Mass versus superconducting-gap generation: The operator formulation for $N=2$

L. V. Belvedere
Institut für Theoretische Physik, der Universität Heidelberg, Philosophenweg 16, D-6900 Heidelberg, West Germany
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We consider the operator realization of dynamical mass versus superconducting-gap generation in a two-dimensional O(2)-symmetric model. We show the existence of two phases which are characterized by a topologically conserved charge. The equivalence to the massive/superconducting Thirring model is established.

I. INTRODUCTION

In recent years, the discovery of many applications of two-dimensional quantum-field-theory models in realistic systems of condensed-matter physics showed that these theories were more than just theoretical laboratories, but genuine theories capable of making experimental predictions.[1] Thus, the recent and increasing role played by two-dimensional field theories in the physics of condensed matter and the fact that many of them may be exactly solved[2] enhance the interest in the study of models which are potentially relevant from the application point of view.

Following this philosophy, in the present paper we will consider the O(N)-symmetric model which was introduced in Ref. 3, and is defined by the Lagrangian density

$$\mathcal{L}(x)=\sum_{a=1}^{N} i\bar{\psi}_a(x)\gamma_\mu\partial^\mu\psi_a(x) +\sum_{a,b}^{N}[g\psi_a(x)_{(1)}\psi_a(x)_{(2)}\psi_b^*(x)_{(2)}\psi_b^*(x)_{(1)} +\tilde{g}\psi_a^*(x)_{(1)}\psi_a(x)_{(2)}\psi_b^*(x)_{(2)}\psi_b(x)_{(1)}] , \tag{1.1}$$

where (1) and (2) are spinor indices. In addition to the O(N) symmetry, the model (1.1) exhibits U(1) and chiral-U(1) invariance.

For $g=\tilde{g}$, the model (1.1) reduces to the model studied in the Ref. 4 and can be written as

$$\mathcal{L}(x)=\sum_{a=1}^{N} i\bar{\psi}_a(x)\gamma_\mu\partial^\mu\psi_a(x)+\frac{g}{8}\sum_{a,b}^{N}J(x)_{\mu ab}J(x)^\mu_{ab} \tag{1.2}$$

with the O(N) currents given by

$$J(x)_{\mu ab}=\bar{\psi}_a(x)\gamma_\mu\psi_b(x)-\bar{\psi}_b(x)\gamma_\mu\psi_a(x) . \tag{1.3}$$

The Lagrangian density (1.2) is invariant under the chiral conjugation $\mathbb{C}_5$, which transforms charge into chirality and is defined by

$$\mathbb{C}_5:\begin{cases}\psi_a(x)_{(1)}\to\psi_a^*(x)_{(1)},\\ \psi_a(x)_{(2)}\to\psi_a(x)_{(2)}.\end{cases} \tag{1.4}$$

When $\tilde{g}=0$, it was shown that in the large-N limit the model given by (1.1) exhibits dynamical generation of a superconducting gap.[5] It was conjectured that this mechanism would occur for all values of N, as in the analogous case of mass generation in the chiral Gross-Neveu model ($g=0$) (Ref. 6).

When $g\neq 0$ and $\tilde{g}\neq 0$, the model given by (1.1) exhibits in the large-N limit the dynamical mass versus superconducting-gap generation mechanism.[3] For $g=\tilde{g}$ and large N, the chiral conjugation symmetry is dynamically broken.[3,4] In this paper we will consider the case $N=2$. When $g=\tilde{g}$, the model given by (1.1) is O(2)×chiral O(2) invariant.

When $\tilde{g}=0$, it was proved in Ref. 5 that the equivalence of the O(2)-symmetric model given by (1.1) to the superconducting Thirring model at $g_{\mathrm{Th}}=-\pi/2$ $(\beta^2=8\pi)$ (Ref. 7).

