

Constraints on t -quark mass and quark mixings from $K_L \rightarrow \mu\bar{\mu}$, and relations to other rare decays

C. Q. Geng and John N. Ng

TRIUMF, Theory Group, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3

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Incorporating the recent measurements of the branching ratio $(K_L^0 \rightarrow \mu\bar{\mu})/(K_L^0 \rightarrow \text{all})$ we have extracted newer constraints on the t -quark mass and the unknown Kobayashi-Maskawa quark-mixing parameters. This is used to estimate the branching ratio of $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ in the minimal standard model; it is found to lie in the tighter range of $(1-2.5) \times 10^{-10}$. Effects of a lower branching ratio for $K_L^0 \rightarrow \mu\bar{\mu}$ have a minimal impact on other rare kaon and B -meson decays.

The study of kaon rare decays has played a pivotal role in formulating the standard model (SM) of electroweak interactions. It is now attracting renewed interest, partly due to the prospect of significantly improved experiments now underway. Another important recent development is that the lower limit on the t -quark mass m_t is being steadily pushed upwards. Within the context of the SM, the lower limit is $m_t > 78 \text{ GeV}/c^2$. This implies that the short-distance contributions due to virtual t -quark exchanges in rare decays of kaons have become increasingly important.

In this paper we focus on the decay

$$K_L^0 \rightarrow \mu^+ \mu^- \tag{1}$$

and the information on m_t and quark-mixing parameters that one can extract from the branching ratio of (1). This in turn will constrain the branching ratio of decays such as

$$K^+ \rightarrow \pi^+ \nu\bar{\nu}, \tag{2}$$

where short-distance effects are believed to be dominant. Recently there are two new measurements² of (1) which lowered its previous value.³ Taking an average of all these measurements we find

$$R \equiv \frac{\Gamma(K_L^0 \rightarrow \mu\bar{\mu})}{\Gamma(K_L^0 \rightarrow \text{all})} = (7.34 \pm_{-0.66}^{+0.71}) \times 10^{-9}. \tag{3}$$

If we denote the real and imaginary parts of the amplitude for the decay (1) by $\text{Re}A$ and $\text{Im}A$, respectively, it is well known⁴ that $\text{Im}A$ is dominated by the two-photon in-

termediate state [see Fig. 1(a)]. This contribution to R , denoted by $R_{2\gamma}$, can be reliably calculated⁵ from the measured branching ratio³ of $\Gamma(K_L^0 \rightarrow \gamma\gamma)/\Gamma(K_L^0 \rightarrow \text{all}) = (5.70 \pm 0.23) \times 10^{-4}$ and gives the following unitarity bound for (1):

$$R \geq R_{2\gamma} = (6.83 \pm 0.28) \times 10^{-9}. \tag{4}$$

The difference between R and $R_{2\gamma}$ is

$$\Delta R = R - R_{2\gamma} = (0.51 \pm_{-0.94}^{+0.99}) \times 10^{-9}. \tag{5}$$

While Eq. (5) indicates that the decay (1) is consistent with the unitarity bound, it also implies that other possible contributions must be small. Among these are contributions from intermediate states such as $2\pi, \pi\pi\gamma$, etc. However, these states all make small contributions⁶ to $\text{Im}A$ compared to $\text{Im}A_{\gamma\gamma}$. The most interesting contribution comes from short-distance electroweak effects involving the exchange of virtual heavy quarks, in particular the t quark. This is depicted in Fig. 1. Short-distance (SD) physics is important for $\text{Re}A$. We write

$$\text{Re}A = \text{Re}A_{\text{LD}} + \text{Re}A_{\text{SD}}, \tag{6}$$

where LD represents the contribution from long-distance physics coming from intermediate states such as $2\gamma, 2\pi$, etc. We shall assume that no accidental cancellation exists between $\text{Re}A_{\text{LD}}$ and $\text{Re}A_{\text{SD}}$, and ΔR will then give an upper bound on $\text{Re}A_{\text{SD}}$. We know of no reliable way of calculating $\text{Re}A_{\text{LD}}$, and the above assumption appears to us to be a natural one to make. Explicitly, the branching ratio due to the short-distance contribution is^{7,8}

$$R_{\text{SD}} = \frac{\alpha^2}{4\pi^2 \sin^4 \theta_W} \frac{(1 - 4m_\mu^2/M_K^2)^{1/2}}{(1 - m_\mu^2/M_K^2)^2} \frac{|\text{Re} \sum_{i=c,t} \eta_i V_{is}^* V_{id} C(x_i)|^2}{|V_{us}|^2}.$$

