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Inflation as a dynamical effect of higher dimensions

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The possibility of explaining the inflationary phase of the evolution of the Universe as a dynamical effect of extra dimensions is discussed. Methods of qualitative dynamical systems have been applied to determine the regions of the phase space in which inflation takes place. The problem is analyzed within the class of multidimensional Friedmann-Robertson-Walker$\times T^D$ models with a hydrodynamical energy-momentum tensor, material fields satisfying the Freund-Rubin ansatz, and quantum effects arising from massless scalar fields at the low-temperature approximation. The stability of inflation is also demonstrated.

I. INTRODUCTION

An inflationary phase at an early stage of the evolution of our Universe has become a paradigm of modern cosmology. The horizon problem can be solved if the inflationary phase lasts long enough. At the same time inflationary cosmology could explain the flatness, age, homogeneity, and isotropy of our Universe.[1] Inflation could also describe the transition from higher-dimensional cosmology to the ordinary four-dimensional regime[2] (for alternative ideas in this context, see Ref. 3). In such a case inflation could be a purely (classical) gravitational mechanism. Inflation is usually understood as an exponential expansion of the scale factor $R(t)$ of physical space.

The condition

$$\ddot{R}>0 \tag{1}$$

is necessary to solve the horizon, flatness, and isotropy problems. Condition (1) would be a sufficient condition for the solution of these problems if it can be shown that the correct energy-momentum tensor, for matter fields present in the early Universe, guarantees that condition (1) is valid for a sufficiently long period of time.[4] In addition to the standard class of models with an exponential evolution, condition (1) is satisfied by the power-law or "generalized" inflationary models[5] in which $R(t)\sim t^A$, $A>1$ (Ref. 5). In the case of $(1+3)$-dimensional Friedmann models condition (1) implies the breakdown of the strong energy condition $\rho+3p<0$, where ρ and p are the energy density and pressure, respectively. In this paper we characterize the class of multidimensional Friedmann-Robertson-Walker (FRW)$\times T^D$ models satisfying condition (1). We shall denote the scale of physical space by $R(t)$.

The method of dynamical systems was employed by Skea and Stein-Schabes[6] to investigate multidimensional cosmological models [FRW$(k=0)\times T^D$] with the energy-momentum tensor $T^\mu_\nu=(\rho,\cdots m\rho\cdots,\cdots n\rho\cdots)$. In the work by Wiltshire[7] multidimensional world models (FRW$\times S^D$) with the Freund-Rubin ansatz and the cosmological constant were studied by using the dynamical system method. Our choice of the variables x, y, and τ enables the discussion of the curvature effects of the physical space and reduces the dynamics to a two-dimensional dynamical system (as in the case investigated by Wiltshire[7]), but additionally it allows us to analyze the case of an arbitrary dimension of internal space. The obtained results show that the topological structure of the phase space is independent of dimension D of the internal space (with the exception of $D=1$). Skea and Stein-Schabes[6] have also reduced the system to two dimensions and shown the structure of phase space is independent of D.

II. MULTIDIMENSIONAL COSMOLOGICAL MODELS AS DYNAMICAL SYSTEMS

We assume the metric of the FRW$\times T^D$ model in the form

$$ds^2=dt^2-R^2(t)d\Omega_3^2-r^2(t)dx_4^2-\cdots-r^2(t)dx_{D+3}^2\ , \tag{2}$$

where $R(t)$ and $r(t)$ are the scale factors of the physical and internal space, respectively, $d\Omega_3^2$ is the line element of the unit maximally symmetric space.

The following three cases will be discussed.

