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Gluonic contribution to g_1 and its relationship to the spin-dependent parton distributions

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We examine the suggestion of Altarelli and Ross and of Carlitz, Collins, and Mueller that there is a hard gluonic contribution to the first moment of the proton's spin-dependent structure function g_1. We find that if the soft (collinear) divergence in the gluonic contribution is regulated dimensionally or with a quark mass, then the first moment vanishes. More generally, we suggest that the hard gluonic contribution to g_1 be identified by subtracting certain contributions that are attributable to the spin-dependent quark distributions. We show that the first moment of the resulting hard gluonic contribution vanishes, provided that the UV regulator for the spin-dependent quark distributions respects gauge invariance, Lorentz invariance, and the analyticity structure of the unregulated distributions. However, by relaxing the Lorentz-invariance requirement, we are able to construct quark distributions such that the hard gluonic contribution to the first moment of g_1 is nonzero. The corresponding quark distributions are related to matrix elements of Lorentz-variant operators. Hence, they have no analogue in the standard operator-product expansion and do not satisfy the usual forms of the quark sum rules. We conclude that the size of the gluonic contribution to the first moment of g_1 is entirely a matter of the convention used in defining the quark distributions.

I. INTRODUCTION

The current-current correlation function $W_{\mu\nu}$ that appears in deep-inelastic lepton-hadron scattering has a spin-dependent part $\Delta W_{\mu\nu}$, which can be written in terms of two structure functions g_1 and g_2:

$$\Delta W_{\mu\nu}=i4\pi\frac{M_p}{P\cdot q}\epsilon_{\mu\nu\lambda\sigma}q^\lambda \times\left[S^\sigma g_1(x,Q^2)+\left[S^\sigma-\frac{S\cdot q}{P\cdot q}P^\sigma\right]g_2(x,Q^2)\right], \quad (1.1)$$

where q is the four-momentum of the virtual photon, and P, M_P, and $S(P)$ are the four-momentum, mass, and spin-polarization vector of the target hadron. As usual, $Q^2=-q^2$, and $x=Q^2/(2P\cdot q)$. In the parton model, $g_1(x,Q^2)$ can be related to the spin-dependent distributions of quarks in the hadron:

$$g_1(x,Q^2)=\sum_i(e_i^2/2)\Delta q_i(x,Q^2)+O(\alpha_s(Q^2)), \quad (1.2a)$$

where the subscripts i denote the quark flavor, e_i is the quark charge, and $\Delta q_i(x,Q^2)$ is the distribution of quarks and antiquarks with spin parallel to the target's spin minus the distribution of quarks and antiquarks with spin antiparallel to target's spin. In the case of a proton target, one usually considers the u, d, and s quarks. Then, the first moment of the proton structure function g_1^P is given by

$$\int_0^1 dx\, g_1^P(x,Q^2)=\tfrac{1}{2}[\tfrac{4}{9}\Delta u(Q^2)+\tfrac{1}{9}\Delta d(Q^2)+\tfrac{1}{9}\Delta s(Q^2)]+O(\alpha_s(Q^2)), \quad (1.2b)$$

where

$$\Delta q_i(Q^2)=\int_0^1 dx\, \Delta q_i(x,Q^2).$$

Usually the Δq_i are assumed to be related to the expectation values in the proton state of the axial-vector current:

$$2M_P\Delta q_i S^\mu(P)=\langle P|j_{5_i}^\mu|P\rangle, \quad (1.3)$$

where $j_{5_i}^\mu=\bar\psi_i\gamma^\mu\gamma_5\psi_i$ is the part of the axial-vector current that arises from the quark and antiquark of flavor i, and $\bar\psi_i$ and ψ_i are the quark fields. As a consequence of (1.3), the combination $\Delta u-\Delta d$ can be related to g_A/g_V through the Bjorken sum rule,[1] and the combination $\Delta u+\Delta d-2\Delta s$ can be related to the F/D value[2] obtained from semileptonic hyperon decays. By making use of these relations and the measured value of the first moment of g_1^P (Ref. 3), one can determine the SU(3)-singlet combination of the spin-dependent quark distributions $\Delta\Sigma=\Delta u+\Delta d+\Delta s$ (Ref. 4). The result is that $\Delta\Sigma$ differs significantly from unity.

The most straightforward interpretation of $\Delta\Sigma$ is that it represents the fraction of the proton's spin that is carried by the spin of the quarks. In fact, in a static quark model one would find that $\Delta\Sigma=1$. The deviation of the measured value of $\Delta\Sigma$ from unity has led to a good deal of discussion in the literature.[5-8] In particular, Altarelli and Ross[9] (AR) and Carlitz, Collins, and Mueller[10] (CCM) have suggested that (1.2b) could receive important contributions in $O(\alpha_s)$ from the spin-dependent gluon distribution Δg. In this paper we address the issue of the compatibility of this suggestion of AR and CCM with the interpretation of the Δq_i as quark probability distributions.

The remainder of the paper is organized as follows. In

Sec. II A we discuss the origin of the gluonic contributions to g_1 and their relationship to the quark distributions. In Sec. II B we examine the lowest-order gluonic contribution to g_1. We argue that the methods proposed by AR and CCM for regulating the collinear divergence in this contribution are not satisfactory as a means for isolating the contribution's hard part. We then describe two soft regulators, namely, dimensional and quark-mass regulators, that lead to satisfactory expressions for the hard part. An explicit calculation shows that these regulators yield a vanishing contribution to the first moment of g_1. In Sec. II C we present a general method for identifying the hard gluonic contribution to g_1 that is based on subtracting certain soft contributions that are attributable to the spin-dependent distributions of quarks in the proton. This section also contains a demonstration that, for several choices of regulator, the subtraction method leads to a vanishing contribution to the first moment of g_1. In Sec. III we argue that any method for extracting a hard gluonic contribution to g_1 that respects gauge invariance, Lorentz invariance, and certain analyticity properties necessarily yields a vanishing contribution to the first moment of g_1. Section IV contains a discussion of the suitability of some alternative definitions of the hard part of g_1. We show that, by relaxing the requirement that the method for extracting the hard part be Lorentz invariant, it *is* possible to obtain a hard gluonic contribution to g_1 that has a nonvanishing first moment. Hence, we conclude that the size of the hard gluonic contribution to g_1 is not fixed, but depends on the convention used in defining the hard part and, implicitly, the quark distributions. The approach that we present is related to a suggestion of CCM that the hard gluonic contribution to g_1 be identified by observing two-jet production in deep-inelastic lepton scattering. In this approach, the moments of the corresponding quark distributions are matrix elements of Lorentz-variant operators, so they are not related to the operator matrix elements that appear in the standard operator-product expansion. Consequently, the quark distributions do not satisfy the usual sum rules. Finally, in Sec. V, we summarize our results and discuss their relationship to the partonic interpretation of the proton's spin.

