, for $M_{1+}^{\pi^0}(0)$ in units of $10^{-3}m_{\pi^+}^{-1}$. In the present calculation we find $M_{1+}(0)=27.2$ in the same units which is closer to the experimental value[16] of 25.5 ± 0.2. The main reason for the improvement is very simple; we have used the relativized-quark-model light-quark (constituent) mass $m_{u,d}=0.22$ GeV, whereas the NRQM value[17] is approximately 0.33 GeV. There is also some slight increase resulting from the use of the CI wave functions.

In Fig. 2 we show the results of using the Isgur-Karl-Koniuk (IKK) model parameters[18] and wave functions for the multipoles E_{1+}, L_{1+}, and S_{1+}. We have adopted the procedure of Rondon-Aramayo[13] and calculated for $-K^2$ less than zero (in the unphysical region) to demonstrate the convergence of these amplitudes in the limit that $k=|\mathbf{k}|\to 0$. Note that the Siegert theorem $E_{1+}(-K^2)\simeq S_{1+}(-K^2)$ at $-K^2=0$ is roughly satisfied here, although with other wave functions this may not be the case, and we will see that it fails in our calculation also.

Also plotted in Fig. 2 is $(L_{1+})_\rho$, the L_{1+} multipole calculated by assuming current conservation. If current conservation holds for the electromagnetic current at the quark-photon vertex then we should have that $k_0 S_{1+}=kL_{1+}$, so that an alternate way of calculating L_{1+} is $(L_{1+})_\rho=k_0S_{1+}/k$. We can see that this relation is approximately satisfied for small $-K^2$, but becomes less and less valid as $-K^2$ becomes larger. Also note that[13] all of these multipoles start to fall off rapidly by $-K^2\simeq 2.0$ GeV^2, and are effectively suppressed by 4.0 GeV^2, due to the exponential form factor $e^{-k^2/6\alpha^2}$ present in all of the transition amplitudes from a given oscillator substate to another. This decrease is faster than that of the expected dipole form factor.[13,19]

In Fig. 3 we have plotted these multipoles calculated with the CI wave functions expanded up to $N=6$ for both the nucleon and Δ. In the spectral calculation the wave functions were calculated for a coarse grid of α values so that the energies could be roughly minimized in α (one test of the convergence of that calculation is that the energies were soft functions of α). Accordingly, to get some idea of the sensitivity of the calculation to the wave functions we have calculated for $\alpha=0.6$ GeV (where the Δ energy was minimized with a grid of 0.1 GeV) and for $\alpha=0.7$ GeV (where the nucleon energy was minimized). Although the general shape of the multipoles as a function of $-K^2$ is roughly the same as for the IKK model, we see that the dependence on $-K^2$ is now much softer, with the multipoles falling off slowly after $-K^2\simeq 2$–3 GeV^2. This is as expected, since in this basis the spatial wave functions have the form of a sixth-order polynomial in ρ and λ times a Gaussian with a larger α; this means that the resulting amplitudes are sixth-order polynomials in k^2/α^2 multiplied by a Gaussian form factor which falls off less rapidly (of the form $e^{-k^2/6\alpha^2}$). We see from Fig. 3 that current conservation is satisfied to a better approximation in this calculation up to about 1.5 GeV^2. The Siegert theorem does not hold (even approximately) for these wave functions, with $E_{1+}(0)$ and $S_{1+}(0)$ differing roughly by a factor of $E_{1+}(0)/S_{1+}(0)=2.5$. Of these three multipoles, E_{1+} is

