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Quantum chromodynamics and Bloom-Gilman duality

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We study the quantum-chromodynamic explanation of the long-established empirical connection, called Bloom-Gilman duality, between scaling and resonance regimes of the inelastic structure function $\nu W_2(\omega')$, where ν and ω' are the usual kinematic variables. We show how QCD expectations for the baryon transition form factors lead to the observed constancy with momentum transfer of the resonance/"background" ratio. We can also understand why the resonance contribution follows with changing Q^2, a curve whose shape is the same as the scaling-limit curve. We comment on the longitudinal response function and on possible contrasts in exciting resonances with different isospins. These await experimental scrutiny in newer-generation electron facilities.

Bloom and Gilman[1] observed some twenty years ago that the prominent resonances in inelastic electron-proton scattering do not disappear with increasing q^2 relative to a "background" under them, but instead fall at roughly the same rate as any background. Further, the smooth scaling limit seen at high Q^2 and W for the structure function $\nu W_2(\omega')$, where $\omega' = (2m_N\nu + m_N^2)/Q^2 = 1 + W^2/Q^2$, Q^2 and ν are the invariant mass squared and laboratory energy of the photon, m_N is the nucleon mass, and W is the invariant mass of the hadronic final state, is an accurate average for the resonance bumps seen at lower Q^2 and W, but at the same ω'. These two related observations have come to be known as Bloom-Gilman duality. As these authors pointed out, the connection between the behavior of the resonances and the scaling limit hints at a common origin for both. A QCD explanation of why the resonance bumps must average to the curve was given by De Rújula, Georgi, and Politzer.[2] They showed that at moderate Q^2 corrections to the lower moments of the structure function due to higher-twist effects (e.g., final-state interactions) are small while corrections to the higher moments are large. Hence at moderate Q^2, the average value of the structure function cannot be much different from its high-Q^2 value, but at any given point the change could be considerable. The main objective of our work is to explain, using the QCD predictions for the transition form factors in resonance electroproduction, the constancy of the resonance-to-background ratio with Q^2. This also explains why the resonance contributions to νW_2 tracks, with changing Q^2, a curve whose shape is the same as the scaling-limit curve. The QCD explanations of Bloom-Gilman duality also apply to the longitudinal structure function, for which precise data do not exist now but may become available in the future.

Let us begin with the resonances. The double-differential inelastic-electron-scattering cross section is

$$\frac{d\sigma}{d\Omega_e E_e'} = \sigma_M\left(W_2 + 2W_1\tan^2\frac{\theta}{2}\right), \qquad (1)$$

where θ is the electron scattering angle and σ_M is the Mott cross section

$$\sigma_M = \frac{4\alpha^2 E_e'^2\cos^2(\theta/2)}{Q^4}. \qquad (2)$$

At a resonance peak we have

$$W_2 = \frac{1}{1+\nu^2Q^2}\frac{m_N}{\pi m_R\Gamma_R}(2G_0^2 + G_+^2 + G_-^2). \qquad (3)$$

Here $G_{\pm,0}$ are the helicity amplitudes evaluated in the Breit frame for electroproduction of a resonance, characterized by mass m_R and width Γ_R, and are labeled by the virtual-photon helicity.[3] We have approximated the resonances as having a Breit-Wigner shape with a width independent of W. Note that with increasing Q^2, $\nu^2/Q^2 \rightarrow \nu/2m_N$.

The QCD predictions for $G_{\pm,0}$ at high Q^2 are based on calculations of Fig. 1. The gluon exchanges ensure that the final quarks, like the initial ones, have low relative momenta, so that no powers of Q^2 come from the initial or final wave functions. The counting proceeds by having one factor of Q for each unbroken fermion line, a factor of $1/Q$ for each internal fermion propagator, a factor of $1/Q^2$ for each gluon propagator, and an additional factor of $1/Q$

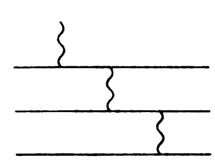

FIG. 1. Leading-order diagram for the transition amplitude G_+. The external wavy line is a photon and the internal wavy lines are gluons. Solid lines are quarks. The amplitudes G_0 and G_- require flipping one or two quark helicities, respectively.