(A) Metric (2) satisfying Einstein's equations in $D+4$ dimensions with the hydrodynamical energy-momentum tensor

$$T^\mu_\nu=\text{diag}\|\rho,-p,\ldots,-p\|\ . \tag{3}$$

(B) Metric (2) satisfying the $(D+4)$-dimensional Einstein equations in supergravity theory with field strength

$$F_{\mu_4\mu_5\cdots\mu_{D+3}}=D\partial_{[\mu_4}A_{\mu_5\cdots\mu_{D+3}]} \tag{4}$$

associated with an Abelian gauge field $A_{\mu_4\mu_5\cdots\mu_{D+2}}$. We shall assume that $F_{\mu_4\cdots\mu_{D+3}}$ is given by the Freund-Rubin ansatz[7] so that

$$F\sim\frac{1}{r^D}dx^4\wedge dx^5\wedge\cdots\wedge dx^{D+3}\ . \tag{5}$$

The last condition automatically satisfies the Maxwell-type equation derived from the corresponding action.[7] The remaining Einstein equations are reduced to equations with the source term

$$T^{\mu}_{\nu}=\mathrm{diag}\|\rho,-p,-p,-p,-p',\ldots,-p'\| \;, \tag{6}$$

where $p=-\rho$, $p'=\rho$, $\rho=\rho_0/V^2_{\mathrm{micro}}$, $V_{\mathrm{micro}}=r^D$, $\rho_0=\mathrm{const}>0$.

These equations read

$$3\frac{\ddot{R}}{R}+D\frac{\ddot{r}}{r}=-\frac{\rho(D+1)+3p+Dp'}{D+2} \;, \tag{7a}$$

$$\left[\frac{\dot{R}}{R}\right]^{\cdot}+\frac{\dot{R}}{R}\left[3\frac{\dot{R}}{R}+D\frac{\dot{r}}{r}\right]=-\frac{2k}{R^2}-\frac{-\rho+(1-D)p+Dp'}{D+2} \;, \tag{7b}$$

$$\left[\frac{\dot{r}}{r}\right]^{\cdot}+\frac{\dot{r}}{r}\left[3\frac{\dot{R}}{R}+D\frac{\dot{r}}{r}\right]=-\frac{3p-2p'-\rho}{D+2} \;, \tag{7c}$$

where $k=0,\pm 1$ is the curvature of the physical space, p and p' are the pressures in the physical and internal spaces, correspondingly. Einstein's equations for case (A) also assume the form (7) with $p=p'=\gamma\rho$ $(0\le\gamma\le 1)$ and $\rho=\rho_0/V^{1+\gamma}_{\mathrm{micro}}V^{1+\gamma}_{\mathrm{macro}}$, $\rho_0=\mathrm{const}$, $V_{\mathrm{macro}}=R^3$.

(C) Metric (2) satisfies the Einstein equations with quantum corrections arising from massless scalar fields at the low-temperature approximation,[8] In this case the problem of the metric back reaction is thermodynamically equivalent to the effect of the energy-momentum tensor (6) with $p=-\rho$, $p'=(4/D)\rho$. By eliminating the energy density ρ from constraint condition (7a), we can reduce the dynamics of the above models to a two-dimensional system defined on the plane x,y; where $x=HR$, $y=hR$, and H,h are the Hubble functions of the macro- and microspaces, respectively. These systems are defined on the region of the plane x,y allowed by the constraint condition. In cases (A)–(C) we assume the following form [in case (A) the equation of state $p=\rho/(D+3)$ of the radiative matter is assumed].

Case (A)

$$x'=\frac{dx}{d\tau}=-\frac{2D+3}{D+3}x^2-\frac{D^2}{D+3}xy+\frac{D(D-1)}{2(D+3)}y^2-\frac{2D+3}{D+3}k \;,$$
$$y'=\frac{dy}{d\tau}=\frac{3}{D+3}x^2+\frac{D-6}{D+3}xy-\frac{D(D+7)}{2(D+3)}y^2+\frac{3k}{D+3} \;, \tag{8}$$

in the region

$$3x^2+3Dxy+\frac{D(D-1)}{2}y^2+3k=\rho R^2\ge 0 \;.$$

Case (B)

