PHYSICAL REVIEW D VOLUME 41, NUMBER 7 1 APRIL 1990

The equivalence theorem

H. Veltman
Department of Physics, University of California, Berkeley, California 94720
and Theoretical Physics Group, Physics Division, Lawrence Berkeley Laboratory, 1 Cyclotron Road, Berkeley, California 94720
(Received 21 August 1989)

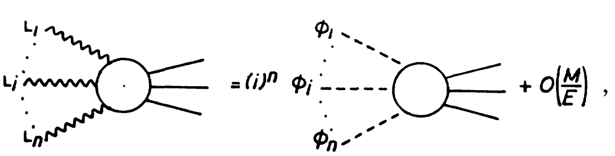
The equivalence theorem states that, at an energy E much larger than the vector-boson mass M, the leading order of the amplitude with longitudinally polarized vector bosons on mass shell is given by the amplitude in which these vector bosons are replaced by the corresponding Higgs ghosts. We prove the equivalence theorem and show its validity in every order in perturbation theory. We first derive the renormalized Ward identities by using the diagrammatic method. Only the Feynman–'t Hooft gauge is discussed. The last step of the proof includes the power-counting method evaluated in the large-Higgs-boson-mass limit, needed to estimate the leading energy behavior of the amplitudes involved. We derive expressions for the amplitudes involving longitudinally polarized vector bosons for all orders in perturbation theory. The fermion mass has not been neglected and everything is evaluated in the region $m_f \approx M \ll E \ll m_{\rm Higgs}$.

I. INTRODUCTION

Lately the equivalence theorem[1-4] has gained some interest.[5,6] The equivalence theorem states that for a process which takes place at a center-of-mass energy much larger than the mass M of the vector boson the longitudinally polarized vector boson behaves like the Higgs ghost. For large energy E, the polarization vector ϵ_L^μ of the longitudinally polarized vector boson W_L may be approximated as

$$\epsilon_L^\mu = \frac{k^\mu}{M} + O\left[\frac{M}{E}\right], \tag{1.1}$$

with k^μ the momentum vector of W_L. The equivalence theorem can be expressed in diagrammatic form as



(1.2)

where the outgoing straight lines denote physical particles other than W_L. Thus the leading energy term for a process containing external W_L's is obtained by calculating the amplitude where these W_L's are replaced by the corresponding Higgs ghosts. In the above equation "$O(M/E)$" means that the equation is good up to order M/E compared to the leading energy term. The equivalence theorem is very useful since it is much easier to calculate amplitudes involving on-mass-shell Higgs ghosts.

The equivalence theorem was first proven at the tree level in the renormalizable standard model by Cornwall, Levin, and Tiktopoulos[3] using a Stükelberg-type gauge. Lee, Quigg, and Thacker[2] subsequently sketched a proof to higher orders, using the functional method in the Feynman–'t Hooft gauge, but with only one external longitudinal leg. Chanowitz and Gaillard,[1] followed by Gounaris *et al.*,[4] claimed to extend the proof to all orders in perturbation theory and with any number of external W_L's in a general R_ξ gauge. In Ref. 1 the functional method was being used and started with the assumption that the renormalized functional for one-particle-irreducible Green's functions is invariant under the (appropriately renormalized) Becchi-Rouet-Stora-Tyutin (BRST) transformations. In Ref. 4 a simpler proof was given using the same starting assumption. It is this starting assumption that has been questioned in the literature.[5] In all cases only the renormalizable standard model has been considered.

The object of this paper is twofold.

(i) We give a new proof of the equivalence theorem in the renormalizable standard model in the Feynman–'t Hooft gauge by using the diagrammatic method. The proof is given to all orders in perturbation theory and with any numbers of external W_L's. For simplicity only the pure SU(2) model is considered and the weak mixing angle θ_W has been set to 0. We would like to stress the fact that we do not intend to give a proof in the general R_ξ gauge and we restrict ourselves to the Feynman–'t Hooft gauge. The reason is that the problem is greatly simplified, as will be made clear later in this section.

(ii) When applied, the equivalence theorem only determines the leading energy term of the amplitude for a process involving longitudinally polarized vector bosons. In a renormalizable theory the amplitude of any process will be at most a constant in the limit of large energy; unitarity is not violated. However, for an effective theory this is not the case and amplitudes will grow large for large energies. For the standard model with the σ model as the Higgs sector this happens in the large-Higgs-boson-mass limit and also in the large-top-quark-mass limit. In this paper we assume the top-quark mass to be of the same order of magnitude as the vector-boson mass and we extend our proof of the equivalence theorem in the limit

$$m_{\rm top} \approx M \ll E \ll m_{\rm Higgs} \quad (1.3)$$

by using the power-counting method. As a result we derive the leading energy behavior for processes involving longitudinally polarized vector bosons in all orders of perturbation theory.

The proof of the equivalence theorem can be described as follows. In the Feynman–'t Hooft gauge the gauge-fixing term in the Lagrangian in the SU(2) model is given by $\mathcal{L}_{\rm gf} = -\frac{1}{2}\mathcal{C}^2$, with $\mathcal{C}_a = -\partial^\mu W_a^\mu + M\phi_a$, where W_a^μ is the vector boson with isospin index a and mass M; ϕ_a is the Higgs ghost, $a = 1,2,3$. The Fourier transform of $\mathcal{C}_a$ is $ik^\mu W_a^\mu + M\phi_a$. The direction of the momentum vector is defined such that we get an extra minus sign in front of k^μ. This point will be made clear in Sec. II. The first step is to show that an amplitude which contains external $\mathcal{C}$ lines is equal to zero in any order of perturbation theory. Consider for the moment an amplitude which has just one external $\mathcal{C}$ line. We have to show that

$$\mathcal{C}_a A_a = 0$$

or

$$ik^\mu A^\mu(W) + MA(\phi) = 0 \; . \quad (1.4)$$

Here $A^\mu(W)$ is the amplitude with one external vector boson for which the polarization vector ϵ^μ is replaced by its momentum vector k^μ and $A(\phi)$ is the amplitude in which the vector boson is replaced by the corresponding Higgs ghost. Equation (1.4) is derived by considering Ward identities on mass shell.

We now see the advantage of the Feynman–'t Hooft gauge. When considering the general R_ξ gauge where the gauge-fixing term is given by

$$C_a(\xi) = -\partial^\mu W_a^\mu + \frac{1}{\xi} M\phi_a \; ,$$

we still have to show the validity of Eq. (1.4). In the Feynman–'t Hooft gauge the gauge-fixing term $C_a(1)$ is precisely the C_a given in Eq. (1.4).

In lowest nonzero order this has already been derived.[1–5] But what happens if we go to higher order in perturbation theory? We then have to renormalize the external lines. If $-\partial^\mu W_a^\mu$ is renormalized differently from $M\phi_a$ we see that the relation of Eq. (1.4) is destroyed and is no longer true. The answer to this problem is to consider the renormalization procedure as described in the Feynman–'t Hooft gauge in Refs. 7 and 8. With this particular renormalization procedure the gauge-fixing term is unchanged after renormalization.[7,8] In other words $\mathcal{C}_a = -\partial^\mu W_a^\mu + M\phi_a$ remains the same before and after renormalization and therefore the relation of Eq. (1.4) is still true.

Our renormalization procedure manifestly respects weak SU(2) for the renormalized Lagrangian. Our result in the Feynman–'t Hooft gauge thus provides an explicit example of how the symmetries of the classical theory may be preserved after renormalization and it therefore supports the starting assumption[9] of Refs. 1 and 4.

In Sec. II we will derive the above equation for any process with any number of external $\mathcal{C}$ lines in every order in perturbation theory. In Ref. 5, where no conclusion was reached about the validity of the equivalence theorem, this same problem of renormalization of the gauge-fixing term is studied for any gauge. This makes the problem much more nontrivial.

The second step in the proof of the equivalence theorem is to somehow modify the above equation to an equation involving the physical vector boson for which $k^\mu \epsilon^\mu = 0$ with k^μ the momentum vector and ϵ^μ the polarization vector. From Eq. (1.1) we see that at an energy much larger than the mass M of the vector boson the longitudinal-polarization vector is equal to the momentum vector plus a vector of order M/E:

$$\frac{ik^\mu}{M} = i\epsilon_L^\mu - iv^\mu, \quad v^\mu = O\left[\frac{M}{E}\right] .$$

Substituting this relation into Eq. (1.4) we arrive at

$$\epsilon_L A(W) = iA(\phi) + vA(W) \; . \quad (1.5)$$

If we can show that $vA(W)$ is indeed at most of order M/E compared to $A(\phi)$ then we have arrived at the equivalence theorem [see Eq. (1.2)] and everything is fine. But things are not quite that simple if we consider multiple vector-boson interaction. For instance, in the standard model in which we do not take the large-Higgs-boson-mass limit and which is therefore renormalizable, we know that the amplitude of any process cannot grow large if the energy grows large. This means that, for example, the amplitude for four-W_L interaction and the amplitude for six-W_L interaction are both at most a constant in the large-energy limit, in spite of the fact that the polarization vectors are proportional to the corresponding momentum vectors (in leading order). This implies that somehow cancellations must occur. Because of the structure of the Yang-Mills vertices this is indeed exactly what happens and this was first noted in Ref. 1. This of course is still the case after we take the large-Higgs-boson-mass limit. Formulating the problem differently, if the amplitude for four-W_L interaction is given by

$$\begin{aligned} A &= \epsilon_{L1}^\mu \epsilon_{L2}^\nu \epsilon_{L3}^\alpha \epsilon_{L4}^\beta A^{\mu\nu\alpha\beta}(W,W,W,W) \\ &= \left[\frac{k_1^\mu}{M} + v_1^\mu\right]\left[\frac{k_2^\nu}{M} + v_2^\nu\right]\left[\frac{k_3^\alpha}{M} + v_3^\alpha\right]\left[\frac{k_4^\beta}{M} + v_4^\beta\right] \\ &\quad \times A^{\mu\nu\alpha\beta}(W,W,W,W) \; , \end{aligned} \quad (1.6)$$

then the above reasoning would imply that, for example,

$$\frac{k_1^\mu}{M}\frac{k_2^\nu}{M}\frac{k_3^\alpha}{M}\frac{k_4^\beta}{M} A^{\mu\nu\alpha\beta}(W,W,W,W) \quad (1.7)$$

is of the same order as

$$\frac{k_1^\mu}{M}\frac{k_2^\nu}{M}\frac{k_3^\alpha}{M} v_4^\beta A^{\mu\nu\alpha\beta}(W,W,W,W) \; . \quad (1.8)$$

The question is, thus, how can we determine the leading order in energy of the amplitude involving longitudinally polarized vector bosons? We cannot apply the power-counting method, since this method does not give information on the occurring cancellations and it thus overestimates the leading order in energy. One way of solving this problem is to find an expression for the amplitude which is of the form (for the purpose of clarity the various indices have not been written down)

$$\epsilon_L\epsilon_L \cdots \epsilon_L A(W,W,\ldots,W) = A(\phi,\phi,\ldots,\phi)+v\cdots vA(\phi,\ldots,\phi,W,\ldots,W) + vv\cdots vA(W,W,\ldots,W)\ , \quad (1.9)$$

where $A(\phi,\phi,\ldots,\phi)$ is the amplitude where all the W_L's are replaced by the unphysical Higgs ghosts, $v\cdots vA(\phi,\ldots,\phi,W,\ldots,W)$ is the amplitude for which a certain number of the W_L's are replaced by the unphysical Higgs ghosts, while for the remaining number of W_L's the longitudinal-polarization vectors are replaced by the subleading vectors v. $vv\cdots vA(W,W,\ldots,W)$ is the amplitude where all the longitudinal-polarization vectors are replaced by the vectors v. In Sec. III we will indeed arrive at this desired expression, obtained by considering the on-shell Ward identities derived in Sec. II. The above equation was first derived in Ref. 1.

