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Angular correlation of charged leptons from heavy-lepton pair produced in e^-e^+ annihilation

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The differential cross section of the $e^-+e^+\rightarrow\gamma,Z\rightarrow L^-+L^+$, $L^-\rightarrow\nu_L+l^-+\bar{\nu}_l$, $L^+\rightarrow\bar{\nu}_L+l^++\nu_l$ process is derived. The angular correlation of charged leptons (l^-,l^+) near the threshold is studied.

I. INTRODUCTION

Charged leptons are important signals of heavy particles in particle physics. In this paper we study the angular correlation of charged leptons (l^- and l^+) from heavy-charged-lepton pairs (L^-L^+) produced in electron-positron annihilation,

$$e^-+e^+\rightarrow\gamma,Z\rightarrow L^-+L^+ ,$$
$$L^-\rightarrow\nu_L+l^-+\bar{\nu}_l,\quad L^+\rightarrow\bar{\nu}_L+l^++\nu_l , \tag{1}$$

by assuming that the heavy leptons decay through the standard $V-A$ weak interaction and that the heavy lepton (mass m) is lighter than the W boson ($m<m_W$). We derive the differential cross section of process (1) by using a modified version of the helicity-projection technique[1] in order to factorize the cross section into production and decay parts.

The helicity-projection technique invented by Barger, Ohnemus, and Phillips[1] is powerful in deriving compact expressions for the complete cross sections of heavy-fermion production and decay via W bosons ($ab\rightarrow Lx_1,\ldots,x_n, L\rightarrow\nu_Ly_1y_2$). They have noticed that the spinor outer products can be represented by the following substitutions:

$$u(L,\pm)\bar{u}(L,\pm)\rightarrow i\gamma\cdot(-L\pm mw)/2 ,$$
$$u(L,+)\bar{u}(L,-)\rightarrow i\gamma\cdot(imB+k)/2 , \tag{2}$$
$$u(L,-)\bar{u}(L,+)\rightarrow i\gamma\cdot(-imB+k)/2 ,$$

$$v(\bar{L},\pm)\bar{v}(\bar{L},\pm)\rightarrow i\gamma\cdot(\bar{L}\pm mw')/2 ,$$
$$v(\bar{L},-)\bar{v}(\bar{L},+)\rightarrow i\gamma\cdot(-imB-k')/2 , \tag{3}$$
$$v(\bar{L},+)\bar{v}(\bar{L},-)\rightarrow i\gamma\cdot(imB-k')/2 ,$$

where

$$L_\mu=(px\cos\phi,px\sin\phi,pz;E) ,$$
$$\bar{L}_\mu=(-px\cos\phi,-px\sin\phi,-px;E) , \tag{4}$$

$$w_\mu=(Ex\cos\phi/m,Ex\sin\phi/m,Ez/m;p/m) ,$$
$$w'_\mu=(-Ex\cos\phi/m,-Ex\sin\phi/m ,\; -Ez/m;p/m) , \tag{5}$$

$$B_\mu=(-\sin\phi,\cos\phi,0;0) , \tag{6}$$

$$k_\mu=\epsilon_{\mu\beta\gamma\delta}B_\beta w_\gamma L_\delta =(mz\cos\phi,mz\sin\phi,-mx;0) ,$$
$$k'_\mu=\epsilon_{\mu\beta\gamma\delta}B_\beta w'_\gamma \bar{L}_\delta =(-mz\cos\phi,-mz\sin\phi,mx;0) , \tag{7}$$

$x=\sin\theta$ and $z=\cos\theta$. C and C^* in Ref. 1 are $mC=k+imB$ and $mC^*=k-imB$, respectively.

The substitutions (2) and (3) are allowed only when the heavy leptons are produced and decay through the $V-A$ weak interaction. In such cases the squared matrix element of a heavy-lepton production and decay sequence averaged over initial and summed over final spins and colors takes the form

$$|M|^2=N_i^{-1}[(X\cdot L)(Y\cdot L)-m^2(X\cdot Y)/2]/D_L^2 \tag{8}$$

if the spin-averaged squared matrix elements of the production and decay of L are given by

$$|M(\text{prod})|^2=N_i^{-1}X\cdot L , \tag{9}$$

$$|M(\text{decay})|^2=Y\cdot L , \tag{10}$$

where $D_L=L^2-m^2+im\Gamma_L$.

