Color gauge group and new low-energy phenomena

Robert Foot and Oscar F. Hernández
Department of Physics, University of Wisconsin-Madison, Madison, Wisconsin 53706
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The unbroken color gauge group $SU(3)_c$ may arise as a consequence of the spontaneous symmetry breaking of some larger group. The possible uniqueness of the color gauge group is examined under a set of minimal assumptions. Under these assumptions it is shown that the color groups $G_c = SU(4)$, SU(5) (and perhaps E_6) are essentially uniquely singled out as possible alternatives of the standard $SU(3)_c$ assignment. The symmetry breaking to $SU(3)_c$ can take place without the introduction of a gauge hierarchy. The consequences of $G_c = SU(5)$ are discussed, including new phenomena which occur between 100 GeV and 1 TeV, and agreement with the standard model at energies below 100 GeV.

I. INTRODUCTION

Experimentally, it appears clear that the forces of nature are described by a Yang-Mills theory with the gauge group

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_y \ . \tag{1}$$

This defines the standard model (SM) of elementary-particle physics. This theory is in good agreement with present experiments (i.e., for $\sqrt{s} \lesssim 100$ GeV). What is not so clear is what lies beyond this theory. We would like to consider things that could lead to new physics at energies of 100 GeV to 1 TeV (versus energies above 10^{15} GeV) without changing the experimental successes of the SM. One possibility is that the strong force is a remnant of a larger group G_c; i.e., the SM is embedded in the group

$$G_c \otimes SU(2)_L \otimes U(1)_{y'} \ . \tag{2}$$

Recently, the possibility that $G_c = SU(4)$ has been proposed.[1] In this theory, the quarks transform under the fundamental (four-dimensional) representation of the color group SU(4). It was shown that one additional Higgs multiplet could give the exotic quarks [the $SU(3)_c$-singlet quarks] a large mass ($M \gtrsim 100$ GeV), while at the same time break $SU(4) \otimes SU(2)_L \otimes U(1)_{y'}$ to $SU(3)_c \otimes SU(2)_L \otimes U(1)_y$. This may occur without the introduction of a gauge hierarchy. The purpose of this paper is to examine the uniqueness of this construction under a certain set of assumptions. Under these assumptions we will show that there is only one new possibility, namely, that $G_c = SU(5)$. This possibility [along with the $G_c = SU(4)$ model] predicts new phenomena which naturally occur in the energy range $\sqrt{s} \approx 100$ GeV to 1 TeV. In addition it leaves the electroweak couplings almost equal (to within 0.3%) to the SM values.

II. UNIQUENESS OF THE COLOR GAUGE GROUP

We assume that the quarks transform under the fundamental representation of G_c. We also assume that G_c is simple. If the fundamental representation of G_c is N dimensional then the quarks have N colors. Anomaly cancellation conditions and classical gauge invariance of the Yukawa Lagrangian imply that the SM generation has the following structure under $G_c \otimes SU(2)_L \otimes U(1)_{y'}$:

$$\begin{aligned} &f_L \sim (1,2,-1), \quad e_R \sim (1,1,-2) \ , \\ &Q_L \sim \left[N,2,\frac{1}{N}\right], \quad u_R \sim \left[N,1,\frac{1}{N}+1\right] \ , \\ &d_R \sim \left[N,1,\frac{1}{N}-1\right] \ , \end{aligned} \tag{3}$$

where we have normalized the hypercharge of the SM Higgs boson to one, i.e., $\phi \sim (1,2,1)$ under Eq. (2). We also assume that we break G_c down to $SU(3)_c$ by introducing only one colored Higgs multiplet χ. We wish to use the same Higgs field χ to give the $2(N-3)n_g$ exotic quarks (n_g is the number of generations) a large mass, i.e., to separate their masses from the ordinary quark masses. Thus, χ analogues the ordinary Higgs-doublet field ϕ, which gives the ordinary fermions their mass, as well as breaking $SU(2)_L \otimes U(1)_y \to U(1)_Q$. We will also assume that χ transforms $(R,1,y)$; i.e., we will restrict χ to transform as a singlet under SU(2). This will allow us to separate the scale of G_c-symmetry breaking from that of $SU(2)_L$-symmetry breaking. Let us now restrict the discussion to one generation of fermions; generalization to three or more generations should be clear. The representations R and y will be determined from the Yukawa Lagrangian