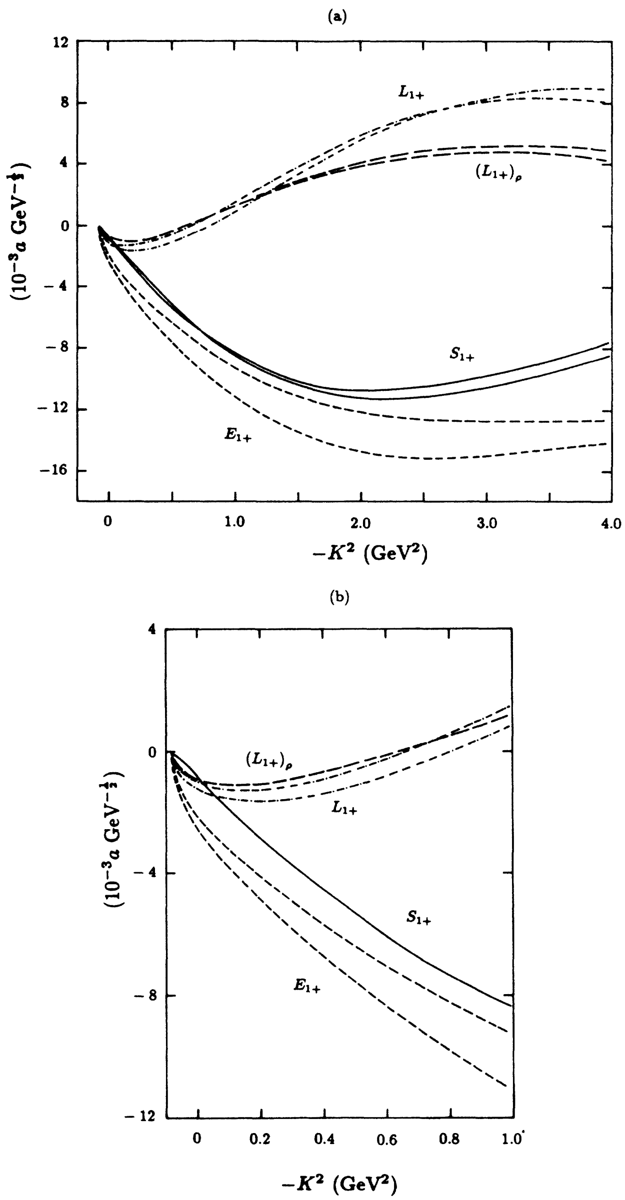

FIG. 3. (a) The multipoles E_{1+}, L_{1+}, S_{1+}, and $(L_{1+})_\rho$ with the wave functions of Ref. 15, expanded to $N \le 6$. In all cases the line with the largest absolute value at $-K^2=4.0$ GeV^2 is the calculation for $\alpha=0.7$ GeV, and that with the smallest $\alpha=0.6$ GeV. We have used $m_{u,d}=0.22$ GeV, and the multipoles are given in terms of the πN scattering phase factor a. (b) As in (a) but with the low $-K^2$ region expanded. For L_{1+} and E_{1+} the upper line is for $\alpha=0.6$ GeV and the lower for $\alpha=0.7$ GeV in both cases. For $(L_{1+})_\rho$ and S_{1+} the lines for these two values of α are coincident on the scale shown.

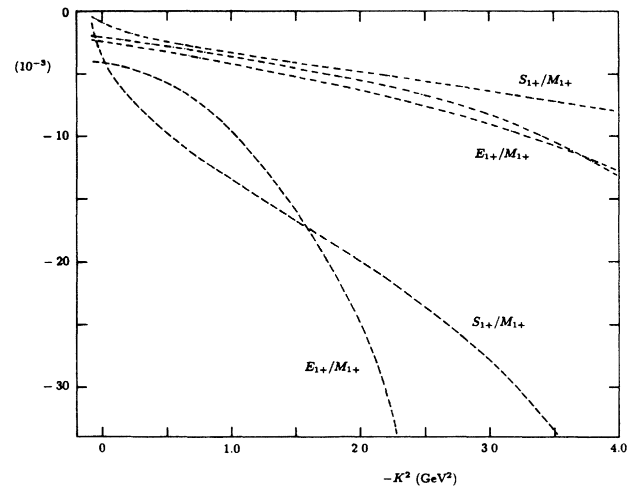

FIG. 4. The multipole ratios E_{1+}/M_{1+} and S_{1+}/M_{1+} for the IKK model (dashed lines) and with the relativized wave functions (dotted-dashed lines). The upper E_{1+}/M_{1+} line is for $\alpha=0.7$ GeV, the lower for $\alpha=0.6$ GeV; the two lines for S_{1+}/M_{1+} are coincident on the scale shown.

the most sensitive to the details of the wave functions.