We now consider the renormalizable standard model and we therefore do not take the large-Higgs-boson-mass limit. This is the region considered by Chanowitz and Gaillard[1] and we follow their argument. The amplitude of any process is at most a constant in the large-energy limit. Let us assume that the amplitude $\epsilon_L\epsilon_L\cdots\epsilon_L A(W,W,\ldots,W)$ is given by a constant. From Eq. (1.9) we see that this constant is given by the amplitude $A(\phi,\phi,\ldots,\phi)$, since the other amplitudes on the right-hand side of Eq. (1.9) are multiplied by the vectors v, which are $O(M/E)$.

If the amplitude $\epsilon_L\epsilon_L\cdots\epsilon_L A(W,W,\ldots,W)$ is $O(M/E)$ or higher, the statement of the equivalence theorem is that the W_L amplitude vanishes at least as M/E at high energy. For example, for the process $W_L^0W_L^0\to\nu\bar{\nu}$ in lowest nonzero order the corresponding $\phi^0\phi^0\to\nu\bar{\nu}$ does not even exist. As a matter of fact,

$$\epsilon_L\epsilon_L A(W_L^0W_L^0\to\nu\bar{\nu})=vvA(W_L^0W_L^0\to\nu\bar{\nu})=O\left(\frac{M^2}{E^2}\right)$$

and as in agreement with the equivalence theorem, the leading energy term (the ~ 1 term) is zero.

In an effective theory we cannot use this argument, but with the help of Eq. (1.9) the leading energy term can be determined as follows. Consider thus the case where we take the large-Higgs-boson-mass limit. The advantage is now that we can apply the power-counting method to the amplitudes on the right-hand side of Eq. (1.9). To find the leading order in energy for the amplitude $A(\phi,\phi,\ldots,\phi)$ we do not even have to consider Yang-Mills vertices and the problem of cancellations is not there. Intuitively this is easy to see, since even if we did allow for vector-boson loops and Yang-Mills vertices could occur there, they are immediately accompanied by vector-boson propagators. In the Feynman–'t Hooft gauge the vector-boson propagator is of order $1/E^2$ since it is given by

$$\frac{\delta^{\mu\nu}}{k^2+M^2-i\epsilon}$$

and contains no $k^\mu k^\nu$ term in the numerator.

For the amplitudes $v\cdots vA(\phi,\ldots,\phi,W,\ldots,W)$ and $vv\cdots vA(W,W,\ldots,W)$, with the help of the power-counting method, we are able to put an upper bound for the leading order in energy, since we know that $v^\mu=\epsilon^\mu-k^\mu/M$ is of order M/E. We only need to show that the leading energy term is given by $A(\phi,\phi,\ldots,\phi)$ and that the leading order of $v\cdots vA(\phi,\ldots,\phi,W,\ldots,W)$ and of $vv\cdots vA(W,W,\ldots,W)$ is always less than that of $A(\phi,\phi,\ldots,\phi)$. In Sec. IV this is done by applying the power-counting method to these amplitudes in the large-Higgs-boson-mass limit.

This paper is organized as follows. In Sec. II the renormalized on-shell Ward identities for any order in perturbation theory are derived using the diagrammatic approach, in Sec. III an expression for the amplitude involving longitudinally polarized vector bosons is derived, and in Sec. IV the power-counting method in the large-Higgs-boson-mass limit is discussed through which finally the equivalence theorem is established. Section V contains the summary and discussion of the results. Appendix A gives an example of the renormalization of external lines for physical particles in any order in perturbation theory. In Appendix B the equivalence theorem is worked out at the tree level for two different processes, one of them being WW scattering. The metric is such that $p^2=-m^2$ for a particle on mass shell with mass m and momentum p.

II. THE WARD IDENTITIES

This section deals with the first step in establishing the equivalence theorem, namely, the derivation of the renormalized Ward identities. We begin with the definition of the Ward identity satisfied by the Green's functions between sources off mass shell. Consider the complete Lagrangian, in the SU(2) model it is given by

$$\mathcal{L}=\mathcal{L}_{\text{inv}}-\tfrac{1}{2}\mathcal{C}^2+\mathcal{L}_{\text{FP}}\ , \quad (2.1)$$

where $-\frac{1}{2}\mathcal{C}^2$ is the gauge-fixing term and $\mathcal{L}_{\text{FP}}$ the corresponding Faddeev-Popov ghost Lagrangian. $\mathcal{L}_{\text{inv}}$ is invariant for the following infinitesimal gauge transformations of the fields:

$$\begin{aligned} &W_a^\mu\to W_a^\mu+g\epsilon_{abc}\Lambda_b W_c^\mu-\partial^\mu\Lambda_a\ ,\\ &\phi_a\to\phi_a+\tfrac{1}{2}g\epsilon_{abc}\Lambda_b\phi_c-\tfrac{1}{2}gH\Lambda_a-M\Lambda_a\ ,\\ &H\to H+\tfrac{1}{2}g\Lambda_a\phi_a\ ,\\ &l^+\to(1-\tfrac{1}{2}ig\Lambda_a\tau_a)l^+,\quad l^-\to l^-\ , \end{aligned} \quad (2.2)$$

with W_a^μ the vector boson with isospin index a, $a=1,2,3$, ϕ_a the Higgs ghost, and H the Higgs boson. $l^\pm=\frac{1}{2}(1\pm\gamma^5)l$, with l^+ a left-handed fermion doublet and l^- a right-handed fermion singlet. In the Feynman–'t Hooft gauge we have

$$\mathcal{C}_a=-\partial^\mu W_a^\mu+M\phi_a\ . \quad (2.3)$$

Of course $\mathcal{C}$ is not gauge invariant. It transforms under (2.2) as

$$\mathcal{C}_a \to \mathcal{C}_a + (\hat{m}_{ab} + g\hat{l}_{ab})\Lambda_b \ . \tag{2.4}$$

The caret on both m_{ab} and l_{ab} indicates that derivatives may occur. Furthermore $\hat{m}_{ab}$ is field independent while $\hat{l}_{ab}$ may depend on the fields. The Faddeev-Popov Lagrangian is then defined as

$$\mathcal{L}_{\rm FP} = \psi_a^*(\hat{m}_{ab} + g\hat{l}_{ab})\psi_b \ , \tag{2.5}$$

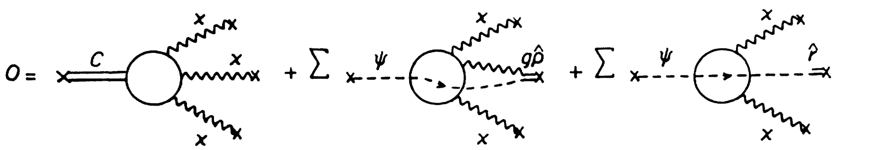
with ψ_a the Faddeev-Popov ghost. From the gauge transformations (2.2) the expressions for $\hat{m}_{ab}$ and $\hat{l}_{ab}$ are found to be

$$\begin{aligned} \hat{m}_{ab} &= \delta_{ab}(\partial^2 - M^2) = -P^{-1}\delta_{ab} \ , \\ g\hat{l}_{ab} &= -g\epsilon_{abc}\partial^\mu W_c^\mu - g\epsilon_{abc} W_c^\mu \partial^\mu \\ &\quad + \tfrac{1}{2}Mg\epsilon_{abc}\phi_c - \tfrac{1}{2}MgH\delta_{ab} \ , \end{aligned} \tag{2.6}$$

where P is the propagator of the Faddeev-Popov ghost. Now consider any field χ (indices are not explicitly written down). Under the infinitesimal gauge transformation the field χ transforms as

$$\chi \to \chi + (\hat{r} + g\hat{\rho})\Lambda \ . \tag{2.7}$$

As before the carets on both r and ρ denote the fact that derivatives may occur and that $\hat{\rho}$ may depend on the fields. The Ward identity for the Green's functions between sources is



(2.8)

where

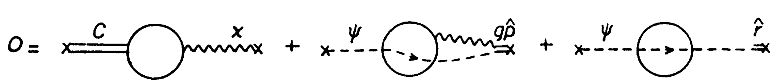
and the Faddeev-Popov ghost line is denoted by - - -➤- - . The short double line stands for ik^μ.

The direction of the momenta is such that they go into the sources, but instead we define them to be coming out of the sources and into the blob and we therefore get an extra minus sign in front of k^μ. The blobs in Eq. (2.8) contain connected, disconnected, reducible, and irreducible graphs. For a nice derivation of the Ward identity using the diagrammatic method see Ref. 10.

There are no restrictions imposed on the fields χ. They can be physical, unphysical, or a combination of fields. Thus χ can represent, for example, the physical vector boson and the fermion, but also the unphysical field combination $\mathcal{C}_a = -\partial^\mu W_a^\mu + M\phi_a$. In Sec. II A we consider the case where in the Ward identity of Eq. (2.8) all the external χ lines are replaced by the $\mathcal{C}$ lines. In doing this we arrive at an expression for the Ward identity for the Green's functions with many external $\mathcal{C}$ lines. It is at this point that we go from the Green's functions to the S matrix and thus arrive at the renormalized on-shell Ward identity. In Sec. II B we include physical fields as well by replacing some of the χ fields by the physical fields, while the remaining χ fields are replaced by the unphysical $\mathcal{C}$ fields.

A. The special case $\chi = \mathcal{C}$

In this section we derive an expression for the Ward identity of Eq. (2.8) in which all the fields χ are replaced by the unphysical fields $\mathcal{C}$. Following Ref. 7, we start out with the simplest case, namely, the two-point function. This is the Ward identity of Eq. (2.8) with only one outgoing field χ:

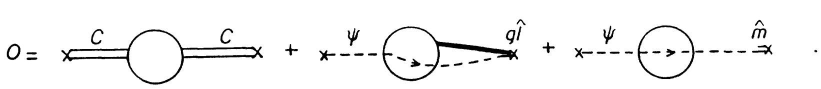


(2.9)

Again the various indices have not been written down. In the case of $\chi = \mathcal{C}$ we have

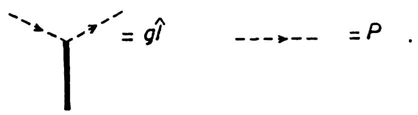
$$\hat{r} = \hat{m} = -P^{-1}, \quad g\hat{\rho} = g\hat{l} \ . \tag{2.10}$$

The Ward identity becomes



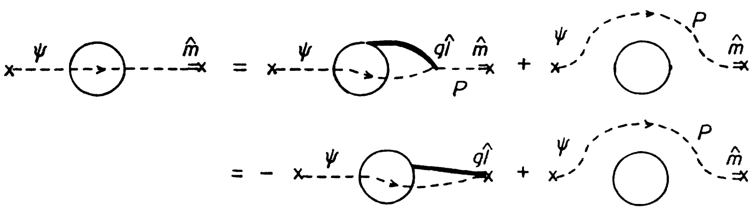
(2.11)

We can simplify the above equation considerably by carrying out some manipulations for the last two diagrams, which have Faddeev-Popov ghost external lines. For the Faddeev-Popov ghost we derive from the Lagrangian $\mathcal{L}_{\rm FP} = \psi_a^*(\hat{m}_{ab} + g\hat{l}_{ab})\psi_b$ the Feynman rules



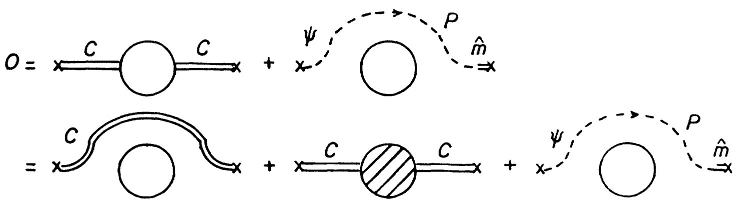
(2.12)