$$x'=\frac{dx}{d\tau}=-\frac{2(4D-1)}{D+2}x^2-\frac{D(7D-4)}{D+2}xy-\frac{D(D-1)^2}{D+1}y^2-\frac{2(4D-1)}{D+2}k \;,$$
$$y'=\frac{dy}{d\tau}=\frac{4(4D-1)}{D+2}xy+\frac{18}{D+2}x^2+\frac{D(2D-5)}{D+2}y^2+\frac{18k}{D+2} \;, \tag{9}$$

in the region

$$\rho R^2=3x^2+3Dxy+\frac{D(D-1)}{2}y^2+3k\ge 0 \;.$$

Case (C)

$$x'=\frac{dx}{d\tau}=-5x^2-4Dxy-\frac{D(D-1)}{2}y^2-5k \;,$$
$$y'=\frac{dy}{d\tau}=\frac{12}{D}x^2+10xy+(D-2)y^2+\frac{12}{D}k \;, \tag{10}$$

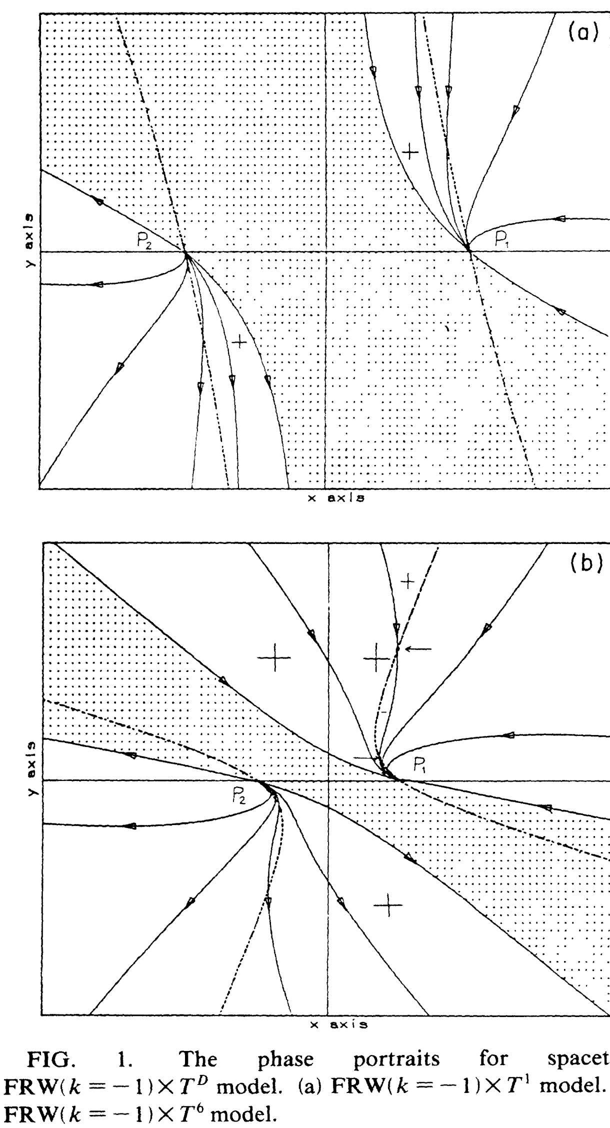

FIG. 1. The phase portraits for spacetime FRW$(k=-1)\times T^D$ model. (a) FRW$(k=-1)\times T^1$ model. (b) FRW$(k=-1)\times T^6$ model.

in the region

$$\rho R^2=3x^2+3Dxy+\frac{D(D-1)}{2}y^2+3k\le 0 ,$$

where τ is the new time parameter along the trajectory of the dynamical system, defined as $d\tau=dt/R$.

Phase portraits corresponding to the above cases are presented in Figs. 1–5. Condition (1), in variables x,y, reduces to

$$\frac{dx}{d\tau}>0 . \qquad (11)$$

The boundaries of (11), i.e., $dx/d\tau=0$ (dotted line) together with the sign of $dx/d\tau$, are also presented in phase portraits. The dotted area denotes the region forbidden by the appropriate constraints.