$$B(K^+ \rightarrow \mu^+ \nu_\mu) \frac{\tau(K_L)}{\tau(K^+)} = 7.4 \times 10^{-5} |\eta_c C_\mu(x_c) + A^2 \lambda^4 (1 - \rho \cos \delta) \eta_t C_\mu(x_t)|^2, \tag{7}$$

where η_i are the QCD correction factors, $x_i = m_i^2/M_W^2$, and

$$C_\mu(x_i) = \frac{4x_i - x_i^2}{4(1 - x_i)} + \frac{3x_i^2 \ln x_i}{4(1 - x_i)^2}. \tag{8}$$

The lifetimes of $\tau(K_L)$ and $\tau(K^+)$ are 5.18×10^{-8} and 1.237×10^{-8} sec, respectively.³ In Eq. (7) we have used the Miani-Wolfenstein parametrization⁹ for the Kobayashi-Maskawa (KM) matrix¹⁰ for the quark mixings:

yashi-Maskawa (KM) matrix¹⁰ for the quark mixings:

$$V \approx \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\rho\lambda^3 e^{i\delta} \\ -\lambda(1 + A^2\rho\lambda^4 e^{-i\delta}) & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho e^{-i\delta}) & -A\lambda^2 & 1 \end{pmatrix}, \tag{9}$$

with $A = 1.05$ and $\lambda = 0.221$. It is straightforward to show

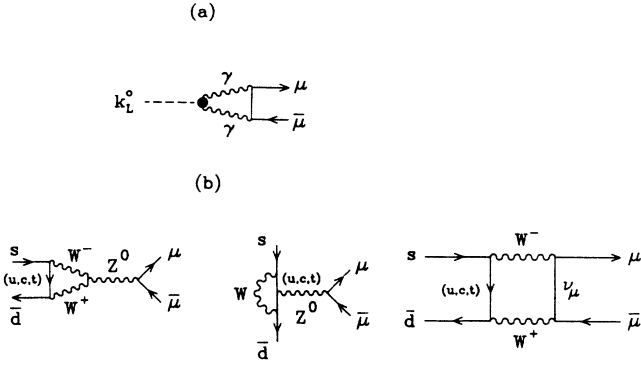


FIG. 1. Feynman diagrams depicting $K_L^0 \rightarrow \mu\bar{\mu}$ decays. (a) depicts two-photon intermediate-state contribution and (b) gives the short-distance contribution from the standard model.

that the dominant contribution in Eq. (7) arises from t -quark exchanges for a large t -quark mass ($m_t \sim M_W$). The QCD correction to Eq. (7) is negligible since there are no large logarithms of $\ln(M_W^2/m_t^2)$.^{11,12} Thus $\eta_t = 1$. Using Eqs. (7), (9), and (5), we obtain the constraint re-

$$B(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = 1.8 \times 10^{-6} \frac{|\sum_{i=c,t} \eta_i V_{is}^* V_{id} C_\nu(x_i)|^2}{|V_{us}|^2} = 1.8 \times 10^{-6} |\eta_c C_\nu(x_c) + A^2 \lambda^4 (1 - \rho e^{i\delta}) \eta_t C_\nu(x_t)|^2, \quad (12)$$

where

$$C_\nu(x_i) = \frac{x_i}{4} \left(\frac{3(x_i - 2)}{(x_i - 1)^2} \ln x_i + \frac{x_i + 2}{x_i - 1} \right). \quad (13)$$

In the case of Eq. (1) the t -quark contribution is most important. Here both c and t quarks are of equal importance. This is because, for $m_t \sim M_W$,