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for each quark helicity flip. This gives

$$G_+ = \frac{g_+}{Q^3},\ G_0 = \frac{g_0}{Q^4},\ \text{and}\ G_- = \frac{g_-}{Q^5}, \tag{4}$$

where $g_{\pm,0}$ are constants up to factors of $\log Q^2$ and dependent on the wave functions of the nucleon and resonance. Thus, at high Q^2 for a given resonance,

$$\nu W_2 = \frac{2m_N^2}{\pi m_R \Gamma_R}\frac{g_+^2}{Q^6} + O\left(\frac{1}{Q^8}\right). \tag{5}$$

Finally, noting that $1/Q^2 = (\omega'-1)/W^2$, we get, with $W = m_R$,

$$\nu W_2 = \frac{2m_N^2}{\pi m_R \Gamma_R}\frac{g_+^2}{m_R^6}(\omega'-1)^3 + O((\omega'-1)^4). \tag{6}$$

This equation applies to each resonance. The $(\omega'-1)^3$ dependence is precisely the same as what is seen[1] as the $\omega' \to 1$ scaling behavior of νW_2 at high Q^2 and W. It is also what is obtained[4] using lowest-order perturbation theory and QCD to calculate the $\omega' \to 1$ behavior of inelastic electron scattering from a proton, neglecting final-state interactions among the quarks. Hence, with changing Q^2 a given resonance peak will move along a curve with the same $(\omega'-1)^3$ shape as the high-Q^2 scaling curve. Also, assuming, as we find natural, that the background under the resonance peak has the same form obtained from neglecting final-state interactions in the QCD calculation of the distribution function, the resonance-to-background ratio will be independent of Q^2. We can also note that if the background is made from the tails of many resonances,[5] then the above argument *a fortiori* gives an $(\omega'-1)^3$ dependence for the background. We shall return below to examine how constant the resonance-to-background ratio actually is empirically for each resonance; the $\Delta(1232)$ and other $I = \frac{3}{2}$ resonances will generally merit some special discussion.

Next we discuss the longitudinal inelastic distribution function, which is harder to access experimentally. The longitudinal structure function is

$$W_L = \left(1+\frac{\nu^2}{Q^2}\right)W_2 - W_1 = \frac{2m_N}{\pi m_R \Gamma_R}G_0^2, \tag{7}$$

the latter again being valid at a resonance peak. In terms of the longitudinal cross section

$$\sigma_L = \frac{8\pi^2 \alpha m_N}{m_R^2 - m_N^2} W_L, \tag{8}$$

we get

$$\sigma_L = \frac{16\pi \alpha m_N^2}{(m_R^2 - m_N^2) m_R \Gamma_R}\frac{g_0^2}{m_R^8}(\omega'-1)^4. \tag{9}$$

The QCD result[3] for the scaling behavior of σ_L is given as $(\sigma_L/\sigma_T) \sim m^2/Q^2$, where m is some characteristic mass or Fermi-momentum scale for the quarks, so that

$$\sigma_L \sim \frac{m^2}{W^2}(\omega'-1)\sigma_T \sim (\omega'-1)^4, \tag{10}$$

for $\omega' \to 1$. Again, the resonance peak moves with changing Q^2 along a curve with the same shape as the scaling curve, and the signal-to-background ratio does not change with Q^2. This is a distinct manifestation of Bloom-Gilman duality. There are currently insufficient data to test this prediction. The resonance peak and resonance-to-background ratio aspects of this can be tested by data to be available from the Continuous Electron Beam Accelerator Facility (CEBAF), now under construction, and the higher-Q^2 scaling curve must be measured at a higher-energy facility.

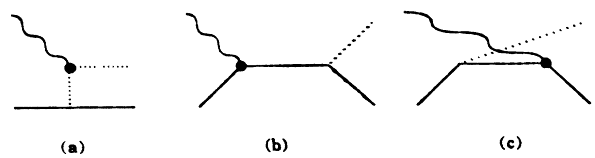

FIG. 2. Leading tree-level diagrams for the background mechanisms in the pseudoscalar effective Lagrangian theory to electroproduce pions (broken line). Wavy lines are virtual photons and solid lines are nucleons. Large solid circles represent photon-hadron interaction vertices. The three contributions shown here are (a) t-channel pion exchange, (b) s-channel nucleon exchange, and (c) u-channel nucleon exchange.