III. A MECHANISM FOR INFLATION

By using the phase portraits, in a similar manner to those in Figs. 1–5 we can draw some conclusions concerning generalized inflation, satisfying condition (11), in a certain time interval τ. We shall discuss cases (A)–(C) separately.

Case (A) FRW$(k=-1)\times T^D$ models with the radiative matter, Figs. 1(a) and 1(b).

(1) These models satisfy condition (11), for $-\infty<\tau<+\infty$ when the internal space expands to a constant size, or (2) they have the property of double (for $D>1$) or single inflation, or (3) they do not implement this idea.

The critical point P_1 (a stable node) represents the solution with static internal space and with the Milne phase of the evolution of physical space $-R\sim t$. This point lies on the boundary of the constraint condition corresponding to vacuum models. One can notice that the models with contracting microspace $(y<0)$ do not implement the idea of inflation as an effect of extra dimensions. The topological structure of phase space for $D>1$ is independent of the number of dimensions of internal space as well as of the equation of state. Thus

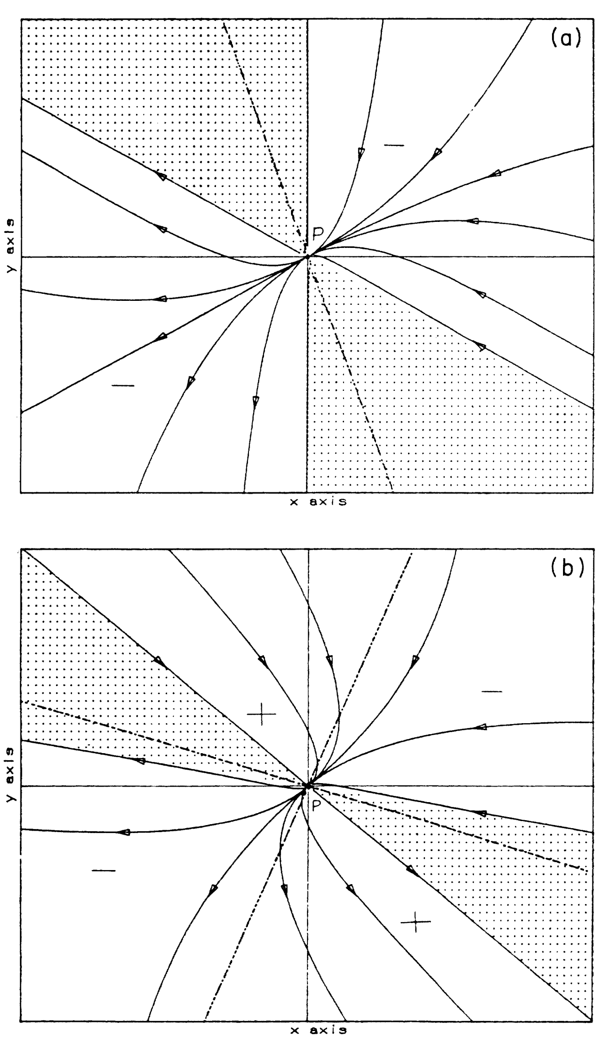

FIG. 2. The phase portraits for spacetime FRW$(k=0)\times T^D$ model. (a) FRW$(k=0)\times T^1$ model. (b) FRW$(k=0)\times T^D$ model.