Substituting the above Feynman rules explicitly in the third diagram of Eq. (2.11),



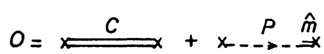
(2.13)

since $\hat{m}=-P^{-1}$. Equation (2.11) becomes



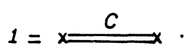
(2.14)

The shaded blob indicates connected diagrams only. We thus explicitly write down the connected and disconnected pieces for the blob connecting two $\mathcal{C}$ lines. The Ward identity must also hold at the tree level. Thus,



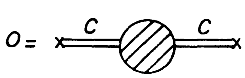
(2.15)

or with $\hat{m}=-P^{-1}$



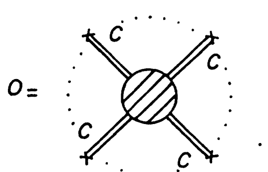
(2.16)

We see that the direct $\mathcal{C}$ line is equal to 1. The diagram of Eq. (2.14), where the blob connecting the two $\mathcal{C}$ lines contains interaction, is thus zero in any order in perturbation theory:



(2.17)

The above manipulations can be carried through for many $\mathcal{C}$ lines and we finally arrive at the equation



(2.18)

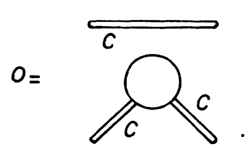
This is the Ward identity for the connected Green's functions containing many external $\mathcal{C}$ lines. Now the important question is, what happens when the above equation is put on mass shell? Two things have to be done. First of all the external $\mathcal{C}$ lines have to be multiplied by the inverse of the propagator $P_i^{-1}=(p_i^2+M^2)$ after which the external momenta are put on mass shell and $p_i^2=-M^2$; second, the external $\mathcal{C}$ lines get renormalized. For this consider the unrenormalized and renormalized Lagrangian in the Feynman–'t Hooft gauge

$$\mathcal{L}=\mathcal{L}_{\text{inv}}-\tfrac{1}{2}\mathcal{C}^2+\mathcal{L}_{\text{FP}}\ ,$$
$$\mathcal{L}^{\text{ren}}=\mathcal{L}^{\text{ren}}_{\text{inv}}-\tfrac{1}{2}\mathcal{C}^2+\mathcal{L}^{\text{ren}}_{\text{FP}}\ . \tag{2.19}$$

$\mathcal{L}^{\text{ren}}$ is obtained from the Lagrangian $\mathcal{L}$ by shifting the fields, coupling constants, and masses in the invariant part of the Lagrangian by an amount δ. Thus

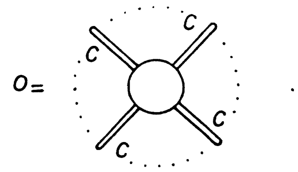
$$W_a^\mu\rightarrow W_a^\mu(1+\delta_W),\quad \phi_a\rightarrow\phi_a(1+\delta_\phi)\ ,$$
$$H\rightarrow H(1+\delta_\phi)+\frac{M}{g}\delta_t,\quad M\rightarrow M(1+\delta_M),\quad \text{etc.}\ , \tag{2.20}$$

while leaving the gauge-fixing term $\mathcal{L}_{\text{gf}}=-\frac{1}{2}\mathcal{C}^2$ unchanged. The renormalized Faddeev-Popov ghost Lagrangian is obtained by using the renormalized gauge transformations and at the same time the ghost fields get shifted by an amount δ. Detailed discussions concerning the renormalization procedure can be found in Refs. 7 and 8. It is thus clear that Eq. (2.18) is also true on mass shell, since the external $\mathcal{C}$ lines are left unchanged. Also, disconnected-type diagrams are zero on mass shell, since the direct $\mathcal{C}$ line contains no pole. Thus on mass shell



(2.21)

The final result can be written as

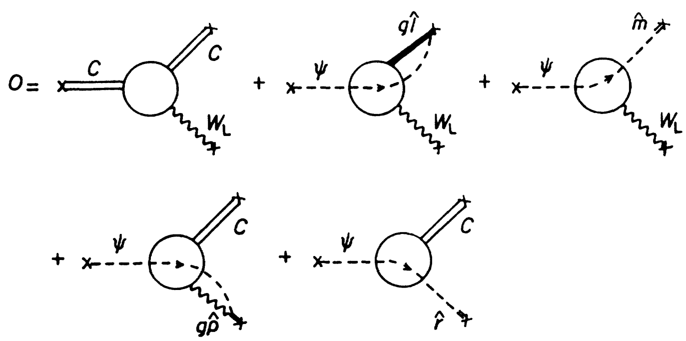


(2.22)

The above equation with external renormalized $\mathcal{C}$ lines on mass shell holds in any order of perturbation theory, including the lowest nonzero order.

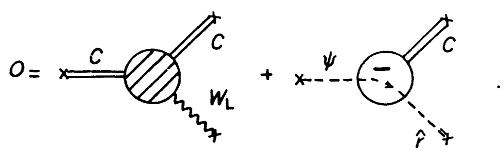
B. Inclusion of physical particles

Here we derive the renormalized Ward identity satisfied by the amplitude containing any number of external $\mathcal{C}$ lines and physical particles. As an example, without losing generality, consider the case of two $\mathcal{C}$ lines and one physical particle, the longitudinally polarized vector boson W_L. The Ward identity of Eq. (2.8) can be written as



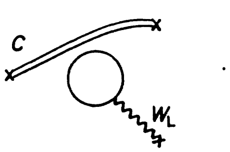
(2.23)

Since the external source J^μ for the vector boson is physical and $J^\mu k^\mu = 0$, the last diagram of Eq. (2.23) does not contribute. The fourth diagram can be split up into a piece containing a pole and a piece containing no pole. The piece containing the pole is proportional to k^μ (Ref. 7) and it therefore does not contribute since it is multiplied with the physical source J^μ. The piece containing no pole survives. The first three diagrams can be treated exactly as shown in Sec. II A. We are left with



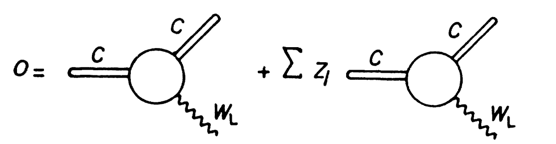
(2.24)

Note that the minus sign in the blob of the second diagram indicates that this diagram has no pole. When going on mass shell the external lines get multiplied by the inverse of the propagator and the external momenta are put on mass shell, plus the external lines get renormalized. The second diagram of Eq. (2.24) contains no pole and on mass shell it is therefore equal to zero. The same can be said for disconnected types of diagram:



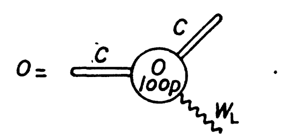
(2.25)

The direct $\mathcal{C}$ line is equal to 1 and thus contains no pole. Now consider the first diagram of Eq. (2.24). As was shown in the previous section, the external $\mathcal{C}$ lines remain fixed. Renormalization of the W_L line amounts to multiplication of the external source J_a^μ by a factor Z_l, where l stands for loop order. The factor Z_l is determined by the requirement that the residue of the pole of the propagator is equal to 1 (Ref. 7) and is thus different in each order in perturbation theory. In lowest nonzero order $Z_0 = 1$. Appendix A shows exactly how this works in the case of the physical vector boson. We so arrive at the expression for the Ward identity on mass shell:



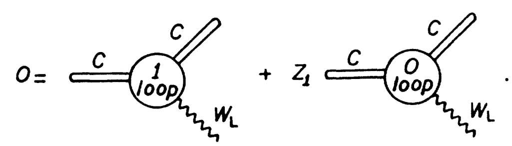
(2.26)

The above equation should also hold at the tree level:



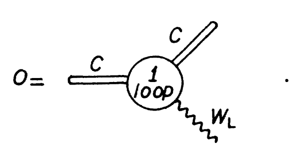
(2.27)

At one loop we have



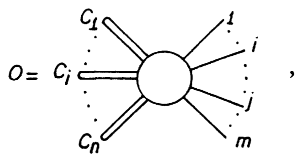
(2.28)

From Eq. (2.27) we see that the second diagram in Eq. (2.28) is zero. Equation (2.28) reduces to



(2.29)

This argument can be carried through for all higher orders in perturbation theory. Finally, we arrive at the renormalized on-shell Ward identity valid in every order in perturbation theory

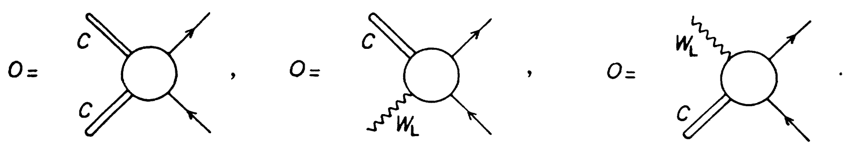


(2.30)

where the straight external lines denote any physical particle, including the longitudinally polarized vector boson. There is the requirement that there is at least one $\mathcal{C}$ line; there is no restriction for the physical particles. Thus $m \geq 0$, $n \geq 1$.

III. THE LONGITUDINALLY POLARIZED VECTOR BOSON

From Eq. (2.30) it seems plausible that some kind of relation can be derived between the longitudinally polarized vector boson W_L and the Higgs ghost ϕ, since the k^μ term in the $\mathcal{C}$ line is the polarization vector of W_L up to order M/E as can be seen from Eq. (1.1). One has to be very careful in this, because in W_L scattering the leading order in E often cancels; see Appendix B. Reference 1 took this possibility into account and we follow their subtraction scheme. Consider, for example, the process $W_L W_L \to f\bar{f}$. From Eq. (2.30) we can write down three Ward identities:

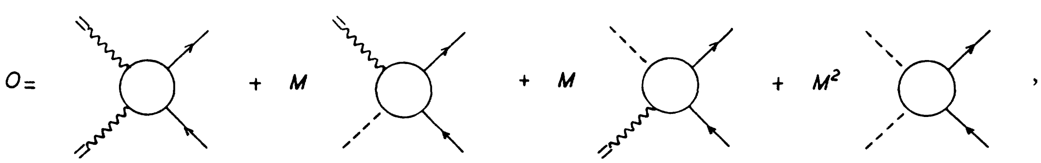


(3.1)

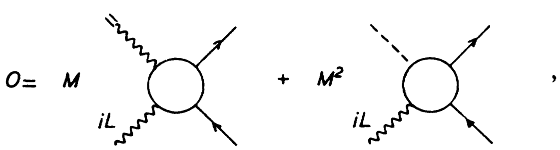
After substituting the relation

(3.2)

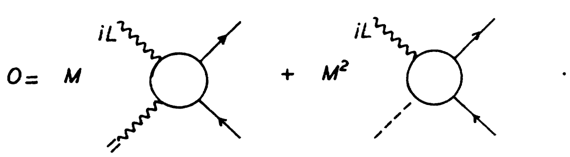
(remember that the short double line stands for ik^μ) the diagrams of Eq. (3.1) can be written as



(3.3)

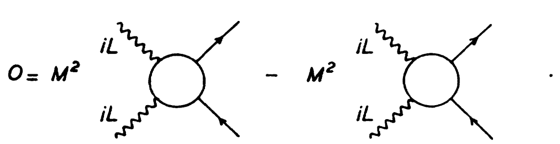


(3.4)



