Simple relativistic quark-model analysis of flavored pseudoscalar mesons

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Using a simple relativistic quark model suggested by Dziembowski *et al.*, we study the open flavored pseudoscalar meson, i.e., $Q\bar{q}$, where Q is s or c and q is u or d. Our analysis, which is a straightforward extension of their previous treatment for the pion, focuses on the kaon to provide further tests of this approach. Our results support the utility of this model as the predictions for the kaon charge radius $\langle r_K^2 \rangle^{1/2}$, the kaon form factor F_K, and the decay constant f_K compare favorably with experimental data.

I. INTRODUCTION

Because a rigorous QCD calculation for hadronic wave functions is currently not feasible, it is important to have QCD-inspired models which, at a minimum, realistically describe hadron static properties. Such models, if they are to be seriously regarded, also should possess the same underlying symmetries that would be present in the exact solution, have explicit quark degrees of freedom, provide quark confinement and be properly gauge and Lorentz invariant. Recently, Dziembowski *et al.*[1] presented a simple hadron model which contains these fundamental ingredients. In their model, they utilized a harmonic-oscillator wave function to govern the behavior, including confinement, of interacting valence quarks and they implemented the light-cone formalism to provide a proper relativistic treatment. In view of the simplicity of this model, their numerical results for the hadron static properties and electromagnetic form factors are rather impressive. Even more remarkable, the hadronic quark distribution amplitudes generated in this model exhibit the "bump-dip" momentum-fraction signature similar to that provided by the QCD sum-rule method.[2] More recently, they[3] also used this model to analyze new European Muon Collaboration (EMC) data concerning the proton spin asymmetry.

In this Brief Report we generalize this approach by extending the model to flavored pseudoscalar mesons in which the constituent quarks have unequal masses, i.e., $Q\bar{q}$, where Q is s or c and q is u or d. We present numerical predictions for static properties of the strange, K, and charmed, D, mesons and find that the extended model, without any additional parameters, can consistently reproduce the rms radii and decay constants for both the pion and kaon (data are very limited for the heavier flavored mesons). In anticipation of future kaon electroproduction measurements we also make predictions for the kaon form factor.

In the next section we extend the model of Ref. 1 and derive general analytic formulas for the electromagnetic form factor and the decay constant which are valid for any pseudoscalar meson. In Sec. III we present our numerical results and comparison with the available experimental data. Finally, we conclude this paper in Sec. IV with a summary.

II. MODEL EXTENSION AND ANALYTIC RESULTS

Even though the model details have been previously specified,[1] we briefly summarize the essential aspects of this approach before presenting new analytic results of the extended model. The model is based upon the light-cone quantization method[4] which provides an improved Fock-state expansion for hadron states since in the light-cone formalism the vacuum and hadronic states are rigorously orthogonal.[5] The key approximation in this approach is to truncate the expansion by retaining only the lowest Fock state. For example, the open flavor meson state $|M\rangle$ considered in this paper is represented by

$$|M\rangle = \psi^M_{Q\bar{q}} |Q\bar{q}\rangle \,, \tag{2.1}$$

where $|Q\bar{q}\rangle$ is the two-body Fock state for a heavy quark Q and a light antiquark $\bar{q}$. Here the model wave function $\psi^M_{Q\bar{q}}$ is given by the product of the light-cone harmonic-oscillator wave function Φ, which is prescribed by Brodsky, Huang, and Lepage,[6] and the light-cone spin wave function χ, which is Melosh[7] transformed from the equal-t static spin wave function. These wave functions are functions of the Lorentz-invariant variables $x_i = P_i^+/P^+$, $\mathbf{k}_{\perp i} = \mathbf{p}_{\perp i} - x_i \mathbf{P}_\perp$, and λ_i, where $P^\mu = (P^+, P^-, \mathbf{P}_\perp) = (P^0 + P^3, (m_M^2 + \mathbf{P}_\perp^2)/P^+, \mathbf{P}_\perp)$ is the momentum of the meson M, and P_i^μ and λ_i are the momentum and the helicity of constituent quarks, respectively. The particle masses are specified by m_M for the meson and m_i $(i=1,2)$ for the two quarks. Thus, in our case, $\psi^M_{Q\bar{q}}$ is given by

$$\psi^M_{Q\bar{q}} = \psi(x_i, \mathbf{k}_{\perp i}, \lambda_i) = \Phi(x_i, \mathbf{k}_{\perp i}) \chi_M(x_i, \mathbf{k}_{\perp i}, \lambda_i) \,, \tag{2.2}$$

where

$$\Phi(x_i, \mathbf{k}_{\perp i}) = A \exp\left[-\sum_{i=1}^{2} \frac{\mathbf{k}_{\perp i}^2 + m_i^2}{x_i} \Big/ 8\beta^2 \right] \tag{2.3}$$

and

$$\chi_M(x_i, \mathbf{k}_{\perp i}, \lambda_i) = \bar{u}_{\lambda_1} (m_M + \not{P}) \gamma_5 v_{\lambda_2} \,. \tag{2.4}$$