Via an operator formulation, we will show the existence of two phases: one presenting dynamical mass generation and another presenting dynamical superconducting-gap generation. The phases are characterized by a topologically conserved charge and the two mechanisms being competitive for duality reasons. In the massive phase ($\tilde{g}>g$), the O(2) charge appears as the U(1) charge of the massive Thirring model.[8,9] In the superconducting phase ($g>\tilde{g}$) the O(2) charge appears as the chiral U(1) charge of the superconducting Thirring model.[7] When $g=\tilde{g}$, the model is free and gapless. As a consequence, for $N=2$, chiral conjugation symmetry is not dynamically broken.

II. THE O(2) BOSONIZATION SCHEME

To begin with, consider the O(2) transformations acting on the field ψ_a:

$$\begin{aligned}\psi_1'(x)&=\psi_1(x)\cos\Omega+\psi_2(x)\sin\Omega ,\\ \psi_2'(x)&=-\psi_1(x)\sin\Omega+\psi_2(x)\cos\Omega\end{aligned} \tag{2.1}$$

which can also be written as

$$\begin{aligned}[\psi_1'(x)+i\psi_2'(x)]&=e^{i\Omega}[\psi_1(x)+i\psi_2(x)] ,\\ [\psi_1'(x)-i\psi_2'(x)]&=e^{-i\Omega}[\psi_1(x)-i\psi_2(x)] ,\end{aligned} \tag{2.2}$$

where 1 and 2 are O(2) indices. Following Ref. 5, in order to obtain fields transforming with irreducible representation of O(2), we introduce the two independent fields $\chi(x)$ and $\Theta(x)$ as

$$\chi(x)=\frac{1}{\sqrt{2}}[\psi_1(x)+i\psi_2(x)]\ ,$$
$$\Theta(x)=\frac{1}{\sqrt{2}}[\psi_1(x)-i\psi_2(x)]\ . \tag{2.3}$$

The original O(2) transformations given by (2.1) acts on the χ and Θ fields as gauge transformations of the first kind:

$$\chi'(x)=e^{i\Omega}\chi(x),\quad \Theta'(x)=e^{-i\Omega}\Theta(x)\ . \tag{2.4}$$

From (2.4) we see that the χ and Θ fields have opposite Abelian (O(2)) charges and transforms irreducibly under O(2).

In terms of these new variables, the conserved currents corresponding to the U(1), chiral U(1), and O(2) symmetries are given by

$$J(x)_\mu=\sum_{a=1}^{2}\bar{\psi}_a(x)\gamma_\mu\psi_a(x)$$
$$=\bar{\chi}(x)\gamma_\mu\chi(x)+\bar{\Theta}(x)\gamma_\mu\Theta(x)\ , \tag{2.5}$$

$$J^5(x)_\mu=\epsilon_{\mu\nu}J(x)^\nu\ , \tag{2.6}$$

$$\tilde{J}(x)\mu=\bar{\psi}_1(x)\gamma_\mu\psi_2(x)-\bar{\psi}_2(x)\gamma_\mu\psi_1(x)$$
$$=\bar{\chi}(x)\gamma_\mu\chi(x)-\bar{\Theta}(x)\gamma_\mu\Theta(x)\ . \tag{2.7}$$

Following Ref. 3, we introduce the mass (π) and superconducting (σ) gap operators by

$$\pi(x)=\sum_{a=1}^{2}\psi_a^*(x)_{(1)}\psi_a(x)_{(2)}\ , \tag{2.8}$$

$$\sigma(x)=\sum_{a=1}^{2}\psi_a(x)_{(1)}\psi_a(x)_{(2)}\ . \tag{2.9}$$

In terms of the fields χ and Θ, these operators can be written as

$$\pi(x)=\chi^*(x)_{(1)}\chi(x)_{(2)}+\Theta^*(x)_{(1)}\Theta(x)_{(2)}\ , \tag{2.10}$$

$$\sigma(x)=\chi(x)_{(1)}\Theta(x)_{(2)}+\Theta(x)_{(1)}\chi(x)_{(2)}\ . \tag{2.11}$$