For reaction (1) the helicity-projection technique in the original form[1] is not convenient since the substitutions (2) and (3) are not allowed for the electroweak production part of L and $\bar{L}$, while they are allowed for the weak decay parts of L and $\bar{L}$. Hence, we use a modified version of the helicity-projection technique, which is explained in Sec. II. In Sec. II we derive the differential cross section of process (1). In Sec. III we study the angular correlation of charged leptons l^- and l^+ near the threshold. Discussions and conclusions are given in Sec. IV. In the following we use the particle symbols L, $\bar{L}$, N, $\bar{N}$, l, $\bar{l}$, ν, and $\bar{\nu}$ instead of L^-, L^+, ν_L, $\bar{\nu}_L$, l^-, l^+, ν_l, and $\bar{\nu}_l$.

II. THE DIFFERENTIAL CROSS SECTION

Let us express the amplitude of process (1) as

$$A_{\lambda\lambda'}=\sum_{hh'}A_{\lambda\lambda'hh'}A_h\bar{A}_{h'} , \tag{11}$$

where λ, λ', h, and h' are the helicities of the electron, positron, heavy lepton (L), and its antiparticle ($\bar{L}$), respectively. In (11) $A_{\lambda\lambda' hh'}$ is the helicity amplitude of $e^- + e^+ \to L + \bar{L}$, and A_h and $\bar{A}_{h'}$ are the decay amplitudes of L with helicity h and $\bar{L}$ with helicity h', respectively. We assume that the heavy leptons in the intermediate state are on mass shell and decay through the standard $V-A$ weak interaction and that the mass of the heavy lepton m is smaller than that of the W boson ($m < m_W$).

We find that it is more convenient to rewrite (11) as

$$A_{\lambda\lambda'} = \sum_{hh'} (e_{\lambda\lambda' hh'} E_{2h,2h'} + o_{\lambda\lambda' hh'} O_{2h,2h'}) , \qquad (12)$$

where

$$E_{2h,2h'} = (A_h \bar{A}_{h'} + A_{-h} \bar{A}_{-h'})/\sqrt{2} ,$$
$$O_{2h,2h'} = (A_h \bar{A}_{h'} - A_{-h} \bar{A}_{-h'})/\sqrt{2} , \qquad (13)$$

$$e_{\lambda\lambda' hh'} = (A_{\lambda\lambda' hh'} + A_{\lambda\lambda', -h-h'})/\sqrt{2} ,$$
$$o_{\lambda\lambda' hh'} = (A_{\lambda\lambda' hh'} - A_{\lambda\lambda', -h-h'})/\sqrt{2} . \qquad (14)$$

We prefer form (12) to (11) since the number of terms contained in $e_{\lambda\lambda' hh'}$ or $o_{\lambda\lambda' hh'}$ is one-half of the number of terms contained in the corresponding $A_{\lambda\lambda' hh'}$ in the tree approximation.

By straightforward calculation of the helicity amplitudes $A_{\lambda\lambda' hh'}$ of the $e^- + e^+ \to \gamma, Z \to L + \bar{L}$ process in the tree approximation, we obtain the following amplitudes $A_{\lambda\lambda'}$ in the center-of-mass frame:

$$A_{1/2,-1/2} = e^2[-xmV_R O_{1,1} + (-zEV_R + pA_R)E_{1,-1} + (-EV_R + zpA_R)O_{1,-1}](\sqrt{2}/E)e^{i\phi}$$

and (15)

$$A_{-1/2,1/2} = e^2[-xmV_L O_{1,1} + (-zEV_L - pA_L)E_{1,-1} + (EV_L + zpA_L)O_{1,-1}](\sqrt{2}/E)e^{-i\phi} ,$$

where

$$V_R = 1/s + (x_W - \tfrac{1}{4})/(1-x_W)(s - m_Z^2 + im_Z\Gamma_Z) ,$$
$$V_L = 1/s + (x_W - \tfrac{1}{2})(x_W - \tfrac{1}{4})/x_W(1-x_W)(s - m_Z^2 + im_Z\Gamma_Z) , \qquad (16)$$
$$A_R = -1/4(1-x_W)(s - m_Z^2 + im_Z\Gamma_Z), \quad A_L = -(x_W - \tfrac{1}{2})/4x_W(1-x_W)(s - m_Z^2 + im_Z\Gamma_Z) ,$$

$x_W = \sin^2\theta_W$, E and p are the energy and momentum of the heavy leptons, respectively, $s = 4E^2 = 4(p^2 + m^2)$, and the four-momenta of the electron, positron, heavy lepton, and its antiparticle, e^-, e^+, L, and $\bar{L}$, are chosen as

$$e^- = (0,0,E;E), \quad e^+ = (0,0,-E;E), \quad L = (px\cos\phi, px\sin\phi, pz;E) ,$$
$$\bar{L} = (-px\cos\phi, -px\sin\phi, -pz;E) . \qquad (17)$$

($x = \sin\theta$ and $z = \cos\theta$.)