Notice that β, the oscillator parameter, is the only dynamical parameter entering the model. The constant

A is fixed by normalizing $\psi^M_{Q\bar{q}}$ to one. Using this wave function, we have extended the pion-form-factor result[8] by deriving a general open flavored pseudoscalar-meson form-factor formula:

$$F_M(Q^2)=e_1G_M(\xi^2)+e_2G_M(\xi'^2)\ , \tag{2.5}$$

with

$$G_M(\xi^2)=N\int_0^1\frac{dx}{x(1-x)}h(x)\times\exp\left[\frac{-[\xi^2+m_1^2+x(m_2^2-m_1^2)]}{x(1-x)}\right],$$

$$N=2\left[\frac{2\beta^3A}{\pi P^+}\right]^2,$$

$$h(x)=(a_1a_2)^2-2a_1a_2[\xi^2+x(1-x)]-(a_1+a_2)^2[\xi^2-x(1-x)]+\xi^4+2(x-x^2)^2\ ,$$

$$\xi^2=\frac{(1-x)^2Q^2}{16\beta^2},\quad \xi'^2=\left[\frac{x}{1-x}\right]^2\xi^2\ ,$$

$$a_1=\frac{xm_M+m_1}{2\beta},\quad a_2=\frac{(1-x)m_M+m_2}{2\beta}\ .$$

The constant A introduced in Eqs. (2.2) and (2.3) ensures the correct meson total charge $F_M(0)=e_1+e_2$. Also notice that Eq. (2.5) is properly symmetric under the exchange of 1 and 2 since the results must be independent of the quark number assignment. For example, for the K^+ meson one can assign either $e_1=e_u=\frac{2}{3}$, $e_2=e_{\bar{s}}=\frac{1}{3}$, $m_1=m_u$, $m_2=m_{\bar{s}}$, or $e_1=e_{\bar{s}}=\frac{1}{3}$, $e_2=e_u=\frac{2}{3}$, $m_1=m_{\bar{s}}$, $m_2=m_u$. Finally, for the pion case, assuming $m_u=m_d$, the two integrations in Eq. (2.5) are identical and our result reduces to the previous expression for the pion given by Dziembowski.[8]

The charge radius of the meson can be determined by

$$\langle r^2\rangle_M=-6\frac{dF_M(Q^2)}{dQ^2}\bigg|_{Q^2=0}\ , \tag{2.6}$$

where the electromagnetic form factor $F_M(Q^2)$ is given by Eq. (2.5). Likewise, we have also extended the quark distribution amplitude[9] formula for the pion to a more general form:

$$\phi(x_i)=\frac{4\beta^4A}{\pi^2P^+}\exp\left[-\frac{x_1m_2^2+x_2m_1^2}{8x_1x_2\beta^2}\right]\times(a_1a_2-2x_1x_2)\ . \tag{2.7}$$

Again for $m_1=m_2$, Eq. (2.7) also exactly reproduces the results given by Dziembowski *et al.* Finally, the general expression for the pseudoscalar-meson decay constant is

$$f_M=\frac{8\sqrt{3}\beta^4A}{\pi^2P^+}\int_0^1 dx[a_1a_2-2x(1-x)]\times\exp\left[-\frac{m_1^2+x(m_2^2-m_1^2)}{8x(1-x)\beta^2}\right]. \tag{2.8}$$

In the next section these formulas are applied to generate numerical results to describe the flavored-meson static properties.

III. NUMERICAL RESULTS

For the numerical calculation, we used the following constituent-quark masses:

$$m_u=m_d=330\text{ MeV}\ ,\quad m_s=450\text{ MeV}\ ,\quad m_c=1.5\text{ GeV}\ . \tag{3.1}$$

The u- and d-quark masses were taken from a recent analysis of nucleon and pion electromagnetic form factors by Dziembowski.[8] For the heavier s and c quarks we have taken mass assignments from Ref. 10 and, because these values are less certain, we have also detailed below the numerical sensitivity to variation in these parameters. As argued by Dziembowski *et al.*,[1] we also used spin-averaged meson masses:

$$M_\pi=612.4\text{ MeV},\quad M_K=792.5\text{ MeV}\ ,\quad M_D=1.9749\text{ GeV}\ . \tag{3.2}$$

To constrain the model parameter β, Dziembowski *et al.* argued that it should be related to the value of the quark transverse momentum and they used $\beta\approx320$ MeV for the pion. In our analysis, we simply show results for several β values to document the overall sensitivity of the model predictions. In Table I our results are summarized and compared with data.[11] Notice that there is some sensi-

TABLE I. Summary of numerical results for the static properties of various mesons.

