Radiative corrections to Higgs-boson-mass sum rules in the minimal supersymmetric extension to the standard model

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(Received 8 August 1989)

The mass sum rules involving Higgs bosons that arise in supersymmetric extensions to the standard model are renormalized. The general procedure for calculating radiative corrections to these relations is presented. The radiative corrections from loop contributions of quarks, leptons, and their supersymmetric partners to the mass relation $M_h^2+M_H^2=M_A^2+M_Z^2$ of the minimal supersymmetric extension to the standard model are derived. The results indicate that large corrections to the sum rules may arise from heavy matter fields. Squarks significantly heavier than their fermionic partners contribute large contributions when mixing occurs in the squark sector. These large corrections result from squark–Higgs-boson couplings that become large in this limit. Contributions to individual Higgs-boson masses that are quadratic in the squark masses cancel in the sum rule. Thus the naturalness constraint on Higgs-boson masses is hidden in the sum rule.

I. INTRODUCTION

Supersymmetry has been studied extensively in recent years as a way to solve the naturalness problem. At the moment it is the only known way to reconcile the vast difference between the electroweak and grand-unified-theory (GUT) scales while still retaining scalars as fundamental fields. Quadratic divergences that would otherwise appear in scalar mass corrections do not arise because of the chiral invariance of their fermionic superpartners. In this paper we calculate radiative corrections from quark and squark loops to Higgs-boson-mass relations that arise in the minimal supersymmetric extension to the standard model. The radiative corrections arising from loops containing neutralinos and charginos have been considered in Ref. 1. No large corrections to the mass relations were found. We find that large corrections can occur for quark and squark loops, but only if significant mixing occurs between the left- and right-handed squarks. In addition we develop a formalism for calculating radiative corrections to Higgs-boson mass relations in a supersymmetric extension with an arbitrary number of Higgs doublets.

In this paper we are primarily concerned with extensions of the standard model that have two Higgs doublets only. The two-Higgs-doublet model has eight degrees of freedom in the Higgs sector which become three neutral Higgs bosons (H,h,A), two charged Higgs boson (H^+,H^-), and the usual three Goldstone bosons (G,G^+,G^-) that are absorbed by the W and the Z. H and h are CP-even eigenstates while A is CP odd. We follow the usual practice of calling these scalars and pseudoscalars, respectively. We consider the supersymmetric version of the two-Higgs-doublet extension to the standard model.[2] The restrictions imposed by supersymmetry tightly constrain the couplings in the Higgs sector. This leads to mass relations for the physical Higgs boson. In addition, at the tree level the lightest neutral Higgs boson h must be lighter than the Z, and the heaviest neutral Higgs boson H must be heavier than the Z. In fact this conclusion remains true for supersymmetric extensions of the standard model containing an arbitrary number of Higgs doublets (containing no Higgs singlets or other representations).[3]

We shall refer to the two-Higgs-doublet supersymmetric extension of the standard model as the minimal supersymmetry extension (MSE). In this model, there exist the tree-level mass relations

$$M_H^2+M_h^2=M_A^2+M_Z^2 \tag{1.1}$$

and

$$M_{H^\pm}^2=M_A^2+M_W^2 \;. \tag{1.2}$$

We explicitly calculate the $O(\alpha)$ corrections to the relation (1.1) arising from the quark and lepton sectors. The corrections to (1.1) and (1.2) will all be $O(\alpha)$ for the one-loop calculation since in supersymmetric models the cubic and quartic couplings in the Higgs potential are related to the gauge couplings g and g'. There is no arbitrary coupling in supersymmetric extensions of the standard model such as the quartic coupling λ in the standard model. The philosophy is, therefore, slightly different in the renormalization of the mass relation in (1.1) of the MSE. The sum rule in (1.1) involves physically measurable masses, without any reference to couplings. So we can take these masses as the parameters that define the Higgs sector, and find radiative corrections to (1.1) in terms of these parameters. We find that large corrections to the mass relation in (1.1) can arise from matter loops but only if the significant mixing occurs between the squark fields.