$$L_{\text{Yuk}} = \lambda_1 \bar{Q}_L \chi (Q_L)^c + \lambda_2 \bar{u}_R \chi (d_R)^c + \text{H.c.} + L_0 \ , \tag{4}$$

where L_0 comprises the usual mass terms:

$$L_0 = \lambda_3 \bar{f}_L \phi e_R + \lambda_4 \bar{Q}_L \phi d_R + \lambda_5 \bar{Q}_L \phi^c u_R + \text{H.c.} \tag{5}$$

L_{Yuk} gives Dirac mass terms to the exotic fermions [the conjugate acts on the $SU(2)_L$ indices also]. From Eq. (4), we see that the quantum numbers of χ are given by

$$\chi \sim \left[R,1,\frac{2}{N}\right] \tag{6}$$

and R is an irreducible representation contained in the tensor product $N\times N$ of G_c. By assumption, the G_c representation of χ must contain a $\mathrm{SU}(3)_c$-singlet piece which develops a vacuum expectation value (VEV) which breaks G_c spontaneously. Consider the case when the representation G_c is self-conjugate. In this case, since the 3 of $\mathrm{SU}(3)_c$ is complex,

$$R \supset 3+\bar{3}+\text{extra bits} . \qquad (7)$$

For the theory to be consistent with experiment, the exotic quark fields must gain a large mass (i.e., $M \gtrsim 50$ GeV). However, the mass terms in Eq. (4) cannot couple $\bar{3}$ to $\bar{3}$, but 3 to $\bar{3}$ (as well as singlets to singlets). An examination of the fermion mass matrix shows that two things can happen. Either the exotic color-triplet quarks gain a large mass and the ordinary quarks will gain a large mass, or there will be light quarks and heavy quarks, but these mass-eigenstate quarks will have approximately vectorlike couplings to the W and Z gauge bosons. Either way, such a theory is inconsistent with experiment. Thus the groups G_c which have a self-conjugate fundamental representation can be excluded. This rules out the following candidate groups for G_c: SO(N), SP(N), G_2, F_4, E_7, and E_8 (Ref. 2). The remaining possibility is that $G_c=\mathrm{SU}(N)$ and E_6. We consider the case of E_6 an unlikely possibility since it would be difficult to break the fundamental 27-dimensional representation of E_6 down to the fundamental three-dimensional representation of SU(3) in a consistent way and with only one Higgs boson. We will subsequently focus on SU(N). In this case the $\mathrm{SU}(N)_c$ representation of χ is contained in the tensor product $N\times N$. The tensor product $N\times N$ contains the symmetric $N(N+1)/2$ and antisymmetric $N(N-1)/2$ representation. Spontaneous symmetry breaking of SU(N) was studied a long time ago. It was shown in Ref. 3 that the symmetric representation can break SU(N) to SU($N-1$), while the antisymmetric representation can break SU(N) down to $\mathrm{SU}(N-2)\otimes\mathrm{SU}(2)$. This implies that SU(4) and SU(5) are the only possible SU(N) generalizations of the color group (with our assumptions that G_c is simple, the quarks transform under the fundamental representation, and we have only one extra Higgs boson).

III. THE $\mathrm{SU(5)}_c$ MODEL

At this stage we have not yet shown that the SU(4) and SU(5) theories are consistent with experiment, we have only excluded the other gauge groups. The possibility that $G_c=\mathrm{SU}(4)$ was examined in Ref. 1. In that paper it was shown that with χ in the symmetric 10 representation of SU(4), with the quantum numbers given in Eq. (6) with $N=4$, rotated the unbroken electric charge generator to

$$Q=I_3+\frac{Y'}{2}+\frac{T}{8} , \qquad (8)$$

where T is the SU(4) generator with the fundamental representation normalized as

$$T=\mathrm{diag}(\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},-1) . \qquad (9)$$

It is straightforward to show that $Y=Y'+T/4$ gives the familiar hypercharge quantum numbers for the ordinary color-triplet quarks. Thus the electric charges agree with experiment.