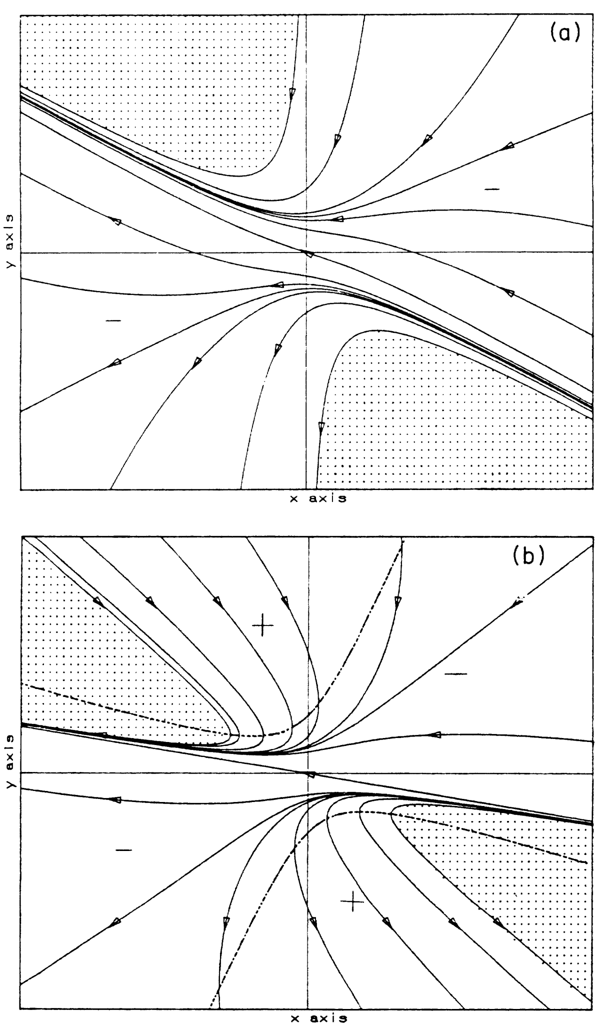

FIG. 3. The phase portraits for spacetime FRW$(k=+1)\times T^D$ model. (a) FRW$(k=+1)\times T^1$ model. (b) FRW$(k=+1)\times T^6$. Sign of $dx/d\tau$ (+) denotes regions of the phase space in which inflation takes place.