In Fig. 4 we have plotted the multipole ratios E_{1+}/M_{1+} and S_{1+}/M_{1+} as a function of $-K^2$, for both the IKK model and with the CI wave functions. We see that these ratios are again much softer functions of $-K^2$, and that E_{1+}/M_{1+} and S_{1+}/M_{1+} are smaller than in the IKK model. Again we have plotted the ratios for $\alpha=0.6$ and 0.7 GeV to illustrate their sensitivity to the details of the wave functions, and we see that E_{1+}/M_{1+} is more sensitive than S_{1+}/M_{1+}, as we expect from the sensitivity of E_{1+}. We should point out here that these ratios are much less model dependent than the multipoles themselves, for instance, they do not depend at all on our ansatz for the "wave-function normalization" for virtual photons.

IV. DISCUSSION

The analyses of the experimental data for these multipoles concentrate on extraction of the ratio E_{1+}/M_{1+} at the photon point (from photopion experiments). This process is very complicated, although it would appear[9] that they support a small negative ratio there of about -1%. Our calculation (see Table I) gives a somewhat smaller value of approximately -0.2%. The analyses of the rather poor data away from the photoproduction point are not of sufficient precision that they can make a meaningful test of these models. From Fig. 4 and Table I we can see that the planned experiments need to be able to measure these ratios at low $-K^2$ to an accuracy of a few tenths of 1% if they are to distinguish the various models.

In our calculation the use of relativized wave functions has not removed the problem of lack of current conservation, but only improved it, as evidenced by the improved agreement of $(L_{1+})_\rho$ with L_{1+}. This does not indicate a problem with the quark model. Rather it indicates that, as shown by many authors,[20,21] we have not used a transition operator with the correct physical behavior. As pointed out by Close and Li,[20] it is inconsistent to use the nonrelativistic limit of the transition operator, but to treat correctly the problem of configuration mixing, as these corrections are of the same order. We can also see that the lack of current conservation may in part be due to integrating the electromagnetic current four-vector over the three-space wave functions. Even if this was done without taking the nonrelativistic limit of the current operator, the result would not transform as a four-vector and current conservation would be lost.

There are also other reasons to be cautious about making conclusions about the quark model based on the breakdown of current conservation. It is interesting that our results show fairly good agreement between $(L_{1+})_\rho$ and L_{1+} for smaller $-K^2$ which breaks down above $-K^2 \simeq 1.5$ GeV^2. This is a signal that the entire calculation can be trusted only below this scale, and in particular indicates that one cannot[22] simply boost the center-of-mass wave functions by multiplying by a plane wave $\exp(i\mathbf{P}\cdot\mathbf{R})$. Once the momentum of the recoiling baryon exceeds its mass (in the center-of-momentum frame of the Δ, $k \simeq 1$ GeV for $-K^2 = 1.0$ GeV^2) this simple prescription must start to break down. And, of course, our calculation has little light to shed on Carlson's perturbative QCD (high $-K^2$) limit[23] $E_{1+}/M_{1+} = 1$.

One worrying feature of the results which is best illustrated in the analytic calculations of Refs. 1, 2, and 7 is that the E_{1+} moment, which is an electric quadrupole transition, vanishes in the limit of small k like k (with a factor $e^{-k^2/6\alpha^2}$), and not like k^2 which we would expect for quadrupole radiation. However, this behavior has been calculated for k fixed by the mass difference of the Δ and nucleon, which in these models is itself fixed by the strength of the contact part of the hyperfine interaction. The same strength multiplies the tensor part of the hyperfine interaction which is responsible for the D-wave mixings which have led to a nonzero coefficient of k. If the hyperfine strength is reduced by reducing α_s the Δ-nucleon splitting and so k are reduced roughly linearly, as is this nonzero coefficient induced by the tensor interactions. In the limit of small α_s and so k, the product of this coefficient and k will go to zero like k^2. We see that there is implicit dependence on k in E_{1+} which could allow for E_{1+} to have the correct scaling behavior in the long-wavelength limit.