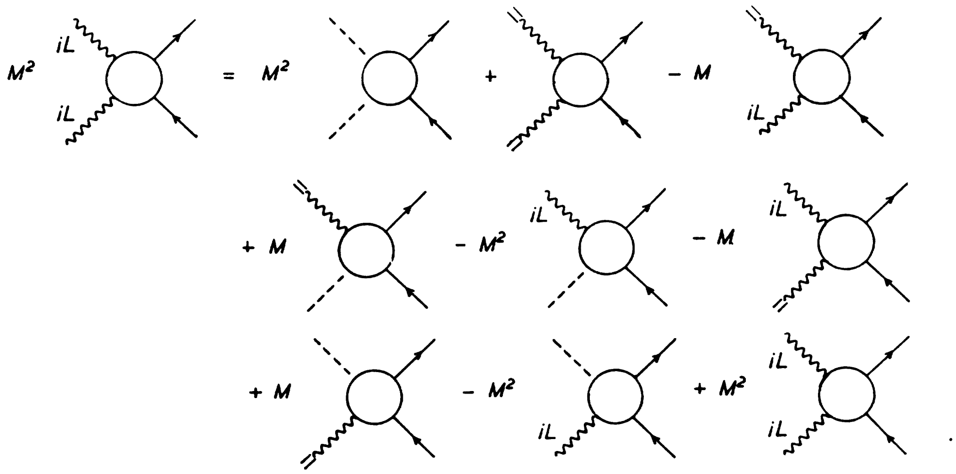
(3.5)

Note that L stands for ϵ_L. Equations (3.4) and (3.5) have been multiplied by an overall factor iM. Subtract Eqs. (3.4) and (3.5) from Eq. (3.3) and add



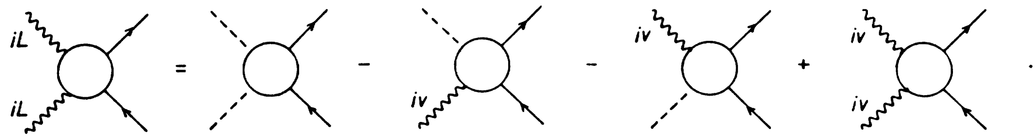
(3.6)

We get



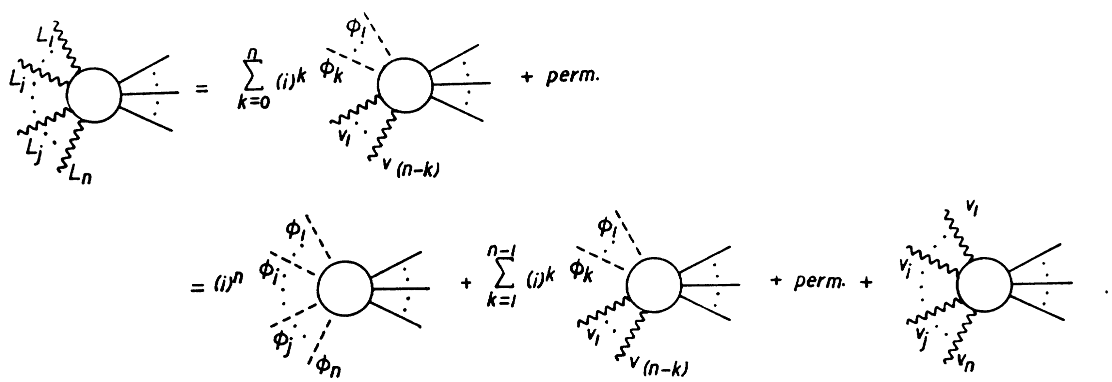
(3.7)

Now make use of Eq. (1.1); $\epsilon_L^\mu = k^\mu/M + v^\mu$ with $v^\mu = O(M/E)$. Equation (3.7) is reduced to (dividing by an overall factor M^2)



(3.8)

This is still an exact equation. For example, the amplitude for the process $W_L^0 W_L^0 \to \nu\bar{\nu}$ corresponds to the very last diagram of Eq. (3.8). The only thing that remains to be shown to establish the equivalence theorem is that the first diagram on the right-hand side of Eq. (3.8) indeed gives the leading energy term. Appendix B gives some examples. The generalization of Eq. (3.8) is



(3.9)

In the above equation the straight lines denote any physical particle other than W_L.

We are now able to prove the validity of the equivalence theorem in the renormalizable standard model for amplitudes that are proportional to a constant in the large-energy limit. As was argued in the Introduction and in Ref. 1, for a renormalizable theory the amplitude of any process is at most a constant in the large-energy limit. We see from Eq. (3.9) that this constant term is given by the amplitude $A(\phi,\phi,\ldots,\phi)$, since the other amplitudes on the right-hand side of Eq. (3.9) are multiplied by the subleading vectors v. For amplitudes that are $O(M/E)$ or higher, for example, $W_L^0 W_L^0 \to \nu\bar{\nu}$, the statement of the equivalence theorem is that the amplitude vanishes at least as $O(M/E)$ at high energy.

For the proof of the equivalence theorem in the large-Higgs-boson-mass limit we need the power-counting method described below.

IV. POWER COUNTING IN THE LARGE-HIGGS-BOSON-MASS LIMIT

As was argued in the Introduction [compare Eq. (3.9) with Eq. (1.9)], we are able to apply the power-counting method to the amplitudes on the right-hand side of Eq. (3.9) to determine their leading order in energy.

In Sec. IV A we establish the equivalence theorem in the large-Higgs-boson-mass limit. We show that the leading order of the amplitude for a process involving W_L's is given by the amplitude in which the W_L's are replaced by the corresponding Higgs ghosts. In Sec. IV B we discuss W_L scattering an in Sec. IV C we study the amplitude involving one external fermion pair for which the fermion mass cannot be neglected. Everything is evaluated in the region

$$m_t \approx M \ll E \ll m_{\text{Higgs}} .$$

In order to use the power-counting method, we need the explicit form of the Lagrangian. In the SU(2) model we have

$$\mathcal{L} = \mathcal{L}_{\text{inv}} - \tfrac{1}{2}\mathcal{C}^2 + \mathcal{L}_{\text{FP}} ,$$

with

$$\begin{aligned}\mathcal{L}_{\text{inv}} = & -\tfrac{1}{4}G_a^{\mu\nu}G_a^{\mu\nu} - \tfrac{1}{2}M^2W^2 - \tfrac{1}{2}(\partial^\mu H)^2 - \tfrac{1}{2}m^2H^2 - \tfrac{1}{2}(\partial^\mu\phi_a)(D_a^\mu\phi) + \tfrac{1}{2}gW_a^\mu(H\partial^\mu\phi_a - \phi_a\partial^\mu H) \\ & -\tfrac{1}{8}g^2W^2(\phi^2+H^2) - \tfrac{1}{2}gMW^2H - \tfrac{1}{2}\lambda vH(\phi^2+H^2) - \tfrac{1}{8}\lambda(\phi^2+H^2)^2 - \beta\left[\tfrac{1}{2}(H^2+\phi^2) + \frac{2M}{g}H\right] - M\phi_a\partial^\mu W_a^\mu \\ & -(\bar f\gamma^\mu\partial^\mu f) - m_f(\bar f f) + \frac{ig}{4}[f\gamma^\mu(1+\gamma^5)\tau_a f]W_a^\mu - \frac{m_f}{v}H(\bar f f) - \frac{m_f}{v}\phi_a[\bar f(\epsilon_{abc}\tau_b s_c + i\tau_a\gamma^5 - is_a\gamma^5)f] . \end{aligned} \tag{4.1}$$

In here

$$\begin{aligned} & W^2 = W_a^\mu W_a^\mu, \quad \phi^2 = \phi_a\phi_a , \\ & G_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu + g\epsilon_{abc}W_b^\mu W_c^\nu , \\ & D_a^\mu\phi = \partial^\mu\phi_a + g\epsilon_{abc}W_b^\mu\phi_c , \end{aligned} \tag{4.2}$$

λ is the Higgs self-coupling, with $\lambda = m^2/v^2$ and v is the vacuum expectation value (VEV), with $M = \frac{1}{2}gv$. M is the vector-boson mass, m is the Higgs-boson mass, f is a fermion doublet, τ_a are the Pauli spin matrices, and s is a spurion with $s_1 = s_2 = 0$, $s_3 = 1$. β is a constant and is fixed such that the total tadpole contribution is zero. In lowest order $\beta = 0$. In the Feynman–'t Hooft gauge

$$\mathcal{C}_a = -\partial^\mu W_a^\mu + M\phi_a$$

and for the Faddeev-Popov ghost Lagrangian we have

$$\mathcal{L}_{\text{FP}} = \psi_a^*(\hat m_{ab} + g\hat l_{ab})\psi_b ,$$

with

$$\begin{aligned} \hat m_{ab} &= \delta_{ab}(\partial^2 - M^2) = -P^{-1}\delta_{ab} , \\ g\hat l_{ab} &= -g\epsilon_{abc}\partial^\mu W_c^\mu - g\epsilon_{abc}W_c^\mu\partial^\mu \\ & \quad + \tfrac{1}{2}Mg\epsilon_{abc}\phi_c - \tfrac{1}{2}MgH\delta_{ab} . \end{aligned}$$

A. The equivalence theorem

In this section, we will first derive the general expression for the amplitude containing any number of external Higgs ghosts, vector bosons, and fermions. Then it will be shown that the amplitude with all longitudinally polarized replaced by the unphysical Higgs ghost gives the amplitude in leading order. For the moment only allow for internal ϕ and Higgs lines. The case in which also internal vector bosons and fermions are allowed will be discussed later. See Fig. 1. For the Feynman diagram of Fig. 1 the amplitude will be of the form

$$A \sim \epsilon^1\epsilon^2 \cdots \epsilon^{E_W} A^{E_W} , \tag{4.3}$$

with

$$A^{E_W} \sim f(\lambda, v, g, \mu) I_F \tag{4.4}$$

and E_W stands for the number of external vector bosons. For the time being we do not worry about gamma-matrices or the wave functions u and $\bar u$ of the external fermions. As can be seen from the Lagrangian, $f(\lambda, v, g, \mu)$ is some function of the Higgs self-coupling λ, the VEV v, the coupling constant g and of the fermion mass m_f ($\mu = m_f/v$). I_F is defined as the Feynman integral for L loops and is of the form

$$\int d^4k_1 d^4k_2 \cdots d^4k_L \frac{k_1k_2\cdots k_l}{(k_1^2+m_1^2)(k_2^2+m_2^2)\cdots(k_L^2+m_L^2)} . \tag{4.5}$$

The momentum dependence in the numerator I_F is due to the corresponding derivative $W\phi\phi$ and $WH\phi$ coupling in the Lagrangian.

First look at $f(\lambda, v, g, \mu)$. Listing all the relevant vertices with their corresponding coupling strength occurring in the Lagrangian, we can construct Table I. Furthermore, define

$$V_d = V_d^1 + V_d^2, \quad V_W = V_W^1 + V_W^2, \quad V_3 = V_3^{\phi\phi H} + V_3^{HHH} ,$$
$$V_4 = V_4^{\phi\phi\phi\phi} + V_4^{\phi\phi HH} + V_4^{HHHH}, \quad V_f = V_f^{\phi f\bar f} + V_f^{Hf\bar f} .$$

The V_i^j terms in the last column denote the number of vertices occurring in the Feynman diagram considered, of the type specified in the first column. The subscript d in V_d^1 and V_d^2 stands for the fact that one derivative occurs for these vertices. We now easily read off the form of $f(\lambda, v, g, \mu)$. It is given by

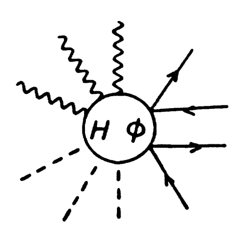


FIG. 1. Feynman diagram containing internal Higgs and ϕ lines only.

TABLE I. Column 1: type of vertex. Column 2: corresponding coupling strength. Column 3: number of times it occurs in the considered Feynman diagram.