The chiral conjugation symmetry is realized by

$$\mathbb{C}_5{:}\chi_{(1)}\to\Theta^*_{(1)};\chi_{(2)}\to\chi_{(2)};\Theta_{(2)}\to\Theta_{(2)}\ ; \tag{2.12}$$

that is,

$$\sigma(x)\leftrightarrow\pi(x)\ . \tag{2.13}$$

Because of the "Abelian" structure of the conserved currents (2.5)–(2.7), we can introduce the Mandelstam representation for the χ and Θ fields.[10] In order to ensure the correct anticommutation relations between χ and Θ, Klein factors are also introduced:

$$\chi(x)=\left[\frac{m}{2\pi}\right]^{1/2}\mathbb{K}(\phi_2){:}e^{i\sqrt{\pi}[\gamma^5\phi_1(x)+\eta_1(x)]}{:}\ , \tag{2.14}$$

$$\Theta(x)=\left[\frac{m}{2\pi}\right]^{1/2}\mathbb{K}(\phi_1){:}e^{i\sqrt{\pi}[\gamma^5\phi_2(x)+\eta_2(x)]}{:}\ , \tag{2.15}$$

where m is an arbitrary finite mass scale,

$$\eta_a(x)=\int_{x_1}^{\infty}\partial_0\phi_a(x^0,z^1)dz^1\ . \tag{2.16}$$

The Klein factors are defined by

$$\mathbb{K}_{(\alpha)}(\phi_a)=\exp\left[-i\frac{\pi}{4}\gamma^5_{\alpha\alpha}\right]{:}\exp\left[i\frac{\sqrt{\pi}}{2}K_a\right]{:}\ , \tag{2.17}$$

where

$$K_a=q_a\int_{-\infty}^{+\infty}\partial_1\phi_a(z)dz^1 \tag{2.18}$$

with $q_a=+1\ (-1)$ for $a=1\ (2)$, and

$$[\eta_a(x),K_b]=i\delta_{ab}\ . \tag{2.19}$$

Introducing the two independent scalar fields

$$\phi=\frac{1}{\sqrt{2}}(\phi_1+\phi_2)\ , \tag{2.20}$$

$$\hat{\phi}=\frac{1}{\sqrt{2}}(\phi_1-\phi_2) \tag{2.21}$$

which satisfy canonical commutation relations, and using the point-splitting prescription, we obtain the conserved currents as

$$J(x)_\mu=-\left[\frac{2}{\pi}\right]^{1/2}\epsilon_{\mu\nu}\partial^\nu\phi(x)\ , \tag{2.22}$$

$$J^5(x)_\mu=-\left[\frac{2}{\pi}\right]^{1/2}\partial_\mu\phi(x)\ , \tag{2.23}$$

$$\tilde{J}(x)_\mu=-\left[\frac{2}{\pi}\right]^{1/2}\epsilon_{\mu\nu}\partial^\nu\hat{\phi}(x)\ . \tag{2.24}$$

As a consequence of the continuous U(1) and chiral U(1) symmetries, $\phi(x)$ is a free massless field. From Eqs. (2.22) and (2.24) we see that the conserved current introduced in the Klein factors (2.18),

$$K(x)_{\mu a}=\epsilon_{\mu\nu}\partial^\nu\phi_a(x)\ , \tag{2.25}$$

corresponds to the combinated U(1) and O(2) transformations:

$$K(x)_{\mu a}=J(x)_\mu+q_a\tilde{J}(x)_\mu\ . \tag{2.26}$$

Writing the fermion fields χ and Θ in terms of the field ϕ and $\hat{\phi}$, they factorize as

$$\chi(x)=\mathbb{K}\psi_0(x)\tilde{\mathbb{K}}\hat{\chi}(x)\ , \tag{2.27}$$

$$\Theta(x)=\mathbb{K}\psi_0(x)\tilde{\mathbb{K}}\hat{\Theta}(x)\ , \tag{2.28}$$

where

$$\hat{\Theta}(x)=\hat{\chi}^*(x)\ . \tag{2.29}$$

$\psi_0(x)$ is a noncanonical free massless fermion field with spin $\frac{1}{4}$:

$$\psi_0(x)={:}e^{i\sqrt{\pi/2}[\gamma^5\phi(x)+\eta(x)]}{:} \tag{2.30}$$

with

$$\epsilon_{\mu\nu}\partial\nu\phi(x)=\partial_\mu\eta(x) \tag{2.31}$$

and

$$\hat{\chi}(x)=\left[\frac{m}{2\pi}\right]^{1/2}:e^{i\sqrt{\pi/2}[\gamma^5\hat{\phi}(x)+\hat{\eta}(x)]}: \tag{2.32}$$

which also carries Lorentz spin $\frac{1}{4}$. The factorized Klein factors are given by

$$\mathbb{K}=:e^{i\sqrt{\pi/8}Q}: , \tag{2.33}$$

$$\tilde{\mathbb{K}}=\exp\left[-i\frac{\pi}{4}\gamma^5\right]:\exp(i\sqrt{\pi/8}\tilde{Q}): , \tag{2.34}$$

where Q and $\tilde{Q}$ are, respectively, the U(1) and O(2) conserved charges.

The chiral conjugation symmetry is now given by

$$\mathbb{C}_5=\mathbb{C}_5^0\times\tilde{\mathbb{C}}_5 . \tag{2.35}$$

The conjugation $\mathbb{C}_5^0$ corresponds to the free chiral conjugation symmetry and is given by

$$\mathbb{C}_5^0:\begin{cases}\psi_0(x)_{(1)}\rightarrow\psi_0^*(x)_{(1)},\\ \psi_0(x)_{(2)}\rightarrow\psi_0(x)_{(2)}\end{cases} \tag{2.36}$$

which means that

$$\phi(x)\leftrightarrow\eta(x) . \tag{2.37}$$

The chiral conjugation $\tilde{\mathbb{C}}_5$ leaves $\hat{\chi}$ invariant and acts only on the operators $\tilde{\mathbb{K}}$ as

$$\tilde{\mathbb{C}}_5:\begin{cases}\tilde{\mathbb{K}}_{(1)}\rightarrow\tilde{\mathbb{K}}^*_{(1)},\\ \tilde{\mathbb{K}}_{(2)}\rightarrow\tilde{\mathbb{K}}_{(2)}.\end{cases} \tag{2.38}$$

The mass and superconducting-gap operators (2.10) and (2.11) are obtained by the point-splitting limit procedure and are given by

$$\pi(x)=\pi_0(x):\cos[\sqrt{2\pi}\hat{\phi}(x)]: , \tag{2.39}$$

$$\sigma(x)=\sigma_0(x):e^{i\sqrt{\pi/2}\tilde{Q}}::\sin[\sqrt{2\pi}\hat{\phi}(x)]: , \tag{2.40}$$

where $\sigma_0(x)$ and $\pi_0(x)$ carry the free U(1) and chiral U(1) charges and are given by

$$\sigma_0(x)=:e^{i\sqrt{2\pi}\eta(x)}: , \tag{2.41}$$

$$\pi_0(x)=:e^{i\sqrt{2\pi}\phi(x)}: . \tag{2.42}$$

In terms of the σ and π operators, the Lagrangian density (1.1) can be written as

$$\mathcal{L}(x)=\sum_{a=1}^{2}i\bar{\psi}_a(x)\gamma_\mu\partial^\mu\psi_a(x)+g:\sigma(x)\sigma^*(x):+\tilde{g}:\pi(x)\pi^*(x): . \tag{2.43}$$

Using (2.39) and (2.40), we obtain by the point-splitting limit prescription

$$:|\sigma(x)|^2:=\frac{1}{\pi}:[\partial_\mu\phi(x)]^2:+\frac{1}{\pi}:[\partial_\mu\hat{\phi}(x)]^2:-(m/\pi)^2:\cos[\sqrt{8\pi}\hat{\phi}(x)]: , \tag{2.44}$$

$$:|\pi(x)|^2:=\frac{1}{\pi}:[\partial_\mu\phi(x)]^2:+\frac{1}{\pi}:[\partial_\mu\hat{\phi}(x)]^2:+(m/\pi)^2:\cos[\sqrt{8\pi}\hat{\phi}(x)]: \tag{2.45}$$