II. THE GLUONIC CONTRIBUTION TO $g_1(x,Q^2)$

A. Preliminary considerations

Contributions to deep-inelastic scattering can arise from single gluonic constituents of the proton through diagrams of the type shown in Fig. 1. Here, the gluon's momentum p is a variable of integration. It is only for those values of p such that the gluon is nearly on its mass shell that we can give the gluon a partonic interpretation. That is, we can regard Fig. 1 as a contribution that arises from the distribution of gluons inside a proton only if $|p^2| \lesssim \mu_{\text{fact}}^2$, where μ_{fact} is a "factorization scale" that we use to distinguish between "hard" and "soft" processes. Even if $|p^2| \lesssim \mu_{\text{fact}}^2$, Fig. 1 may yield contributions that cannot be regarded as arising from the distribution of gluons in the proton. The reason is that the subdiagram

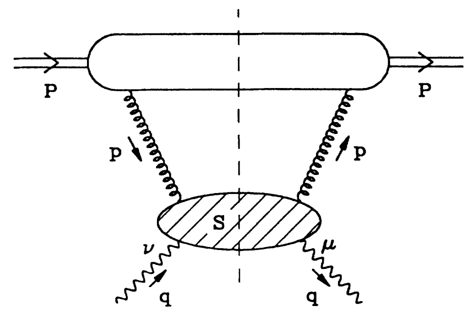

FIG. 1. Contributions to the forward photon-proton deep-inelastic scattering process that arise from the gluonic constituents of the proton. The dashed line represents the final-state cut. Curly lines denote gluons. Wavy lines denote photons. The open blob at the top of the diagram represents the proton.

labeled S can itself contain soft (nearly on-shell) partons. Such soft subprocesses are not perturbatively calculable. Their interpretation is, in general, quite complicated and is the subject of factorization theorems. It may turn out that a soft subprocess has an interpretation in terms of the distribution to find a soft parton internal to S in the proton. Consider, for example, the leading-order (in α_s) subdiagrams of the type S, which are given by the box diagrams of Fig. 2. When the virtual quarks (or antiquarks) in these box diagrams are nearly on their mass shells, then the diagrams must be interpreted in terms of the distribution of quarks and antiquarks in a proton, rather than in terms of the distribution of gluons in a proton.

We denote the spin-dependent contribution of the dia-

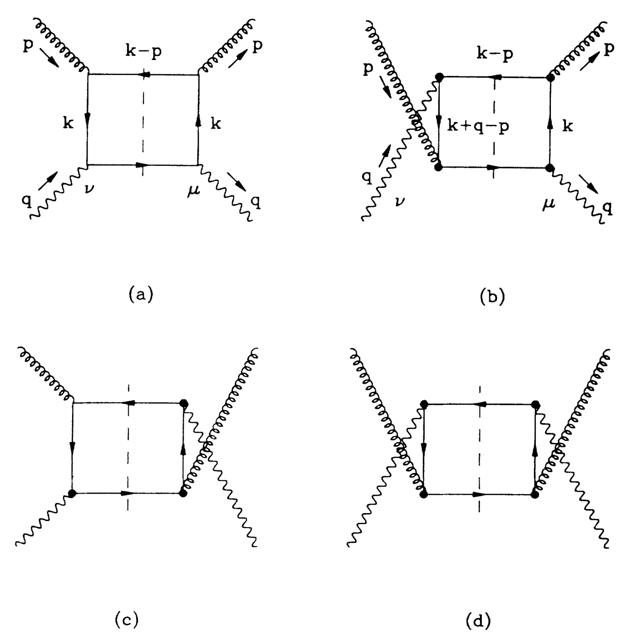

FIG. 2. The box diagrams that give the leading gluonic contributions to the spin-dependent structure function $g_1(x,Q^2)$. Solid lines denote quarks (or antiquarks).

grams of Fig. 2 (defined in terms of suitable projectors below) by A. In defining A, we truncate the external legs and omit the factors of the electromagnetic coupling e. The part of A that comes from the region of integration in which all the virtual quarks are far off their mass shells we call $A^{\rm hard}$. The remainder we call $A^{\rm soft}$. The separation of A into its hard and soft parts is the central issue in this paper.

$A^{\rm hard}$ contains the short-distance part of the lowest-order subprocess and yields the following contribution to g_1^P:

$$[g_1^P(x,Q^2)]_{\rm gluonic}=\langle e^2/2\rangle A^{\rm hard}(x,Q^2/\mu_{\rm fact}^2)\otimes\Delta g(x,\mu_{\rm fact}^2)\ , \tag{2.1}$$

where Δg is one-half the difference between the distribution of gluons of positive helicity and the distribution of gluons of negative helicity. The convolution $\otimes$ is defined by

$$B(x)\otimes C(x)\equiv\int_x^1\frac{dy}{y}B\left[\frac{x}{y}\right]C(y)\ , \tag{2.2}$$

from which it follows that the mth moment of the gluonic contribution to $g_1^P(x)$ is proportional to the mth moment of $A^{\rm hard}(x)$ times the mth moment of $\Delta g(x)$:

$$\int_0^1 dx\,x^{m-1}g_1^P(x)=\langle e^2/2\rangle\left[\int_0^1 dx\,x^{m-1}A^{\rm hard}(x)\right]\times\left[\int_0^1 dx\,x^{m-1}\Delta g(x)\right]\ . \tag{2.3}$$

$A^{\rm soft}$ is interpreted as arising from the distribution of quarks and antiquarks in a gluon:

$$A^{\rm soft}(x,\mu_{\rm fact}^2)=\sum_i\sigma_i^{q-\gamma}(x)\otimes(\Delta q_i^g)^{(1)}(x,\mu_{\rm fact}^2)\ , \tag{2.4}$$

where $\sigma_i^{q-\gamma}(x)$ is the Born cross section for the quark of flavor i with momentum fraction x to scatter from a photon (we omit factors of the electromagnetic charge e), and $(\Delta q_i^g)^{(1)}(x,\mu_{\rm fact}^2)$ is the $O(\alpha_s)$ contribution to the spin-dependent distribution for finding a quark in a gluon with longitudinal-momentum fraction x at scale $\mu_{\rm fact}$.

B. The computation of the leading-order contribution

Now let us discuss the computation of the leading-order gluonic contribution to $g_1^P(x,Q^2)$. That is, we examine the quantity A, which arises from the spin-dependent part of the box diagrams of Fig. 2. It is convenient in evaluating these diagrams to choose the frame in which the gluon's momentum is given by $p=(p^+,p^-,0_\perp)$ and the virtual photon's momentum is given by $q=(q^+,q^-,0_\perp)$, with $2q^+q^-=-Q^2$. We assume that the gluon is not far off its mass shell: $|p^2|\ll Q^2$. We can obtain the contribution to A by the following procedure. First we project out the tensor structure of the type associated with g_1 in (1.1) by contracting the virtual-photon Lorentz indices μ and ν into the tensor $(i/4\pi)\epsilon_{\mu\nu}$, where $\epsilon_{\mu\nu}$ is antisymmetric and has nonzero components only in the transverse direction ($\epsilon_{12}=+1$). The resulting expression is diagonal in the gluon helicity basis $\varepsilon=(1/\sqrt{2})(0,0,1,\pm i)$, and antisymmetric under a helicity flip. Hence, the contribution to g_1 that arises from this expression is proportional to Δg. We obtain the contribution to A, normalized according to (2.1), by taking one-half the difference between the negative- and positive-helicity expressions. Then, using the energy-momentum-conserving δ functions to carry out the k^+ and k^- integrations, and evaluating the Dirac and color traces, we obtain

$$A(x)=-\frac{\alpha_s N_f T}{\pi}\int_0^{K^2}\frac{dk_T^2}{\sqrt{1-k_T^2/K^2}}\left[\frac{(1-2x)(k_T^2+m^2)-2m^2(1-x)}{[k_T^2+m^2-p^2x(1-x)]^2}-\frac{1}{2K^2}(1-2x)\right]\ , \tag{2.5}$$

where m is the quark mass, $K^2=Q^2[(1-x)/4x]$, $T=\frac{1}{2}$, and we have dropped higher-twist terms of $O(p^2/Q^2)$ and $O(m^2/Q^2)$.

If we were to set m^2 and p^2 to zero, then the expression in (2.5) would be logarithmically divergent in the region k_T near zero. That is, a collinear singularity would arise when the quark and antiquark are moving parallel to each other because the virtual quark (or antiquark) goes on its mass shell. As we mentioned earlier, such soft (collinear) processes are not perturbatively calculable and should not be included in $A^{\rm hard}$. Instead we partition A into a hard contribution $A^{\rm hard}(x)$, for which $k_T^2\gtrsim\mu_{\rm fact}^2$, and a soft contribution $A^{\rm soft}(x)$ for which $k_T^2\lesssim\mu_{\rm fact}^2$. We assume that the factorization scale $\mu_{\rm fact}$ is much larger than any typical hadronic scale Λ, so that $A^{\rm hard}$ can be calculated reliably in perturbation theory. $A^{\rm hard}$ yields a gluonic contribution to $g_1^P(x,Q^2)$, as given in (2.1). $A^{\rm soft}$ contains the collinear singularity and is interpreted in terms of the distribution of quarks and antiquarks in the gluon according to (2.4).