β (MeV)	300	320	340	360	Data
$\langle r^2\rangle_{\pi^+}$ (fm^2)	0.44	0.41	0.38	0.36	0.44±0.05
f_π (MeV)	97	93	88	82	93
$\langle r^2\rangle_{K^+}$	0.41	0.38	0.35	0.33	0.34±0.05
$\langle r^2\rangle_{K^0}$	−0.055	−0.050	−0.046	−0.042	−0.054±0.026
f_K	121	122	121	120	113
$\langle r^2\rangle_{D^+}$	0.32	0.29	0.26	0.24	
f_D	112	122	131	141	< 183

tivity to different values for β=300, 320, 340, and 360 MeV. For the pion, we reproduced the published results of Dziembowski[8] and also confirmed that the value β=320 MeV generates the best agreement with the experimental pion data. For the kaon, however, the best value to reproduce the measured kaon charge radius is β=340 MeV for the K^+ and β=300 MeV for the K^0. Thus, the "average" value of β for the kaon system is also 320 MeV. We also present results for the D meson even though there is limited experimental data.

We have also investigated the sensitivity of all calculated K and D observables to variations in the s- and c-quark masses, respectively. Changing m_s by $\pm 10\%$ produces a 1 to 2% variation in the decay constant and in the rms radius for the K^+. For the K^0, which would have a zero charge radius if $m_u = m_s$, the calculated rms charge radius is, not surprisingly, more sensitive to m_s since a $\pm 10\%$ variation in m_s produces a 10 to 15% change in $\langle r^2 \rangle_{K^0}^{1/2}$. Because m_c is better known[12] than m_s we have varied m_c only to determine if our model has the proper asymptotic behavior. In particular for large m_c one can show, in the nonrelativistic limit, that f_D falls as $(m_c)^{-1/2}$. For m_c variations up to 5 GeV our calculations approximately reproduce this result and we attribute numerical deviations to m_c not yet being asymptotically large. This also explains in part why, for small β values, f_K is actually larger than f_D in Table I.

Finally, using β=340 MeV, we have calculated the K^+ form factor for Q^2 between 0 and 2 GeV^2. This result is given in Fig. 1. For comparison, we also show the result based on vector-meson dominance $[F_K = (1+Q^2/m_V^2)^{-1}$, m_V=vector meson mass]. Unfortunately the kaon form factor is not accurately known and therefore it is not currently possible to distinguish between our predictions and those provided by other groups[13] using alternative models. However, future kaon electroproduction experiments, as envisioned at CEBAF, will permit stringent, definitive model tests.

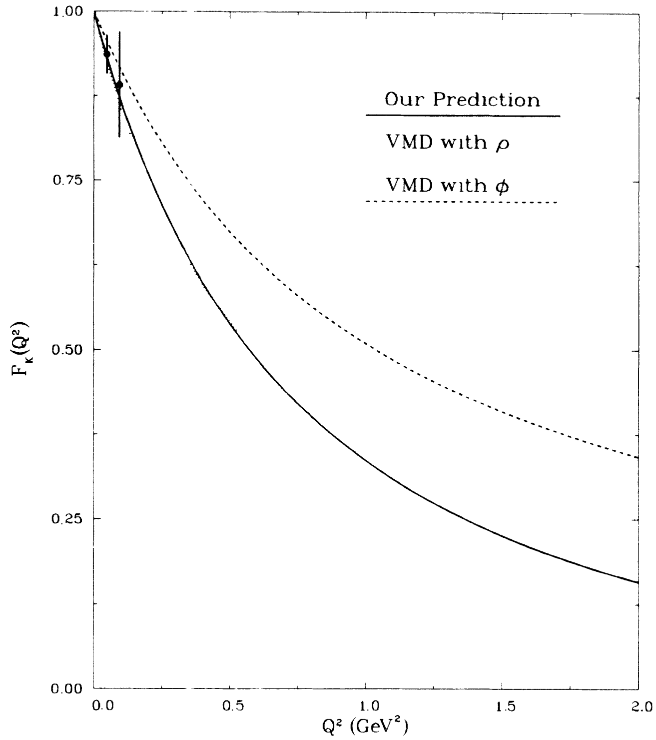

FIG. 1. Kaon electromagnetic form factor $F_K(Q^2)$ at $0<Q^2<2$ GeV^2. Experimental data are from Ref. 10.

IV. CONCLUSION

In this paper, we extended the simple relativistic quark model proposed by Dziembowski *et al.*[1] to investigate the static properties of the open-flavored mesons. It is significant to note that the extended model also entails only one parameter β. While calculations are somewhat sensitive to different β values, the value $\beta \simeq 320$ MeV can describe both pion and kaon static properties. Because this model provides a good description of the pion form factor at low Q^2 we have also calculated the kaon form factor in the region $0<Q^2<2$ GeV^2. We look forward to future kaon electroproduction experiments to test our predictions.

The validity of this model can be further tested through applications to other systems such as mesons having nonzero angular momentum, baryon octets[14] and also by reproducing the nonstatic properties given by quark distribution amplitudes of mesons and baryons. Such studies are in progress and will be reported in a future communication. In conclusion, this model, while both conceptually and computationally simple, provides a remarkably good description of static properties for the N, π, ρ, K hadrons and clearly merits further investigation.

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