The bosonized Lagrangian density is given by

$$\alpha\mathcal{L}(x)=L(x)+\hat{L}(x) , \tag{2.46}$$

where

$$\alpha=\left[1+\frac{2}{\pi}(g+\tilde{g})\right]^{-1} , \tag{2.47}$$

$$L(x)=\tfrac{1}{2}:[\partial_\mu\phi(x)]^2: \tag{2.48}$$

and

$$\hat{L}(x)=\tfrac{1}{2}:[\partial_\mu\hat{\phi}(x)]^2:+\frac{m^2}{2\pi}G:\cos[\sqrt{8\pi}\hat{\phi}(x)]: \tag{2.49}$$

with the effective coupling constant G given by

$$G=\frac{\tilde{g}-g}{\pi/2+g+\tilde{g}} . \tag{2.50}$$

When $g\neq\tilde{g}$, the physical content of the model is given by a sine-Gordon theory with $\beta^2=8\pi$. When $g=\tilde{g}$, the model is invariant under the chiral conjugation (1.4) and is free and gapless ($G=0$). It corresponds to the massless Thirring model according to Eqs. (1.2) and (1.24).

III. VACUUM STRUCTURE AND CONSERVED CHARGES

The mass operator $\pi_a(x)$ breaks the chiral $[\mathrm{U}(1)]^N$ symmetry and the superconducting gap operator $\sigma_a(x)$ breaks the $[\mathrm{U}(1)]^N$ invariance. As a consequence, the currents

$$J(x)_{\mu a}=\bar{\psi}_a(x)\gamma_\mu\psi_a(x) , \tag{3.1}$$

$$J^5(x)_{\mu a}=\bar{\psi}_a(x)\gamma_\mu\gamma^5\psi_a(x) \tag{3.2}$$

are not conserved at the classical level and one has

$$\partial_\mu J(x)_a^\mu=g\sum_{b\neq a}[\sigma_a(x)\sigma_b^*(x)-\sigma_b(x)\sigma_a^*(x)] , \tag{3.3}$$

$$\partial_\mu J^5(x)_a^\mu=\tilde{g}\sum_{b\neq a}[\pi_a(x)\pi_b^*(x)-\pi_b(x)\pi_a^*(x)] . \tag{3.4}$$

In terms of the χ and Θ fields, the currents (3.1) and (3.2) are given by

$$J(x)_{\mu a}=J(x)_\mu+q_a\hat{J}(x)_\mu , \tag{3.5}$$

$$J^5(x)_{\mu a}=\epsilon_{\mu\nu}J(x)_a^\nu , \tag{3.6}$$

where

$$\hat{J}(x)_\mu=\bar{\chi}(x)\gamma_\mu\Theta(x)+\bar{\Theta}(x)\gamma_\mu\chi(x) . \tag{3.7}$$

The divergence of the currents $J_{\mu a}$ and $J^5_{\mu a}$ are given by

$$\partial_\mu\hat{J}(x)^\mu=\partial_-\hat{J}(x)_+-\partial_+\hat{J}(x)_- , \tag{3.8}$$

$$\partial_\mu\hat{J}^5(x)^\mu=-\partial_-\hat{J}(x)_+-\partial_+\hat{J}(x)_- \tag{3.9}$$

with

$$\hat{J}(x)_{\pm}=\frac{m}{\pi}:\sin\sqrt{2\pi}(\hat{\phi}(x)\mp\hat{\eta}(x)): . \tag{3.10}$$

When $g=\tilde{g}$, both currents are conserved. This follows from the fact that

$$\partial_{\mp}(\hat{\phi}(x)\mp\hat{\eta}(x))=\mp\int_{x_1}^{\infty}\Box\hat{\phi}(n_0,z_1)dz^1 \tag{3.11}$$

and in this case the field $\hat{\phi}$ is free and massless. Thus, for $N=2$, the chiral conjugation symmetry is not dynamically broken and both currents $J_{\mu a}$ and $J^5_{\mu a}$ are conserved at the quantum level.