The differential cross section of the $e^- + e^+ \to \gamma, Z \to L + \bar{L} \to (N + l + \bar{\nu}) + (\bar{N} + \bar{l} + \nu)$ process is given by

$$d\sigma = \sum_{\text{final helicities}} (pE/16\pi^2)(2\pi)^{-10}\delta^4(L - N - l - \bar{\nu})\delta^4(\bar{L} - \bar{N} - \bar{l} - \nu)$$
$$\times dz\, d\phi (d^3N/2N_0)(d^3l/2l_0)(d^3\bar{\nu}/2\bar{\nu}_0)(d^3\bar{N}/2\bar{N}_0)(d^3\bar{l}/2\bar{l}_0)$$
$$\times (d^3\nu/2\nu_0)(2m\Gamma_L)^{-2}(|A_{1/2,-1/2}|^2 + |A_{-1/2,1/2}|^2)/4 , \qquad (18)$$

where L, N, l, $\bar{\nu}$, $\bar{L}$, $\bar{N}$, $\bar{l}$, and ν stand for the four-momenta of L^-, ν_L, l^-, $\bar{\nu}_l$, L^+, $\bar{\nu}_L$, l^+, and ν_l, respectively.

The differential cross sections for the polarized electron and positron beams are obtained by replacing the factor $(|A_{1/2,-1/2}|^2 + |A_{-1/2,1/2}|^2)/4$ in (18) by an appropriate $|\text{amplitude}|^2$ such as $|A_{1/2,-1/2}|^2$ or $|A_{-1/2,1/2}|^2$ for linearly polarized beams.

In order to calculate the differential cross sections by using (18) we have to use the relations

$$
\begin{aligned}
(|O_{1,1}|^2)&=(64G_F^2)^2(D\bar{D})^2(N\cdot l)(\bar{N}\cdot\bar{l})[(L\cdot\bar{v})(\bar{L}\cdot v)-m^2(w\cdot\bar{v})(w'\cdot v)-m^2(B\cdot\bar{v})(B\cdot v)+(k\cdot\bar{v})(k'\cdot v)]\ ,\\
(|E_{1,-1}|^2)&=(64G_F^2)^2(D\bar{D})^2(N\cdot l)(\bar{N}\cdot\bar{l})[(L\cdot\bar{v})(\bar{L}\cdot v)+m^2(w\cdot\bar{v})(w'\cdot v)-m^2(B\cdot\bar{v})(B\cdot v)-(k\cdot\bar{v})(k'\cdot v)]\ ,\\
(|0_{1,-1}|^2)&=(64G_F^2)^2(D\bar{D})^2(N\cdot l)(\bar{N}\cdot\bar{l})[(L\cdot\bar{v})(\bar{L}\cdot v)+m^2(w\cdot\bar{v})(w'\cdot v)+m^2(B\cdot\bar{v})(B\cdot v)+(k\cdot\bar{v})(k'\cdot v)]\ ,\\
(E^*_{1,-1}O_{1,1}+\text{H.c.})&=-2m\,(64G_F^2)^2(D\bar{D})^2(N\cdot l)(\bar{N}\cdot\bar{l})[(w\cdot\bar{v})(k'\cdot v)+(k\cdot\bar{v})(w'\cdot v)]\ ,\\
(E^*_{1,-1}O_{1,-1}+\text{H.c.})&=-2m\,(64G_F^2)^2(D\bar{D})^2(N\cdot l)(\bar{N}\cdot\bar{l})[(L\cdot\bar{v})(w'\cdot v)+(w\cdot\bar{v})(\bar{L}\cdot v)]\ ,\\
(O^*_{1,1}O_{1,-1}+\text{H.c.})&=2(64G_F^2)^2(D\bar{D})^2(N\cdot l)(\bar{N}\cdot\bar{l})[(L\cdot\bar{v})(k'\cdot v)+(k\cdot\bar{v})(\bar{L}\cdot v)]\ ,\\
(E^*_{1,-1}O_{1,1}-\text{H.c.})&=2im\,(64G_F^2)^2(D\bar{D})^2(N\cdot l)(\bar{N}\cdot\bar{l})[(L\cdot\bar{v})(B\cdot v)-(B\cdot\bar{v})(\bar{L}\cdot v)]\ ,\\
(E^*_{1,-1}O_{1,-1}-\text{H.c.})&=2im\,(64G_F^2)^2(D\bar{D})^2(N\cdot l)(\bar{N}\cdot\bar{l})[(k\cdot\bar{v})(B\cdot v)-(B\cdot\bar{v})(k'\cdot v)]\ ,\\
(O^*_{1,1}O_{1,-1}-\text{H.c.})&=2im^2(64G_F^2)^2(D\bar{D})^2(N\cdot l)(\bar{N}\cdot\bar{l})[(w\cdot\bar{v})(B\cdot v)-(B\cdot\bar{v})(w'\cdot v)]\ ,
\end{aligned}
\tag{19}
$$