V. CONCLUSIONS

We have seen from the above that we expect our results to be generally reliable below approximately 1.5–2.0 GeV^2. The extension of the nonrelativistic calculation performed here, that of using wave functions expanded in a large harmonic-oscillator basis (which are solutions of a realistic "relativized" Hamiltonian), is a necessary first step, and has shown that the truncation of the wave functions does contribute to the lack of current conservation, but there are many reasons why current conservation should fail. It has also shown that extension of the wave function basis has little effect on the adherence to the Siegert theorem, and that using this theorem to evaluate E_{1+}/M_{1+}, at least for this set of wave functions, would lead to error. The next step[20] is to correct, consistently to $O(p^2/m^2)$, the transition operator. The result of this first step is that the ratios E_{1+}/M_{1+} and S_{1+}/M_{1+} are quite small and negative at $-K^2=0$, and have soft dependence on $-K^2$ away from the photon point.

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APPENDIX: OSCILLATOR SUBSTATE TRANSITION AMPLITUDES

In this appendix we give the formulas for the transition amplitudes between a pair of oscillator substates of the wave functions for the Δ and nucleon.

1. Helicity-1 amplitudes

We decompose the amplitude $(A_\lambda)_{ab} = 3\langle a(\frac{3}{2},\lambda)|H_3^t|b(\frac{1}{2},\lambda-1)\rangle$ into a derivative term $(A_\lambda^\rho)_{ab}$ and a spin term $(A_\lambda^s)_{ab}$. The states $|a(\frac{3}{2},\lambda)\rangle$ and $|b(\frac{1}{2},\lambda-1)\rangle$ are given by the sums (for example, for $|a\rangle$)

$$|a(\tfrac{3}{2},\lambda)\rangle = C_A(uud)\sum_M C(L_a,S_a,M,\lambda-M;\tfrac{3}{2},\lambda)\Psi_{L_a,M,n_{\rho_a},l_{\rho_a},n_{\lambda_a},l_{\lambda_a}}(\rho,\lambda)\chi_{S_a,\lambda-M} \tag{A1}$$

so that

$$\begin{aligned}(A_\lambda^s)_{ab} &= 3\left[-\frac{1}{3}\right]\left[\frac{k}{m}\right]\left[\frac{\pi}{k}\right]^{1/2}\left\langle a(\tfrac{3}{2},\lambda)\left|e^{-ik\sqrt{2/3}\lambda_z}\frac{\sigma_{3+}}{2}\right|b(\tfrac{1}{2},\lambda-1)\right\rangle \\ &= -\frac{k}{m}\left[\frac{\pi}{k}\right]^{1/2}\sum_M C(L_a,S_a,M,\lambda-M;\tfrac{3}{2},\lambda)C(L_b,S_b,M,\lambda-1-M;\tfrac{1}{2},\lambda-1) \\ &\quad\times\langle\Psi_{L_a,M,n_{\rho_a},l_{\rho_a},n_{\lambda_a},l_{\lambda_a}}|e^{-ik\sqrt{2/3}\lambda_z}|\Psi_{L_b,M,n_{\rho_b},l_{\rho_b},n_{\lambda_b},l_{\lambda_b}}\rangle \\ &\quad\times C(S_b,1,\lambda-1-M,1;S_a,\lambda-M)\left\langle S_a\left|\left|\frac{\sigma_{3+}}{2}\right|\right|S_b\right\rangle .\end{aligned} \tag{A2}$$