Large corrections $[O(\alpha m_q^4/M_W^2)$ where m_q is a quark mass] to the Higgs-boson masses arise as they do in the standard model. The squark $\tilde{q}$ corrections to Higgs-boson masses that are $O(\alpha m_{\tilde{q}}^2)$ are quadratic in the supersymmetry-breaking scale. If they become large, they destroy the stability of the electroweak scale to radiative corrections, necessitating large subtractions that require unnatural fine-tuning order by order in perturbation theory. These contributions cancel exactly in the renormalization of the sum rule. Therefore, the naturalness constraint is "hidden" in the sum rule. Mixing between left- and right-handed squarks occurs in general. If the off-diagonal entries in the left-right squark-quark mass matrix are large, then large squark–Higgs-boson couplings can arise and result in large corrections to the mass relation.

In Sec. II we review the aspects of the model that are needed for this work. In Sec. III we explain in detail the formalism for renormalizing the Higgs sector of the MSE. We discuss the results of actual calculations we have performed in the MSE in Sec. IV. Since the physical masses of the Higgs bosons (H,h,A) and the Z are measurable, the $O(\alpha)$ correction to the mass relation in (1.1) is a physically measurable quantity. In Appendix A we display some Feynman vertices that are needed to calculate the Higgs self-energy diagrams in the MSE. In Appendix B we display the full result for the correction to (1.1) arising from the up-quark and up-squark loops. This result is easily generalized to all contributions from other loops involving quarks, leptons, and their supersymmetric partners. In Appendix C we show that the tadpole contributions cancel in the MSE. Finally in Appendix D we discuss how the formalism developed in Sec. III can be generalized to models with more than two Higgs doublets.

After this work was completed we became aware of the work of Gunion and Turski[4] in which they calculate sfermion corrections to the Higgs-boson mass relation in (1.2). They also find that large corrections arise when the squark–Higgs-boson coupling becomes large. They discuss radiative corrections in general in Ref. 5. Their general analysis concludes that large corrections to mass sum rules arise when large Yukawa couplings are present. Our results agree with the conclusions of this general analysis.

II. THE MINIMAL SUPERSYMMETRIC EXTENSION OF THE STANDARD MODEL

We shall follow the notation of Gunion and Haber[6] with the one exception that they refer to the neutral Higgs boson H, h, A, and G as H_1^0, H_2^0, H_3^0, and G^0, respectively. Throughout this paper any mass without a subscript will be a *physical* mass (e.g., M_H, M_h, etc.). Any subscript on a mass parameter [e.g., $(M_H)_b$, $(M_H)_r$, etc.] indicates that this parameter is in general different from the physical mass. The definitions of these mass parameters will be given when they arise. Our review will be brief, and the interested reader is urged to consult Refs. 2, 3, and 6 for more details on the MSE.

Call the two-complex-doublet scalar fields ϕ_1 and ϕ_2. The Higgs potential develops an asymmetric minimum, giving rise to spontaneous symmetry breaking. Then ϕ_1 gives mass to the d-type quarks and squarks, and ϕ_2 gives mass to the u-type quarks and squarks.

Supersymmetry constrains the otherwise independent quartic couplings to be combinations of the gauge couplings g and g'. The superpotential contains the following pieces:

$$W=\epsilon_{ij}(\mu H_1^i H_2^j + f H_1^i \tilde{L}^j \tilde{R} + f_1 H_1^i \tilde{Q}^j \tilde{D} + f_2 H_2^i \tilde{Q}^i \tilde{U}) , \tag{2.1}$$

where $\tilde{Q}$ and $\tilde{L}$ are the weak SU(2)-doublet quark and lepton superfields, $\tilde{U}$ and $\tilde{D}$ are the weak SU(2)-singlet quark superfields, and $\tilde{R}$ is the SU(2)-singlet lepton superfield. The scalar potential receives contributions from the so-called D terms and F terms. These are

$$V=\tfrac{1}{2}[D^a D^a+(D')^2]+F_i^* F_i , \tag{2.2}$$

where

$$D^a=\tfrac{1}{2} g A_i^* \sigma_{ij}^a A_j , \tag{2.3a}$$

$$D'=\tfrac{1}{2} g' y_i A_i^* A_i + \xi , \tag{2.3b}$$

$$F_i=\frac{\partial W}{\partial A_i} . \tag{2.3c}$$

Here A_i denotes a generic scalar field appearing in the superpotential. ξ is the Fayet-Iliopoulos term[7] that may arise for U(1) gauge groups. The hypercharge assignments of the two Higgs doublets are $y_1=-1$ and $y_2=1$, ensuring anomaly cancellation.