AR and CCM suggest procedures for regulating the collinear divergence in (2.5) so as to arrive at a finite expression, which they interpret as a contribution to $A^{\rm hard}$. AR propose setting $p^2=0$ but keeping $m^2\neq 0$ in the denominator and first term in the numerator of (2.5). They drop the term $-2m^2(1-x)$ in the numerator. The result is

$$A^{\rm AR}(x)=-\frac{\alpha_s N_f T}{\pi}(1-2x)\left[\ln\frac{Q^2}{m^2}+\ln\frac{1-x}{x}-1\right]\ , \tag{2.6}$$

where we have dropped terms of $O(m^2/Q^2)$. CCM propose setting $m^2=0$ but keeping $-p^2>0$. In this case the result is

$$A^{\mathrm{CCM}}(x)=-\frac{\alpha_s N_f T}{\pi}(1-2x)\left[\ln\frac{Q^2}{-p^2}+\ln\frac{1}{x^2}-2\right], \tag{2.7}$$

where we have dropped terms of $O(p^2/Q^2)$. In either case, the contribution to the first moment is nonzero:

$$\int_0^1 dx\ A^{\mathrm{AR}}(x)=\int_0^1 dx\ A^{\mathrm{CCM}}(x)=-\frac{\alpha_s N_f}{2\pi}, \tag{2.8}$$

and it arises from the region of integration $k_T^2\sim Q^2$.

Since the first moment of $A(x)$ as computed by AR and CCM [Eq. (2.8)] is independent of the soft-regulator scale, it is tempting to say that this entire quantity should be counted as a contribution to A^{hard}. However, such a procedure is of questionable validity, since $A(x)$ *does* receive contributions that depend logarithmically on the soft cutoff. It happens that these contributions cancel when one takes the first mommemt of $A(x)$ because of an antisymmetry under $x\rightarrow 1-x$. But a cancellation of soft contributions from different regions of x space is unreliable: the symmetry that produces that cancellation could be an artifact of perturbation theory. In fact, the antisymmetry of the logarithmic contributions under $x\rightarrow 1-x$ is a consequence of chiral symmetry (helicity conservation), which we expect to be broken for scales of order of Λ. One way to eliminate contributions to $A(x)$ from scales of order Λ would be to identify the soft cutoff with $\mu_{\mathrm{fact}}\gg\Lambda$. We now examine this possibility.

If one follows the AR procedure for computing (2.5) and identifies m with the factorization scale μ_{fact}, then the result is a purely hard contribution, since only momenta for which $k_T^2\gtrsim\mu_{\mathrm{fact}}^2$ contribute. However, by neglecting some of the numerator terms proportional to m^2 in (2.5), AR have, in effect, changed the quark propagator. Consequently, the graphical Ward identity (Feynman identity) no longer holds [see (3.2)], and the expression is no longer invariant with respect to gauge transformations of the photon or gluon fields. Hence, we conclude that the AR procedure is not satisfactory as a means for identifying a hard contribution in (2.5).

In the CCM procedure, the quantity $-p^2$, which provides the infrared cutoff, is actually an integration variable, since the box diagrams are ultimately embedded in a larger process in which the gluon emerges from the target hadron (see Fig. 1). Consequently, $-p^2$ ranges over values on the order of Λ^2, and there are contributions to (2.5) from $k_T^2\sim\Lambda^2$. Of course, one could, as a technical device for regulating the box diagrams, set $-p^2=\mu_{\mathrm{fact}}^2$ within the integrand of (2.5). But the resulting expression would be invariant with respect to gauge transformations of the gluon field only for values of the gluon momentum p such that $-p^2=\mu_{\mathrm{fact}}^2$. We conclude, then, that the CCM procedure is also unsatisfactory as a means for identifying a hard contribution in (2.5).

One gauge-invariant procedure for defining the hard part of (2.5) is to identify m with the factorization scale, but now retain all the m^2 terms in the integrand. That is, in (2.5) we set $m^2=\mu_{\mathrm{fact}}^2$, with $Q^2\gg\mu_{\mathrm{fact}}^2\gg\Lambda^2, -p^2$. The result is

$$A^{\mathrm{mass}}(x)=-\frac{\alpha_s N_f T}{\pi}\left[(1-2x)\left[\ln\frac{Q^2}{\mu_{\mathrm{fact}}^2}+\ln\frac{1-x}{x}-1\right]-2(1-x)\right]. \tag{2.9}$$

The last term in brackets comes from the m^2 term that was omitted by AR, and it arises from the region of integration $k_T^2\sim\mu_{\mathrm{fact}}^2$. The first moment of (2.9) vanishes:

$$\int_0^1 dx\ A^{\mathrm{mass}}(x)=0\ . \tag{2.10}$$

The contribution to the first moment that arises from the region $k_T^2\sim Q^2$ has been canceled by a contribution from the region $k_T^2\sim\mu_{\mathrm{fact}}^2$. This canceling contribution appears because, by using the quark mass as a soft cutoff, we have broken the chiral symmetry and, thus, violated helicity conservation. In fact, will turn out that the regularization of (2.5) is closely related to the regularization of the triangle diagram, for which, as is well known, the chiral-symmetry properties of the regulator play a crucial role.

An alternative gauge-invariant method for defining the hard part of (2.5) is to cut off the integral by continuing to $4+2\epsilon$ dimensions with $\epsilon>0$. Here we regard ϵ as a small ($\epsilon\ll 1$), but fixed, cutoff, which we choose in such a way as to ensure that the integral is insensitive to contributions from the region of integration $k_T^2\lesssim\Lambda^2$. It turns out that this amounts to requiring that $(K^2/\Lambda^2)^\epsilon\gg 1$. Then, since $K^2\gg m^2, |p^2|$, we drop terms that are subleading in m^2/K^2 and p^2/K^2. We also drop terms of $O(\epsilon)$. The result is

$$A^{\mathrm{dim}}(x)=-\frac{\alpha_s N_f T}{\pi}\times\left[(1-2x)\left[\frac{1}{\epsilon}+\ln\frac{Q^2}{4\pi\mu_{\mathrm{MS}}^2}+\gamma_E+\ln\frac{1-x}{x}-1\right]-2(1-x)\right]. \tag{2.11}$$

Here μ_{MS} is the usual scale that is introduced to preserve the engineering dimensions of the dimensionally regulated quantity. (MS denotes the minimal-subtraction scheme.) Making the identification $\mu_{\mathrm{fact}}^2=4\pi\mu_{\mathrm{MS}}^2\exp(-\gamma_E-1/\epsilon)$, we see that expression (2.11) is equivalent to expression (2.9). (Since ϵ is fixed, we can absorb it into the definition of the cutoff scale μ_{fact}.) The last term in brackets in (2.11) is identical to the last term in (2.9) and it also arises from the region $k_T^2\sim\mu_{\mathrm{fact}}^2$. This term appears in the case of dimensional regularization because continuation to $4+2\epsilon$ dimensions breaks chiral invariance by violating the property $\{\gamma_5,\gamma_\mu\}=0$. Obviously, the first moment again vanishes:

$$\int_0^1 dx\ A^{\mathrm{dim}}(x)=0\ . \tag{2.12}$$

C. The subtraction method

The two gauge-invariant methods that we have examined thus far for defining A^{hard} both give a vanishing contribution to the first moment of $g_1(x)$. In order to investigate the extent to which this is a universal property of gauge-invariant methods, it is convenient to introduce a general approach, which is based on subtracting the part of A that can be interpreted in terms of the distribution of quarks inside a gluon. This approach is closely related to the formal subtraction procedure that is used in proving QCD factorization theorems, and it is essentially the same as the approach that is usually employed in the spin-averaged case. Our strategy is as follows. First we arrive at an approximate expression for the box diagrams that is valid for $k_T^2 \ll Q^2$. We impose a UV cutoff on the integration in this expression so that only the region $k_T^2 \lesssim \mu_{\text{fact}}^2$ contributes. The resulting quantity, is $A^{\text{soft}}(x,\mu_{\text{fact}}^2)$, which has the interpretation of the convolution of the quark-photon cross section with the distribution to find a quark in a gluon, as given in (2.4). As usual, we take $\mu_{\text{fact}}^2 \gg \Lambda^2$. We then subtract $A^{\text{soft}}(x,\mu_{\text{fact}}^2)$ from $A(x,Q^2)$ to obtain the hard contribution $A^{\text{hard}}(x,Q^2/\mu_{\text{fact}}^2)$:

$$\begin{aligned} A^{\text{hard}}(x,Q^2/\mu_{\text{fact}}^2) &= A(x,Q^2) - A^{\text{soft}}(x,\mu_{\text{fact}}^2) \\ &= A(x,Q^2) - \sum_i \sigma_i^{q-\gamma}(x) \\ &\quad \otimes (\Delta q_i^g)^{(1)}(x,\mu_{\text{fact}}^2) \; . \end{aligned} \tag{2.13}$$

In this procedure, μ_{fact} is the scale associated with the UV cutoff on $A^{\text{soft}}(x,\mu_{\text{fact}}^2)$, whereas, in our previous discussion of Sec. II B, μ_{fact} was the scale associated with the soft cutoff on A^{hard}. The two procedures are, of course, entirely equivalent, provided that μ_{fact} satisfies $Q^2 \gg \mu_{\text{fact}}^2 \gg \Lambda^2$.

Now we give the specifics of the calculation of A^{hard} by this method. The quark-photon cross section is given by

$$\sigma_i^{q-\gamma}(x) = \frac{e_i^2}{\langle e^2 \rangle}\delta\left[x - \frac{Q^2}{2p\cdot q}\right] . \tag{2.14}$$

The Feynman diagrams corresponding to $(\Delta q_i^g)^{(1)}$ are shown in Fig. 3.

In the region $k_T^2 \ll Q^2$, each of the diagrams of Fig. 2 actually contributes twice to the diagrams of Fig. 3. The reason is that the on-mass-shell conditions of Fig. 2 yield two solutions. In the region $k_T^2, |p^2| \ll Q^2$ these solutions are given approximately by $k^+ = (p^+/2)[1+x \mp (1-x)]$, $k^- = -(Q^2/4xp^+)(1 \mp 1)$. The upper solution taken with the diagrams of Figs. 2(a)–2(d) yields the diagrams of Figs. 3(a)–3(d), respectively, while the lower solution taken with the diagrams of Figs. 2(a)–2(d) yields the diagrams that one obtains by reversing the arrows of the fermion lines in the diagrams of Figs. 3(d)–3(a), respectively.

The diagrams of Fig. 3 contain double "eikonal" lines, which correspond to quarks moving with infinite momentum along the light-cone direction $n = (0,1,0_T)$ (Ref. 11). They are the remnants of the quark lines that have disappeared in passing from Fig. 2 to Fig. 3. The Feynman

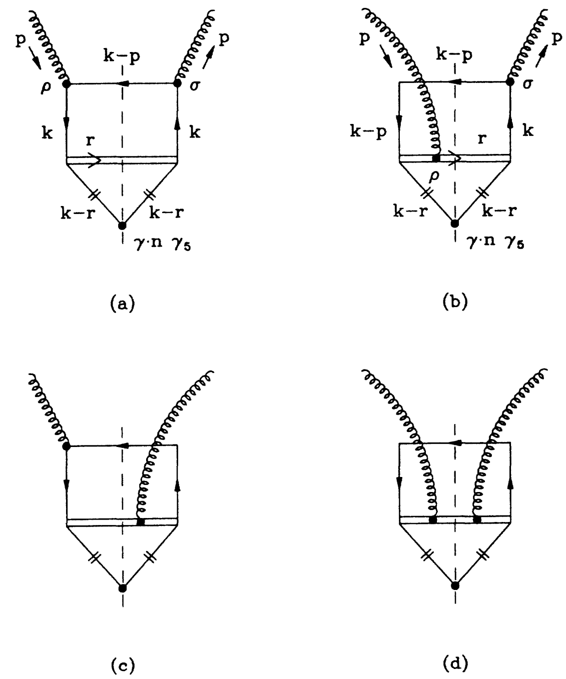

FIG. 3. The diagrams that give the quark contribution to $(\Delta q_i^g)^{(1)}$. In addition there are antiquark diagrams, which are obtained by reversing the arrows on the fermion lines in the quark diagrams.

rules for the double "eikonal" lines are shown in Fig. 4. The factors associated with propagators and vertices to the left of the cut are, as usual, just the complex conjugates of the factors associated with propagators and vertices to the right of the cut. The cut vertex[12] at the bottom of each diagram represents a factor $\frac{1}{2}\gamma\cdot n\gamma_5\delta(p\cdot n\, x - k\cdot n)$. Hash marks on a quark propagator indicate that the diagrammatic factor associated with it is to be omitted. Hence, the diagrams of Fig. 3 (including the antiquark diagrams) yield

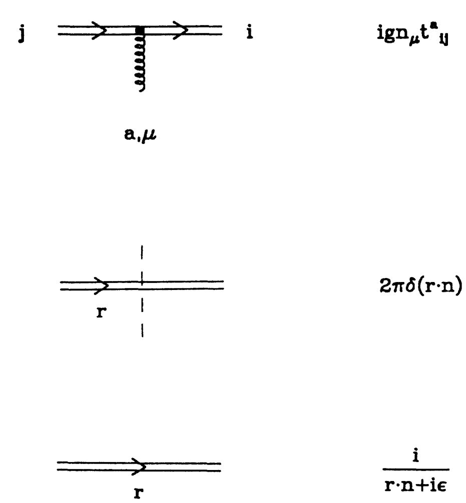

FIG. 4. The Feynman rules for the double "eikonal" lines.

$$(\Delta q_i^g)^{(1)}(x)=2Tg^2\frac{\mu_{\rm MS}^{4-d}}{(2\pi)^d}\int d^dk\,\delta\left[x-\frac{k^+}{p^+}\right]\frac{1}{2p^+}2\pi\delta((p-k)^2-m^2)$$
$$\times\frac{\epsilon^{\sigma\rho}}{2}{\rm Tr}\left[\frac{\gamma_\sigma(\gamma\cdot k+m)\gamma^+\gamma_5(\gamma\cdot k+m)\gamma_\rho[\gamma\cdot(p-k)-m]}{(k^2-m^2+i\epsilon)^2}\right.$$
$$+\frac{\gamma_\sigma(\gamma\cdot k+m)\gamma^+\gamma_5[\gamma\cdot(p-k)-m]n_\rho}{(p\cdot n+i\epsilon)(k^2-m^2+i\epsilon)}$$
$$\left.+\frac{n_\sigma\gamma^+\gamma_5(\gamma\cdot k+m)\gamma_\rho[\gamma\cdot(p-k)-m]}{(p\cdot n-i\epsilon)(k^2-m^2+i\epsilon)}+\frac{n_\sigma n_\rho\gamma^+\gamma_5[\gamma\cdot(p-k)-m]}{(p\cdot n+i\epsilon)(p\cdot n-i\epsilon)}\right]. \tag{2.15}$$

The four terms in large parentheses correspond to the diagrams of Figs. 3(a)–3(d), respectively.