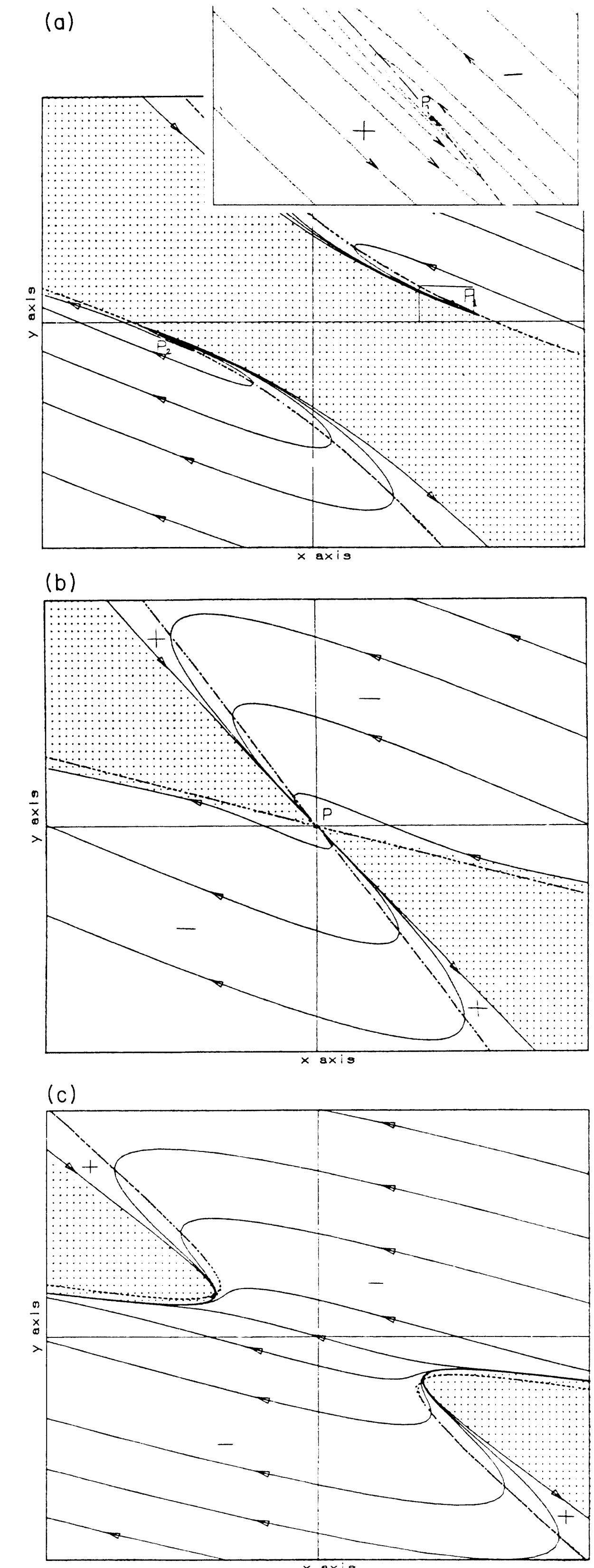

FIG. 4. Behavior of trajectories of the dynamical system for spacetime: FRW$(k=0,\pm1)\times T^7$ model in $N=1$, $d=11$ theory of supergravity. (a) FRW$(k=-1)\times T^7$ model. (b) FRW$(k=0)\times T^7$ model. (c) FRW$(k=+1)\times T^7$ model.