In general we add all possible soft supersymmetry-breaking terms.[8] The Higgs potential is then given by [we assume that the Fayet-Iliopoulos term associated with $U(1)_Y$ is small and neglect it]

$$V=\tfrac{1}{4} g^2 \sum_{a=1}^{3} |\phi_1^\dagger \sigma^a \phi_1 - \phi_2^\dagger \sigma^a \phi_2|^2 + \frac{g'^2}{8}(\phi_1^\dagger\phi_1-\phi_2^\dagger\phi_2)^2 + |\mu|^2(\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2)+V_{\text{soft}} \tag{2.4a}$$

which can be written

$$\begin{aligned} V=&\tfrac{1}{8} g^2[4|H_1^{i*}H_2^i|^2-2(H_1^{i*}H_1^i)(H_2^{i*}H_2^i) \\ &+(H_1^{i*}H_1^i)^2+(H_2^{i*}H_2^i)^2] \\ &+\tfrac{1}{8} g'^2(H_2^{i*}H_2^i-H_1^{i*}H_1^i)^2 \\ &+|\mu|^2(H_1^{i*}H_1^i+H_2^{i*}H_2^i)+V_{\text{soft}} , \end{aligned} \tag{2.4b}$$

where

$$V_{\text{soft}}=m_1^2 H_1^{i*}H_1^i + m_2^2 H_2^{i*}H_2^i - (m_{12}^2 \epsilon_{ij} H_1^i H_2^j + \text{H.c.}) . \tag{2.4c}$$

We are using the notation[6]

$$\phi_1^\dagger\phi_1=H_1^{i*}H_1^i , \tag{2.4d}$$

$$\phi_2^\dagger\phi_2=H_2^{i*}H_2^i , \tag{2.4e}$$

$$\phi_1^\dagger\phi_2=\epsilon_{ij}H_1^i H_2^j . \tag{2.4f}$$

In this notation H_1^1 and H_2^2 are the neutral components of

H_1 and H_2, respectively, while H_1^2 and H_2^1 are the charged components. The quantities m_1, m_2, and m_{12} are arbitrary mass parameters. This Higgs potential has a minimum away from $H_1=H_2=0$ so spontaneous symmetry breaking occurs. It is possible through a choice of phase to choose the vacuum expectation values to be real and non-negative. We define v_1 and v_2 to be the vacuum expectation values of H_1 and H_2, respectively, so that

$$\langle H_1\rangle=\begin{bmatrix} v_1 \\ 0 \end{bmatrix}, \quad \langle H_2\rangle=\begin{bmatrix} 0 \\ v_2 \end{bmatrix} . \tag{2.5}$$

To obtain the correct tree-level mass $M_W^2=\frac{1}{2}g^2v^2$, we require $v_1^2+v_2^2=v^2$.

The masses of the Higgs bosons can be obtained from (2.4) using the vacuum expectation values in (2.5). The mass matrices must be diagonalized to obtain M_H^2, M_h^2, and M_A^2. In the MSE there is the tree-level mass relation given in (1.1) where $M_h<M_Z$ and $M_H>M_Z$. Beyond the tree level this relation is no longer exact but receives $O(\alpha)$ corrections. To implement the renormalization procedure, we fix M_H, M_A, and M_Z to be the physical masses. Then the physical mass of the other neutral Higgs boson h is given by a relation

$$M_h^2=M_A^2+M_Z^2-M_H^2+\Delta , \tag{2.6}$$

where Δ is a correction that is $O(\alpha)$. There are two free parameters that characterize the three-level masses in the Higgs sector if M_Z is fixed at its experimentally measured value. We shall take M_H and M_A to be the two free parameters.

III. FORMALISM FOR RADIATIVE CORRECTIONS

We use an on-shell scheme for renormalization. External lines are evaluated with momenta on shell. The physical mass is defined as the position of the pole in the propagator. The ultimate results of this section are the relations (3.27) and (3.32) below. These equations indicate that at the one-loop level the wave-function-renormalization factors do not enter, and the corrections to the mass sum rules are given entirely by combinations of Higgs-boson and vector-boson self-energies.