Here the notation and conventions for the spatial wave functions Ψ, and the spin wave functions χ is as in Ref. 15. The spin-reduced matrix element in (A2) is given by

$$\left\langle S_a \left|\left| \frac{\sigma_{3+}}{2} \right|\right| S_b \right\rangle = \begin{bmatrix} -\sqrt{5/6} & 0 & \sqrt{2/3} \\ 0 & -\sqrt{3/2} & 0 \\ -\frac{2}{\sqrt{3}} & 0 & \frac{1}{\sqrt{6}} \end{bmatrix}, \tag{A3}$$

where the ijth entry in the matrix corresponds to

$$\left\langle \chi_i \left|\left| \frac{\sigma_{3+}}{2} \right|\right| \chi_j \right\rangle ,$$

with $\chi_1 = \chi^S_{3/2}$, $\chi_2 = \chi^\rho_{1/2}$, and $\chi_3 = \chi^\lambda_{1/2}$. The spatial matrix elements are given by

$$\begin{aligned} \langle \Psi_{L_a,M,n_{\rho_a},l_{\rho_a},n_{\lambda_a},l_{\lambda_a}} | e^{-ik\sqrt{2/3}\lambda_z} | \Psi_{L_b,M,n_{\rho_b},l_{\rho_b},n_{\lambda_b},l_{\lambda_b}} \rangle &= \delta_{n_{\rho_a},n_{\rho_b}} \delta_{l_{\rho_a},l_{\rho_b}} \sum_{K=|l_{\lambda_a}-l_{\lambda_b}|}^{l_{\lambda_a}+l_{\lambda_b}} \\ &\quad \times (2K+1) C(L_b,K,M,0;L_a,M)(-1)^{L_b+l_{\rho_a}-K-l_{\lambda_a}} \sqrt{2L_b+1} \\ &\quad \times W(l_{\lambda_a},l_{\lambda_b},L_a,L_b;K,l_{\rho_a}) \sqrt{2l_{\lambda_b}+1}\, C(K,l_{\lambda_b},0,0;l_{\lambda_a},0) \\ &\quad \times \left\langle n_{\lambda_a} l_{\lambda_a} \left|\left| \left[\frac{z^K d^K}{(zdz)^K} \left[\frac{\sinh z}{z} \right] \right] \right|_{z=-ik\sqrt{2/3}\lambda} \right| n_{\lambda_b} l_{\lambda_b} \right\rangle , \end{aligned} \tag{A4}$$

where the last factor in (A4) is a radial integral, with

$$\langle n_a l_a | f(\lambda) | n_b l_b \rangle = \mathcal{N}_{n_a l_a} \mathcal{N}_{n_b l_b} \int_0^\infty d(\alpha\lambda)(\alpha\lambda)^{2+l_a+l_b} e^{-\alpha^2\lambda^2} L_{n_b}^{l_b+1/2}(\alpha\lambda) f(\lambda) L_{n_a}^{l_a+1/2}(\alpha\lambda) \tag{A5}$$

with the definition of the normalization coefficients $\mathcal{N}_{nl}$ and the Laguerre polynomials $L_n^{l+1/2}$ also given in Ref. 15.

The calculation of the derivative term $(A^\rho_\lambda)_{ab}$ proceeds similarly, except that the spin matrix element is trivial and we must make an insertion of $1 = \sum_{n_{\lambda_c},l_{\lambda_c}} | n_{\lambda_c} l_{\lambda_c} \rangle \langle n_{\lambda_c} l_{\lambda_c} |$ between the phase factor and derivative operator in this part of H_3^t [see Eq. (6)]. We have, therefore,