When $g\neq\tilde{g}$, some care must be taken in considering the divergences (3.8) and (3.9). In order to be able to use the differentation formulas for exponentials of "bona fide" operators, we use the methods given in the Refs. 11 and 12 and obtain

$$\partial_{\mp}\hat{J}(x)_{\pm}=\int_{-\infty}^{+\infty}[:\cos\sqrt{8\pi}\hat{\phi}(z):,:\sin\sqrt{8\pi}(\hat{\phi}(x)\mp\hat{\eta}(x)):]_{x_0=z_0}dz^1 . \tag{3.12}$$

The nonzero contributions to the commutator (3.12) come from the neighborhood of the point $z_1=x_1$ (Refs. 11 and 12). Using the equal-time commutators

$$[\hat{\phi}^{(+)}(x),\hat{\phi}^{(-)}(z)]=[\hat{\eta}^{(+)}(x),\hat{\eta}^{(-)}(z)]=-\frac{1}{4\pi}[\ln m(x_1-z_1+i\epsilon)+\ln m(x_1-z_1-i\epsilon)] , \tag{3.13}$$

$$[\hat{\eta}^{(+)}(x),\hat{\phi}^{(-)}(z)]=\frac{1}{4\pi}[\ln m(x_1-z_1-i\epsilon)-\ln m(x_1-z_1+i\epsilon)] , \tag{3.14}$$

we obtain, from (3.2),

$$\partial_\mu\hat{J}(x)^\mu=\frac{G}{2\pi}\partial_1\hat{J}(n)^1 , \tag{3.15}$$

$$\partial_\mu\hat{J}^5(x)^\mu=\frac{G}{2\pi}\partial_1\hat{J}^5(x)^1 . \tag{3.16}$$

Integrating over x^1 and using the fact that

$$\hat{\eta}(x_0,x_1=+\infty)\equiv 0 \tag{3.17}$$

we obtain the time derivative of the corresponding charges as

$$\dot{\hat{Q}}=c[2:\sin\sqrt{2\pi}\hat{\phi}(x_0,+\infty):-:\sin\sqrt{2\pi}(\tilde{\phi}(x_0,-\infty)-\hat{\eta}(x_0,-\infty)):-:\sin\sqrt{2\pi}(\hat{\phi}(x_0,-\infty)+\hat{\eta}(x_0,-\infty)):] , \tag{3.18}$$

$$\dot{\hat{Q}}^5=c[-:\sin\sqrt{2\pi}(\hat{\phi}(x_0,-\infty)-\hat{\eta}(x_0,-\infty)):+:\sin\sqrt{2\pi}(\hat{\phi}(x_0,-\infty)+\hat{\eta}(x_0,-\infty)):] , \tag{3.19}$$

where

$$\hat{\eta}(x_0,-\infty)=\int_{-\infty}^{+\infty}\partial_0\hat{\phi}(x^0,z^1)dz^1 . \tag{3.20}$$

Considering finite-energy solutions, the values of $\hat{\phi}(x_0,x_1=\pm\infty)$ must correspond to the vacuum configurations. The physical potential is given by

$$U[\hat{\phi}]=-G\frac{m^2}{2\pi}:\cos[\sqrt{8\pi}\hat{\phi}(x)]: . \tag{3.21}$$

When $\tilde{g}>g$, the model given by (1.1) exhibits dynamical mass generation. In the massive phase the minima of the potential (3.21) are given by the constant configurations

$$\langle\hat{\phi}\rangle_M=n\sqrt{\pi/2},\quad n=0,\pm1\pm2,\ldots \tag{3.22}$$

and we obtain, from (3.18) and (3.19),

$$\dot{\hat{Q}}=0 , \tag{3.23}$$

$$\dot{\hat{Q}}^5=2c:\sin[\sqrt{2\pi}\hat{\eta}(x_0,-\infty)]: . \tag{3.24}$$

In the limit when $g\to 0$, we obtain the O(2) sector of the pure chiral Gross-Neveu model.[6]