Vertex	Coupling strength	Total number
$W\phi\phi$	g, deriv.	V_d^1
$WH\phi$	g, deriv.	V_d^2
$WW\phi\phi$	g^2	V_W^1
$WWHH$	g^2	V_W^2
WWH	$g^2 v$	V_3^b
$\phi\phi H$	λv	$V_3^{\phi\phi H}$
HHH	λv	V_3^{HHH}
$\phi\phi\phi\phi$	λ	$V_4^{\phi\phi\phi\phi}$
$\phi\phi HH$	λ	$V_4^{\phi\phi HH}$
$HHHH$	λ	V_4^{HHHH}
$\phi f\bar{f}$	m_f/v	$V_f^{\phi f\bar{f}}$
$Hf\bar{f}$	m_f/v	$V_f^{Hf\bar{f}}$

$$f(\lambda,v,g,\mu)\sim\lambda^{V_3+V_4}v^{V_3}g^{V_d+2V_W+2V_3^b}\frac{m_f^{V_f}}{v^{V_f}}v^{V_3^b} . \tag{4.6}$$

We can already make one simple observation. Since there are no internal vector-boson lines or fermion lines, we derive from Table I the following expressions for the total number of external vector bosons E_W and fermions E_f:

$$E_W=V_d+2V_W+2V_3^b , \tag{4.7}$$

$$E_f=2V_f . \tag{4.8}$$

Substituting Eq. (4.7), Eq. (4.8), and $\lambda=m^2/v^2$ in the expression for $f(\lambda,v,g,\mu)$, we find

$$f(\lambda,v,g,\mu)\sim\frac{m^{2(V_3+V_4)}}{v^{(V_3+2V_4+V_f-V_3^b)}}g^{E_W}m_f^{E_f/2} . \tag{4.9}$$

Consider now the Feynman integral I_F. The mass dimension D of I_F is

$$D=4L-2I+V_d , \tag{4.10}$$

where L = number of loops and $I=I_H+I_\phi$ = number of internal Higgs bosons and ϕ lines.

In the heavy-Higgs-boson-mass limit the scale of the Feynman integral I_F is set completely by the mass of the Higgs boson. Therefore the integral must be expanded in powers of p/m, where p is a typical external momentum and m is the Higgs-boson mass:

$$I_F=\left[a_0m^D+a_1\left[\frac{p}{m}\right]m^D+\cdots+a_N\left[\frac{p}{m}\right]^N m^D\right]+\cdots . \tag{4.11}$$

Depending on V_d, if D is odd we need an odd expansion in p and if D is even we need a quadratic expansion in p. The series expansion is always a function of the Higgs-boson mass squared. The expansion terminates for some value of N, which gives the leading energy behavior. N is determined from the condition that the next terms in the expansion contribute to the amplitude of the Feynman diagram $A^{E_W}\sim f(\lambda,v,g,\mu)I_F$ to order $1/m^2$ or higher. Thus the Feynman diagram has the expansion

$$A^{E_W}\sim\frac{m^{2(V_3+V_4)}}{v^{(V_3+2V_4+V_f-V_3^b)}}g^{E_W}m_f^{E_f/2}\times\left[a_0m^D+a_1\left[\frac{p}{m}\right]m^D+\cdots+a_N\left[\frac{p}{m}\right]^N m^D\right]+O\left[\frac{1}{m^2}\right] . \tag{4.12}$$

Note that the coefficients a_k can in general be functions of $\ln(m^2)$. N is found by requiring that for this particular term in the expansion of Eq. (4.12) the power of m^2 is zero:

$$D+2(V_3+V_4)=N . \tag{4.13}$$

We now use the well-known identity

$$L=I-V+1 , \tag{4.14}$$

where, for the total number of vertices V and internal lines I,

$$V=V_3+V_4+V_d+V_W+V_b^3+V_f , \tag{4.15}$$

$$I=I_H+I_\phi . \tag{4.16}$$

It follows that

$$N=2L+2-V_d-2(V_W+V_b^3+V_f) . \tag{4.17}$$

Expressed in terms of E_W and E_f we find

$$N=2L+2-(E_W+E_f) . \tag{4.18}$$

The only thing that is left to be done is to simplify the term $v^{(V_3+2V_4+V_f-V_3^b)}$ outside the brackets of Eq. (4.12). For this consider E_ϕ, the number of external ϕ lines. The total number of internal and external ϕ lines is

$$E_\phi+2I_\phi=2V_3^{\phi\phi H}+2V_4^{\phi\phi HH}+4V_4^{\phi\phi\phi\phi}+2V_d^1+V_d^2+2V_W^1+V_f^{\phi f\bar{f}} . \tag{4.19}$$

Since there are no external Higgs lines, we have, for the total number of internal Higgs lines,

$$2I_H=3V_3^{HHH}+V_3^{\phi\phi H}+2V_4^{\phi\phi HH}+4V_4^{HHHH}+V_d^2+2V_W^2+V_3^b+V_f^{Hf\bar{f}} . \tag{4.20}$$

There is a factor of 2 in front of I_ϕ and I_H, since two lines are needed for every loop. Adding the above two equations we get

$$E_\phi+2(I_\phi+I_H)=3V_3+4V_4+2V_d+2V_W+V_3^b+V_f . \tag{4.21}$$

Substituting $I=L+V-1$, we find

$$V_3+2V_4+V_f-V_3^b=E_\phi+E_f+2(L-1) . \tag{4.22}$$

Finally, with the help of Eq. (4.3) and with $1/v \sim g/M$, we arrive at the equation

$$A \sim \left[\frac{g}{M} \right]^{2(L-1)+E_\phi+E_f} g^{E_W} m_f^{E_f/2} \epsilon^1 \epsilon^2 \cdots \epsilon^{E_W} \times (a_0 m^N + a_1 p m^{N-1} + \cdots + a_N p^N) ,$$
$$N = 2L + 2 - (E_W + E_f) . \tag{4.23}$$

As was mentioned before, the coefficients a_k can be functions of $\ln(m^2)$. For the polarization vector ϵ^i we can either substitute v^i, with $v^i = \epsilon_L^i - k^i/M = O(M/E)$, or the transverse polarization vector $\epsilon_T^i = O(1)$.

A few observations can be made. Since in the Feynman–'t Hooft gauge the vector-boson propagator has no $k^\mu k^\nu$ term in the numerator, we see that including vector bosons inside the blob in Fig. 1 will not give rise to new leading terms in energy. Furthermore, vertices with vector bosons coupled to the Higgs boson or the ϕ have no m^2 dependence. Consequently, for such diagrams we do not have to expand up to the same order in m^2 as for internal ϕ lines in the considered loop order. The same can be said when allowing fermion lines inside the blob of Fig. 1, since this corresponds to having a propagator for each internal fermion line which is at least proportional to the inverse momentum,

$$P_f = \frac{-i\not{p} + m_f}{p^2 + m_f^2} \tag{4.24}$$

and will thus also not give rise to new leading terms in energy. Therefore the expansion of Eq. (4.23) indeed gives the leading terms in energy for the amplitude A. Define

$$E_W = (n-l) + m ,$$

where $(n-l)$ = number of external W_L's, each with a polarization vector v^i, and m = number of external W_T's, each with a polarization vector ϵ_T^i. Then the amplitude of Eq. (3.9) is, in leading order,

$$A(W_L^1, W_L^2, \ldots, W_L^n, W_T^1, W_T^2, \ldots, W_T^m, f^1, \bar{f}^1, f^2, \bar{f}^2, \ldots, f^j, \bar{f}^j)$$
$$\sim \sum_{l=0}^{n} (i)^l \left[\frac{g}{M} \right]^{2(L-1)+l+2j} g^{(n-l)+m} \epsilon_T^1 \epsilon_T^2 \cdots \epsilon_T^m v^1 v^2 \cdots v^{(n-l)} m_f^j \{ a_0 m^N + a_1 p m^{N-1} + \cdots + a_N p^N \} ,$$
$$N = 2L + 2 - (n-l) - m - 2j . \tag{4.25}$$

We see immediately that the series for which $n = l$ indeed gives the leading order, since for $l < n$ the expression between the curly brackets is multiplied with

$$v^1 v^2 \cdots v^{(n-l)} = \left[O\left[\frac{M}{p} \right] \right]^{(n-l)} .$$

This is in agreement with the equivalence theorem [see Eq. (1.2)] and in leading order we have

$$A(W_L^1, W_L^2, \ldots, W_L^n, W_T^1, W_T^2, \ldots, W_T^m, f^1, \bar{f}^1, f^2, \bar{f}^2, \ldots, f^j, \bar{f}^j)$$
$$\sim (i)^n \left[\frac{g}{M} \right]^{2(L-1)+n+2j} g^m \epsilon_T^1 \epsilon_T^2 \cdots \epsilon_T^m m_f^j \{ a_0 m^{2L+2-2j-m} + a_1 p m^{2L+1-2j-m} + \cdots + a_{2L+2-2j-m} p^{2L+2-2j-m} \}$$
$$\sim (i)^n A(\phi^1, \phi^2, \ldots, \phi^n, W_T^1, W_T^2, \ldots, W_T^m, f^1, \bar{f}^1, f^2, \bar{f}^2, \ldots, f^j, \bar{f}^j) . \tag{4.26}$$

If there is an even number of external W_T's, we need a quadratic expansion in p. If there is an odd number of W_T's, the expansion is odd in p. The expansion is always a function of the Higgs-boson mass squared. Note that the expression inside the curly brackets of Eq. (4.26) is independent of the number of external ϕ lines and with every external vector boson (for which at most $\epsilon \sim 1$) or fermion pair the power for each term in the series expansion goes down one or two units. In the next two sections we will discuss Eq. (4.26) for the two most important types of processes: namely, W_L scattering and the production of one fermion pair (more fermion pairs will decrease the power of the expansion for the amplitude even more).

B. W_L scattering

See Fig. 2. In the case of W_L interaction, in which fermions and transverse polarized vector bosons are not considered, the amplitude of Eq. (3.9) in leading order is, from Eq. (4.26),

$$A(W_L^1, W_L^2, \ldots, W_L^n) \sim (i)^n \left[\frac{g}{M} \right]^{2(L-1)+n} \times \{ a_0 m^{2L+2} + a_2 p^2 m^{2L} + \cdots + a_{2L+2} p^{2L+2} \}$$
$$\sim (i)^n A(\phi^1, \phi^2, \ldots, \phi^n) . \tag{4.27}$$

As was noted before, the expression inside the curly brackets of Eq. (4.27) is independent of the number of external ϕ lines. This means that, for example, at the tree level the amplitudes for $W_L W_L \to W_L W_L$ and $W_L W_L \to W_L W_L W_L W_L$ are both proportional to E^2 in leading order, although the polarization vectors are each proportional to E. Furthermore not all coefficients a_k are nonzero. The term $a_0 (m^2)^{L+1}$ actually cancels. This is a

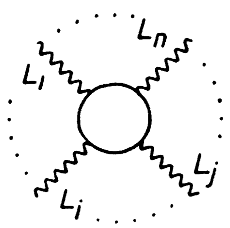


FIG. 2. Longitudinal vector-boson interaction.

consequence of the low-energy theorem. For example, at the tree level for the process $\phi\phi \to \phi\phi$ the m^2 term from the diagrams of Fig. 3, is exactly canceled by the m^2 term from the diagram of Fig. 4, and the tree-level amplitude is $\sim (g/M)^{E_\phi - 2} a_2 p^2$ (Ref. 1). We thus put

$$a_0 = 0 \; . \tag{4.28}$$

Then there is the screening theorem, which states that the leading order in m^2 is not observable. For an illustration consider the amplitude at one loop:

$$A_{(1\ \text{loop})} \sim \left[\frac{g}{M} \right]^{E_\phi} (a_2 p^2 m^2 + a_4 p^4) \; . \tag{4.29}$$

According to the screening theorem, the $a_2 p^2 m^2$ term is unobservable and we may put

$$a_2 = 0, \quad L \geq 1 \; . \tag{4.30}$$

At one loop we thus expect

$$A_{(1\ \text{loop})} \sim \left[\frac{g}{M} \right]^{E_\phi} a_4 p^4 \; . \tag{4.31}$$

Indeed this result was obtained in Ref. 6. Let us see what happens at the two-loop level. With both a_0 and a_2 equal to zero,

$$A_{(2\ \text{loop})} \sim \left[\frac{g}{M} \right]^{E_\phi + 2} (a_4 p^4 m^2 + a_6 p^6) \; . \tag{4.32}$$

The a_4 term gives a quadratic dependence on the Higgs-boson mass, as was obtained in Ref. 8.