We note that the quark and antiquark distributions have definitions to all orders in α_s in terms of the expectation value of an operator in the initial hadronic state (in our case a gluon state):[11]

$$\Delta q_i^g(x,\mu^2)=\frac{1}{4\pi}\int dy^-(e^{-ixp^+y^-}+e^{ixp^+y^-})\langle p|\bar\psi_i(0,y^-,0_T)\gamma^+\gamma_5$$
$$\times\mathcal{P}\left[\exp\left[ig\int_0^{y^-}dz^-\mathcal{A}_a^+(0,z^-,0_T)t_a\right]\right]\psi_i(0,0,0_T)|p\rangle_c\,, \tag{2.16}$$

where $\mathcal{A}_a$ is the gluon field of color a, t_a is an SU(3)-color matrix, $\mathcal{P}$ denotes path ordering, the subscript c indicates that only connected diagrams are to be included, and μ is the scale of the UV regulator that one must impose in order to define the operator matrix element. The first term in parentheses in (2.16) is associated with the quark contribution, and the second term is associated with the antiquark contribution. (Here we have used the fact that the spin-dependent quark distribution at x is equal to the spin-dependent antiquark distribution at $-x$.) The operator expression for the quark distributions is manifestly gauge invariant, and it is easy to verify that this property also holds for the complete set of diagrams in Fig. 3. (Note that the gauge-invariance property holds at the level of the integrands: no shift of the integration variable is necessary.) If one works in a gauge in which the gluon polarization is transverse to the vector n, then the diagrams involving gluon connections to the eikonal lines [Figs. 3(b)–3(d)] vanish, and the remaining diagram [Fig. 3(a)] is identical to the one that appears in the cut vertex formalism.[12]

In arriving at the definition of $A^{\rm soft}$, we have neglected terms in A of higher order in k_T^2/Q^2. Consequently, the unregulated expression for $A^{\rm soft}$ contains a UV divergence (in the quark distribution factor) that was not present in A. Hence, we *must* provide a UV regulator for $A^{\rm soft}$ if the expression is to make sense. On the other hand, the fact that $p^2\neq 0$ in a hadron means that $A^{\rm soft}$ contains no soft divergences. However, we can choose, if we wish, to impose an additional soft cutoff. Since $A^{\rm soft}$ is a good approximation to A for $k_T^2\ll Q^2$, any dependence on the cutoff will cancel in the difference $A-A^{\rm soft}=A^{\rm hard}$, provided that we choose the same soft cutoff procedure for both A and $A^{\rm soft}$ and take the cutoff scale to be much less than Q.

In order to provide some illustrative examples, we have calculated A and $A^{\rm soft}$ using $p^2\neq 0$, $m^2\neq 0$, and dimensional soft-cutoff procedures and using dimensional and Pauli-Villars (PV) UV regulators for $A^{\rm soft}$. (In the case in which dimensional continuation is used to cut off both the soft and UV regions, we partition the k_T^2 integration into two regions, continuing to $4-2\epsilon$ dimensions in the UV region and $4+2\epsilon$ dimensions in the soft region.) We have already presented the results for A in (2.7), (2.9), and (2.11). These results and the results for $A^{\rm soft}$ are summarized in Table I. As expected, $A^{\rm hard}$ is independent of the choice of soft cutoff procedure. For the UV regulators we have employed, $A^{\rm hard}$ is also independent of the choice of UV regulator, apart from definition of the factorization scale:

$$A^{\rm hard}(x,Q^2/\mu_{\rm fact}^2)=-\frac{\alpha_sN_fT}{\pi}\left[(1-2x)\left[\ln\frac{Q^2}{\mu_{\rm fact}^2}+\ln\frac{1-x}{x}-1\right]-2(1-x)\right], \tag{2.17}$$

where $\mu_{\rm fact}^2=4\pi\mu_{\rm MS}^2\exp(-\gamma_E-1/\epsilon)$ in the case of the dimensional regularization, and $\mu_{\rm fact}^2=M^2$ in the case of Pauli-Villars regularization. Here again we regard ϵ as a fixed cutoff. (Equivalently, one could "renormalize" $A^{\rm soft}$ by discarding the UV poles in ϵ.) These results for $A^{\rm hard}$ are also equivalent, through a redefinition of the factorization scale, to the results of (2.9) and (2.11), which were derived by applying dimensional and mass soft cutoffs to the box diagrams. As in that previous analysis, we find that the first moment of $A^{\rm hard}(x)$ [and hence the contri-

TABLE I. (a) Contributions to $A(x)$ (from the box diagrams of Fig. 2) that result from the use of various soft cutoffs; (b) contributions to $A^{\text{soft}}(x)$ (from diagrams of Fig. 3) that result from the use of various soft cutoffs and UV regulators.

(a)

Soft	$A(x)$
$m^2\neq 0$	$-\frac{\alpha_s N_f T}{\pi}\left[(1-2x)\left[\ln\frac{Q^2}{m^2}+\ln\frac{1-x}{x}-1\right]-2(1-x)\right]$
$p^2\neq 0$	$-\frac{\alpha_s N_f T}{\pi}(1-2x)\left[\ln\frac{Q^2}{-p^2}+\ln\frac{1}{x^2}-2\right]$
Dim.	$-\frac{\alpha_s N_f T}{\pi}\left[(1-2x)\left[\frac{1}{\epsilon}+\ln\frac{Q^2}{4\pi\mu_{\text{MS}}^2}+\gamma_E+\ln\frac{1-x}{x}-1\right]-2(1-x)\right]$

(b)

Soft	UV	$A^{\text{soft}}(x)$
$m^2\neq 0$	Dim.	$-\frac{\alpha_s N_f T}{\pi}(1-2x)\left[-\frac{1}{\epsilon}+\ln\frac{4\pi\mu_{\text{MS}}^2}{m^2}-\gamma_E\right]$
$p^2\neq 0$	Dim.	$-\frac{\alpha_s N_f T}{\pi}\left[(1-2x)\left[-\frac{1}{\epsilon}+\ln\frac{4\pi\mu_{\text{MS}}^2}{-p^2}-\gamma_E+\ln\frac{1}{x(1-x)}-1\right]+2(1-x)\right]$
Dim.	Dim.	0
$m^2\neq 0$	PV	$-\frac{\alpha_s N_f T}{\pi}(1-2x)\ln\frac{M^2}{m^2}$
$p^2\neq 0$	PV	$-\frac{\alpha_s N_f T}{\pi}\left[(1-2x)\left[\ln\frac{M^2}{-p^2}+\ln\frac{1}{x(1-x)}-1\right]+2(1-x)\right]$
Dim.	PV	$-\frac{\alpha_s N_f T}{\pi}(1-2x)\left[\frac{1}{\epsilon}+\ln\frac{M^2}{4\pi\mu_{\text{MS}}^2}+\gamma_E\right]$

bution to the first moment of $g_1(x)$] vanishes:

$$\int_0^1 dx\ A^{\text{hard}}(x,Q^2/\mu_{\text{fact}}^2)=0\ . \tag{2.18}$$

Again chiral symmetry plays a role in the vanishing of the first moment of $A^{\text{hard}}(x)$: a detailed examination of the calculation reveals that the contribution from $A(x)$ is canceled by chiral-symmetry-breaking terms in $A^{\text{soft}}(x)$. These chiral-symmetry-breaking terms arise from the regulator mass in the case of a Pauli-Villars UV regulator and from the property $\{\gamma_5,\gamma_\mu\}\neq 0$ in the case of a dimensional UV regulator.

III. GENERAL ARGUMENT FOR THE VANISHING OF $\int_0^1 dx\ A^{\text{hard}}(x)$

At this stage it is not clear whether result (2.18) is merely an artifact of the particular UV regulators we have employed or, rather, a universal feature that arises in any satisfactory separation of the hard and soft contributions to A. We now explore this issue by giving a general argument that reveals a set of sufficient conditions for the vanishing of the first moment of $A^{\text{hard}}(x)$.