$$\begin{aligned} (A^\rho_\lambda)_{ab} &= -3 \left[-\frac{1}{3} \right] \frac{1}{m} \left[\frac{\pi}{k} \right]^{1/2} \langle a(\tfrac{3}{2},\lambda) | e^{-ik\sqrt{2/3}\lambda_z} (\tfrac{2}{3})^{1/2} p_{\lambda+} | b(\tfrac{1}{2},\lambda-1) \rangle \\ &= \frac{1}{m} \left[\frac{\pi}{k} \right]^{1/2} \delta_{S_a,S_b} \sum_M C(L_a,S_a,\lambda-M,M;\tfrac{3}{2},\lambda) C(L_b,S_b,\lambda-1-M,M;\tfrac{1}{2},\lambda-1) \\ &\quad \times \langle \Psi_{L_a,\lambda-M,n_{\rho_a},l_{\rho_a},n_{\lambda_a},l_{\lambda_a}} | e^{-ik\sqrt{2/3}\lambda_z} (\tfrac{2}{3})^{1/2} p_{\lambda+} | \Psi_{L_b,\lambda-1-M,n_{\rho_b},l_{\rho_b},n_{\lambda_b},l_{\lambda_b}} \rangle . \end{aligned} \tag{A6}$$

The last factor in (A6) has the value

$$\begin{aligned} &\langle \Psi_{L_a,\lambda-M,n_{\rho_a},l_{\rho_a},n_{\lambda_a},l_{\lambda_a}} | e^{-ik\sqrt{2/3}\lambda_z} (\tfrac{2}{3})^{1/2} p_{\lambda+} | \Psi_{L_b,\lambda-1-M,n_{\rho_b},l_{\rho_b},n_{\lambda_b},l_{\lambda_b}} \rangle \\ &\quad = \delta_{n_{\rho_a},n_{\rho_b}} \delta_{l_{\rho_a},l_{\rho_b}} \sum_m C(l_{\rho_a},l_{\lambda_a},m,\lambda-M-m;L_a,\lambda-M) C(l_{\rho_a},l_{\lambda_b},m,\lambda-1-M-m;L_b,\lambda-1-M) \\ &\qquad \times \sum_{n_{\lambda_c},l_{\lambda_c}} \langle n_{\lambda_a} l_{\lambda_a} \lambda-M-m | e^{-ik\sqrt{2/3}\lambda_z} | n_{\lambda_c} l_{\lambda_c} \lambda-M-m \rangle \\ &\qquad \times \langle n_{\lambda_c} l_{\lambda_c} \lambda-M-m | (\tfrac{2}{3})^{1/2} p_{\lambda+} | n_{\lambda_b} l_{\lambda_b} \lambda-1-M-m \rangle . \end{aligned} \tag{A7}$$

Here the phase-factor expectation value is given by a formula similar to (A4),

$$\langle n_{\lambda_a} l_{\lambda_a} \lambda - M - m | e^{-ik\sqrt{2/3}\lambda_z} | n_{\lambda_c} l_{\lambda_c} \lambda - M - m \rangle$$
$$= \sum_{K=|l_{\lambda_a}-l_{\lambda_c}|}^{l_{\lambda_a}+l_{\lambda_c}} (2K+1) C(l_{\lambda_c}, K, \lambda - M - m, 0; l_{\lambda_a}, \lambda - 1 - M) \left[\frac{2l_{\lambda_c}+1}{2l_{\lambda_a}+1} \right]^{1/2}$$
$$\times C(K, l_{\lambda_c}, 0, 0; l_{\lambda_a}, 0) \left\langle n_{\lambda_a} l_{\lambda_a} \left| \left[\frac{z^K d^K}{(z dz)^K} \left[\frac{\sinh z}{z} \right] \right]_{z=-ik\sqrt{2/3}\lambda} \right| n_{\lambda_c} l_{\lambda_c} \right\rangle , \quad \text{(A8)}$$

and the momentum operator integral is given by

$$\langle n_{\lambda_c} l_{\lambda_c} \lambda - M - m | (\tfrac{2}{3})^{1/2} p_{\lambda+} | n_{\lambda_b} l_{\lambda_b} \lambda - 1 - M - m \rangle$$
$$= -\sqrt{2}\alpha C(l_{\lambda_b}, 1, \lambda - 1 - M - m, 1; 1_{\lambda_c}, \lambda - M - m) C(l_{\lambda_c}, 1, 0, 0; l_{\lambda_b}, 0) [i(\tfrac{2}{3})^{1/2}] \left\langle n_{\lambda_c} l_{\lambda_c} \left| i\frac{p}{\alpha} \right| n_{\lambda_b} l_{\lambda_b} \right\rangle . \quad \text{(A9)}$$

