# Consistent Fujikawa regularization of the spinning string in superspace from Pauli-Villars regularization

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We use Pauli-Villars (PV) regularization of the spinning string in superspace to identify those Fujikawa regulators which produce consistent anomalies. We use a general background gauge, and obtain, after elimination of the nonpropagating fields and using a super-Einstein and Lorentz-invariant mass term, a diagonal Fujikawa regulator with only super-Weyl anomalies. The PV extended action has extra "twisted" Becchi-Rouet-Stora-Tyutin (BRST) symmetries; dropping these, the BRST transformations of the PV fields turn into off-shell superconformal transformations plus field equations. Thus, the transformations of superconformal field theory are just on-shell BRST transformations.

Anomalies can be calculated in Fujikawa's approach as the trace of the Jacobian of the path-integral measure, suitably regulated.[1] A question which was recently settled is which regulators give consistent anomalies? The answer[2] was obtained by first using Pauli-Villars (PV) regularization, computing the anomalies due to the noninvariances of the mass term, and reading off from this result the regulator and the quantum variables to be used in Fujikawa's approach in order to reproduce these anomalies. (Other quantum variables can be used, but in this case one must add extra terms to the Jacobian.[3] An open, and fundamental, problem is to justify Fujikawa's approach by regulating the measure itself instead of the Jacobians, for example, on a lattice, and deducing the Jacobians and regulators *ab initio*, instead of using other regularization schemes for this purpose.) Since the Pauli-Villars method yields consistent anomalies, being a Feynman-graph calculation, one obtains in this way consistent regulators. As an application of these ideas, the consistent regulator for the bosonic string was determined.[2] As a mass term one took

$$M_1(\det g)\epsilon_{\mu\nu}T^\mu T^\nu + M_2 g_{\mu\rho}\epsilon_{\nu\sigma}(S^{\mu\nu}-\tfrac{1}{2}g^{\mu\nu}g_{\kappa\lambda}S^{\kappa\lambda})(S^{\rho\sigma}-\tfrac{1}{2}g^{\rho\sigma}g_{\kappa\lambda}S^{\kappa\lambda}) , \tag{1}$$

where $T_\mu$ are the coordinate PV ghosts and the symmetric $S^{\mu\nu}$ are the coordinate PV antighosts. Since $T^\mu$ transforms as a coordinate tensor under Becchi-Rouet-Stora-Tyutin (BRST) transformations, and $S^{\mu\nu}$ as a coordinate tensor density, this mass term is coordinate BRST invariant, but not Weyl BRST invariant. Eliminating all the nonpropagating fields, and using the conformal gauge, the PV antighosts transformed as coordinate tensor densities, while the ordinary antighosts transformed into the stress tensor of matter, PV matter, PV ghost-antighost, and ordinary ghost-antighost. According to the last contribution, also the ordinary antighosts transform as tensor densities under BRST coordinate transformations. As consistent regulators for the bosonic string, the operators $D_+^\dagger D_-$, $D_-^\dagger D_+$, $D_- D_+^\dagger$, $D_+ D_-^\dagger$ in the ghost and antighost sectors were found, where $D_\pm=\rho^{1/2}\partial_\pm\rho^{-1}$ are the kinetic operators of the ghosts and $D_\pm^\dagger=\rho^{-1}\partial_\pm\rho^{1/2}$ are the kinetic operators of the antighosts in the conformal gauge $g_{+-}=\rho$.

In this paper we shall extend these results to the spinning string, and work in a general gauge, rather than in the conformal gauge. We shall use superspace methods since this leads to simpler formulas, which extend several results of the bosonic string in a natural way. The quantization of the spinning string in superspace has been discussed by several authors. In particular, Martinec[4] obtained the ghost action [our result in Eq. (8)] by constructing a metric on the space of deformations of the inverse vielbein, and identifying which part leads to propagating ghosts. Brooks and Gates[4] obtained the same result by generalizing the $x$-space result of Ref. 5. They, and Lauer,[4] studied its background symmetries. We will rederive the result of Ref. 4, but using Faddeev-Popov quantization and keeping track of the nonpropagating fields. As a consequence, the BRST symmetry is obvious at the starting point, but the final form of the BRST laws for (8) is interesting: they are (a slight generalization of) the superconformal transformation proposed in Ref. 6. Although this dynamical derivation is amusing, our main result is the form of the consistent Fujikawa regulator.