where

$$D=m_W^2/[m_W^2-(l+\bar{v})^2]$$

and (20)

$$\bar{D}=m_W^2/[m_W^2-(\bar{l}+v)^2]\ .$$

The relations (19) can be derived easily by using the substitutions (2) and (3) and the fact that Y (and $\bar{Y}$) defined by (10) are

$$Y=128G_F^2D^2(N\cdot l)\bar{v}$$

and (21)

$$\bar{Y}=128G_F^2\bar{D}^2(\bar{N}\cdot\bar{l})v\ .$$

The covariant spin vector of the heavy lepton (L^-), w_μ, and that of its antiparticle (L^+), w'_μ, B_μ, k_μ, and k'_μ are given by (5), (6), and (7). In left-hand sides of (19) $(\overline{AB})$ indicates that the sum over the helicities of the decay products, N, l, $\bar{v}$, $\bar{N}$, $\bar{l}$, and v, is taken on AB.

The differential cross section of the $e^-+e^+\to\gamma,Z\to L^-+L^+\to(\nu_L+q+\bar{q}')+(\bar{\nu}_L+\bar{l}+\nu_l)$ process is obtained from (18) and (19) by replacing l by q and $\bar{v}$ by $\bar{q}'$ and by multiplying the right-hand sides of the relations (19) by the color factor 3.

III. THE DIFFERENTIAL CROSS SECTION NEAR THE THRESHOLD

In order to study the angular correlation of the charged leptons l^+ and l^- we have to substitute (19) into (18) and integrate over d^3N, $d^3\bar{v}$, $d^3\bar{N}$, d^3v, dz, and $d\phi$. Unfortunately it is not easy to carry out the integration analytically for $p\neq0$.

Therefore, in this paper we study the angular correlation of charged leptons near the threshold. Since the L^- and L^+ leptons are heavy, the cross section in this approximation is valid for wide energy range, $m<E\lesssim\frac{5}{4}m$.

We obtain the following angular correlations of the charged leptons near the threshold:

$$
\begin{aligned}
d\sigma=&F(\pi\alpha^2pm)(G_F^2m_W^2/16\pi^4m^2\Gamma_L)^2(d^3l/l_0^4)(d^3\bar{l}/\bar{l}_0^4)\\
&\times(\{[m_W^2(l_0^2+l_0l_3-ml_3)+m(m-2l_0)l_0(l_0+l_3)][2ml_0+(m_W^2-2ml_0)\ln(1-2ml_0/m_W^2)]\\
&\quad-2m^2l_0^2(l_0^2+l_0l_3-ml_3)\}\\
&\quad\times\{[m_W^2(\bar{l}_0^2-\bar{l}_0\bar{l}_3+m\bar{l}_3)+m(m-2\bar{l}_0)\bar{l}_0(\bar{l}_0-\bar{l}_3)][2m\bar{l}_0+(m_W^2-2m\bar{l}_0)\ln(1-2m\bar{l}_0/m_W^2)]\\
&\quad-2m^2\bar{l}_0^2(\bar{l}_0^2-\bar{l}_0\bar{l}_3+m\bar{l}_3)\}|\bar{V}_R|^2\\
&\quad+\{[m_W^2(l_0^2-l_0l_3+ml_3)+m(m-2l_0)l_0(l_0-l_3)][2ml_0+(m_W^2-2ml_0)\ln(1-2ml_0/m_W^2)]\\
&\quad-2m^2l_0^2(l_0^2-l_0l_3+ml_3)\}\\
&\quad\times\{[m_W^2(\bar{l}_0^2+\bar{l}_0\bar{l}_3-m\bar{l}_3)+m(m-2\bar{l}_0)\bar{l}_0(\bar{l}_0+\bar{l}_3)][2m\bar{l}_0+(m_W^2-2m\bar{l}_0)\ln(1-2m\bar{l}_0/m_W^2)]\\
&\quad-2m^2\bar{l}_0^2(\bar{l}_0^2+\bar{l}_0\bar{l}_3-m\bar{l}_3)\}|V_L|^2)+O(p^2)\ ,
\end{aligned}
\tag{22}
$$

where F is the factor due to the Coulomb interaction between L^- and L^+,

$$F=2\pi\alpha m/\{p[1-\exp(-2\pi\alpha m/p)]\}\ , \tag{23}$$

in the nonrelativistic approximation, and $\bar{V}_R$ and $\bar{V}_L$ are V_R and V_L given in (16) with $s=4m^2$. In deriving (22) we have neglected the masses of the produced leptons ν_L, l, and ν_l.