Since A^{hard} is independent of the choice of soft cutoff, for convenience we take the $m^2\neq 0$ soft cutoff (for both A and A^{soft}). Now suppose we take moments of $A^{\text{soft}}(x)$ with respect to x. We use the factor $\delta(p\cdot nx-k\cdot n)$ from each cut vertex to carry out the x integration. Also, we use the following identity for the factor associated with the cut on the final-state quark line:

$$2\pi\delta((p-k)^2-m^2)=\frac{i}{(p-k)^2-m^2+i\epsilon}+\frac{i}{-[(p-k)^2-m^2]+i\epsilon}\ . \tag{3.1}$$

For the second term on the right-hand side of (3.1), we can deform the k^- contour into the upper half plane, avoiding the k^- pole in that term and all of the k^- poles in the propagators. Here we assume that the UV regulator respects the analyticity structure of the original diagrams in the sense that it does not introduce any poles in the upper half of the complex k^- plane—except for poles that originate from the cut of the final-state quark line, as in the first term on the right-hand side of (3.1). For the diagrams of Figs. 3(a)–3(c), the contour at infinity vanishes; the diagrams of Fig. 3(d) give no contribution because the Dirac trace vanishes. Thus, only the first term on the right-hand side of (3.1) contributes. Combining it with the final-state sum over spinors $\gamma\cdot(p-k)+m$, we obtain the quark propagator. Hence, by taking moments, we have converted the set cut diagrams in Fig. 3 to a set of uncut diagrams (see Fig. 5). These uncut diagrams correspond to the expectation values of local operators in the gluon state.

Now we establish the connection between the first moment of $A^{\text{soft}}(x)$ and matrix elements of the axial-vector current. We first make use of the gauge invariance of the uncut diagrams of Fig. 5 to specialize, temporarily, to a gauge $n\cdot A=0$. Here we assume that the UV cutoff

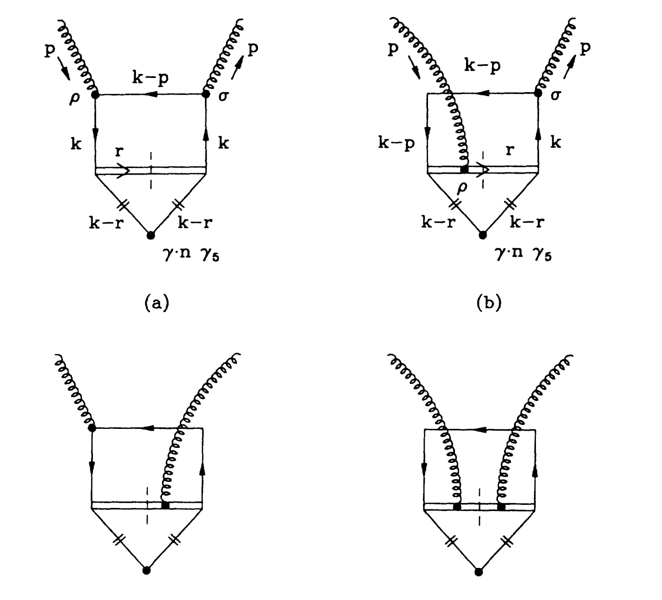

FIG. 5. The uncut diagrams that are equivalent to the cut diagrams of Fig. 3. In addition to the quark diagrams shown, there are antiquark diagrams, which are obtained by reversing the arrows on the fermion lines.

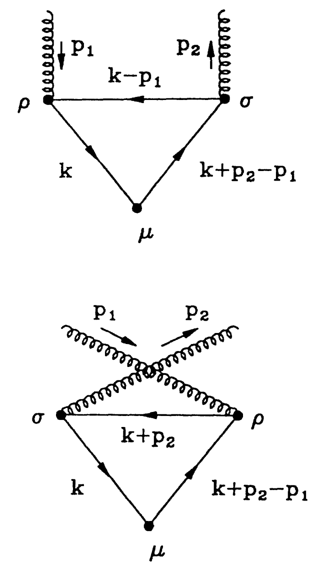

FIG. 6. The triangle diagrams that correspond to the matrix element of the singlet axial-vector current between gluon states of momentum p.

respects gauge invariance. Then the diagrams involving connections to the eikonal lines [Figs. 5(b)–5(c)] vanish. (A closer examination of these diagrams reveals that they actually vanish in an arbitrary gauge, provided that the regulator is symmetric under $k \to -k$.) In the remaining diagrams [Fig. 5(a)], we drop the eikonal lines, since, after integration over the loop momentum r, these contribute a factor unity. The result is the set of triangle diagrams of Fig. 6. These diagrams represent the matrix element of the singlet axial-vector current between gluon states of momentum p. Hence, we have arrived at the usual relationship (1.3) between the singlet spin-dependent quark distributions and matrix elements of the axial-vector current.

Now we investigate the effect on the diagrams of Fig. 6 of a gauge transformation of the gluon field of leading order in g, that is, the transformation $A_\rho \to A_\rho + \partial_\rho \phi$. In order to check whether the term proportional to ϕ gives a vanishing contribution, we dot one of the gluon momenta p into the corresponding gluon-quark vertex and apply the graphical Ward identity (Feynman identity)

$$\frac{1}{\gamma\cdot l - m + i\epsilon}\, p\cdot\gamma\, \frac{1}{\gamma\cdot(l-p) - m + i\epsilon} = \frac{1}{\gamma\cdot(l-p) - m + i\epsilon} - \frac{1}{\gamma\cdot l - m + i\epsilon}\,, \quad (3.2)$$

where l is the momentum flowing out of the vertex. Applying (3.2) to the diagrams of Fig. 6, we find that the contributions from those diagrams cancel each other, provided that we can shift the loop momentum k. We assume that the regulator permits such a shift. That is, we assume that the regulator is Lorentz invariant and, hence, does not single out any particular Lorentz frame. Then, the diagrams of Fig. 6 constitute a gauge-invariant set and yield the complete contribution to the first moment of $A^{\rm soft}(x)$.

Finally, we can use the gauge-invariance property of the diagrams of Fig. 6 to show that their contribution is independent of the choice of UV regulator. First, we consider a slight generalization of these diagrams in that we allow a nonzero momentum to flow into the lower (axial-vector) vertices. We denote the sum of these generalized diagrams by $\Gamma_{\rho\sigma\mu}(p_1,p_2)$, where p_1 and p_2 are the momenta of gluons 1 and 2, ρ and σ are the Lorentz indices associated with their polarizations, and μ is the Lorentz index associated with the axial-vector vertex. Then, by gauge invariance, we have

$$p_1^\rho \Gamma_{\rho\sigma\mu}(p_1,p_2) = 0\,, \quad (3.3a)$$

and

$$p_2^\sigma \Gamma_{\rho\sigma\mu}(p_1,p_2) = 0\,. \quad (3.3b)$$

We differentiate (3.3a) with respect to p_1^ρ and evaluate it at $p_1=0$; and we differentiate (3.3b) with respect to p_2^σ and evaluate it at $p_2=0$. The result is

$$\Gamma_{\rho\sigma\mu}(0,p_2) = \Gamma_{\rho\sigma\mu}(p_1,0) = \Gamma_{\rho\sigma\mu}(0,0) = 0\,. \quad (3.4)$$

Here, we have used the fact that, because of the $m^2 \neq 0$ soft cutoff, $\Gamma_{\rho\sigma\mu}(p_1,p_2)$ and its first derivatives with respect to p_1 and p_2 are finite at $p_1=0$ and $p_2=0$ (Ref. 13). Now we can use the result (3.4) to write $\Gamma_{\rho\sigma\mu}(p_1,p_2)$ in a doubly subtracted form:

$$\Gamma_{\rho\sigma\mu}(p_1,p_2) = \Gamma_{\rho\sigma\mu}(p_1,p_2) - \Gamma_{\rho\sigma\mu}(p_1,0) - [\Gamma_{\rho\sigma\mu}(0,p_2) - \Gamma_{\rho\sigma\mu}(0,0)]\,. \quad (3.5)$$

Since the triangle diagrams are superficially linearly divergent in the UV, the right-hand side of (3.5) is convergent in the UV. Hence, we can remove the UV regulator. That is, the result is independent of the choice of UV regulator. Then, explicit calculation yields

$$\Gamma_{\rho\sigma\mu}(p,p)=0 \ . \tag{3.6}$$

Apparently, the chiral-symmetry-violating contribution that arises from the $m^2\neq 0$ soft cutoff is precisely canceled by a chiral-symmetry-violating contribution from the UV regulator whenever the UV regulator respects gauge invariance. This property is reminiscent of the behavior of the triangle anomaly. The divergence of the axial-vector current receives an anomalous contribution that is uniquely determined, provided that the UV regulator respects Lorentz invariance and vector-current conservation.[14] That anomalous contribution is precisely canceled by the contribution that appears when one introduces a nonzero fermion mass. Note, however, that the chiral-symmetry-breaking contributions to A^{soft} are not identical to the anomaly: the anomaly vanishes when $p_1=p_2$.