The background may also be considered from a meson-baryon point of view.[6] Considering electroproduction of pions, with a pseudoscalar[7] pion-nucleon coupling, some leading tree diagrams are shown in Fig. 2. Assuming the pion to have a monopole electromagnetic form factor and the nucleon a dipole electromagnetic form factor, and giving the form factors the same behavior even when one of the hadron legs is off shell, leads to

$$\begin{aligned} M_a &\sim Q F_{\pi\gamma}(Q^2)\frac{1}{Q^2}F_{\pi NN}(Q^2) \sim \frac{1}{Q^3}F_{\pi NN}(Q^2), \\ M_b &\sim Q F_{N\gamma}(Q^2)\frac{1}{Q}F_{\pi NN}(Q^2) \sim \frac{1}{Q^4}F_{\pi NN}(Q^2), \\ M_c &\sim \frac{1}{Q^4}F_{\pi NN}(Q^2), \end{aligned} \tag{11}$$

where M_j are the amplitudes corresponding to diagrams 2(a)–2(c) [the normalization is such that $\gamma^* N \to \pi N$ would have $d\sigma/dt = (1/s^2)|M_i|^2 = (1/W^4)|M_i|^2$]. Thus for high Q^2, the t-channel pion exchange, Fig. 2(a), dominates. However for any falling $F_{\pi NN}$, this single diagram falls faster than the $1/Q^3$ expected from the QCD background calculation. But at high Q^2 one can expect that other mesons including pion recurrences will be exchanged. What this will do depends very much on the density of states and the couplings of the higher states. With only pion recurrences and a Regge-like behavior, the nth recurrence has a mass squared $M_n^2 \simeq n$ in $(\text{GeV})^2$ units. Putting in a dipole falloff also with a mass M_n for the $\pi_n NN$ vertex, and including the mass in the propagator, gives

$$\begin{aligned} M_a &\sim \sum_n \frac{1}{Q}\frac{1}{(Q^2+M_n^2)^2} \\ &\simeq \int_0^\infty dn \frac{1}{Q}\frac{1}{(Q^2+n)^2} \\ &= \frac{1}{Q^3}. \end{aligned} \tag{12}$$

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Including an M_n dependence at the photon vertex does not change the power of this result, and the power is in accord[8] with QCD. We should note that a Hagedorn-type density of states,[9] $\rho(m) \sim e^{m/T_0}$, leads to a divergent result unless some couplings to higher states are cut off. We may believe that meson-baryon theory must lead to the same scaling results as QCD, but the foregoing discussion indicates that this result does not emerge trivially.

Two comments need to be made about the $I=\frac{3}{2}$ resonances, i.e., the Δ's. The first is that, experimentally, the Δ(1232) bump falls noticeably with Q^2 relative to its background. This is in great contrast to the second resonance bump, at about 1520 MeV, which is very steady in Q^2 relative to its background.[10] Our earlier analysis of course assumed that there was no accident from the hadron structure that made the leading helicity amplitude anomalously small. This need not be true. If, in our earlier notation, $g_+ \simeq 0$, then the data over the whole measured Q^2 range would be dominated by next-to-leading amplitudes which fall more quickly than the background. This leads to the second comment, that at least for one nucleon wave function (or more properly, "distribution amplitude"), that of Chernyak and Zhitnitsky,[11] there is a seemingly accidental cancellation that makes g_+ for the Δ(1232) quite small.[12] The cancellation is not strongly dependent on the Δ wave function except for the Δ being $I=\frac{3}{2}$ gives characteristic weight to contributions from different parts of the nucleon wave function that cancel each other.

The second resonance region, at about 1520 MeV, is dominated by an $I=\frac{1}{2}$ resonance. Let us then turn our attention to the third resonance region, at about 1670 MeV, where the resonance-to-background ratio falls somewhat[10] with Q^2, but not as dramatically as the Δ(1232) does. This bump contains a number of resonances, some $I=\frac{1}{2}$ and some $I=\frac{3}{2}$. We may speculate that the bump is falling because the $I=\frac{3}{2}$ resonances are falling with respect to the background, while the $I=\frac{1}{2}$ resonances are not. New data separating the various contributions to the third resonance region would be very valuable and can be expected from CEBAF.