C. The case of only one external fermion pair

Since the top-quark mass is assumed to be of the same order of magnitude as the vector-boson mass, the Yukawa-coupling cannot be neglected. Therefore in this section we treat the role of m_f in more detail and derive an expression for the amplitude with m_f as a parameter. Consider thus the case of just one external fermion pair and, in addition to internal Higgs and ϕ lines, allow for at the most one internal fermion line or one internal vector-boson line. See Fig. 5.

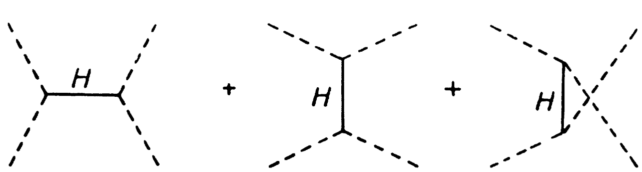


FIG. 3. Higgs propagator diagrams for ϕ-ϕ scattering.

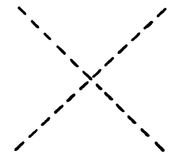


FIG. 4. ϕ four-vertex.

As we noted before, having more internal or external fermion and vector-boson lines causes the power of the expansion series to go down accordingly. Thus consider

$$A(W_L^1, W_L^2, \ldots, W_L^n, f, \bar{f}) \; . \tag{4.33}$$

For the amplitude we can write

$$A = \bar{u}[f_0(\gamma) A_0 + f_W(\gamma) A_W + f_f(\gamma) A_f] u \; . \tag{4.34}$$

In here $f_0(\gamma)$, $f_W(\gamma)$, and $f_f(\gamma)$ are functions of the γ matrices. They are only written down here to remind us that γ matrices may occur. u and $\bar{u}$ are the wave functions of the external fermion pair. A_0 corresponds to the amplitude for which the Feynman diagrams do not contain internal fermion or vector-boson lines and $A_0 \sim m_f$. A_f corresponds to the case where there is one internal fermion line and thus $A_f \sim m_f, m_f^2$. A_W corresponds to the amplitude for which the Feynman diagram contains one internal vector-boson line coupled to $f\bar{f}$ through the $Wf\bar{f}$ vertex and A_W is thus independent of m_f. The form of A_0 is in fact already known. Substituting $E_f = 2$ in Eq. (4.26) we have, for the amplitude of Eq. (3.9) in leading order,

$$\begin{aligned} A_0(W_L^1, W_L^2, \ldots, W_L^n, f, \bar{f}) & \\ \sim (i)^n \left[\frac{g}{M} \right]^{2L+n} m_f & \\ \times \{ a_0 m^{2L} + a_2 p^2 m^{2L-2} + \cdots + a_{2L} p^{2L} \} & \\ \sim (i)^n A_0(\phi^1, \phi^2, \ldots, \phi^n, f, \bar{f}) \; . & \end{aligned} \tag{4.35}$$

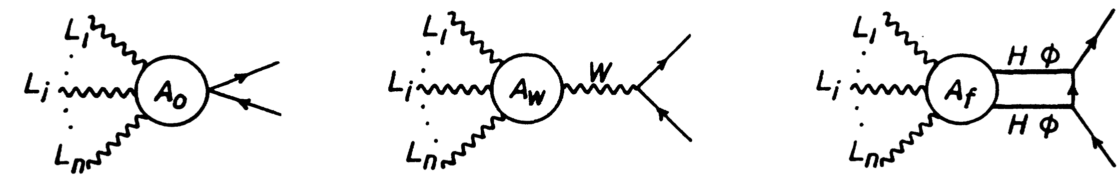
Note that just as in the case for W_L scattering, only the $l = n$ series has to be kept. For A_W and A_f it can be shown, using the method described in Sec. IV A, that the leading-order expression for the amplitude is also given by the amplitude for which all the external W_L's are replaced by the corresponding ϕ's, as in agreement with the equivalence theorem. Thus consider

$$\begin{aligned} (i)^n A_W(\phi^1, \ldots, \phi^n, f, \bar{f}) &\sim (i)^n f(\lambda, v, g) I_W \; , \\ (i)^n A_f(\phi^1, \ldots, \phi^n, f, \bar{f}) &\sim (i)^n f(\lambda, v, \mu) I_f \; , \end{aligned} \tag{4.36}$$

where

$$\begin{aligned} f(\lambda, v, g) &\sim \frac{m^{2(V_3+V_4)}}{v^{V_3+2V_4}} g^{V_d + V^{Wf\bar{f}}} \; , \\ f(\lambda, v, \mu) &\sim \frac{m^{2(V_3+V_4)}}{v^{V_3+2V_4+V_f}} m_f^{V_f} \; . \end{aligned} \tag{4.37}$$

We first derive the series expansion for A_W. For $f(\lambda, v, g)$ the extra vertex is the $Wf\bar{f}$ vertex, with



FG. 5. A_0: Feynman diagrams without internal vector-boson or fermion lines. A_W: Feynman diagrams containing one internal vector-boson line. A_f: Feynman diagrams containing one internal fermion line.

$V^{Wf\bar{f}}=1$ it raises g to the first power and since $V_f=0$ there is no m_f dependence. The expression for I_W is that of Eq. (4.9), but now with $I=I_H+I_\phi+I_W$. With $V_d=1$ the mass dimension of I_W is

$$D_W=4L-2I+1\ .$$

It then follows that

$$I_W\sim\left[b_1\left[\frac{p}{m}\right]m^{D_W}+b_3\left[\frac{p}{m}\right]^3 m^{D_W}+\cdots+b_{N_W}\left[\frac{p}{m}\right]^{N_W} m^{D_W}\right]+\cdots \tag{4.38}$$

and

$$f(\lambda,v,g)\sim\frac{m^{2(V_3+V_4)}}{v^{V_3+2V_4}}g^2\ . \tag{4.39}$$

The expansion terminates for the value N_W, such that the next term in the expansion for $A_W\sim f(\lambda,v,g)I_W$ is of order $1/m^2$ or higher. Thus

$$N_W=D_W+2(V_3+V_4)\ . \tag{4.40}$$

Using the identity

$$L=I-V+1\ , \tag{4.14}$$

but now with

$$V=V_3+V_4+V_d+V^{Wf\bar{f}},\quad V^{Wf\bar{f}}=1,\quad V_d=1\ ,$$
$$I=I_H+I_\phi+I_W,\quad I_W=1\ , \tag{4.41}$$

we find, for N_W,

$$N_W=2L-1 \tag{4.42}$$

and with

$$E_\phi+2(I_H+I_\phi)=3V_3+4V_4+2V_d,\quad V_d=1\ , \tag{4.43}$$

it follows that

$$V_3+2V_4=E_\phi+2L-2\ . \tag{4.44}$$

The amplitudes A_W can now be written down in known quantities:

$$A_W\sim(i)^n\left[\frac{g}{M}\right]^{E_\phi+2L-2}g^2 \times\{b_1pm^{2L-2}+b_3p^3m^{2L-4}+\cdots+b_{2L-1}p^{2L-1}\}+O\left[\frac{1}{m^2}\right]\ . \tag{4.45}$$

For $A_f\sim I_f f(\lambda,v,\mu)$ the situation is a bit more complicated. This is due to the form of the fermion propagator P_f:

$$P_f=\frac{-i\not{p}+m_f}{p^2+m_f^2}\ .$$

There are thus two different expansions needed for I_f. They are defined as follows. I_{f1} corresponds to the case where the momentum in the numerator of the propagator P_f survives,

$$P_f\sim\frac{-i\not{p}}{p^2+m_f^2}\sim\frac{1}{p}\ ,$$

and the mass dimension of I_{f1} is $D_1=4L-2(I_H+I_\phi)-1$. I_{f2} corresponds to the case where the momentum in the numerator of the propagator P_f cancels,

$$P_f\sim\frac{m_f}{p^2+m_f^2}\sim\frac{1}{p^2}\ ,$$

and the mass dimension of I_{f2} is $D_2=4L-2(I_H+I_\phi)-2$. We arrive at

$$I_{f1}\sim\left[c_1\left[\frac{p}{m}\right]m^{D_1}+c_3\left[\frac{p}{m}\right]^3m^{D_1}+\cdots+c_{N_1}\left[\frac{p}{m}\right]^{N_1}m^{D_1}\right]+\cdots\ ,$$
$$I_{f2}\sim m_f\left[d_0m^{D_2}+d_2\left[\frac{p^2}{m^2}\right]m^{D_2}+\cdots+d_{N_2}\left[\frac{p^2}{m^2}\right]^{N_2}m^{D_2}\right]+\cdots\ . \tag{4.46}$$

The expression for the amplitude is

$$A_f\sim(i)^nf(\lambda,v,\mu)(I_{f1}+I_{f2})\ . \tag{4.47}$$

We need to find N_1 and N_2 for which each of the series expansions for the amplitude A terminates. We find

$$N_1=D_1+2(V_3+V_4),\quad 2N_2=D_2+2(V_3+V_4)\ , \tag{4.48}$$

with

$$L=I_\phi+I_H+I_f-V+1,\quad V=V_3+V_4+V_f\ . \tag{4.49}$$

For the total number of Higgs bosons, ϕ, and fermion lines we have

$$E_\phi+2(I_\phi+I_H)=3V_3+4V_4+V_f\ ,$$
$$E_f+2I_f=2V_f,\quad E_f=2,\quad I_f=1\ . \tag{4.50}$$

It then follows that

$$N_1=2L-1,\quad 2N_2=2L-2\ ,$$
$$V_3+2V_4+V_f=2L+E_\phi\ . \tag{4.51}$$

The expression for the amplitude A_f becomes

$$A_f\sim(i)^n\left[\frac{g}{M}\right]^{E_\phi+2L} m_f^2 \times[(c_1pm^{2L-2}+c_3p^3m^{2L-4}+\cdots+c_{2L-1}p^{2L-1}) +m_f(d_0m^{2L-2}+d_2p^2m^{2L-4} +\cdots+d_{2L-2}p^{2L-2})]\ . \tag{4.52}$$

The amplitude $A=\bar{u}[f_0(\gamma)A_0+f_W(\gamma)A_W+f_f(\gamma)A_f]u$ is thus, with $E_\phi=n$,

$$A\sim(i)^n\left[\frac{g}{M}\right]^{n+2L}\bar{u}[f_0(\gamma)m_f(a_0m^{2L}+a_2p^2m^{2L-2}+\cdots+a_{2L}p^{2L}) +f_W(\gamma)M^2(b_1pm^{2L-2}+b_3p^3m^{2L-4}+\cdots+b_{2L-1}p^{2L-1}) +f_1(\gamma)m_f^2(c_1pm^{2L-2}+c_3p^3m^{2L-4}+\cdots+c_{2L-1}p^{2L-1}) +f_2(\gamma)m_f^3(d_0m^{2L-2}+d_2p^2m^{2L-4}+\cdots+d_{2L-2}p^{2L-2})]u\ . \tag{4.53}$$

The coefficients a_i, b_i, c_i, and d_i can in general be functions of $\ln(m^2)$. Adding diagrams with more internal fermion lines results in decreasing the power in the series expansion and raising m_f to that same power. As an example consider the process $\phi\phi\to t\bar{t}$ at the tree level and at one loop. According to Eq. (4.53),

$$A(\phi\phi\to t\bar{t})_{0\text{ loop}}\sim(i)^2\left[\frac{g}{M}\right]^2\bar{u}f_0(\gamma)(m_fa_0)u\ ,$$
$$A(\phi\phi\to t\bar{t})_{1\text{ loop}} \sim(i)^2\left[\frac{g}{M}\right]^4\bar{u}\{f_0(\gamma)m_f(a_0m^2+a_2p^2) +f_W(\gamma)M^2b_1p+f_1(\gamma)m_f^2c_1p +f_2(\gamma)m_f^3d_0\}u\ . \tag{4.54}$$

From the screening theorem we expect a_0 for the one-loop amplitude to be zero. Thus

$$A(\phi\phi\to t\bar{t})_{0\text{ loop}}\sim(i)^2\left[\frac{g}{M}\right]^2\bar{u}f_0(\gamma)(m_fa_0)u\ ,$$
$$A(\phi\phi\to t\bar{t})_{1\text{ loop}} \sim(i)^2\left[\frac{g}{M}\right]^4\bar{u}[f_0(\gamma)m_fa_2p^2+f_W(\gamma)M^2b_1p +f_1(\gamma)m_f^2c_1p+f_2(\gamma)m_f^3d_0]u\ . \tag{4.55}$$

Indeed this result has been obtained in Ref. 11 for the process $W_L^aW_L^b\to t\bar{t}$.