When $g>\tilde{g}$, the model presents dynamical generation of a superconducting gap. In this phase, the minima of the potential are given by

$$\langle\hat{\phi}\rangle_{sc}=\frac{2n+1}{2}\left[\frac{\pi}{2}\right]^{1/2},\quad n=0\pm1,\pm2,\ldots \tag{3.25}$$

and we find

$$\dot{\hat{Q}}=2c\{1+:\cos[\sqrt{2\pi}\hat{\eta}(x_0,-\infty)]:\} , \tag{3.26}$$

$$\dot{\hat{Q}}^5=0 . \tag{3.27}$$

In the limit when $\tilde{g}\to 0$, we obtain the O(2) model discussed in Ref. 5.

The decoupling of U(1) and chiral U(1) degrees of freedom implies that the physical content of the model must be solely in the interacting fields $\hat{\chi}$ and $\hat{\Theta}$, which satisfy neither Fermi or Bose statistics, but have Lorentz spin $\frac{1}{4}$. These two interacting objects are not independent, according to (2.29), which is the analogue for $N=2$ of the larger-N feature of antiparticles being bound states of $N-1$ particles in similar systems.[13] As long as the physical content of the model can be described in terms of just one of those objects, say, $\hat{\chi}$, the two currents $\hat{J}_{\mu 1}$ and $\hat{J}_{\mu 2}$ are also not independent:

$$\hat{J}(x)_{\mu a}=q_a\hat{J}(x)_\mu . \tag{3.28}$$

The conservation of the charges $\hat{Q}$ and $\hat{Q}^5$ emerges as a quantum effect obeying the topology of the dual (competitive) vacuum structure generated by the dynamical mass versus superconducting-gap generation mechanism.

The massive and the superconducting phases can be described in terms of a spin-$\frac{1}{2}$ Thirring field. In terms of the field $\hat{\chi}$, the interacting piece of the Lagrangian density can be written as

$$\hat{\mathcal{L}}(x)=i:\bar{\hat{\chi}}(x)\gamma_\mu\partial^\mu\hat{\chi}(x):+\frac{G}{2}[:\hat{\chi}^*(x)_{(1)}\hat{\chi}(x)_{(2)}\hat{\chi}(x)_{(2)}\hat{\chi}^*(x)_{(1)}:+\text{H.c.}]\;. \tag{3.29}$$

In the massive phase we can make the identification

$$\hat{\chi}(x)_{(1)}=:\psi(x)_{(1)}e^{i\sqrt{\pi/2}\hat{\phi}(x)}:\;, \tag{3.30}$$

$$\hat{\chi}(x)_{(2)}=:\psi(x)_{(2)}e^{-i\sqrt{\pi/2}\hat{\phi}(x)}:\;, \tag{3.31}$$

where the spin-$\frac{1}{2}$ fermion field operator $\psi(x)$ is given by

$$\psi(x)=\left[\frac{m}{2\pi}\right]^{1/2}\exp\left[-i\frac{\pi}{4}\gamma^5\right]:\exp\{i[\sqrt{2\pi}\gamma^5\hat{\phi}(x)+\sqrt{\pi/2}\hat{\eta}(x)]\}:\;. \tag{3.32}$$

Thus, we may relate the O(2) current given by Eq. (2.24) with U(1) current for $\psi(x)$,

$$\tilde{J}(x)_\mu=-\sqrt{2/\pi}\epsilon_{\mu\nu}\partial^\nu\hat{\phi}(x)\equiv:\bar{\psi}(x)\gamma_\mu\psi(x):\;. \tag{3.33}$$

Inserting (3.32) into (3.29), we may express the physical Lagrangian density in the massive phase in terms of ψ as

$$\begin{aligned}\hat{\mathcal{L}}(x)=&\,i:\bar{\psi}(x)\gamma_\mu\partial^\mu\psi(x):+\frac{m}{\pi}G:\bar{\psi}(x)\psi(x):\\&+g_{\text{Th}}:[\bar{\psi}(x)\gamma_\mu\psi(x)][\bar{\psi}(x)\gamma^\mu\psi(x)]:\end{aligned} \tag{3.34}$$

which corresponds to the massive Thirring model with coupling constant $g_{\text{Th}}=-(\pi/2)(\beta^2=8\pi)$ (Refs. 10 and 14).