In Eq. (A9),

$$\left\langle nl \left| i\frac{p}{\alpha} \right| n'l' \right\rangle$$

is a radial integral [similar to (A5) but over the Fourier-transformed radial wave functions] which is nonzero only when $l' = l - 1$, where it takes the values

$$\left\langle nl \left| i\frac{p}{\alpha} \right| n'l-1 \right\rangle = \begin{cases} -\sqrt{n+l+1/2} & \text{if } n'=n , \\ -\sqrt{n+1} & \text{if } n'=n+1 , \end{cases} \quad \text{(A10)}$$

and when $l' = l + 1$, where

$$\left\langle nl \left| i\frac{p}{\alpha} \right| n'l+1 \right\rangle = \begin{cases} \sqrt{n} & \text{if } n'=n-1 , \\ \sqrt{n+l+3/2} & \text{if } n'=n . \end{cases} \quad \text{(A11)}$$

Note that (A7) [and so $(A_\lambda^p)_{ab}$] is always real since the last factor in (A8) is pure imaginary for the odd values of $l_{\lambda_a} + l_{\lambda_c}$ which apply there [and real for even $l_{\lambda_a} + l_{\lambda_b}$ which apply in (A4)].

2. Helicity-0 amplitudes

The helicity-0 amplitude $(A_l)_{ab} = 3\langle a(\tfrac{3}{2},\tfrac{1}{2}) | H_3^l | b(\tfrac{1}{2},\tfrac{1}{2}) \rangle$ is calculated in a similar way to the helicity-1 amplitude; we decompose $(A_l)_{ab}$ into a derivative term $(A_l^p)_{ab}$ and a constant term $(A_l^c)_{ab}$. An analysis similar to that yielding (A6) gives, for the derivative term,

$$(A_l^p)_{ab} = 3 \left[-\frac{1}{3} \right] \left[\frac{1}{m} \right] \left[\frac{2\pi}{k} \right]^{1/2} \langle a(\tfrac{3}{2},\tfrac{1}{2}) | (\tfrac{2}{3})^{1/2} p_{\lambda z} e^{-ik\sqrt{2/3}\lambda_z} | b(\tfrac{1}{2},\tfrac{1}{2}) \rangle$$
$$= -\frac{1}{m} \left[\frac{2\pi}{k} \right]^{1/2} \delta_{S_a,S_b} \sum_M C(L_a, S_a, \tfrac{1}{2}-M, M; \tfrac{3}{2}, \tfrac{1}{2}) C(L_b, S_b, \tfrac{1}{2}-M, M; \tfrac{1}{2}, \tfrac{1}{2})$$
$$\times \langle \Psi_{L_a, 1/2-M, n_{\rho_a}, l_{\rho_a}, n_{\lambda_a}, l_{\lambda_a}} | (\tfrac{2}{3})^{1/2} p_{\lambda z} e^{-ik\sqrt{2/3}\lambda_z} | \Psi_{L_b, 1/2-M, n_{\rho_b}, l_{\rho_b}, n_{\lambda_b}, l_{\lambda_b}} \rangle . \quad \text{(A12)}$$