We conclude then that the first moment of $A^{\text{soft}}(x)$ vanishes. Consequently the first moment of $A^{\text{hard}}(x)$ is given by the first moment of $A^{\text{mass}}(x)$. Hence, there is no hard gluonic contribution in order α_s to the first moment of $g_1(x)$—provided that the UV regulator used in defining the quark distributions is both gauge invariant and Lorentz invariant and respects the analyticity structure of the unregulated quark distributions in the sense discussed above. Jaffe and Manohar[15] have reached a similar conclusion via an analysis based on the operator-product expansion.

IV. SOME ALTERNATIVE DEFINITIONS OF $\Delta q(x,Q^2)$

Keeping in mind the constraints imposed by our preceding analysis, we now examine some alternative definitions of the quark distributions.

Both AR and CCM have suggested that the first moment of the singlet spin-dependent quark distributions be defined by replacing the axial-vector current on the right-hand side of (1.3) by

$$\tilde{j}_5^{\mu}=j^{\mu}-k^{\mu} \ , \tag{4.1a}$$

where

$$k^{\mu}=\frac{\alpha_s N_f}{2\pi}\epsilon^{\mu\nu\rho\sigma}A_{\nu}^{a}(\partial_{\rho}\mathcal{A}_{\sigma}^{a}-\tfrac{1}{3}gf^{abc}\mathcal{A}_{\rho}^{b}\mathcal{A}_{\sigma}^{c}) \ . \tag{4.1b}$$

Such a redefinition removes the anomalous contribution from the matrix elements of the axial-vector current and results in a nonzero gluonic contribution to the first moment of $g_1(x)$. This does not contradict our previous result, however, since k^{μ} is not gauge invariant. It is true that the current k_{μ} is gauge invariant to leading order in g for forward matrix elements.[16] However, as has been pointed out by Jaffe and Manohar,[15] k_{μ} is not gauge invariant, even in the forward direction, beyond the leading order in g. Thus, (4.1) does not lead to a satisfactory definition of the quark distributions.

A more acceptable alternative definition of the quark distributions can be obtained by relaxing the requirement that the UV regulator be Lorentz invariant. In particular, one could regulate the distributions by imposing a "brute-force" upper limit on the k_T^2 integration in the cut diagrams of Fig. 3. This approach yields the result

$$A^{\text{soft}}(x,\mu_{\text{UL}}^2)=-\frac{\alpha_s N_f T}{\pi}\int_0^{\mu_{\text{UL}}^2}\frac{dk_T^2}{[k_T^2+m^2-p^2x(1-x)]^2}\times[(1-2x)(k_T^2+m^2)-2m^2(1-x)] \ . \tag{4.2}$$

Here we have assumed that $\mu_{\text{UL}}^2 \ll Q^2$. If we use the $m^2\neq 0$ soft cutoff and integrate over k_T^2, we obtain

$$A^{\text{soft}}(x,\mu_{\text{UL}}^2)=-\frac{\alpha_s N_f T}{\pi}\times\left[(1-2x)\ln\left[\frac{\mu_{\text{UL}}^2}{m^2}\right]-2(1-x)\right] , \tag{4.3}$$

where we have assumed that $|p^2| \ll m^2 \ll \mu_{\text{UL}}^2$. This yields, for the first moment of $A^{\text{soft}}(x,\mu_{\text{UL}}^2)$,

$$\int_0^1 dx\ A^{\text{soft}}(x,\mu_{\text{UL}}^2)=\frac{\alpha_s N_f}{2\pi} \ . \tag{4.4}$$

Now, with an $m^2\neq 0$ soft cutoff, the first moment of $A(x)$ vanishes. Hence, we see that, with a brute-force UV cutoff on A^{soft}, $A^{\text{hard}}(x)$ has the first moment advocated by AR and CCM:

$$\int_0^1 dx\ A^{\text{hard}}(x,\mu_{\text{UL}}^2)=-\frac{\alpha_s N_f}{2\pi} \ . \tag{4.5}$$

More generally, one could define the spin-dependent distributions of quarks and antiquarks in a gluon to all orders in α_s by

$$\Delta q_i^g(x,\mu_{\text{UL}}^2)=\int d^2k_T\Delta q_i^g(x,k_T) \ , \tag{4.6}$$

where $\Delta q_i^g(x,k_T)$ is the k_T-dependent quark distribution obtained by fixing both x and k_T at the cut vertex, and the integration over k_T is cut off at $k_T\sim\mu_{\text{UL}}$. This distribution has the following definition in terms of operator matrix elements:[11]

$$\Delta q_i^g(x,k_T)=\int \frac{dy^- d^2y_T}{2(2\pi)^3}\left(e^{-i(xp^+y^- - k_T\cdot y_T)}+e^{i(xp^+y^- - k_T\cdot y_T)}\right) \times \left\langle p \left| \bar{\psi}_i(0,y^-,y_T)\gamma^+\gamma_5 \mathcal{P}\left[\exp\left[ig\int_C dz_\nu \mathcal{A}_a^\nu(z)t_a\right]\right]\psi_i(0,0,0_T)\right| p\right\rangle_c , \tag{4.7}$$

where the line integral is along a contour C linking the point $(0,0,0_T)$ with the point $(0,y^-,y_T)$. [Note that $\Delta q_i^g(x,k_T)$ depends implicitly on a scale μ associated with the UV regularization of divergent subdiagrams.] One can measure $\Delta q_i(x,k_T)$ in a number of ways. If $\Delta q_i(x,k_T)$ is obtained from deep-inelastic lepton scattering by measuring the transverse-momentum dependence of the associated two-jet production, then the appropriate contour C is predominantly lightlike and passes through $x_0=+\infty$. CCM have suggested such a measurement as a means for identifying the hard gluonic contribution to g_1. On the other hand, if $\Delta q_i(x,k_T)$ is obtained from the Drell-Yan process by measuring the transverse momentum of the lepton pair, then the contour C is predominantly spacelike and passes through $x_3=-\infty$ (Refs. 17 and 18). The expressions for $\Delta q_i(x,k_T)$ based on the two contours C we have described are, in general, unequal. However, the integrals of these expressions over k_T up to $|k_T|\gg\Lambda$ are related through calculable perturbative expansions.[17,18]

The first moments of the alternative quark distributions (4.6) involve the matrix elements of an operator that is effectively local, since, after integration of (4.7) over x and k_T, the size of the operator is $1/p^+ \ll 1/\Lambda$ in the longitudinal direction and $1/\mu_{\rm UL} \ll 1/\Lambda$ in the transverse direction.[19] However, because the brute-force cutoff singles out a special frame, the quark distributions are not related to the forward matrix elements of Lorentz-invariant operators. Rather, the distributions are given by matrix elements of Lorentz-*variant* operators that depend on the vector n (Ref. 20). That is why the alternative definition of the quark distributions (4.6) has no analogue in the operator-product analysis of Jaffe and Manohar.[15]

It is important to note that, because the first moments of the alternative quark distributions (4.6) are not given by matrix elements of the axial-vector current, these unorthodox distributions do not satisfy the usual sum rules, such as (1.2b) and the Bjorken sum rule. Since the alternative quark distributions are related to the standard distributions (2.16) through calculable perturbative expansions, this is not a serious drawback. However, it is clear that one cannot simply apply standard sum-rule-based expressions to the alternative quark distributions.