A well-known complication of the superspace approach is that local $d=2$, $N=1$ superspace is a constrained formalism: the 16 components of the inverse supervielbein $E_A{}^M$ are subject to constraints, leading to 6 prepotentials.[4,7] Since there are also 6 local gauge symmetries (4 super reparametrizations, 1 Lorentz, and 1 Weyl) we fix the following 6 components of $E_A{}^M$:

$$E_+{}^{\pm\pm}=F_+{}^{\pm\pm},\quad E_-{}^{\pm\pm}=F^{\pm\pm},$$
$$E_+{}^+=F_+{}^+,\quad E_-{}^-=F_-{}^- , \tag{2}$$

where we have used $++,--$ as bosonic indices and $+,-$ as fermionic indices, and the upper (lower-indices are curved (flat). As a result of the constraints, all $E_A{}^M$

 

are then equal to $F_A{}^M$. Decomposing $A=(a,\alpha)$ where $a=++,--$ is bosonic and $\alpha=+,-$ is fermionic, the action for the spinning string a general gauge reads

$$S=\int d^2Z[E\epsilon^{\beta\alpha}E_\alpha{}^M\partial_M X E_\beta{}^N\partial_N X -(-1)^A(E_A{}^M-F_A{}^M)F_M{}^B\pi_B{}^A -(-1)^A(\delta E_A{}^M)F_M{}^B B_B{}^A]\ , \tag{3}$$

where $E=\text{sdet}E_M{}^A$ and the only nonzero components of $\pi_B{}^A$ and $B_B{}^A$ correspond to the index structures shown in Eq. (2), namely, $B_a{}^\alpha$, $B_+{}^+$, and $B_-{}^-$. Note that the upper and lower indices of $\pi$ and $B$ are flat and background flat, respectively, a distinction which disappears after putting $E_A{}^M=F_A{}^M$.

The symbol $\delta E_A{}^M$ denotes the complete BRST variations of $E_A{}^M$, with the BRST parameter $\Lambda$ removed from the left. The BRST transformation laws are

$$\delta E_\alpha{}^M=-C^N\partial_N E_\alpha{}^M-E_\alpha{}^N\partial_N C^M+\tfrac{1}{2}L\tau_{3\alpha}{}^\beta E_\beta{}^M+WE_\alpha{}^M\ ,$$
$$\delta E_a{}^M=-C^N\partial_N E_a{}^M+E_a{}^N\partial_N C^M+L\epsilon_a{}^b E_b{}^M+2WE_a{}^M -i\gamma_a{}^{\alpha\beta}\nabla_\alpha W E_\beta{}^M\ . \tag{4}$$

The law $\delta E_a{}^M$ follows from $\delta E_\alpha{}^M$ as a result of the constraints. The objects $\delta E_A{}^M E_M{}^B=H_A{}^B$ can be straightforwardly evaluated. One finds

TABLE I. We list here the nonvanishing components of the torsion where $T_{AB}{}^C=(-1)^{AB}T_{BA}{}^C$ and $\{\nabla_+,\nabla_-\}=RM$.

| |
|---|
| $T_{+,+}{}^{++}=2i$ |
| $T_{-,-}{}^{--}=-2i$ |
| $T_{\pm,\mp\mp}{}^{\mp}=-\frac{i}{2}R$ |
| $T_{++,--}{}^{\mp}=-\frac{1}{2}\nabla_\pm R$ |

$$\Lambda H_A{}^B=\nabla_A\Lambda C^B+\Lambda C^C\phi_C M_A{}^B-\Lambda C^C T_{CA}{}^B+\Lambda L M_A{}^B +\delta_A{}^\alpha(\Lambda W\delta_\alpha{}^B) +\delta_A{}^a[2\Lambda W\delta_a{}^B+i\gamma_a{}^{\alpha\beta}\nabla_\alpha(\Lambda W)\delta_\beta{}^B]\ , \tag{5}$$

where $\nabla_A=E_A{}^M\partial_M+\phi_A M$, with $\phi_A$ the Lorentz connection, $MV_A=M_A{}^B V_B$ with $M_a{}^b=\epsilon_a{}^b$ and $M_\alpha{}^\beta=\frac{1}{2}\tau_{3\alpha}{}^\beta$, $C^B=C^M E_M{}^B$, $C^M$ is the supercoordinate ghost, $L$ the Lorentz ghost, and $W$ the Weyl ghost, and the torsion is defined by $T_{AB}{}^C=[\nabla_A E_B{}^M-(-1)^{AB}\nabla_B E_A{}^M]E_M{}^C$. It satisfies the constraints $T_{\alpha\beta}{}^a=2i\gamma_{\alpha\beta}{}^a$, $T_{bc}{}^a=T_{ab}{}^\alpha=T_{\alpha\beta}{}^\gamma=0$; see Table I.