$$|C_\mu(x_c)/C_\mu(x_t)| \sim 5 \times 10^{-4} \quad (14)$$

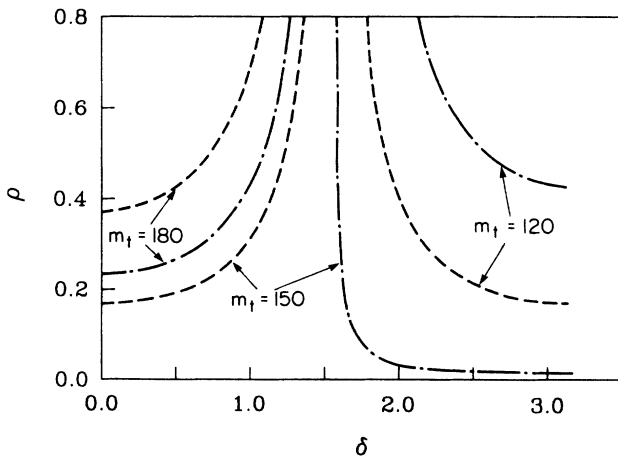


FIG. 2. Constraint on the ρ and δ parameters from $K_L^0 \rightarrow \mu\bar{\mu}$ decays for different values of m_t . The dot-dashed curves are for $\Delta R = 1.5 \times 10^{-9}$ and the short-dashed curves are for $\Delta R = 1.0 \times 10^{-9}$.

lations

$$1 + \frac{1.4}{|C_\mu(x_t)|} > \rho \cos \delta > 1 - \frac{1.4}{|C_\mu(x_t)|} \quad (10a)$$

and

$$1 + \frac{1.7}{|C_\mu(x_t)|} > \rho \cos \delta > 1 - \frac{1.7}{|C_\mu(x_t)|}, \quad (10b)$$

where the relations above hold for

$$\Delta R < 1.0 \times 10^{-9} \quad (11a)$$

and

$$\Delta R < 1.5 \times 10^{-9}, \quad (11b)$$

respectively. The lower bounds of (10a) and (10b) are automatically satisfied. Also $\rho < 0.8$ from the measurement of semileptonic b -quark decays.¹³ Figure 2 shows the region of the $\rho - \delta$ parameter space that is allowed by Eqs. (10a) and (10b) for a given m_t . For example, if $m_t = 150 \text{ GeV}/c^2$ and $\Delta R < 1 \times 10^{-9}$, the allowed values of ρ and δ are to the left of the dot-dashed curve in Fig. 2.

Next we examine the constraint obtained in Eqs. (10a) and (10b) on the kaon rare decays of $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ which is expected to be dominated by short-distance physics. The branching ratio is explicitly given by^{8,14}

whereas

$$|C_\nu(x_c)/C_\nu(x_t)| \sim 3 \times 10^{-3}. \quad (15)$$

The QCD corrections for Eq. (2) have been studied by many authors.^{15,16} These corrections are important for

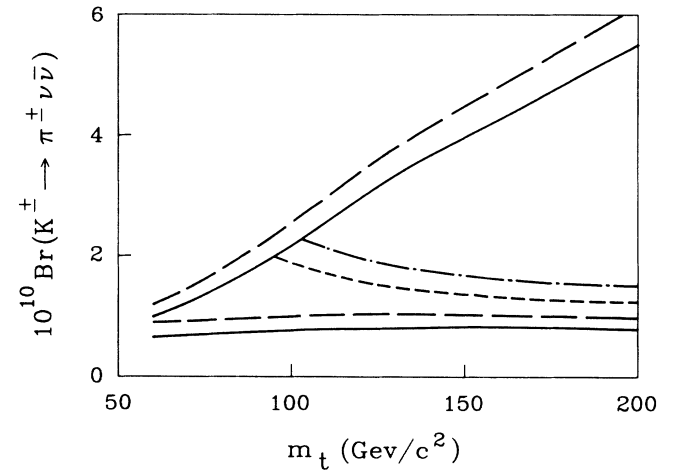


FIG. 3. The allowed branching ratio of $K^\pm \rightarrow \pi^\pm \nu\bar{\nu}$ as a function of m_t . The long-dashed and solid curves are the boundaries for this decay without the input from $K_L^0 \rightarrow \mu\bar{\mu}$. The long-dashed curve is obtained without QCD correction and solid curve has QCD corrections. The dot-dashed and short-dashed curves are boundaries taken into account $K_L^0 \rightarrow \mu\bar{\mu}$ decay. Legend is the same as in Fig. 2.

the c -quark contribution and they are negligible for a larger t -quark mass $m_t \geq M_W$.¹⁶ Such corrections tend to lower the branching ratio of Eq. (2) compared to the uncorrected results (see the dashed and solid curves in Fig. 3). Imposing the constraints of Eqs. (10a) and (10b) lowers the upper bound on the branching ratio of Eq. (11) for large values of m_t . It does not change the results for the lower bound in any significant way. This is displayed in Figs. 3 and 4 with the dot-dashed curves.