In the case of $G_c=\mathrm{SU}(5)$, the electric charge generator couples to

$$Q=I_3+\frac{Y'}{2}+\frac{T}{5} , \qquad (10)$$

where T is normalized so that in the fundamental representation, T has the form

$$T=\mathrm{Diag}(\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3},\tfrac{-1}{2},\tfrac{-1}{2}) . \qquad (11)$$

One can check that $Q\langle\phi\rangle=0, Q\langle\chi\rangle=0$, where Q is the charge operator defined in Eq. (10), and $\langle\phi\rangle$ and $\langle\chi\rangle$ denote the VEV's of the Higgs fields ϕ and χ, respectively. These VEV's can be expressed as

$$\langle\phi\rangle=\begin{bmatrix}0\\ u\end{bmatrix}, \quad \langle\chi\rangle=\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&0\\0&0&0&0&w\\0&0&0&-w&0\end{bmatrix} . \qquad (12)$$

Again, one can show that $Y=Y'+2T/5$ gives the familiar values for the hypercharges of the color-triplet quarks. As in the SU(4) case, the $\mathrm{SU(3)}_c$-singlet quarks have charges $\pm 1/2$. However there is a novel difference between the SU(4) and SU(5) models. In the SU(5) model, there is a $\mathrm{SU(2)}'$ subgroup which remains unbroken. Explicitly, if $g_s w > gu$, then the gauge group breaks as follows:

$$\begin{array}{l}\mathrm{SU}(5)\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_{y'}\\ \qquad\downarrow\ g_s w/\sqrt{2}\\ \mathrm{SU}(3)_c\otimes\mathrm{SU}(2)'\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_y\\ \qquad\downarrow\ gu/\sqrt{2}\\ \mathrm{SU}(3)_c\otimes\mathrm{SU}(2)'\otimes\mathrm{U}(1)_Q .\end{array} \qquad (13)$$

Since the coupling constant g_s is larger than the electroweak couplings g,g', the condition $g_s w > gu$ can be satisfied even when $w\approx u$. (Note that in our theory quarks and leptons are not put in the same irreducible representations of the gauge group, and hence we do not require any hierarchy of the form $w \gg u$ which is characteristic of unified theories.) The group $\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)'\otimes\mathrm{U}(1)_Q$ is unbroken. The $\mathrm{SU}(2)'$ gauge bosons interact with the exotic $\mathrm{SU}(3)_c$-singlet quarks. They do not interact with ordinary matter at the tree level. Furthermore their production cross sections are expected to be small, and once they are produced, they will be difficult to detect.

At this stage several comments are in order.

(1) This model, as well as the SU(4) model, will contain a baryon-number symmetry $B+T/3$ for SU(4) and $B+2T/3$ for SU(5). Here B has charge 1/3 for the quarks and 2/3 for χ. The SU(4) model predicts a new heavy absolutely stable particle. The existence of such a particle is not expected to be a cosmological problem (cf. stable heavy neutrino[4]). However because the exotic par-

ticles in the SU(5) model will be confined by the SU(2)′ force [see comment (5)], they will not be absolutely stable. For example, an exotic hadron made up of two charge 1/2 particles would decay via the exchange of charge 1/6 bosons [see comment (7)] into a π^0 and a π^+.

(2) The Higgs field χ transforms as the **10** representation of SU(5), and it can be represented by an antisymmetric 5×5 matrix. The Higgs potential has the form

$$\begin{aligned}V(\phi,\chi)=&\lambda_1(\phi^\dagger\phi-u^2)^2+\lambda_2(\mathrm{Tr}\chi^\dagger\chi-2w^2)^2\\&+\lambda_3[2\,\mathrm{Tr}\chi^\dagger\chi\chi^\dagger\chi-(\mathrm{Tr}\chi^\dagger\chi)^2]\\&+\lambda_4(\phi^\dagger\phi-u^2)(\mathrm{Tr}\chi^\dagger\chi-2w^2)\ .\end{aligned} \tag{14}$$