Integrating over $\pi_B{}^A$ and $E_A{}^M F_M{}^B$ one finds an action with covariant derivatives which contain only spin connections since we use flat indices. Adding the corresponding PV action, one obtains

$$S=\int d^2Z[F\nabla^\alpha X\nabla_\alpha X-(\nabla_\alpha C^a+2i\gamma_{\alpha\beta}{}^a C^\beta)B_a{}^\alpha+(2W-\nabla_\alpha C^\alpha)B+(L+C^B\phi_B-\nabla_\alpha C^\beta\tau_{3\beta}{}^\alpha)B_3 +F\nabla^\alpha Y\nabla_\alpha Y-(\nabla_\alpha T^a+2i\gamma_{\alpha\beta}{}^a T^\beta)S_a{}^\alpha+(2R-\nabla_\alpha T^\alpha)S+(P+T^B\phi_B-\nabla_\alpha T^\beta\tau_{3\beta}{}^\alpha)S_3 +FMY^2+FM_1T^{++}T^{--}+F^{-1}M_2S_{++}{}^-S_{--}{}^+]\ , \tag{6}$$

where $2B=B_+{}^++B_-{}^-$, $2B_3=B_+{}^+-B_-{}^-$, and similarly for $S$ and $S_3$, while $R$ and $P$ are the PV Weyl and Lorentz ghosts, respectively. The mass parameters $M$, $M_1$, and $M_2$ are independent. The result in (6) generalizes the PV action of Ref. 2 to superspace and to a general gauge, and the superspace action of Ref. 7 to a general gauge and to the PV case.

Before eliminating all nonpropagating fields, we discuss the invariances of the action. On general grounds we note that, after integrating over $\pi_A{}^B$ and $E_A{}^M F_M{}^B$ in the theory without PV fields, and subsequently replacing $X$ by $X+Y$, $C^A$ by $C^A+T^A$, $W$ by $W+R$, $L$ by $L+P$, and $B_A{}^B$ by $B_A{}^B+S_A{}^B$, the part of the action which contains an even number of PV fields is invariant under the following decomposition of the original BRST transformation laws:

$$\delta\phi(\text{non-PV})=(\text{terms even in PV fields})\ ,$$
$$\delta\phi(\text{PV})=(\text{odd in PV fields})\ . \tag{7}$$

The law for $B_A{}^B+S_A{}^B$ follows from the supervielbein equation of motion. The non-PV fields contain now in their transformation rules in (7) terms quadratic in non-PV fields and terms quadratic PV fields. While this is what we expect for the antighost by extrapolating the results of Ref. 2, we do not expect terms quadratic in PV fields in the laws for the matter and ghost fields. We claim that these terms can be dropped since they form an extra symmetry. The reason is that the massless action has an obvious O(2) symmetry between PV and non-PV fields, and the commutator between an ordinary BRST transformation with PV ghosts and this O(2) rotation produces the terms quadratic in PV fields in the matter and ghost laws. [To give an example, in the bosonic string the sum of the $X$ action $-\frac{1}{2}\sqrt{g}\,(\partial X)^2$ and massless $Y$ action and the massless PV action $S^{\mu\nu}(D_\mu T_\nu-g_{\mu\nu}R)$ is separately invariant under the "twisted" BRST laws

$$\delta X=-\Lambda T^\mu\partial_\mu Y,\quad \delta Y=-\Lambda T^\mu\partial_\mu X\ ,$$

and

$$\delta S^{\mu\nu}=-\Lambda\sqrt{g}\,(\partial^\mu X\partial^\nu Y+\partial^\mu Y\partial^\nu X-g^{\mu\nu}\partial^\sigma X\partial_\sigma Y)\ .$$

These laws are the commutator of the SO(2) rotation $\delta X=Y$, $\delta Y=-X$ with

$$\delta X=-\Lambda T^\mu\partial_\mu X,\quad \delta Y=0\ ,$$
$$\delta S^{\mu\nu}=-\Lambda\sqrt{g}\,(\partial^\mu X\partial^\nu X-\tfrac{1}{2}g^{\mu\nu}\partial^\sigma X\partial_\sigma X)$$

which are the ordinary BRST laws, but for $X$ and the PV antighost.] Dropping these separate symmetries—the action in (6) is invariant under the usual laws for the ghosts, the antighosts transform into the sum of the 4 stress tensors, while *the PV fields (ghost and antighosts alike) transform as tensors under BRST transformations.*

After eliminating all nonpropagating fields $B$ and $W$, $S$ and $R$, $B_3$ and $L$, $S_3$ and $P$, $C^\beta$ and the gamma trace of $B_a{}^\alpha$, $T^\beta$ and the gamma trace of $S_a{}^\alpha$, we obtain the following action, using $\epsilon^{+-}\epsilon_{+-}=1$:

$$S=\int d^2Z(2F\nabla_-X\nabla_+X+2F\nabla_-Y\nabla_+Y+FMY^2-B_{++}{}^-\nabla_-C^{++}-B_{--}{}^+\nabla_+C^{--}-S_{++}{}^-\nabla_-T^{++}$$
$$-S_{--}{}^+\nabla_+T^{--}+FM_1T^{++}T^{--}+F^{-1}M_2S_{++}{}^-S_{--}{}^+)\;. \tag{8}$$