In this section, we denote all bare fields and parameters by the subscript b. The absence of this subscript indicates a renormalized field or a renormalized parameter. For example, H_b denotes the bare heavy Higgs field, while H denotes the renormalized field.

In the multi-Higgs-doublet models, renormalization is complicated by mixing of the physical Higgs bosons necessitating rediagonalization at each order. This is analogous to the mixing of the Z and the photon in the renormalization of the standard model.[9] Here we follow the method of Aoki *et al.*[10] for on-shell renormalization of fields when mixing is present.

First define the scalar and pseudoscalar parts of the charge-neutral Higgs-boson fields by

$$H_1^1=v_1+\frac{1}{\sqrt{2}}(S_1+iP_1) , \tag{3.1a}$$

$$H_2^2=v_2+\frac{1}{\sqrt{2}}(S_2+iP_2) . \tag{3.1b}$$

H and h are linear combinations of S_1 and S_2 while A and G are linear combinations of P_1 and P_2. The factor of $\sqrt{2}$ is included so the kinetic energy terms for the physical-Higgs-boson fields will have the canonical form.

Wave-function renormalization now takes a matrix form. Define the matrices

$$Z_S^{1/2}=\begin{bmatrix} Z_{HH}^{1/2} & Z_{Hh}^{1/2} \\ Z_{hH}^{1/2} & Z_{hh}^{1/2} \end{bmatrix} \tag{3.2a}$$

and

$$Z_P^{1/2}=\begin{bmatrix} Z_{GG}^{1/2} & Z_{GA}^{1/2} \\ Z_{AG}^{1/2} & Z_{AA}^{1/2} \end{bmatrix} . \tag{3.2b}$$

In the bare Lagrangian we denote all parameters and fields with the subscript b. In particular the Higgs potential in (2.4) is rewritten in terms of bare fields and masses by attaching a subscript b to all quantities. Then the wave-function renormalization of the Higgs fields can be expressed as

$$\begin{bmatrix} H \\ h \end{bmatrix}_b=Z_S^{1/2}\begin{bmatrix} H \\ h \end{bmatrix} \tag{3.3a}$$

and

$$\begin{bmatrix} G \\ A \end{bmatrix}_b=Z_P^{1/2}\begin{bmatrix} G \\ A \end{bmatrix} . \tag{3.3b}$$

The matrices in (3.2) are not in general symmetric. There are four independent parameters for each matrix. We have that $Z_S^{1/2}=I+O(\alpha)$ so that $Z_{HH}^{1/2}=1+O(\alpha)$, $Z_{hh}^{1/2}=1+O(\alpha)$, $Z_{Hh}^{1/2}=O(\alpha)$, and $Z_{hH}^{1/2}=O(\alpha)$. The kinetic energy terms for the charge neutral pieces are

$$\tfrac{1}{2}\partial^\mu(H\ \ h)(Z_S^{1/2})^T Z_S^{1/2}\partial_\mu\begin{bmatrix} H \\ h \end{bmatrix} + \tfrac{1}{2}\partial^\mu(G\ \ A)(Z_P^{1/2})^T Z_P^{1/2}\partial_\mu\begin{bmatrix} G \\ A \end{bmatrix} . \tag{3.4}$$

Now we proceed to investigate the mass terms. We shift the parameters that occurs in the Higgs-boson-mass terms as follows:

$$(m_1^2)_b=m_1^2+\delta m_1^2 , \tag{3.5a}$$

$$(m_2^2)_b=m_2^2+\delta m_2^2 , \tag{3.5b}$$

$$(m_{12}^2)_b=m_{12}^2+\delta m_{12}^2 , \tag{3.5c}$$

$$(M_Z^2)_b=M_Z^2+\delta M_Z^2 , \tag{3.5d}$$

$$(v_1)_b=v_1+\delta v_1 , \tag{3.5e}$$

$$(v_2)_b=v_2+\delta v_2 . \tag{3.5f}$$

The Higgs potential in (2.4) depends on five parameters, so we can choose five parameters in (3.5) to determine the potential. The parameters we use to define the theory are the physical masses M_H, M_h, M_A, and M_Z as well as the coupling g. The quantities in (3.5) are related to these five in a complicated way determined by the Higgs potential in (2.4). Other parameters such as $|\mu|^2$ and its associated counterterm are determined in terms of the five parameters and counterterms in (3.5). The dependence of μ on the other parameters is given in Eq. (3.25) of Ref. 6. The shifts in v_1 and v_2 reflect the fact that the location of the minimum of the Higgs potential receives $O(\alpha)$ corrections. Our goal then is to formulate renormalization conditions for the physical masses without any references to the unmeasurable parameters that occur in (3.5).

The Higgs-boson-mass terms arise in the potential given by (2.4). The parameters m_1, m_2, and m_{12} are undetermined due to the arbitrariness of the soft supersymmetry-breaking terms. The mass constraints arise because the quartic couplings in (2.4) are determined in terms of the gauge couplings by supersymmetric and gauge invariants. Then the mass terms that arise are of the form

$$\frac{1}{2}(S_1 \quad S_2)_b \begin{bmatrix} A & B \\ B & C \end{bmatrix}_b \begin{bmatrix} S_1 \\ S_2 \end{bmatrix}_b , \tag{3.6}$$

where

$$A_b=(m_1^2)_b+\frac{1}{2}(M_Z^2)_b \left[\frac{3v_1^2-v_2^2}{v_1^2+v_2^2}\right]_b , \tag{3.7a}$$

$$B_b=-(m_{12}^2)_b+\frac{1}{2}(M_Z^2)_b \left[\frac{v_1 v_2}{v_1^2+v_2^2}\right]_b , \tag{3.7b}$$

$$C_b=(m_2^2)_b+\frac{1}{2}(M_Z^2)_b \left[\frac{3v_2^2-v_1^2}{v_1^2+v_2^2}\right]_b . \tag{3.7c}$$

This mass matrix is diagonalized by the real orthogonal matrix characterized by the angle α:

$$O_\alpha=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} , \tag{3.8a}$$

where

$$\tan 2\alpha=\frac{2B_b}{(A-C)_b} . \tag{3.8b}$$

With a redefinition of fields given by

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix}_b = O_\alpha \begin{bmatrix} H \\ h \end{bmatrix}_b \tag{3.9}$$

the mass matrix is diagonalized to give

$$O_{-\alpha}\begin{bmatrix} A & B \\ B & C \end{bmatrix}_b O_\alpha=\begin{bmatrix} M_H^2 & 0 \\ 0 & M_h^2 \end{bmatrix}_b , \tag{3.10a}$$

where

$$(M_H^2)_b=\frac{1}{2}\{(A_b+C_b)+[(A_b-C_b)^2+4B_b^2]^{1/2}\} , \tag{3.10b}$$

$$(M_h^2)_b=\frac{1}{2}\{(A_b+C_b)-[(A_b-C_b)^2+4B_b^2]^{1/2}\} . \tag{3.10c}$$

The shifts in the parameters introduced in (3.5) generate shifts in the parameters A_b, B_b, and C_b that appear in the unrenormalized mass matrix through the definitions in (3.7). We define the renormalized values of these parameters and the associated counterterms as $A_b=A+\delta A$, $B_b=B+\delta B$, and $C_b=C+\delta C$ where A, B, and C are defined just as the bare quantities are defined in (3.7) but in terms of the renormalized quantities. It is unnecessary to retain terms second order in the counterterms because these are higher order in the perturbation theory. The inverse propagator is a matrix due to the mixing of the Higgs bosons, and we denote it by

$$i\Gamma_S(p^2)=\begin{bmatrix} i\Gamma_{HH}(p^2) & i\Gamma_{Hh}(p^2) \\ i\Gamma_{hH}(p^2) & i\Gamma_{hh}(p^2) \end{bmatrix} . \tag{3.11}$$