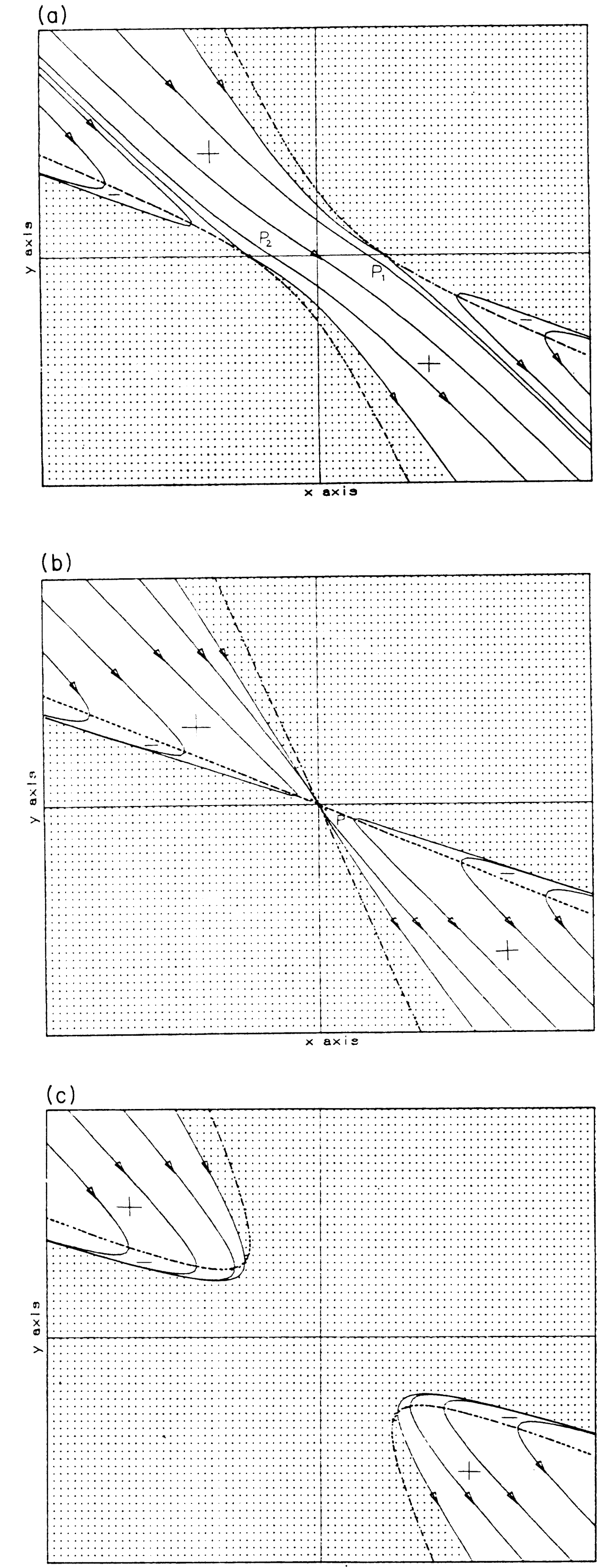

FIG. 5. The phase portraits for spacetime: FRW$(k=0,\pm1)\times T^6$ model with low-temperature quantum effects. (a) FRW$(k=-1)\times T^6$ model. (b) FRW$(k=0)\times T^6$ model. (c) FRW $(k=+1)\times T^6$ model. Sign of $dx/d\tau$ (+) denotes regions of phase space in which inflation takes place.

the case $D=6$ with the radiative matter is a representative one.

$\mathrm{FRW}(k=0)\times T^D$ models, Figs. 2(a) and 2(b), do not satisfy condition (11) for $D=1$, but for $D>1$ they satisfy it (for $-\infty<\tau<\tau_0$) in the class of models with expanding internal space. As in the previous case this effect is stable in the sense that there exist close trajectories with the same property. The critical point P represents a solution {Minkowski} × {static internal space}, and is degenerate. Therefore, the whole system is structurally unstable and even a small change of the right-hand sides of the respective equations changes the topological structure of phase space.

$\mathrm{FRW}(k=+1)\times T^D$ models [Figs. 3(a) and 3(b)] for $D=1$, do not implement the idea of inflation as an effect of extra dimensions. For $D>1$, they satisfy this condition in the interval $-\infty<\tau<\tau_0$.

Case (B) The phase portraits, Figs. 4(a)–4(c), represent the evolution of models originating from the supergravity theory with the Freund-Rubin ansatz. The topological structure of phase space is independent of the dimension D of internal space. Figures 4(a)–4(c) illustrate the dynamics of the respective models.

$\mathrm{FRW}(k=-1)\times T^D$ models in supergravity theory have the property of "infinite inflation;" i.e., there exists an infinite number of intervals $[\tau_0^i,\tau_1^i]$ in which condition (11) is satisfied. This follows from the fact that the critical points P_1 (stable focus) and P_2 (unstable focus), representing the asymptotic states of models lie on the boundary of (11). The critical points P_1 and P_2 correspond to the situation when the curvature term $1/R^2$ is proportional to the material term $1/r^{2D}$.

$\mathrm{FRW}(k=0,+1)\times T^D$ models undergo inflation in the interval $-\infty<\tau<\tau_0$ (or $\tau_0<\tau<+\infty$).

Case (C) The dynamics of $\mathrm{FRW}\times T^D$ models with low-temperature quantum effects is presented in Figs. 5(a)–5(c). These models satisfy condition (11), for $-\infty<\tau<\tau_0$. The topological structure of phase space is independent of the dimension D of interval space for a given k. The critical points P_1 and P_2 [Fig. 5(a)] are both unstable saddles and only a zero-measure set of trajectories can lead to a static internal space.

IV. CONCLUSION

In this paper we have characterized the class of $\mathrm{FRW}\times T^D$ models for which the idea of generalized inflation as a dynamical effect of extra dimensions is realized. Some of these models [e.g., $\mathrm{FRW}(k=-1)\times T^D$ with radiation] additionally implement the idea of dynamical dimensional reduction. We have noticed that the models with expanding microspace $\mathrm{FRW}\times T^D$ with radiation are the most interesting from the point of view of generalized inflation. An interesting effect is that the $\mathrm{FRW}(k=-1)\times T^D$ model, in supergravity theory with the Freund-Rubin ansatz, can pass through an inflationary phase infinitely many times. The above effects are stable in the sense that there exist close trajectories with the property.

Since first submitting this paper, a paper by Mingemi and Wiltshire[9] has recently appeared which is worth noticing in the context of properties of multidimensional world models.

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