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# Brief Reports

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## Fermions on the field of $p$-adic numbers

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Based on a superfield approach we propose a fermionic action valued on a $p$-adic field. The supersymmetric transformations that leave the superaction invariant are also given.

### I. INTRODUCTION

It has been almost a decade since $p$-adic numbers (see, for example, Ref. 1) made its appearance in the physics literature,[2] in the context of the mean field theory of spin glasses. More recently, the formulation of the $p$-adic string[3] and its relation to the Veneziano amplitude through an adelic formula[4] opened the possibility that number theory can also give us a different insight into string theory.

The formulation of $p$-adic bosonic free field theory,[5] and in particular the $p$-adic string action,[6] give us an understanding of $p$-adic string amplitudes in terms of correlation functions. An insight on the geometry of this field theory has recently been given.[7] A first step to the understanding of the interacting theory was given by defining the $p$-adic version of the O($N$) nonlinear $\sigma$ model,[8] and it was shown to possess similar properties to the two-dimensional one on real space. This program was extended by considering a $p$-adic $\sigma$ model on a tachyonic background,[9] with the result that the requirement of discrete scale invariance at the quantum level reproduces the equation of motion of the space-time effective field theory previously studied in Ref. 10. In addition there have been some developments that include the formulation of $p$-adic quantum mechanics,[11,12] $p$-adic conformal field theory,[13] and a possible mechanism to break supersymmetry in the superstring nonperturbatively by using $p$-adic fields.[14] All these approaches involve an underlying bosonic field theory on $p$-adic variables.

In this paper, our purpose is to formulate a fermionic action on a $p$-adic field $Q_p$ and the supersymmetric transformations that leaves the superaction invariant. Section II contains the general formalism for the bosonic and fermionic case and Sec. III deals with the superfield approach.

### II. GENERAL FORMALISM

The four-point amplitude of the $p$-adic string is given [up to an SL(2,$Q_p$) volume factor] by

$$A_4=\int_{Q_p}\prod_{i=1}^{4}dx_i\prod_{i<j}|x_i-x_j|_p^{k_i\cdot k_j}\ , \tag{1}$$

where integration is over the $p$-adic field $Q_p$ and $|x|_p$ denotes the $p$-adic norm. Two of the interesting features of $A_4$ is that it satisfies an adelic formula with the Veneziano amplitude[4] and its corresponding field theory is nonlocal. In fact, the correlation function

$$\left\langle\int_{Q_p}\prod_{i=1}^{4}dx_i\exp(k_i\cdot\phi_i)\right\rangle \tag{2}$$

will reproduce Eq. (1) provided that the two-point function is given by[6]

$$G(x,y)=-\ln|x-y|_p \tag{3}$$

giving a nonlocal bosonic action

$$S_B=-\tfrac{1}{2}\int_{Q_p}dx\ \phi(x)[K_B\phi](x)\ , \tag{4}$$

where the kinetic operator $K_B$ is given by

$$[K_B\phi](x)=\frac{cp^2}{p+1}\int_{Q_p}dx'\frac{\phi(x')}{|x-x'|_p^2} \tag{5}$$

with $c=(1-p^{-1})/\ln p$. Operators of this type have been studied in Ref. 12, in particular (up to a constant) $K_B$ was identified as an analog of the first derivative operator.

The Fourier transform of (5) is simple to perform, with the result

$$S_B=c\int_{Q_p}du\ \tilde{\phi}(-u)|u|_p\tilde{\phi}(u) \tag{6}$$

with the corresponding propagator $\Pi(u)=c^{-1}|u|_p^{-1}$.

In order to formulate a fermionic action we introduce the kinetic operator

$$[K_F^{(\gamma)}\Psi](x)=-A\int_{Q_p}dx'\frac{\operatorname{sgn}_p(x-x')\Psi(x')}{|x-x'|_p^{\gamma}}\ , \tag{7}$$

where $A$ and $\gamma$ are still undetermined constants, and the multiplicative character [the use of $\operatorname{sgn}_p(x)$ for a fermionic action was suggested in Ref. 7; however, we find a disagreement on the value of $\gamma$ as discussed in Sec. III] $\mathrm{sgn}_\tau x$ corresponding to the three quadratic extensions $Q_p(\sqrt{\tau})$, $\tau=\epsilon,\epsilon p,p$, satisfies

$$\mathrm{sgn}_\tau z=\begin{cases}1 & \text{if } z=x^2-\tau y^2 \text{ for } x,y\in Q_p ,\\ -1 & \text{otherwise}\end{cases} \tag{8}$$

and

$$\mathrm{sgn}_\tau(-x)=-\mathrm{sgn}_\tau(x) \quad \text{for } \tau=p,p\epsilon \text{ and } p=3(\mathrm{mod}4) . \tag{9}$$