We used $v^a w_a=v^{++}w_{++}+v^{--}w_{--}$ and $v_\alpha=(v_+,v_-)$ for $\alpha=1,2$. The non-PV part in the conformal gauge agrees with Refs. 4 and 7. This action has a number of symmetries. First of all, it is invariant under super-Einstein and Lorentz transformations, if we also transform the background field $F_A{}^M$. This is obvious since all indices are flat and Lorentz transformation are diagonal on all fields. Under the Weyl rescalings $F_\alpha{}^M\to\Sigma F_\alpha{}^M$, $T^{\pm\pm}\to\Sigma^{-2}T^{\pm\pm}$, $S_{\pm\pm}{}^\mp\to\Sigma S_{\pm\pm}{}^\mp$, and similarly for $C^{\pm\pm}$ and $B_{\pm\pm}{}^\mp$, the massless part of the action is invariant since $\phi_\pm\to\Sigma\phi_\pm\pm2\nabla_\pm\Sigma$, see Howe.[4] The invariance under these background symmetries follows from the fact that we have chosen a background-invariant gauge-fixing term, because we can always achieve invariance by letting the ghost and antighost transform as tensors.

Next we consider BRST transformations. Under these $F_A{}^M$ is inert. The massless part of the action in (8) is invariant under the following rules:

$$\delta X=-\Lambda C^A\nabla_AX,\quad \delta Y=-\Lambda C^A\nabla_AY\;,$$
$$\delta C^a=\Lambda(-C^A\nabla_AC^a-i\gamma_{\alpha\beta}{}^aC^\alpha C^\beta)\;,$$
$$\delta T^a=\Lambda(-C^A\nabla_AT^a-T^A\nabla_AC^a-2i\gamma_{\alpha\beta}{}^aC^\alpha T^\beta)\;, \tag{9}$$
$$\delta\hat S_a{}^\alpha=\Lambda[-FC^A\nabla_A(F^{-1}\hat S_a{}^\alpha)-\tfrac32\nabla_aC^b\hat S_b{}^\alpha]\;,$$
$$\delta\hat B_a{}^\alpha=\Lambda[-2F(\nabla^\alpha X\nabla_aX+\nabla^\alpha Y\nabla_aY)$$
$$-FC^A\nabla_A(F^{-1}\hat B_a{}^\alpha)-\tfrac32\nabla_aC^b\hat B_b{}^\alpha$$
$$-FT^A\nabla_A(F^{-1}\hat S_a{}^\alpha)-\tfrac32\nabla_aT^b\hat S_b{}^\alpha]\;.$$

In this result, $C^\alpha$ [and $T^\alpha$] are to be replaced by their field equations $-(i/4)\nabla_\beta C^b\gamma_b{}^{\beta\alpha}$ [and $-(i/4)\nabla_\beta T^b\gamma_b{}^{\beta\alpha}$] and $\hat B_a{}^\alpha$ [$\hat S_a{}^\alpha$] denotes the gamma traceless parts $B_{++}{}^-$ and $B_{--}{}^+$ [$S_{++}{}^-$ and $S_{--}{}^+$].

In a flat background the $X$ law becomes

$$\delta X=\xi^{++}\partial_{++}X-\frac{i}{2}\nabla_+\xi^{++}\nabla_+X$$

plus a part with $\xi^{--}$, with $\xi^{++}=-\Lambda C^{++}$, which is the usual law for a tensor field with superconformal spin $h=0$ (Ref. 6), but in Minkowski spacetime.[7] The $\gamma$ terms in $\delta C^a$ and $\delta T^a$ are due to the torsion needed to replace $\partial_N$ in $\delta C^N=-\Lambda C^N\partial_NC^M$ by covariant derivatives. The first two terms in $\delta T^a$ themselves look like an ordinary super reparametrization for a curved-indexed supervector, but decomposing $A$ into $(a,\alpha)$, one finds

$$\delta T^a=\Lambda\left[-C^b\nabla_bT^a-T^b\nabla_bC^a+\frac{i}{8}\nabla_\beta C^b\gamma_b{}^{\beta\alpha}\nabla_\alpha T^a\right.$$
$$\left.+\frac{i}{8}\nabla_\beta T^b\gamma_b{}^{\beta\alpha}\nabla_\alpha C^a+\frac{i}{8}\nabla_\alpha C^c\gamma^{a\alpha\beta}\nabla_\beta T_c\right] \tag{10}$$

which is the sum of $h=-1$ superconformal transformations and field equations. Note that all these results hold in a general background. The $\delta T^a$ law has the same BRST Jacobian as the $\delta C^a$ law, in agreement with Ref. 2. To obtain $\delta\hat B_a{}^\alpha$ we proceeded as follows. The variation of $B_A{}^B$ under BRST transformation is equal to $\pi_A{}^B\Lambda$. We eliminate $\pi_A{}^B$ from the $\pi_A{}^B$ and $E_A{}^M$ field equations. For the variation of the supervielbein in the $B$, $C$ ghost action we find

$$\delta S\Lambda=\int d^2Z(-1)^{A+1}H_A{}^B[\pi_B{}^A\Lambda+\Lambda C^D(\partial_DB_B{}^A+\phi_DM_B{}^CB_C{}^A)+\Lambda(\partial_NC^N)B_B{}^A$$
$$+\nabla_B(\Lambda C^C)B_C{}^A-\Lambda C^DT_{DB}{}^CB_C{}^A+\Lambda LB_B{}^CM_C{}^A]$$
$$+H_\alpha^B[\Lambda WB_B{}^\alpha-B_B{}^ai\gamma_a{}^{\alpha\beta}(\nabla_\beta\Lambda W)-(\nabla_B\Lambda W)i\gamma_a{}^{\alpha\beta}B_\beta{}^a]-H_a{}^B(2\Lambda WB_B{}^a)\;. \tag{11}$$