Then we have

$$i\Gamma_S(p^2)=(Z_S^{1/2})^T Z_S^{1/2} p^2-(Z_S^{1/2})^T (M_S^2)_D Z_S^{1/2}-\delta M_S , \tag{3.12a}$$

where

$$(M_S^2)_D\equiv\begin{bmatrix} (M_H^2)_r & 0 \\ 0 & (M_h^2)_r \end{bmatrix} \tag{3.12b}$$

and

$$\delta M_S\equiv\begin{bmatrix} \delta M_H^2 & \delta M_{Hh}^2 \\ \delta M_{hH}^2 & \delta M_h^2 \end{bmatrix} . \tag{3.12c}$$

The subscript D in (3.12b) indicated that the renormalized mass matrix (with subscripts r) is diagonal. In obtaining (3.12) we have dropped terms that are second order in perturbation theory, used (3.10a), and defined

$$\delta M_H^2=\delta A\cos^2\alpha+\delta B\sin 2\alpha+\delta C\sin^2\alpha , \tag{3.13a}$$

$$\delta M_h^2=\delta A\sin^2\alpha-\delta B\sin 2\alpha+\delta C\cos^2\alpha , \tag{3.13b}$$

$$\delta M_{Hh}^2=\delta M_{hH}^2=(\delta C-\delta A)\sin\alpha\cos\alpha+\delta B\cos 2\alpha . \tag{3.13c}$$

The inverse propagator matrix in (3.11) is symmetric as it should be. We have also defined the quantities

$$(M_H^2)_r=\frac{1}{2}[(A+C)+\sqrt{(A-C)^2+4B^2}] , \tag{3.14a}$$

$$(M_h^2)_r=\frac{1}{2}[(A+C)-\sqrt{(A-C)^2+4B^2}] . \tag{3.14b}$$

At this point the renormalized parameters $(M_H^2)_r$ and $(M_h^2)_r$ are not the physical masses M_H^2 and M_h^2. The connection between these quantities must be specified by renormalization conditions.

We have expressed the inverse propagator $i\Gamma_S(p^2)$ in terms of wave-function-renormalization parameters defined in (3.2) and the counterterms defined in (3.5). The actual expression is rather complicated, but fortunately we will only need to know the linear combination $\delta A+\delta C$ to calculate the radiative corrections to the mass relation (1.1). Notice that $\delta M_H^2+\delta M_h^2=\delta A+\delta C$, i.e., the trace of the mass matrix is invariant under the orthogonal transformation. We have that $\delta A+\delta C=\delta m_1^2+\delta m_2^2+\delta M_Z^2$ so that we arrive at the conclusion

$$\delta M_H^2+\delta M_h^2=\delta m_1^2+\delta m_2^2+\delta M_Z^2 \ . \tag{3.15}$$

By repeating the analysis in the pseudoscalar sector in the same way, we also have (in the Landau gauge)

$$\delta M_G^2+\delta M_A^2=\delta m_1^2+\delta m_2^2 \ , \tag{3.16}$$

so that

$$\delta M_H^2+\delta M_h^2=\delta M_G^2+\delta M_A^2+\delta M_Z^2 \ . \tag{3.17}$$

We define the self-energies of the scalars and the vector bosons as shown in Fig. 1 with external legs amputated. The vacuum expectation values v_1 and v_2 are in general renormalized, and tadpole diagrams must be taken into account (Fig. 2). We will argue below that the tadpole contributions to the final result Δ are zero with the renormalization conditions we choose. This will be shown explicitly in Appendix C. The renormalized inverse propagator

$$i\tilde{\Gamma}_S(p^2)=\begin{bmatrix} i\tilde{\Gamma}_{HH}(p^2) & i\tilde{\Gamma}_{Hh}(p^2) \\ i\tilde{\Gamma}_{hH}(p^2) & i\tilde{\Gamma}_{hh}(p^2)\end{bmatrix} \tag{3.18}$$

includes the expression in (3.12) and the self-energy contributions shown in Fig. 1. The inverse propagator matrix in (3.18) is symmetric. We have

$$i\tilde{\Gamma}_{HH}(p^2)=(Z_{HH}+Z_{hH})p^2-(M_H^2)_rZ_{HH}-(M_h^2)_rZ_{hH} -\delta M_H^2+\Pi_{HH}(p^2) \ , \tag{3.19a}$$