Again the last factor in (A12) can be written

$$\langle \Psi_{L_a, 1/2-M, n_{\rho_a}, l_{\rho_a}, n_{\lambda_a}, l_{\lambda_a}} | (\tfrac{2}{3})^{1/2} p_{\lambda z} e^{-ik\sqrt{2/3}\lambda_z} | \Psi_{L_b, 1/2-M, n_{\rho_b}, l_{\rho_b}, n_{\lambda_b}, l_{\lambda_b}} \rangle$$
$$= \delta_{n_{\rho_a}, n_{\rho_b}} \delta_{l_{\rho_a}, l_{\rho_b}} \sum_m C(l_{\rho_a}, l_{\lambda_a}, m, \tfrac{1}{2}-M-m; L_a, \tfrac{1}{2}-M) C(l_{\rho_a}, l_{\lambda_b}, m, \tfrac{1}{2}-M-m; L_b, \tfrac{1}{2}-M)$$
$$\times \sum_{n_{\lambda_c}, l_{\lambda_c}} \langle n_{\lambda_a} l_{\lambda_a} \tfrac{1}{2} - M - m | (\tfrac{2}{3})^{1/2} p_{\lambda z} | n_{\lambda_c} l_{\lambda_c} \tfrac{1}{2} - M - m \rangle$$
$$\times \langle n_{\lambda_c} l_{\lambda c} \tfrac{1}{2} - M - m | e^{-ik\sqrt{2/3}\lambda_z} | n_{\lambda_b} l_{\lambda_b} \tfrac{1}{2} - M - m \rangle . \quad \text{(A13)}$$

The phase factor expectation value can be adapted from (A8) with $\lambda=\frac{1}{2}$, and the momentum operator integral is given by

$$\langle n_{\lambda_a} l_{\lambda_a} \tfrac{1}{2} - M - m | (\tfrac{2}{3})^{1/2} p_{\lambda_z} | n_{\lambda_c} l_{\lambda_c} \tfrac{1}{2} - M - m \rangle = \alpha C(l_{\lambda_c}, 1, \tfrac{1}{2}, \tfrac{1}{2} - M - m, 0; l_{\lambda_a}, \tfrac{1}{2} - M - m)$$
$$\times C(l_{\lambda_a}, 1, 0, 0; l_{\lambda_c}, 0)[i(\tfrac{2}{3})^{1/2}] \left\langle n_{\lambda_a} l_{\lambda_a} \left| i\frac{p}{\alpha} \right| n_{\lambda_c} l_{\lambda_c} \right\rangle , \tag{A14}$$

with the last factor the radial integral of (A10) and (A11).

The constant term $(A_l^c)_{ab}$ has the expansion

$$(A_l^c)_{ab} = 3\left[-\frac{1}{3}\right]\frac{1}{m}\left[\frac{2\pi}{k}\right]^{1/2}\left\langle a(\tfrac{3}{2},\tfrac{1}{2})\left|\frac{k}{2}e^{-ik\sqrt{2/3}\lambda_z}\right|b(\tfrac{1}{2},\tfrac{1}{2})\right\rangle$$
$$= -\frac{k}{2m}\left[\frac{2\pi}{k}\right]^{1/2}\delta_{S_a,S_b}\sum_M C(L_a,S_a,M,\tfrac{1}{2}-M;\tfrac{3}{2},\tfrac{1}{2})C(L_b,S_b,M,\tfrac{1}{2}-M;\tfrac{1}{2},\tfrac{1}{2})$$
$$\times\langle \Psi_{L_a,M,n_{\rho_a},l_{\rho_a},n_{\lambda_a},l_{\lambda_a}} | e^{-ik\sqrt{2/3}\lambda_z} | \Psi_{L_b,M,n_{\rho_b},l_{\rho_b},n_{\lambda_b},l_{\lambda_b}} \rangle , \tag{A15}$$

where the last factor can be found from (A4).

The calculation of the scalar amplitude $(A_s)_{ab} = 3\langle a(\tfrac{3}{2},\tfrac{1}{2})|H_3^s|b(\tfrac{1}{2},\tfrac{1}{2})\rangle$ is trivial once we have the constant term $(A_l^c)_{ab}$ in (A15), since

$$(A_s)_{ab} = -\frac{2}{k}(A_l^c)_{ab} . \tag{A16}$$

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