One recognizes already in the $H_A{}^B$ term the structure of a general supercoordinate transformation on a tensor density $B_M{}^A$ with Lorentz index $A$, but with the lower index made flat, which induces the torsion term. Since the only components of $B_A{}^B$ are $B_\alpha{}^\alpha$, $B_+{}^+$, and $B_-{}^-$, it follows that the last three terms are absent. Moreover, the index $A$ of $H_A{}^B$ is always fermionic. Hence, there are only $H_\alpha{}^\beta$ variations. Because of the constraints not all $H_A{}^B$ are independent, but $H_\alpha{}^a$, $H_+{}^+$, $H_-{}^-$ form an independent set. Furthermore, by variation of the constraints one finds that

$$H_{\mp}{}^{\mp}=\pm\frac{i}{2}(\nabla_+ H_-{}^{\pm\pm}+\nabla_- H_+{}^{\pm\pm}) \,. \qquad (12)$$

From this result one finds $\delta B_A{}^B$. Substituting the field equations

$$B_+{}^+=B_-{}^-=B_a{}^\alpha\gamma^a{}_{\alpha\beta}=0, \quad C^\alpha=-\frac{i}{4}\nabla_\beta C^b\gamma_b{}^{\beta\alpha} \,,$$

$$W=\tfrac{1}{2}\nabla_\alpha C^\alpha, \quad L=-C^B\phi_B+\nabla_\alpha C^\beta\tau_{3\beta}{}^\alpha \,,$$

we find that the $-C^B\phi_B$ term in $L$ makes the transport term to $\Lambda C^D\nabla_C B_B{}^A$ covariant. In this way we obtain the $\hat{B}$-dependent terms for $\delta\hat{B}_a{}^\alpha$ in (9). The $X$-dependent terms are obtained in a similar way. Using (7) we then obtain the complete $\delta\hat{B}_a{}^\alpha$ and $\delta\hat{S}_a{}^\alpha$ laws in (9). We have checked by direct computation that the action in (8) is indeed invariant under (9). The result shows that $\hat{B}$ transforms into the sums of all stress tensors, and in particular, that $\hat{B}$ and $\hat{S}$ have conformal spin $\frac{3}{2}$.

Using the representation where

$$(\gamma^{++})_{++}=1, \quad (\gamma^{--})_{--}=-1 \,,$$

$$\eta_{++,--}=2, \quad \eta^{++,--}=\tfrac{1}{2} \,,$$

hence

$$(\gamma_{++})^{++}=-2, \quad (\gamma_{--})^{--}=2 \,,$$

one finds the transformation rules for the fields in (9) on a light-cone basis; see Table II. (Use $\nabla_+\nabla_+=i\nabla_{++}$.)

Before determining the regulators, we show that the background transformations lead to an expression for the variation of the mass term in (8) which agrees with BRST anomaly. That is, making a classical Weyl rescaling in (8) with infinitesimal parameter $\sigma$ ($\Sigma=e^\sigma$), and replacing $\sigma$ by $\Lambda W$ where $W$ is the Weyl ghost, the action varies into

$$\delta\mathcal{L}=(-2\Lambda W)FMY^2+(-6\Lambda W)FM_1T^{++}T^{--}+(4\Lambda W)F^{-1}M_2S_{++}{}^-S_{--}{}^+ \,. \qquad (13)$$

We claim that the same result for $\delta\mathcal{L}$ is obtained if we vary the action in (8) under the BRST transformation in (9), denoting $\frac{1}{2}\nabla_\alpha C^\alpha$ by $W$. Actually we find two extra terms, coming from the last term in $\delta T^{\pm\pm}$,

$$\begin{aligned}\delta\mathcal{L}=&(-2\Lambda W)FMY^2+(-6\Lambda W)FM_1T^{++}T^{--}\\&+(4\Lambda W)F^{-1}M_2S_{++}{}^-S_{--}{}^+\\&+\frac{i}{2}\Lambda FM_1(T^{--}\nabla_-T^{--}\nabla_-C^{++}\\&\qquad+T^{++}\nabla_+T^{++}\nabla_+C^{--}) \,, \end{aligned}\qquad (14)$$

but since $\nabla_\pm C^{\mp\mp}$, is the $\hat{B}$ field equation, we can add an $M_1$-dependent term to $\delta\hat{B}$ which removes the last two