Since the two cases $\tau=p,p\epsilon$ differ only by normalizations we will work explicitly with only one of them ($\tau=p$). The fermionic action $S_F$ is then

$$S_F=-\tfrac{1}{2}\int_{Q_p} dx\, \Psi(x)[K_F^{(\gamma)}\Psi](x) . \tag{10}$$

In order to obtain $S_F$ in "momentum" space we need to apply the Fourier transformation of the multiplicative character $\pi(x)=|x|_p^\alpha \mathrm{sgn}_p x$ given by

$$\int_{Q_p}|x|_p^\alpha \mathrm{sgn}_p x\, \exp(2\pi i u x)=\Gamma(\pi,\alpha+1)\frac{\mathrm{sgn}_p u}{|u|_p^{\alpha+1}} , \tag{11}$$

where the gamma function $\Gamma(\pi,\alpha)$ is defined[15] by

$$\Gamma(\pi,\alpha)=\int_{Q_p}dx|x|_p^{\alpha-1}\mathrm{sgn}_p x\, \exp(2\pi i x) = \pm i p^{\alpha-1/2} \quad \text{for } p=3(\mathrm{mod}4) . \tag{12}$$

A simple calculation gives

$$S_F=\frac{1}{2}\frac{A}{\Gamma(\pi,\gamma)}\int_{Q_p}du\, \tilde{\Psi}(-u)|u|_p^{\gamma-1}\mathrm{sgn}_p u \tilde{\Psi}(u) . \tag{13}$$

We see that the choice of $\gamma=\frac{3}{2}$ makes it possible to identify the fermionic Lagrangian as a kind of square root of the bosonic one. In what follows we will show that this choice is the only one consistent if we define supersymmetric transformations and a superfield in a way that resembles the ordinary supersymmetric case.

### III. *p*-adic SUPERSYMMETRY

Let us start by defining the superaction $S$ as

$$S=\tfrac{1}{2}\int_{Q_p}dx\int d\theta\, D^2\Phi\, D\Phi , \tag{14}$$

where $\Phi(x,\theta)$ is the superfield

$$\Phi(x,\theta)=\phi(x)+\theta\Psi(x) \tag{15}$$

and the superderivative $D$ is defined by

$$[D\Phi](x,\theta)=\theta[K_F^{(\gamma)}\Phi](x)+\frac{\partial}{\partial\theta}\Phi(x,\theta) \tag{16}$$

such that Eq. (14) explicitly reads

$$S=\tfrac{1}{2}\int_{Q_p}dx\int d\theta\{\theta[K_F^{(\gamma)}\phi](x)[K_F^{(\gamma)}\phi](x) - \theta\Psi(x)[K_F^{(\gamma)}\Psi](x)\} . \tag{17}$$

By use of Eq. (11) and the definition of the Gel'fand gamma function $\Gamma_p(\alpha)$,

$$\Gamma_p(\alpha)=\int_{Q_p}dx|x|_p^{\alpha-1}\exp(-2\pi i x) , \tag{18}$$

it can be shown that the first term gives the bosonic action $S_B$ provided that $\gamma=\frac{3}{2}$ and the constant $A=\sqrt{c}p$. Therefore, the superfield formalism fixes the fermionic action, and the full action in component fields reads

$$S=-\frac{cp^2}{2(p+1)}\int_{Q_p}dx\, dx'\frac{\phi(x)\phi(x')}{|x-x'|_p^2} +\frac{\sqrt{c}p}{2}\int_{Q_p}dx\, dx'\frac{\Psi(x)\mathrm{sgn}_p(x-x')\Psi(x')}{|x-x'|_p^{3/2}} \tag{19}$$

with the corresponding fermionic propagator

$$\Pi_F(u)=\frac{i\,\mathrm{sgn}_p u}{\sqrt{c|u|_p}} . \tag{20}$$

The action is invariant under the transformations

$$\delta\phi(x)=\Psi(x)\xi , \quad \delta\Psi(x)=-[K_F^{(3/2)}\phi](x)\xi, \quad \{\Psi,\xi\}=0 , \tag{21}$$

where we see that $\delta\phi$ is a local transformation but $\delta\Psi$ involves a nonlocal operator that replaces the ordinary derivative of the standard case. The commutator of two such transformations gives

$$[\delta_1,\delta_2]\phi(x)=-2\xi_1\xi_2[K_F^{(3/2)}\phi](x) . \tag{22}$$

The action $S$ is also invariant under transformations $\delta\phi$ and $\delta\Psi$ involving the nonlocal operators $K_F^{(3/2)}$ and $K_B$, respectively, but the algebra would not close either in the operator $K_F^{(3/2)}$ or $K_B$.

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[15]See, for example, Chap. 2 of the second reference in Ref. 1.