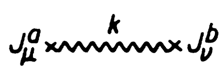


FIG. 6. W source-source transition T in lowest nonzero order.

V. SUMMARY AND DISCUSSION

In Sec. II we derived the renormalized Ward identities in all orders of perturbation theory, with external $\mathcal{C}$ lines on mass shell. Only the Feynman–'t Hooft gauge has been considered.

From the renormalized Ward identities, through the relation $\epsilon_L=k/M+v$ and the subtraction scheme, we derived in Sec. III a relation between amplitudes containing vector bosons multiplied by the vectors ϵ_L or v and the Higgs ghosts [see Eqs. (1.9) and (3.9)].

In the Introduction and at the end of Sec. III we argued that in the renormalizable standard model the equivalence theorem is satisfied for processes for which the amplitude is a constant in the large energy limit.

We then considered the standard model in the large-Higgs-boson-mass limit. In Sec. IV we used the power-counting method to prove the equivalence theorem in the large-Higgs-boson-mass limit. As a result expressions have been derived for amplitudes containing longitudinally polarized vector bosons in all orders of perturbation theory. It is maybe interesting to consider the tree and one-loop amplitudes of a few processes. Define

$$R=\frac{A^1}{A^0}$$

as the ratio of the one loop A^1 and the tree-level amplitude A^0. For W_L scattering we derive from Eqs. (4.27), (4.28), and (4.31),

$$R=\left[\frac{g}{M}\right]^2ap^2\ , \tag{5.1}$$

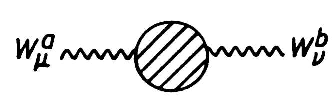


FIG. 7. Irreducible W self-energy diagrams.

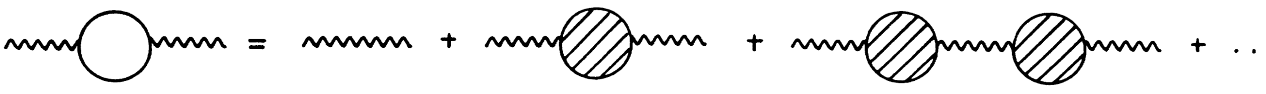


FIG. 8. W dressed propagator.

where p is a typical momentum and a may depend on $\ln(m^2_{\text{Higgs}})$. For the production of one fermion pair, where the fermion mass has not been neglected, we derive from Eq. (4.55) in leading order also the expression of Eq. (5.1). For the production of one fermion pair, where the fermion is considered massless (for example, $W^+_L W^-_L \to \nu\bar{\nu}$), we derive from Eq. (4.53) again Eq. (5.1)

We see that for all of these type of processes involving W_L's the expression for R is proportional to the momentum squared in the large-Higgs-boson-mass limit.

Note added. After completion of this work we received a paper on the same subject by J. Bagger and C. Schmidt [Phys. Rev. D **41**, 264 (1989)].

ACKNOWLEDGMENTS

The author thanks her advisor M. K. Gaillard for her invaluable assistance and helpful discussions. The author also thanks R. Akhoury for his help and important contributions concerning the part on the power-counting method. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and in part by the National Science Foundation under Grant No. PHY-85-15857.

APPENDIX A: RENORMALIZATION OF THE EXTERNAL LINE FOR THE PHYSICAL VECTOR BOSON

The discussion following below can be found in Ref. 7. It is written down here for the purpose of clarity and completeness concerning the discussion of Sec. II B.

When going from the Green's functions between sources to the S matrix, the external momenta are put on mass shell plus the external lines get renormalized, which amounts to multiplying the external sources by a factor Z. The factor Z is found by considering self-energy diagrams. Take for instance the physical vector boson and let us first look at its source J^c_a. In lowest nonzero order the source-source transition T is shown in Fig. 6, where the wiggly line represents the bare propagator and is given by

$$P^{\mu\nu}_{ab} = \frac{\delta_{ab}\delta_{\mu\nu}}{k^2+M^2-i\epsilon} . \tag{A1}$$

For J^a_μ we have $J^a_\mu k_\mu = 0$, since we consider physical sources only. The expression for T is thus

$$\frac{(J^a_\mu)^2}{k^2+M^2-i\epsilon} . \tag{A2}$$

The pole of the propagator is for $k^2=-M^2$. Going to the S matrix we thus multiply with (k^2+M^2) and subsequently put $k^2=-M^2$. Furthermore we require that the residue of the pole be equal to 1. Thus $J^a_\mu = \epsilon^a_\mu$ with $(\epsilon^a_\mu)^2=1$ and $\epsilon^a_\mu k_\mu = 0$. ϵ^a_μ is the well-known polarization vector of the physical vector boson.

In higher order in perturbation theory, we start out with the irreducible W self-energy diagram. See Fig. 7. The corresponding expression is

$$\delta_{ab}[A(k^2)\delta_{\mu\nu} + B(k^2)k_\mu k_\nu] . \tag{A3}$$

We need the expression for the dressed propagator $P^{ab}_{\mu\nu}$ of Fig. 8. It is given by

$$P^{ab}_{\mu\nu} = \delta^{ab}\left[\frac{\delta_{\mu\nu}}{k^2+M^2-A(k^2)} + \frac{k_\mu k_\nu B(k^2)}{[k^2+M^2-A(k^2)][k^2+m^2-A(k^2)-B(k^2)k^2]}\right] \tag{A4}$$

and the source-source transition is shown in Fig. 9. This time the sources are multiplied by a factor Z. Z is of the form $Z_{\mu\nu} = \delta_{\mu\nu}Z_1 + k_\mu k_\nu Z_2$. Since we consider physical sources only and $J^a_\mu k_\mu = 0$, we see that Z_2 and $B(k^2)$ do not contribute. We may write

$$Z_{\mu\nu} = Z\delta_{\mu\nu} \tag{A5}$$

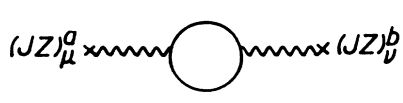


FIG. 9. W source-source transition T in higher order.

and the expression for the source-source transition T is now

$$T = \frac{Z^2(J^a_\mu)^2}{k^2+M^2-A(k^2)} . \tag{A6}$$

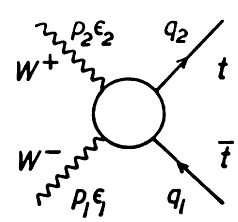


FIG. 10. The process $W^+W^- \to t\bar{t}$.

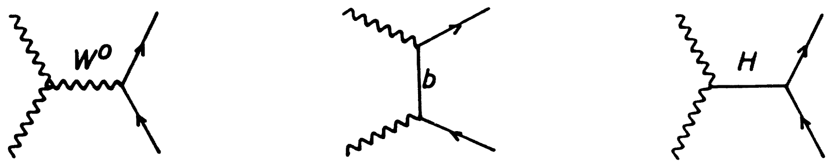


FIG. 11. Feynman diagrams contributing to the amplitude for $W^+W^- \to t\bar{t}$ in lowest nonzero order.

The pole of the propagator is for $k^2 = -M^2 + A_0$. Develop $A(k^2)$ in a Taylor-series expansion around the pole as follows:

$$A(k^2) = A_0 + (k^2+M^2-A_0)\frac{dA(k^2)}{dk^2}\bigg|_{k^2=-M^2+A_0} + (k^2+M^2-A_0)^2\frac{d^2A(k^2)}{d^2k^2}\bigg|_{k^2=-M^2+A_0} + \cdots$$
$$= A_0 + (k^2+M^2-A_0)A_1 + (k^2+M^2-A_0)^2 A_{\text{rest}} . \quad \text{(A7)}$$

T can now be written as

$$T = \frac{Z^2(J^a_\mu)^2}{(k^2+M^2-A_0)[1-A_1-(k^2+M^2-A_0)A_{\text{rest}}]} . \quad \text{(A8)}$$

When going on mass shell we first multiply T with $(k^2+M^2-A_0)$ and then put $k^2+M^2-A_0=0$. The requirement is that the residue equals 1. Thus

$$1 = \frac{Z^2(J^a_\mu)^2}{1-A_1} . \quad \text{(A9)}$$

We already know that $(J^a_\mu)^2 = 1$. Thus for Z we find

$$Z = \sqrt{1-A_1} . \quad \text{(A10)}$$

At one loop we have $A_1 = A_1(g^2)$ and the expression for Z is, in this case,

$$Z_1 = \sqrt{1-A_1(g^2)} = 1 - \frac{A_1(g^2)}{2} + O(g^4) . \quad \text{(A11)}$$

APPENDIX B

Here we will give two examples of the equivalence theorem. First $W_L^+W_L^- \to t\bar{t}$ then $W_L^aW_L^b \to W_L^cW_L^d$ will be discussed. Both are considered only at the tree level and everything is evaluated in the region $m_{\text{top}} \approx M \ll E \ll m_{\text{Higgs}}$.

The process $W_L^+W_L^- \to t\bar{t}$

The process is defined in Fig. 10. All momenta are taken to be ingoing, with $p_1+p_2+q_1+q_2=0$. In lowest nonzero order in perturbation theory there is no cancellation in leading order in energy and we may write

$$A(W_L^+W_L^- \to t\bar{t}) = \epsilon^\mu_{L_1}\epsilon^\nu_{L_2}A_{\mu\nu}$$
$$= \frac{p_1^\mu}{M}\frac{p_2^\nu}{M}A_{\mu\nu}\left[1+O\left[\frac{M^2}{E^2}\right]\right] . \quad \text{(B1)}$$

There are three Feynman diagrams shown in Fig. 11.