TABLE II. BRST transformation rules on a light-cone basis.

$$\delta X=\left[\Lambda(iC^{++}\nabla_+^2X-\frac{i}{2}\nabla_+C^{++}\nabla_+X-iC^{--}\nabla_-^2X+\frac{i}{2}\nabla_-C^{--}\nabla_-X\right]$$

$$\delta Y=\Lambda\left[iC^{++}\nabla_+^2Y-\frac{i}{2}\nabla_+C^{++}\nabla_+Y-iC^{--}\nabla_-^2Y+\frac{i}{2}\nabla_-C^{--}\nabla_-Y\right]$$

$$\delta C^{\pm\pm}=\Lambda\left[iC^{++}\nabla_+^2C^{\pm\pm}\mp\frac{i}{4}\nabla_\pm C^{\pm\pm}\nabla_\pm C^{\pm\pm}-iC^{--}\nabla_-^2C^{\pm\pm}\pm\frac{i}{2}\nabla_\mp C^{\mp\mp}\nabla_\mp C^{\pm\pm}\right]$$

$$\begin{aligned}\delta T^{\pm\pm}=\Lambda\Big[&iC^{++}\nabla_+^2T^{\pm\pm}-\frac{i}{2}\nabla_+C^{++}\nabla_+T^{\pm\pm}+iT^{++}\nabla_+^2C^{\pm\pm}-iC^{--}\nabla_-{}^2T^{\pm\pm}+\frac{i}{2}\nabla_-C^{--}\nabla_-T^{\pm\pm}\\&-iT^{--}\nabla_-^2C^{\pm\pm}\pm\frac{i}{2}\nabla_\mp T^{\mp\mp}\nabla_\mp C^{\pm\pm}\Big]\end{aligned}$$

$$\begin{aligned}\delta S_{\pm\pm}{}^{\mp}=\Lambda\Big[&iFC^{++}\nabla_+^2(F^{-1}S_{\pm\pm}{}^{\mp})-\frac{i}{2}F\nabla_+C^{++}\nabla_+(F^{-1}S_{\pm\pm}{}^{\mp})\pm\tfrac{3}{2}i\nabla_\pm^2C^{\pm\pm}S_{\pm\pm}{}^{\mp}-iFC^{--}\nabla_-^2(F^{-1}S_{\pm\pm}{}^{\mp})\\&+\frac{i}{2}F\nabla_-C^{--}\nabla_-(F^{-1}S_{\pm\pm}{}^{\mp})\Big]\end{aligned}$$

$$\begin{aligned}\delta B_{\pm\pm}{}^{\mp}=\Lambda\Big[&-2iF(\nabla_\pm X\nabla_\pm^2X+\nabla_\pm Y\nabla_\pm^2Y)+iFC^{++}\nabla_+^2(F^{-1}B_{\pm\pm}{}^{\mp})-\frac{i}{2}F\nabla_+C^{++}\nabla_+(F^{-1}B_{\pm\pm}{}^{\mp})\\&\pm\tfrac{3}{2}i\nabla_\pm^2C^{\pm\pm}B_{\pm\pm}{}^{\mp}-iFC^{--}\nabla_-{}^2(F^{-1}B_{\pm\pm}{}^{\mp})+\frac{i}{2}F\nabla_-C^{--}\nabla_-(F^{-1}B_{\pm\pm}{}^{\mp})\\&+iFT^{++}\nabla_+^2(F^{-1}S_{\pm\pm}{}^{\mp})-\frac{i}{2}F\nabla_+T^{++}\nabla_+(F^{-1}S_{\pm\pm}{}^{\mp})\pm\tfrac{3}{2}i\nabla_\pm^2T^{\pm\pm}S_{\pm\pm}{}^{\mp}-iFT^{--}\nabla_-{}^2(F^{-1}S_{\pm\pm}{}^{\mp})\\&+\frac{i}{2}F\nabla_-T^{--}\nabla_-(F^{-1}S_{\pm\pm}{}^{\mp})\Big]\end{aligned}$$

TABLE III. We list here the dependent variations $H_A{}^B=\delta E_A{}^M E_M{}^B$ as functions of the independent ones $H_\alpha{}^a, H_+{}^+, H_-{}^-$.