$$i\tilde{\Gamma}_{hh}(p^2)=(Z_{hh}+Z_{Hh})p^2-(M_h^2)_rZ_{hh}^2-(M_H^2)_rZ_{Hh} -\delta M_h^2+\Pi_{hh}(p^2) \ , \tag{3.19b}$$

$$\begin{aligned} i\tilde{\Gamma}_{Hh}(p^2)=i\tilde{\Gamma}_{hH}(p^2)=&(Z_{HH}^{1/2}Z_{Hh}^{1/2}+Z_{hh}^{1/2}Z_{hH}^{1/2})p^2 \\ &-(M_H^2)_rZ_{HH}^{1/2}Z_{Hh}^{1/2} \\ &-(M_h^2)_rZ_{hh}^{1/2}Z_{hH}^{1/2} \\ &-\delta M_{Hh}^2+\Pi_{Hh}(p^2) \ . \end{aligned} \tag{3.19c}$$

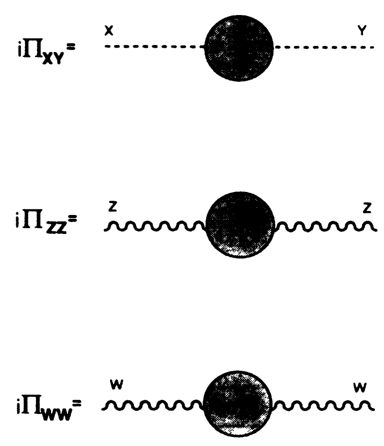

FIG. 1. The self-energy diagrams are defined as shown with the external legs amputated. $X,Y=H,h,A,G$.

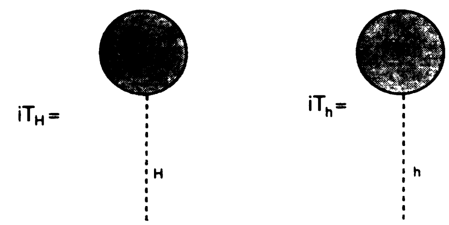

FIG. 2. The two kinds of tadpoles.

In the on-shell scheme we adopt the renormalization conditions[10]

$$i\tilde{\Gamma}_{HH}(M_H^2)=0 \ , \tag{3.20a}$$

$$i\tilde{\Gamma}_{hh}(M_h^2)=0 \ , \tag{3.20b}$$

$$i\tilde{\Gamma}_{Hh}(M_H^2)=i\tilde{\Gamma}_{Hh}(M_h^2)=0 \ , \tag{3.20c}$$

$$i\tilde{\Gamma}'_{HH}(M_H^2)=1 \ , \tag{3.20d}$$

$$i\tilde{\Gamma}'_{hh}(M_h^2)=1 \ , \tag{3.20e}$$

where $i\tilde{\Gamma}'(p^2)$ is the derivative of $i\tilde{\Gamma}(p^2)$.

We choose as an additional renormalization condition that $(M_H)_r$ be set equal to the physical mass M_H of the H (Ref. 11). Then from (3.19) and (3.20) we conclude that

$$\delta M_H^2=\Pi_{HH}(M_H^2) \ . \tag{3.21}$$

The pseudoscalar sector can be treated in the same way. In this case we define $(M_A)_r$ to be the physical mass M_A of the A and require that the Goldstone boson G have zero mass at the one-loop level in the Landau gauge, i.e., $(M_G)_r=M_G=0$. The masslessness of the Goldstone boson at one-loop follows from the Ward identities. Then we obtain

$$\delta M_A^2=\Pi_{AA}(M_A^2) \ , \tag{3.22}$$

$$\delta M_G^2=\Pi_{GG}(0) \ . \tag{3.23}$$

The remaining condition is obtained from (2.6) and (3.20b). This is

$$\delta M_h^2=\Pi_{hh}(M_h^2)+\Delta \ , \tag{3.24}$$

where we have used the fact that $(M_H^2)_r+(M_h^2)_r=(M_A^2)_r+(M_Z^2)_r$. Similarly it can be shown that

$$\delta M_Z^2=-A_{ZZ}(M_Z^2) \ , \tag{3.25}$$

where $A_{ZZ}(p^2)$ is defined as the real part of the coefficient of $g^{\mu\nu}$ in the vacuum-polarization tensor