In Fig. 4 we show the effect of imposing Eq. (11) on the branching ratio of Eq. (2) as a function of the KM phase for four different values of $m_t = 60, 90, 120,$ and $180 \text{ GeV}/c^2$. The previous allowed region is obtained by using the ρ - δ parameters given in Ref. 17 where the authors have taken into account various experimental constraints relevant to the determination of KM parameters. As seen in Fig. 4, the region for the two low values of m_t are unaffected. On the other hand for $m_t = 180 \text{ GeV}/c^2$ a large portion of the previously allowed value of δ is now eliminated leaving only $0.25 < \delta < 0.95$.

The constraints on the short-distance physics from (1) are less dramatic in other rare decays of kaons. In particular, the currently popular $K_L^0 \rightarrow \pi^0 e^+ e^-$ decay is not much affected. The branching ratio of the direct CP -violating piece of the decay is proportional to $A^2 \lambda \rho \sin \delta$. A smaller (1) does not have a strong effect on this part. Obviously the indirect CP -violating piece of the decay which is proportional to ϵ will not be affected by Eqs. (10a) and (10b). Yet another place where the new lower value of R plays a role is in the polarization-asymmetry measurement¹⁸ in $K_L^0 \rightarrow \mu \bar{\mu}$ decays. Here the effect is not due to the short-distance physics but rather that $\text{Re}A$ is now constrained to be lower than before. With the central values in Eqs. (3) and (4), the maximum allowed asymmetry is reduced to 0.5 instead of 0.96 given by Ref. 18. Hence, the possible CP -violating signal is also reduced, although not by much. Furthermore, it is also easy to see that rare decays of a b quark such as $b \rightarrow s \gamma, b \rightarrow s \bar{l} l$ are also not affected much.

Finally, we remark on the literature that dealt with (1) as a test of the standard-model predictions. The use of (1) to restrict m_t and put rough bounds on quark-mixing angles was done in Refs. 7 and 8 and they played a central role in building up the standard model with three quark-lepton families. However, the further use of this procedure has been criticized recently^{19,20} based on model

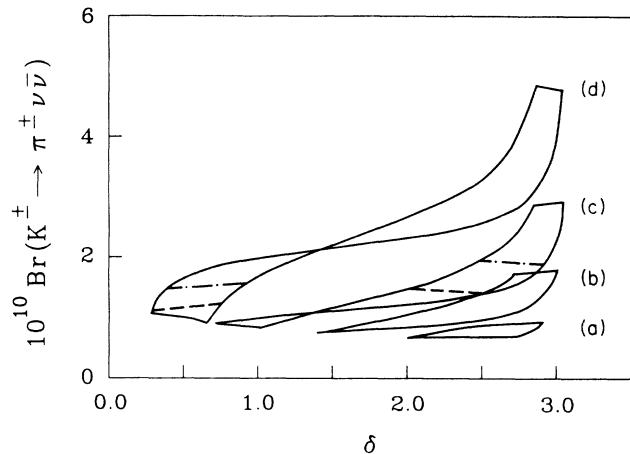


FIG. 4. Allowed branching ratio for $K^\pm \rightarrow \pi^\pm \nu \bar{\nu}$ with $K_L^0 \rightarrow \mu \bar{\mu}$ taken into account is given below the dot-dashed and short-dashed curves with the legend of Figs. 2(a)–2(d) are for $m_t = 60, 90, 120,$ and $180 \text{ GeV}/c^2$, respectively.

considerations of the long-distance effect in (1). Although A_{LD} cannot be reliably calculated we believe that the assumption that the long-distance piece and short-distance physics due to virtual heavy-quark exchanges do not accidentally cancel is a reasonable working hypothesis.²¹ Our work is quantitative analysis inputting the most pertinent data. Relating this information to other kaon rare decays, as far as we know, has not been done before. This result will be of use to experimentalists and serves as a benchmark prediction of SM. The decay (2) will be a stringent test of the standard model if it is done to the level of 1×10^{-10} . On the other hand, if the branching ratio is measured to be larger than 5×10^{-10} it will indicate that either new physics is at work or our assumption of no cancellation between long- and short-distance physics is invalid. This latter possibility can be tested by future precise measurement of the decay $\eta \rightarrow \mu \bar{\mu}$ which is currently underway.²² More theoretical studies are also required.

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