$$H_\pm{}^\mp=\mp\frac{i}{2}(\nabla_+H_-{}^{\mp\mp}+\nabla_-H_+{}^{\mp\mp})$$
$$H_{\pm\pm}{}^{\mp\mp}=\mp i\nabla_\pm H_\pm{}^{\mp\mp}$$
$$H_{\pm\pm}{}^{\pm\pm}=2H_\pm{}^\pm\mp i\nabla_\pm H_\pm{}^{\pm\pm}$$
$$H_{\pm\pm}{}^\pm=\pm\tfrac{1}{2}RH_\mp{}^{\mp\mp}\mp i(\nabla_\pm H_+{}^++\nabla_\pm H_-{}^-)-\tfrac{1}{2}\nabla_\mp^2H_\pm{}^{\mp\mp}-\tfrac{1}{2}\nabla_\mp\nabla_\pm H_\mp{}^{\mp\mp}$$
$$H_{\pm\pm}{}^\mp=\pm\tfrac{1}{2}RH_\pm{}^{\mp\mp}-\tfrac{1}{2}\nabla_\pm^2H_\mp{}^{\mp\mp}-\tfrac{1}{2}\nabla_\pm\nabla_\mp H_\pm{}^{\mp\mp}$$

terms. Since the mass terms are independent of $\hat{B}$, this does not lead to further $M^2$ variations; this explains why there are no terms in (14) with the $\hat{S}$ field equations. [This extra variation of $\hat{B}$ does not contribute to anomalies since the regulator (see below) is diagonal in $BC$ and $ST$ space.]

In fact, the extra $M_1$-dependent term in $\delta B_{++}{}^-$ is just the contribution to the stress tensor from the mass term of the PV ghost fields. To show this, we note that $T^{\pm\pm}=T^ME_M{}^{\pm\pm}$ (we do not use $T^MF_M{}^{\pm\pm}$ in the mass term since we prefer to avoid BRST coordinate anomalies). The variation of $E$ is proportional to $H_a{}^a-H_\alpha{}^\alpha$, and $H_a{}^a-2H_\alpha{}^\alpha$ equals $i\gamma_a{}^{\alpha\beta}\nabla_\alpha H_\beta{}^a$. Hence, from $\delta E$ one does not get a contribution to $\delta B_{\pm\pm}{}^\mp$. From $\delta T^{++}=T^AH_A{}^{++}$ the part $T^\alpha H_\alpha{}^{++}$ precisely yields the extra term in $\delta B_{++}{}^-$ while the part $T^aH_a{}^{++}$ does not contribute since $H_{--}{}^{++}$ multiplies $T^{--}T^{--}$ which obviously vanishes, while $H_{++}{}^{++}$ equals $2H_+{}^+-i\nabla_+H_+{}^{++}$ which is independent of $H_\mp{}^{\pm\pm}$; see Table III. [In fact, the reader might wonder at this point whether the $\delta C^a$ law in Eq. (9) is equal to $\delta(C^MF_M{}^a)$ or $\delta(C^ME_M{}^a)$. The answer is $\delta C^a=\delta(C^MF_M{}^a)=(\delta C^M)F_M{}^a$. It differs from $\delta(C^ME_M{}^a)$ by the variation of the field equation, $\delta E_M{}^a=\delta(E_M{}^a-F_M{}^a)$, which should itself be a field equation. Explicit evaluation of $C^ME_M{}^{++}$ under BRST transformation, using the field equations for $L$, $W$, $C^\alpha$ and the torsion constraints $T_{AB}{}^{++}=\delta_A^+\delta_B^+T_{++}{}^{++}$ leads to

$$C^M\delta E_M{}^{++}=\Lambda\left|iC^{--}\nabla_-^2C^{++}-\frac{i}{2}\nabla_-C^{--}\nabla_-C^{++}\right| . \tag{15}$$

Indeed, only the $B_{\pm\pm}{}^\mp$ field equations appear which is as it should be since the $\hat{B}$ law changes after elimination of $\pi_A{}^B$ if we fix part of $E_M{}^A$ by treating $C^ME_M{}^A$ as independent variables.]

According to the general ideas of Ref. 2, the results in (8) imply that in the Fujikawa method it is most convenient to choose $\tilde{X}=F^{1/2}X$, $\tilde{C}^{\pm\pm}=F^{1/2}C^{\pm\pm}$, and $\tilde{B}_{\pm\pm}{}^\mp=F^{-1/2}B_{\pm\pm}{}^\mp$ as independent variables in the path integral, and as Jacobians for the BRST anomaly one should use a diagonal matrix with entries $-2\Lambda W$, $-6\Lambda W$, $-6\Lambda W$, $4\Lambda W$, $4\Lambda W$ where $W=\frac{1}{2}\nabla_\alpha C^\alpha$. Taking into account that these Jacobians are to be used in a supertrace, these results agree with Ref. 7, but they hold in a general gauge.