The potential has a minimum provided $\lambda_1,\lambda_2>0$, and the minimum is given in Eq. (12) provided $\lambda_3<0$ (Ref. 3). Under the unbroken $\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)'\otimes\mathrm{U}(1)_Q$ subgroup of the gauge group, the Higgs field χ transforms as

$$\mathbf{10}=(3,2,1/6)+(\bar{3},1,1/3)+(1,1,0)\ . \tag{15}$$

The multiplets (3,2,1/6) and the imaginary part of (1,1,0) are Goldstone bosons which are absorbed to give the gauge bosons in the coset $\mathrm{SU}(5)/\mathrm{SU}(3)\otimes\mathrm{SU}(2)$ mass. The physical Higgs bosons are the $(\bar{3},1,1/3)$ and the real part of (1,1,0).

(3) The physical Higgs field $(\bar{3},1,1/3)$ couples to ordinary quarks and the coupling can be obtained from Eq. (4). It is interesting to observe that such a Higgs multiplet and coupling arises in attempts to understand the fermion masses and mixings from radiative mass generation.[5] From the point of view of the SM, these Higgs fields must be put in by hand, while in the SU(5) model, these Higgs fields arise as a natural consequence of symmetry breaking.

(4) In the case where $G_c=\mathrm{SU}(5)$, each generation has an even number of $\mathrm{SU}(2)_L$ doublets. Hence, their is no restrictions on the number of generations from the global anomaly[6] in contrast with the $G_c=\mathrm{SU}(4)$ case[1] which requires an even number of generations.

(5) The SU(2)′ force is expected to be confining, since the beta function is negative at the fixed point $g=0$ (i.e., $g=0$ is an ultraviolet fixed point). We would like to get an idea of how the confinement scale of the exotic hadrons compares to the QCD hadrons. We will see that above the exotic fermion mass threshold several interesting phenomena occur.

For an asymptotically free SU(N) gauge theory, with coupling constant g, the leading-logarithmic behavior of the force constant $\alpha=g^2/(4\pi)$ can be expressed in terms of the renormalization scale μ as[7]

$$\alpha(\mu)=\frac{2\pi}{(\frac{11}{3}N-\frac{2}{3}N_f)\ln\mu/\Lambda}\ , \tag{16}$$

where N_f is the number of Dirac fermions with mass less than or equal to μ transforming under the N-dimensional representation of SU(N) [we have neglected the Higgs contribution to Eq. (16) since it is very small]. Since we will be discussing phenomena which occur at energies greater than or equal to electroweak unification (100 GeV), we will be interested in Λ_{QCD} for six quarks. For definiteness we will work with Λ as defined via the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme. We will take $\Lambda_{\mathrm{QCD}}=0.1$ GeV. Because we are interested in $\mu\geq 100$ GeV, we are safely in a region where the perturbative one-loop approximation given above is valid for QCD.

Let us assume that at the color unification scale the one-loop approximation is also valid for SU(2)′ and then use Eq. (16) to get an idea of how $\Lambda_{\mathrm{SU(2)'}}$ compares with Λ_{QCD}. At the color unification scale ($\mu=\mu_0$) of $\mathrm{SU}(3)_c$ and SU(2)′, we have

$$\alpha_s(\mu_0)=\alpha_{\mathrm{SU(2)'}}(\mu_0)\ . \tag{17}$$

From Eqs. (17) and (16) we can obtain a relation for $\Lambda_{\mathrm{SU(2)'}}$ in terms of Λ_{QCD} and the color unification scale μ_0:

$$\frac{2\pi}{(11-\frac{2}{3}N_f)\ln\mu_0/\Lambda_{\mathrm{QCD}}}=\frac{2\pi}{(\frac{22}{3}-\frac{2}{3}N_f')\ln\mu_0/\Lambda_{\mathrm{SU(2)'}}}\ . \tag{18}$$

Solving Eq. (18), we find that

$$\Lambda_{\mathrm{SU(2)'}}=\Lambda_{\mathrm{QCD}}\left[\frac{\Lambda_{\mathrm{QCD}}}{\mu_0}\right]^{x-1}\ , \tag{19}$$

where

$$x=\frac{33-2N_f}{22-2N_f'} \tag{20}$$

and $N_f=6$, the number of quark flavors in QCD, and N_f' is the number of exotic quarks with mass less than μ_0. Since we are assuming that there are three generations, we know that $0\leq N_f'\leq 6$.