In the large-Higgs-boson-mass limit the third diagram does not contribute. For the first diagram we have

$$M_1 = \epsilon^\mu_+\epsilon^\nu_- \frac{-ig^2}{4}\frac{1}{(p_1+p_2)^2+M^2}$$
$$\times[\bar{u}(-q_2)\gamma^\alpha(1+\gamma^5)u(q_1)]$$
$$\times[\delta_{\alpha\nu}(-2p_1-p_2)_\mu + \delta_{\mu\nu}(p_1-p_2)_\alpha$$
$$+\delta_{\alpha\mu}(2p_2+p_1)_\nu] . \quad \text{(B2)}$$

The expression for the second diagram is

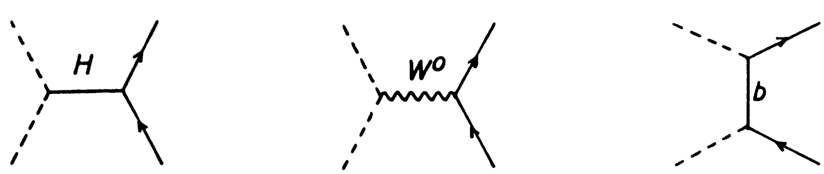


FIG. 12. Feynman diagrams contributing to the amplitude for $\phi^-\phi^- \to t\bar{t}$ in lowest nonzero order.

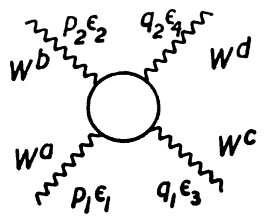


FIG. 13. The process $W^aW^b \to W^cW^d$.

$$M_2=\epsilon_+^{\mu}\epsilon_-^{\nu}\frac{-ig^2}{4}\frac{1}{(p_1+q_1)^2}\times[\bar{u}(-q_2)\gamma^{\mu}(-\not{p}_1-\not{q}_1)\gamma^{\nu}(1+\gamma^5)u(q_1)]\ . \tag{B3}$$

Consider the process in the center-of-mass frame of the two vector bosons, which move along the z axis. We get the four-vectors

$$p_1=\begin{bmatrix}0\\0\\p\\iE\end{bmatrix},\quad p_2=\begin{bmatrix}0\\0\\-p\\iE\end{bmatrix},$$

$$q_1=\begin{bmatrix}-q\sin\theta\\0\\-q\cos\theta\\-iE\end{bmatrix},\quad q_2=\begin{bmatrix}q\sin\theta\\0\\q\cos\theta\\-iE\end{bmatrix}.$$

Substituting

$$\epsilon_-^{\nu}=\frac{1}{M}\begin{bmatrix}0\\0\\E\\ip\end{bmatrix}=\frac{p_1^{\nu}}{M}+O\left[\frac{M}{E}\right],$$

$$\epsilon_+^{\mu}=\frac{1}{M}\begin{bmatrix}0\\0\\-E\\ip\end{bmatrix}=\frac{p_2^{\mu}}{M}+O\left[\frac{M}{E}\right],$$

we find, in leading order [the $O(M/E)$ term is not written down],

$$M_1=\frac{-ig^2}{8(p_1p_2)M^2}[\bar{u}(-q_2)\gamma^{\alpha}(1+\gamma^5)u(q_1)]\times[p_1^{\alpha}(-2p_1p_2)+(p_1-p_2)^{\alpha}(p_1p_2)+p_2^{\alpha}(2p_1p_2)]$$
$$=\frac{g^2}{8M^2}[\bar{u}(-q_2)(i\not{p}_1-i\not{p}_2)(1+\gamma^5)u(q_1)] \tag{B4}$$

and

$$M_2=\frac{-ig^2}{8M^2(p_1q_1)}[\bar{u}(-q_2)\not{p}_2(-\not{p}_1-\not{q}_1)\not{p}_1(1+\gamma^5)u(q_1)]$$
$$=\frac{ig^2}{8M^2(p_1q_1)}[\bar{u}(-q_2)\not{p}_2(2p_1q_1-\not{p}_1\not{q}_1)(1+\gamma^5)u(q_1)]$$
$$=\frac{g^2}{4M^2}[\bar{u}(-q_2)i\not{p}_2(1+\gamma^5)u(q_1)]-\frac{g^2m_t}{4M^2}[\bar{u}(-q_2)(1-\gamma^5)u(q_1)]\ . \tag{B5}$$

The amplitude $A=M_1+M_2$ is given by

$$A(W_L^+W_L^-\to t\bar{t})=-\frac{g^2m_t}{4M^2}[\bar{u}(-q_2)u(q_1)]\ . \tag{B6}$$

In the above manipulations the Dirac equation $(i\not{p}+m_t)u(p)=0$ has been used. The Feynman diagrams for the process $\phi^+\phi^-\to t\bar{t}$ are given in Fig. 12.

Here we see immediately that in the large-Higgs-boson-mass limit the first diagram gives the leading term in energy since the $\phi^+\phi^-H$ coupling is proportional to m_{Higgs}^2. The amplitude A of the first diagram is given by

$$A(\phi^+\phi^-\to t\bar{t})=\frac{g^2m_t}{4M^2}[\bar{u}(-q_2)u(q_1)]\ . \tag{B7}$$

As in agreement with the equivalence theorem we have, in leading order,

$$A(W_L^+W_L^-\to t\bar{t})+A(\phi^+\phi^-\to t\bar{t})=0\ . \tag{B8}$$

The process $W_L^aW_L^b\to W_L^cW_L^d$

For W_LW_L scattering the leading energy term cancels due to the structure of the Yang-Mills vertices. Thus we cannot simply replace the longitudinally polarization vector by the corresponding momentum vector:

$$\epsilon_{\mu}^1\epsilon_{\nu}^2\epsilon_{\alpha}^3\epsilon_{\beta}^4A_{\mu\nu\alpha\beta}\neq\left[\frac{p_{\mu}^1}{M}\frac{p_{\nu}^2}{M}\frac{q_{\alpha}^1}{M}\frac{q_{\beta}^2}{M}\right]A_{\mu\nu\alpha\beta}\left[1+O\left[\frac{M^2}{E^2}\right]\right]. \tag{B9}$$

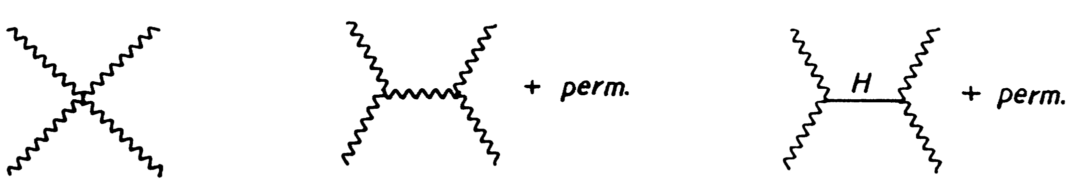


FIG. 14. Feynman diagrams contributing to the amplitude for W_LW_L scattering in lowest nonzero order.

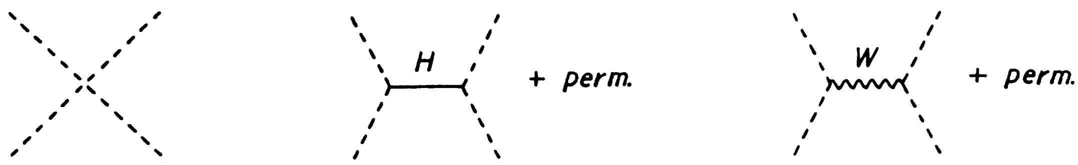


FIG. 15. Feynman diagrams contributing to the amplitude for $\phi\phi$ scattering in lowest nonzero order.

The process is defined in Fig. 13.

All the momenta are taken to be ingoing, thus $p_1+p_2+q_1+q_2=0$. In the center-of-mass frame define the four-vectors as

$$p_1=\begin{bmatrix}0\\0\\p\\iE\end{bmatrix},\quad p_2=\begin{bmatrix}0\\0\\-p\\iE\end{bmatrix},$$

$$q_1=\begin{bmatrix}-p\sin\theta\\0\\-p\cos\theta\\-iE\end{bmatrix},\quad q_2=\begin{bmatrix}p\sin\theta\\0\\p\cos\theta\\-iE\end{bmatrix},$$

$$\epsilon_1^{\nu}=\frac{1}{M}\begin{bmatrix}0\\0\\E\\ip\end{bmatrix},\quad \epsilon_2^{\mu}=\frac{1}{M}\begin{bmatrix}0\\0\\-E\\ip\end{bmatrix},$$

$$\epsilon_3^{\alpha}=\frac{1}{M}\begin{bmatrix}-E\sin\theta\\0\\-E\cos\theta\\-ip\end{bmatrix},\quad \epsilon_4^{\beta}=\frac{1}{M}\begin{bmatrix}E\sin\theta\\0\\E\cos\theta\\-ip\end{bmatrix}.$$

In lowest nonzero order we have Fig. 14.

In the large-Higgs-boson-mass limit the diagrams with the Higgs boson as propagator do not contribute. The amplitude in leading order is found to be

$$A(W_L^aW_L^b\to W_L^cW_L^d)=\frac{g^2}{4M^2}(\delta_{ab}\delta_{cd}s+\delta_{ac}\delta_{bd}t+\delta_{ad}\delta_{bc}u)\,, \quad \text{(B10)}$$

with

$$s=-(p_1+p_2)^2,\quad t=-(p_1+q_1)^2,\quad u=-(p_1+q_2)^2\,.$$

Now consider $A(\phi^a\phi^b\to\phi^c\phi^d)$. See Fig. 15.

We see immediately that the diagrams with the Higgs propagator and the four-vertex give the leading order in energy, since the $\phi\phi H$ and $\phi\phi\phi\phi$ couplings are proportional to the Higgs-boson mass squared. The amplitude for the diagrams with the Higgs propagator is

$$M(H-\text{prop})=\frac{g^2m^4}{4M^2}\left[\delta_{ab}\delta_{cd}\frac{1}{m^2-s}+\delta_{ac}\delta_{bd}\frac{1}{m^2-t}+\delta_{ad}\delta_{bc}\frac{1}{m^2-u}\right] \quad \text{(B11)}$$

and for the four-vertex diagram we have

$$M(4-\text{vertex})=-\frac{g^2m^2}{4M^2}(\delta_{ab}\delta_{cd}+\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc})\,. \quad \text{(B12)}$$

The amplitudes $M(H-\text{prop})$ and $M(4-\text{vertex})$ are each proportional to the Higgs-boson mass squared. Added up, the m^2 term cancels and we have, for the amplitude in leading order,

$$A(\phi^a\phi^b\to\phi^c\phi^d)=\frac{g^2}{4M^2}(\delta_{ab}\delta_{cd}s+\delta_{ac}\delta_{bd}t+\delta_{ad}\delta_{bc}u)\,. \quad \text{(B13)}$$

Comparing the amplitudes of Eq. (B10) and (B13) we see that the equivalence theorem is satisfied.

[1]M. S. Chanowitz and M. K. Gaillard, Nucl. Phys. **B261**, 379 (1985).

[2]B. W. Lee, C. Quigg, and H. Thacker, Phys. Rev. D **16**, 1519 (1977).

[3]J. M. Cornwall, D. N. Levin, and G. Tiktopoulos, Phys. Rev. D **10**, 1145 (1974).

[4]G. J. Gounaris, R. Kögerler, and H. Neufeld, Phys. Rev. D **34**, 3257 (1986).

[5]Y.-P. Yao and C.-P. Yuan, Phys. Rev. D **38**, 2237 (1988).

[6]M. Veltman and F. Yndurain, Nucl. Phys. **B325**, 1 (1989).

[7]G. 't Hooft and M. Veltman, *Renormalization of Yang-Mills Fields and Applications to Particle Physics,* proceedings of the Colloquium, Marseille, France, 1972 (CNRS, Marseille, 1972); Nucl. Phys. **B50**, 318 (1972).

[8]J. v.d. Bij and M. Veltman, Nucl. Phys. **B231**, 205 (1984).

[9]M. K. Gaillard (private communication).

[10]C. M. v.d. Kolk, Phys. Lett. **57B**, 165 (1975).

[11]H. Veltman (in preparation).