We have seen that background symmetry and BRST symmetry lead to the same Jacobians to be used in Fujikawa's approach. We now check that these Jacobians, which PV regularization provides, are the same as the Jacobian's for $\tilde{X}$, $\tilde{C}^{\pm\pm}$, and $\tilde{B}_{\pm\pm}{}^\mp$ in the theory without PV fields. For $\tilde{X}$ one finds easily that this Jacobian equals $1+\mathrm{tr}[(-\frac{1}{2}\Lambda)(\nabla_+C^+ +\nabla_-C^-)]$. For the ghosts and antighosts one finds

$$J(C^{++})=1+\mathrm{str}[(-\tfrac{1}{2}-2)\Lambda\nabla_+C^+-\tfrac{1}{2}\Lambda\nabla_-C^-]$$

and

$$J(B_{++}{}^-)=1+\mathrm{tr}[(-\tfrac{1}{2}+3)\Lambda\nabla_+C^+-\tfrac{1}{2}\Lambda_-C^-] .$$

Hence one reobtains the results in (13), since $J(\hat{C}^{++})$ and $J(\hat{C}^{--})$ [and $J(\hat{B}_{++}{}^-)$ and $J(\hat{B}_{--}{}^+)$] each are multiplied by the same factor due to regularization (see below).

We are now ready to determine the consistent regulators to be used in the Fujikawa approach. In terms of the variables $\phi=(\tilde{Y},\tilde{T}^{++},\tilde{T}^{--},\tilde{S}_{++}{}^-,\tilde{S}_{--}{}^+)$, the PV action takes the form $\frac{1}{2}\phi^T(TO+T\mathcal{M})\phi$, where in the matter sector $T\mathcal{M}=2M$ and $TO=4F^{1/2}\nabla_+\nabla_-F^{-1/2}$, while in the four-dimensional ghost-antighost sector one has

$$T\mathcal{M}=\begin{pmatrix} 0 & M_1 & & \\ -M_1 & 0 & & \\ & & 0 & M_2 \\ & & M_2 & 0 \end{pmatrix} , \quad TO=\begin{pmatrix} & & -F^{1/2}\nabla_-F^{-1/2} & 0 \\ & & 0 & -F^{1/2}\nabla_+F^{-1/2} \\ -F^{1/2}\nabla_-F^{-1/2} & 0 & & \\ 0 & -F^{1/2}\nabla_+F^{-1/2} & & \end{pmatrix} . \tag{16}$$

Following Ref. 2, the regulator $R$ is given by $O^2/M^2$ in the $X$ sector and $O^4/M^4$ in the $B,C$ sector. We find a consistent regulator which is diagonal in the five sectors, and given by

$$\begin{aligned}
R(\tilde{X}) &= 4M^{-2}F^{1/2}\nabla_+\nabla_-\nabla_+\nabla_-F^{-1/2}\,,\\
R(\tilde{C}^{++}) &= (M_1M_2)^{-2}F^{1/2}\nabla_+\nabla_-\nabla_+\nabla_-F^{-1/2}\,,\\
R(\tilde{C}^{--}) &= (M_1M_2)^{-2}F^{1/2}\nabla_-\nabla_+\nabla_-\nabla_+F^{-1/2}\,,\\
R(\tilde{B}_{++}{}^{-}) &= (M_1M_2)^{-2}F^{1/2}\nabla_+\nabla_-\nabla_+\nabla_-F^{-1/2}\,,\\
R(\tilde{B}_{--}{}^{+}) &= (M_1M_2)^{-2}F^{1/2}\nabla_-\nabla_+\nabla_-\nabla_+F^{-1/2}\,.
\end{aligned} \tag{17}$$

Note that $\nabla_\pm$ for different fields contain different connections. Again this result agrees with Ref. 7, but it holds in a general gauge.

We thus see that the Pauli-Villars regularization of the spinning string in superspace leads to a Jacobian and regulator which agree with the result previously obtained in Ref. 7. However, the results here justify the regulator chosen there. The actual value of the anomalies in the various sectors agrees, but our results hold in a general gauge. Moreover we have seen how superconformal transformations are generated dynamically: namely by inserting all field equations (propagating and nonpropagating) into the BRST laws, for the PV fields one finds standard superconformal transformations. Background invariance and BRST symmetry are unrelated; the former is only present when one uses a background-covariant gauge choice. The former is needed for gauge independence, while the latter is used to derive Ward identities. Yet, we have seen that background transformations (under which $F_A{}^M$ varies) and BRST transformations (under which $F_A{}^M$ is inert) lead to the same variation of the mass term in the action, i.e., to the same anomalies.

In $x$ space, the mass term for the Fujikawa variables $\phi$ becomes, in flat space,

$$M_1M_2(M_1\epsilon_{\mu\nu}c^\mu c^\nu + M_2\bar{\beta}_\mu\tau_3\beta_\nu\eta^{\mu\nu}) + M_1\bar{\gamma}\tau_3\gamma + M_2\epsilon_{\mu\nu}\eta_{\rho\sigma}b^{\mu\rho}b^{\nu\sigma}\,, \tag{18}$$

where $c^\mu$ and $\gamma$ ($b^{\mu\nu}$ and $\beta_\mu$) are the coordinate and supersymmetry ghosts (antighosts). The latter are, respectively, traceless and gamma traceless. For the $x$-space action without PV fields, see Ref. 5.

We considered ordinary (1,1) superspace in this paper. Our methods can also be applied to other superspaces, such as (1,0) superspace, in which case we would also obtain Lorentz anomalies.

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