The most natural color unification scale in the model is obtained by assuming that the VEV's [defined in Eq. (12)] satisfy $w\approx u$. Of course, this is not a stringent condition; however, it is useful for the purposes of illustration. Under this assumption, $\mu_0\approx g_sM_Z/g\approx 200$ GeV. This is also approximately the lowest value which is consistent with experiment. Thus Eq. (19) gives

$$\Lambda_{\mathrm{SU(2)'}}=\Lambda_{\mathrm{QCD}}\left[\frac{1}{2000}\right]^{(-1+2N_f')/(22-2N_f')}\ . \tag{21}$$

The value of $\Lambda_{\mathrm{SU(2)'}}$ is very sensitive to N_f'. If N_f' is varied from 0 to 6, then $\Lambda_{\mathrm{SU(2)'}}$ varies from $1.4\Lambda_{\mathrm{QCD}}$ to $10^{-4}\Lambda_{\mathrm{QCD}}$. This is consistent with our perturbative analysis. Because exotic hadrons have not been seen we expect the exotic quarks have masses of at least 50 GeV. Thus $\Lambda_{\mathrm{SU(2)'}}/M_{\text{exotic quarks}}\ll 1$ and the exotic hadrons are nonrelativistically bound states. If $N_f'=6$, the exotic quarks can be separated 10 000 times further than QCD quarks. It is interesting to observe that if four generations exist, and we assume that $N_f=N_f'=8$, then $\Lambda_{\mathrm{SU(2)'}}=10^{-8}\Lambda_{\mathrm{QCD}}$. Thus it may be possible to "see" the tracks of charge 1/2 particles turn into the jets of exotic hadrons.

(6) As in the SU(4) model, one can check that the Z boson couples to the ordinary quarks and leptons like the SM Z boson, with only a very small deviation. The gauge-boson mass matrix arises from the Higgs kinetic term

$$L=(D_\mu\phi)^\dagger(D^\mu\phi)+(D_\mu\chi)^\dagger(D^\mu\chi)\ , \tag{22}$$

where the covariant derivative D_μ, is given by

$$D_\mu=\partial_\mu+igW^0_\mu I_3+ig'B_\mu\frac{Y'}{2}+ig_sC_\mu T_{12}+ig_s\sum_{a=1}^{11}G^a_\mu T_a(+\text{charged-gauge-boson terms})\ , \tag{23}$$

where $T_{12}=\sqrt{3/5}T$ and T is normalized as in Eq. (11). The generators T_a for $a=1,\ldots,11$ are given by Λ_a for $a=1,\ldots,8$ and F_a for $a=9,\ldots,11$, where Λ and F are given by

$$\Lambda_a=\begin{bmatrix}\lambda_a/2 & 0\\ 0 & 0\end{bmatrix}\ ,\quad F_a=\begin{bmatrix}0 & 0\\ 0 & \tau_{a-8}/2\end{bmatrix}\ . \tag{24}$$

Here λ_a are the Gell-Mann matrices and τ_a are the Pauli matrices. The generators Λ_a and F_a are the generators of the unbroken $SU(3)_c$ and $SU(2)'$ subgroups, respectively. It is easy to see that the G_a gauge bosons decouple from the mass matrix, leaving the 3×3 mass matrix:

$$L_{\text{mass}}=\tfrac{1}{2}V^TM^2V\ , \tag{25}$$

where

$$V^T=(W_0,B,C) \tag{26}$$

and

$$M^2=\begin{vmatrix} g^2u^2/2 & -gg'u^2/2 & 0\\ -gg'u^2/2 & g'^2u^2/2+4g'^2w^2/25 & -4\sqrt{3/5}g'g_sw^2/5\\ 0 & -4\sqrt{3/5}g'g_sw^2/5 & 12g_s^2w^2/5\end{vmatrix}\ . \tag{27}$$