In the superconducting phase we identify

$$\hat{\chi}^*(x)_{(1)}=:\Psi(x)_{(1)}e^{-i\sqrt{\pi/2}\hat{\phi}(x)}:\;, \tag{3.35}$$

$$\hat{\chi}(x)_{(2)}=:\Psi(x)_{(2)}e^{-i\sqrt{\pi/2}\hat{\phi}(x)}:\;, \tag{3.36}$$

where

$$\Psi(x)=\left[\frac{m}{2\pi}\right]^{1/2}e^{-i(\pi/4)}:e^{i[\sqrt{2\pi}\hat{\phi}(x)+\sqrt{\pi/2}\gamma^5\hat{\eta}(x)]}: \tag{3.37}$$

and we may relate the O(2) current with the chiral U(1) current for Ψ:

$$\tilde{J}(x)_\mu=-\sqrt{2/\pi}\epsilon_{\mu\nu}\partial^\nu\hat{\phi}(x)\equiv:\bar{\Psi}(x)\gamma_\mu\gamma^5\Psi(x):\;. \tag{3.38}$$

The physical Lagrangian density in the superconducting phase is given by

$$\begin{aligned}\hat{\mathcal{L}}(x)=&\,i:\bar{\Psi}(x)\gamma_\mu\partial^\mu\Psi(x):\\&+\frac{m}{\pi}G[:\Psi(x)_{(1)}\Psi(x)_{(2)}:\\&\qquad+:\Psi^*(x)_{(2)}\Psi^*(x)_{(1)}:]\\&+g_{\text{Th}}:(\bar{\Psi}(x)\gamma_\mu\gamma^5\Psi(x))(\bar{\Psi}(x)\gamma^\mu\gamma^5\Psi(x)):\end{aligned} \tag{3.39}$$

which corresponds to the superconducting Thirring model with coupling constant $g_{\text{Th}}=-\pi/2$ (Ref. 7).

IV. CONCLUDING REMARKS

The model given by (1.1) exhibits dynamical mass versus superconducting-gap generation without the breakdown of he corresponding continuous U(1) and chiral U(1) symmetries, in agreement with the Coleman-Mermin-Wagner theorem.[15] When $g=\tilde{g}$, the two mechanisms cancel each other and the chiral conjugation symmetry is not dynamically broken.

When $\tilde{g}>g$, the discrete γ^5 symmetry is dynamically broken. The model presents mass generation and is equivalent to the massive Thirring model with $\beta^2=8\pi$. From Eqs. (3.12), (3.23), and (3.24) we see that the Hamiltonian operator H satisfies

$$[\hat{Q},H]=0,\quad[\hat{Q}^5,H]\neq0\;, \tag{4.1}$$

where $\hat{Q}$ is the topologically conserved charge characterizing the massive phase. The model is then described in terms of the spin-$\frac{1}{2}$ fermion operator $\psi(x)$ given by (3.32).

When $g>\tilde{g}$, the discrete γ^5 and parity symmetries are dynamically broken. The model exhibits superconducting-gap generation and is equivalent to the superconducting Thirring model with $\beta^2=8\pi$. In this phase, the Hamiltonian operator satifies

$$[\hat{Q},H]\neq0,\quad[\hat{Q}^5,H]=0\;, \tag{4.2}$$

where $\hat{Q}^5$ is the topologically conserved charge characterizing the superconducting phase. The model is then described by the spin-$\frac{1}{2}$ fermion operator $\Psi(x)$, given by (3.37).

The two (dual) phases are related by the chiral conjugation $\mathbb{C}_5$, i.e.,

$$\mathbb{C}_5:\;\psi(x)\leftrightarrow\Psi(x) \tag{4.3}$$

which transforms the mass operator into a superconducting-gap operator.

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