We have focused on the power-law scaling behavior of the resonances form factors and of νW_2. There are also scaling violations involving $\ln(Q^2/\Lambda^2)$. Including these logarithmic terms in the resonance form factors gives for G_+,

$$G_+(Q^2)=\frac{\alpha_s^2(Q^2)}{Q^3}\sum_{m,n} a_{m,n}(\ln Q^2/\Lambda^2)^{-\gamma_m-\gamma_n} ,$$

where γ_i are anomalous dimensions. The $a_{m,n}$ involve the wave functions of the nucleon and the produced resonance,

$$a_{m,n}=b_m^{(N)}b_n^{(R)}\int [dx][dy]\tilde{\phi}_m^*(y)T_H(x,y)\tilde{\phi}_n(x) ,$$

where T_H is the hard-scattering amplitude (Fig. 1), the $\tilde{\phi}_i^*$ are Appel polynomials, x and y represent the momentum fractions of quarks in the incoming and outgoing baryons, and the $b_j^{(R)}$ are coefficients in the expansion of the resonance distribution amplitude:

$$\phi_R(x)=\chi_1\chi_2\chi_3\sum_m b_m^{(R)}\tilde{\phi}_m(x)(\ln Q^2/\Lambda^2)^{-\gamma_m} .$$

It is not entirely clear that the logarithmic corrections are the same for the background as for the resonance peaks. If we view the background as a sum of other resonances's tails, then the relative weighting of the different powers of $\ln Q^2/\Lambda^2$ may change. Hence there can be some change in resonance/background ratio due to the logarithmic scale breaking (as well as due to still incomplete suppression of terms falling with higher powers of $1/Q^2$). For the nucleon elastic form-factor case, most of the scale breaking seems to be due to the logarithms in $\alpha_s(Q^2)$; if the same is true in general for the transition form factor, then the resonance/background ratio would be insensitive to the scale breaking. While the power law is the dominant Q^2 dependence in the high-Q^2 behavior of the form factors, seeking the logarithmic scaling violations would be interesting if the data become precise enough. With or without scaling violations, the resonances must still average to the scaling curve by the arguments of De Rújula, Georgi, and Politzer.[2]

In summary, we have examined the quantum-chromodynamic explanation of the Bloom-Gilman duality, focusing particularly on the constancy of the resonance/background ratio; we have shown how the corresponding arguments work for the longitudinal case; we have shown that meson-baryon theory will not trivially give the same results as QCD at high Q^2; we also have commented on the possibly contrasting behavior of $I=\frac{1}{2}$ and $I=\frac{3}{2}$ resonances at high Q^2. Finally, we have suggested new experimental opportunities. More theoretical work still needs to be done, particularly on the question of the Q^2 regime of validity for Bloom-Gilman duality and questions of absolute normalization.

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*Visitor.

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[7]Using a pseudovector pion-nucleon coupling brings in two difficulties here: First, it is a nonrenormalizable theory; second, it gives a seagull term, after electromagnetic gauging, for which QCD has no prediction yet. However, for low-energy applications in pion physics, pseudovector coupling in effective Lagrangians remains the preferred choice. See N. Dombey and B. J. Read, Nucl. Phys. **B60**, 65 (1973).

[8]We believe that a calculation done with a complete basis will give the correct answer. In particular, calculating a high-momentum-transfer nuclear process with a meson-baryon basis should give the same answer as a quark basis. In practice, such a calculation requires summing over many intermediate states and knowing their couplings and form factors. It is not actually proved that even the scaling behavior of the result obtained in the baryon theory at high momentum transfer will be the same as the QCD result. [For an agreement in the two cases, see the example of electron-deuteron scattering, C. E. Carlson and F. Gross, Phys. Rev. Lett. **53**, 127 (1984). See also R. Woloshyn, *ibid.* **36**, 220 (1976).] The question here is what happens in the present process? If we think that $\gamma^* + N \rightarrow \pi + N$ dominates the background under the resonance bump, we are faced with the observation that summing over a finite number of meson or baryon intermediate states gives too fast a falloff with Q^2, compared with the QCD result. What happens if we sum over all intermediate meson and baryon states than can contribute to a given Q^2? A truncated example of this is given in the text. But the general case has not been studied so far. Finally, we should note that QCD does not support a monopole meson-nucleon form factor.

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