As in the SU(4) model, we can expand out the eigenvalues and coupling in a power series assuming $g_s^2\gg g'^2,g^2$. We obtain

$$M_\gamma^2=0\ ,$$
$$M_Z^2=\tfrac{1}{2}(g^2+g'^2)u^2-\frac{g'^4u^2}{30g_s^2}+O\left[\frac{g'^6u^2}{g_s^4},\frac{g'^6u^4}{g_s^4w^2}\right]\ ,$$
$$M_{Z'}^2=\frac{12g_s^2w^2}{5}\left[1+\frac{g'^2}{15g_s^2}+O\left[\frac{g'^4u^2}{g_s^4}\right]\right]\ . \tag{28}$$

Similarly, to leading order in g'^2/g_s^2, one can check that the Z boson couples to the generator

$$\frac{2e}{\sin2\theta_W}(I_3-\sin^2\theta_WQ)+\Delta\ , \tag{29}$$

where $\tan\theta_W=g'/g$, and e is the electron charge which is given by

$$e=\frac{g\sin\theta_W}{[1+(g'^2\cos^2\theta_W)/(15g_s^2)]^{1/2}}\ . \tag{30}$$

The correction term Δ can be calculated to leading order:[8]

$$\Delta=\frac{2e}{\sin2\theta_W}\frac{g'^2}{30g_s^2}\left[\cos2\theta_WI_3+\sin^2\theta_WQ-\frac{5u^2}{4w^2}T\right]. \tag{31}$$

If the color unification scale μ_0 is of the same order of magnitude as the electroweak unification scale (e.g., $\mu_0\approx200$ GeV), then in Eqs. (27)–(31), the coupling constant g_s is approximately equal to the strong coupling constant at the scale M_Z. Numerically, this approximation corresponds to setting $g'^2/g_s^2\approx1/10$. It is easy to see that under this assumption, the mass and coupling of the Z boson receive only a small modification in the model.

(7) Under the unbroken group $SU(3)_c\otimes SU(2)'\otimes U(1)_Q$ the gauge bosons transform as

$$\mathbf{24}=(8,1,0)+(1,3,0)+(1,1,0)+(3,2,1/6)+(\bar{3},2,-1/6)\ . \tag{32}$$

We identify the eight massless gluons of unbroken $SU(3)_c$, the three massless gauge bosons of unbroken $SU(2)'$, and the new neutral gauge boson (C_μ). In addition there are charged 1/6 gauge bosons, X. These gauge bosons have the mass

$$M_X^2=g_s^2w^2/2\ . \tag{33}$$

Thus the model predicts

$$\frac{M_X^2}{M_{Z'}^2}\simeq\frac{5}{24}\ . \tag{34}$$

(8) Both the SU(4) and SU(5) models preserve the charge quantization feature of the SM (Ref. 9). This means that the existence of the Yukawa Lagrangian and the cancellation of the gauge anomalies imply that the electric charges of the known fermions have their experimentally determined values.

IV. CONCLUSION

In conclusion, we have examined the uniqueness of the color gauge group of the SM. We have shown that two interesting alternatives of the SM can be constructed: $SU(3)_c$ replaced by SU(4) or SU(5). Both these alternatives agree with the SM at energies $\lesssim100$ GeV. Both these alternatives predict the existence of charged $\pm1/2$ fermions. In the case of SU(4), these fermions are not expected to be confined, while in the SU(5) case we have shown that these fermions are expected to be confined into new hadrons. These hadrons have charges 0 and ±1.

The result that the SU(4) model predicts the existence of unconfined charged $\pm 1/2$ particles, while the SU(5) model predicts the existence of integrally charged hadrons, should provide the most direct experimental signature with which to differentiate these two models. Finally, we have argued that the new phenomena predicted by these models would most naturally occur in the energy range 100 GeV to 1 TeV. This is the energy range which will be probed by experiments in the near future.

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