IV. DISCUSSIONS AND CONCLUSIONS

In this paper we have proposed a modified version of the helicity-projection technique in order to derive differential cross sections of productions of heavy fermions through the electroweak and/or strong interactions and their subsequent decays through the $V-A$ weak interaction with full decay correlations, and we have applied it to the $e^-+e^+\to\gamma,Z\to L^-+L^+$, $L^-\to\nu_L+l^-+\bar{\nu}_l$, $L^+\to\bar{\nu}_L+l^++\nu_l$ process.

An alternative expression of the squared matrix element for heavy-lepton pair production with semileptonic decays with full decay correlations has already been obtained by Barger, Han, and Ohnemus[2] by using the helicity-projection technique.[1] They have not calculated the squared matrix elements for polarized electron and positron beams and they have not calculated terms proportional to $M_Z\Gamma_Z/s|D_Z(Z)|^2$ in $\mathrm{Re}(M_Z^*M_\gamma)$, however.

Let us compare the original helicity-projection technique[1] with our modified version. By using the original helicity-projection technique, relatively compact expressions for the squared matrix elements, in which only four-momenta of the initial and final particles appear, are obtained, while the scattering angles of the production process θ and ϕ, and the covariant spin vectors of the heavy leptons w_μ and w'_μ, and the four-vectors k_μ, k'_μ, and B_μ, in addition to the four-momenta of the initial and final particles, appear in the squared matrix elements obtained by using the modified version proposed in this paper. The squared matrix elements obtained in both versions are valid only for L and $\bar{L}$ on mass shell.

On the other hand, the derivation of the squared matrix element for heavy lepton pair production with semileptonic decays with full decay correlations is easier in the modified version since the formula for the squared matrix element in the original version is more complicated than (8) and since the derivation of X and its analogues, which have to be substituted into the formula, is rather complicated when heavy fermions are produced through the electroweak and/or strong interaction and when initial beams are polarized.

If the depolarization of the produced charge $-\frac{1}{3}$ quark pair $(b'+\bar{b}')$ in the hadronization process is negligible, the differential cross section of the $e^-+e^+\to\gamma,Z\to b'+\bar{b}'\to(c+l^-+\bar{\nu}_l)+(\bar{c}+l^++\nu_l)$ process is obtained from (18) and (19) by multiplying (19) by the color factor 3 and by carrying out the replacements

$$L\to b',\quad \bar{L}\to\bar{b}',\quad N\to c,\quad \text{and}\quad \bar{N}\to\bar{c}\ . \tag{24}$$

We have also to replace $V_{R,L}$ given in (16) by

$$\begin{aligned} V_R&=1/3s+(x_W/3-\tfrac{1}{4})/(1-x_W)(s-m_Z^2+im_Z\Gamma_Z)\ ,\\ V_L&=1/3s+(x_W-\tfrac{1}{2})(x_W/3-\tfrac{1}{4})/x_W(1-x_W)(s-m_Z^2+im_Z\Gamma_Z)\ . \end{aligned} \tag{25}$$

The depolarization of the produced b' hadrons (which contain the b' quark) in the hadronization is expected to be negligible only if the mass difference of the pseudoscalar $b'\bar{u}$ meson and the vector $b'\bar{u}$ meson is smaller than their widths. Such an inequality cannot be expected to hold for a b' quark with $m_{b'}<m_W$, however.

In conclusion we have derived the differential cross section of the $e^-+e^+\to\gamma,Z\to L^-+L^+\to(\nu_L+l^-+\bar{\nu}_l)+(\bar{\nu}_L+l^++\nu_l)$ process by using a modified version of the helicity-projection technique, and found a prominent angular correlation of the charged leptons l^- and l^+ near the threshold. In this modified version it is also possible to include multistage cascade decays.

[1]V. Barger, J. Ohnemus, and R. J. N. Phillips, Phys. Rev. D **35**, 158 (1987); **35**, 166 (1987).

[2]V. Barger, T. Han, and J. Ohnemus, Phys. Rev. D **37**, 1174 (1988). See, also, H. Baer, V. Barger, and R. J. N. Phillips, *ibid.* **32**, 688 (1985).