We conclude, then, that it is possible to arrive at satisfactory definitions of the quark distributions such that the hard gluonic contribution to the first moment of g_1 is nonzero. The size of the hard gluonic contribution to the first moment is entirely a matter of the convention chosen in defining the quark distribution. Since the hard gluonic contribution appears in combination with the quark contribution whenever one measures the quark distributions, the choice of convention cannot be fixed by comparison with experiment. In principle, one choice of convention is as valid as another, although one might try to argue that some particular choice is closer to physical intuition than another. Whichever convention is chosen, one must take care to use it consistently for all processes under consideration.

V. SUMMARY AND DISCUSSION

In this paper, we have examined the gluonic contribution to the proton's spin-dependent structure function $g_1^p(x)$. As we have explained, the x moments of this hard gluonic contribution are proportional to the x moments of $A^{\rm hard}(x)$, the hard, perturbatively calculable part of the spin-dependent photon-gluon scattering cross section A. Hence, the identification of $A^{\rm hard}(x)$ has been the central issue in this paper.

Altarelli and Ross (AR) and Carlitz, Collins, and Mueller (CCM) have suggested methods for regulating the collinear divergence in A so as to isolate hard contributions. These methods lead to nonzero contributions to the first moment of $A^{\rm hard}(x)$, and hence to a nonzero contribution to the first moment of $g_1^p(x)$. However, we have argued that the expression of AR is not invariant under gauge transformations of the photon and gluon fields and that the expression of CCM actually contains soft contributions when the gluon is nearly on its mass shell. Since the first moment of $A(x)$ as computed by CCM is independent of the soft regulator scale, it is tempting to say that this quantity should be counted as a contribution to the first moment of $A^{\rm hard}$. But, at any given value of x, $A(x)$ *does* receive contributions that depend logarithmically on the soft regulator. It happens that these contributions cancel when one takes the first moment of $A(x)$ because massless, perturbative QCD is chirally symmetric. However, we would not expect such a cancellation to be reliable in the soft, nonperturbative regime.

It is possible to regulate the collinear divergence in A in a way that admits only hard contributions and preserves gauge invariance. Two examples of such regulators are dimensional continuation and the introduction of a quark mass. We have shown that these two methods lead to a vanishing first moment of $A^{\rm hard}(x)$ because chiral-symmetry-breaking contributions from loop momenta of the order of the regulator scale cancel the contributions from loop momenta of order Q.

We have proposed that, in general, one identify $A^{\rm hard}$ by subtracting from A the part that can be interpreted as arising from the distribution of quarks in a gluon. This is precisely what one does in the spin-independent case. Then, the contribution to $A^{\rm hard}$ depends only on the choice of the UV regulator used to define the quark distributions: it is independent of the method used to regulate the collinear singularities, provided that the same method is used both for A and for the quark distributions. Our calculations show that both Pauli-Villars and dimensional UV regulators yield a contribution to $A^{\text{hard}}(x)$ whose first moment vanishes.

More generally, we have argued that any UV regulator that respects gauge invariance, Lorentz invariance, and the analyticity structure of the unregulated quark distributions leads to an $A^{\text{hard}}(x)$ whose first moment vanishes. The analyticity requirement, which amounts to the optical theorem at the parton level, is crucial in relating the moments of the quark distributions to the forward matrix elements of local operators. The vanishing of the first moment of A^{hard} is closely related to the appearance of an anomaly in the singlet axial-vector current. Any UV regulator that respects Lorentz and gauge invariance necessarily leads to chiral-symmetry-breaking contributions to the triangle diagram. These chiral-symmetry-breaking contributions generate the axial-vector anomaly and also cancel the contributions to A^{hard} from loop momenta of order Q.

Nevertheless, it *is* possible to formulate satisfactory definitions of the quark distributions such that the hard gluonic contribution to the first moment of $g_1^P(x)$ is nonzero. In particular, one can define the quark distributions as integrals over k_T of k_T-dependent quark distributions, thereby relaxing the requirement that the UV regulator respect Lorentz invariance. This approach does not violate any general principles, since it amounts to making use of the special Lorentz vector defined by the proton's momentum. It is appealing in that the quark distributions are directly related to physical observables. The k_T-dependent distributions can be measured, for example, by observing the transverse-momentum dependence of the two-jet production associated with deep-inelastic lepton scattering or by observing the transverse-momentum dependence of lepton-pair production via the Drell-Yan process. In fact, the former measurement has been suggested by CCM as a means for identifying a hard gluonic contribution to $g_1^P(x)$. However, it should be noted that the quark distributions defined by this method correspond to matrix elements of Lorentz-*variant* local operators. That is why the possibility of such unorthodox distributions does not appear in a conventional operator-product analysis.[15] Since the first moments of these alternative spin-dependent quark distributions are not given by forward matrix elements of the axial-vector current, they do not satisfy the usual sum rules, such as (1.2b) and the Bjorken sum rule. However, this is not an insurmountable difficulty, since the alternative quark distributions can be related to the standard quark distributions through calculable perturbative expansions. Also, one has the option of employing the standard definition for the nonsinglet distributions and the alternative definition for the singlet distribution.

We conclude, then, that the size of the hard gluonic contribution to the first moment of $g_1^P(x)$ is purely a matter of the convention used in defining the spin-dependent quark distributions. The choice of convention has no observable consequences, since, in any measurement of the singlet spin-dependent quark distributions, the hard gluonic contribution also appears. Hence, the choice of definition of the spin-dependent quark distributions is more a matter of philosophy than of measurable physics. Of course, none of this implies that the spin-dependent gluon distribution Δg is unmeasurable.[21] In the contribution to g_1, it is the size of the hard gluonic subprocess A^{hard}, and not Δg itself, that depends on the definition of the spin-dependent quark distribution.

Much of the discussion of the possibility of a hard gluonic contribution to the first moment of $g_1^P(x)$ was motivated initially by a desire to reconcile the result of the European Muon Collaboration (EMC) experiment with intuition derived from the static quark model. However, in a bound state of nonrelativistic quarks, the transverse modes of the gluon distribution are suppressed by v/c, So the hard gluonic contribution to $g_1^P(x)$ vanishes in the static limit. Thus, it is not obvious that the inclusion of a hard gluonic contribution to the first moment of $g_1^P(x)$ helps to make the quark distributions compatible with expectations from the static quark model.

Moving beyond the static quark model, one might hope to obtain a sum rule for the total angular momentum of the proton in terms of certain parton distributions. In such a sum rule, the orbital angular momentum, as well as the spins of the quarks and gluons, would be expected to play a role. Unfortunately, as has been pointed out by Jaffe and Manohar,[15] there is no leading-twist parton distribution that measures the orbital angular momentum. This really should come as no surprise. Because the parton distributions are probabilities, rather than amplitudes, it is tempting to think of them as giving an almost classical description of the structure of the proton. However, the parton distributions are merely quantities that appear in the scattering of hadrons in the limit of large momentum transfer. There is no reason to believe that information gained in that limit should yield a complete description of the structure of hadron, and there is no reason to expect that every dynamical variable of the hadronic system should correspond to a parton distribution.

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[20]The brute-force cutoff on k_T leads to an additional (Lorentz-variant) twist-two operator $\mathcal{O}_{\mu_1} \propto \epsilon_{\alpha\beta\gamma\mu_1} G^{\alpha\beta} G^{\mu 2\gamma} n_{\mu_2}$, where G is the gauge-field strength, and we have adopted the notation of Ref. 15. We wish to thank G. P. Lepage for suggesting to us the possibility that such operators might arise from this UV-cutoff procedure.

[21]For a discussion of the measurement of Δg in prompt photon production, see E. L. Berger and J. Qiu, Phys. Rev. D **40**, 778 (1989); **40**, 3128 (1989).