$$\Pi_{ZZ}^{\mu\nu}(p^2)=\mathcal{A}_{ZZ}(p^2)g^{\mu\nu}+\mathcal{B}_{ZZ}(p^2)p^\mu p^\nu \ , \tag{3.26a}$$

$$A_{ZZ}=\text{Re}\mathcal{A}_{ZZ} \tag{3.26b}$$

defined as in Fig. 1. Then using (3.17) and Eqs. (3.21)–(3.25) we find that

$$\Delta=-\Pi_{HH}(M_H^2)-\Pi_{hh}(M_h^2)+\Pi_{AA}(M_A^2) +\Pi_{GG}(0)-A_{ZZ}^*(M_Z^2) \ . \tag{3.27}$$

So the calculation of Δ involves the determination of the Higgs-boson and Z self-energies in (3.27). The final result for Δ must be finite even though the individual self-energies will not be.

The condition that the Goldstone-boson mass be zero at one loop ensures that the tadpole contributions will be zero. This is a consequence of a Ward identity. A discussion of this result in the context of the standard model is given in Refs. 12 and 13. The Goldstone self-energy at zero momentum is related to the tadpole diagrams of the H and h fields as

$$\Pi_{GG}(0)=\frac{-1}{\sqrt{2}v}[\cos(\beta-\alpha)T_H+\sin(\beta-\alpha)T_h] \ . \tag{3.28}$$

The counterterm Lagrangian contains the terms

$$-(\delta M_G^2 G^2+\tau_H H+\tau_h h) \tag{3.29}$$

in which the coefficients satisfy

$$\delta M_G^2=\frac{1}{\sqrt{2}v}\cos(\beta-\alpha)\tau_H+\sin(\beta-\alpha)\tau_h \ . \tag{3.30}$$

So we conclude that (3.23) is equivalent to taking $(T_H+\tau_H)\cos(\beta-\alpha)+(T_h+\tau_h)\sin(\beta-\alpha)=0$. The advantage in calculating $\Pi_{GG}(0)$ rather than the tadpole diagrams T_H and T_h is that the cancellation of divergences is much more obvious in the former case. In terms of the Feynman rules, calculating the Goldstone-boson self-energy is on an equal footing with calculating the Higgs-boson self-energies in the Landau gauge. We have shown explicitly in Appendix C that in the context of the minimal supersymmetry extension (MSE) the tadpole contributions to Δ in (3.27) vanish identically. This result can be proven generally.

Another mass relation that holds at the tree level in the MSE was given in (1.2). It can be shown [in a method analogous to the preceding treatment of the mass relation in (1.1)] that the radiative corrections defined by

$$M_{H^\pm}^2=M_A^2+M_W^2+\tilde{\Delta} \tag{3.31}$$

are given by

$$\tilde{\Delta}=-\Pi_{H^\pm H^\pm}(M_{H^\pm}^2)-\Pi_{G^\pm G^\pm}(0)+\Pi_{AA}(M_A^2)+\Pi_{GG}(0)-A_{WW}(M_W^2) \ . \tag{3.32}$$

Again the tadpole contributions are exactly zero (see Appendix C).

We note that the result in (3.27) continues to hold when a Higgs singlet N is present in certain important cases. The criterion is that N not mix with the other Higgs bosons (H,h,A,G). See Ref. 6 for a discussion of these cases. If the singlet mixes with the Higgs doublet then the mass relation (1.1) is destroyed even at the tree level, and the tree-level constraints $M_h<M_Z$ and $M_H>M_Z$ also disappear. The mass relation (1.2) may be destroyed even if the singlet does not mix with other fields.

IV. RADIATIVE CORRECTIONS

In this section we will discuss the contribution to Δ from quark and squark loops in the MSE. It is necessary to know the Feynman rules for Higgs bosons in the MSE to calculate the self-energy diagrams for the Higgs fields. Many of these have been derived previously in the literature.[2,6,14] We have derived some others that appear in Appendix A.

The calculations involved are somewhat lengthy. Each individual diagram is divergent, and these divergences cancel only when loops involving the fermions and loops involving their superpartners are included. The divergent integrals are evaluated using dimensional regularization with the prescription for γ_5 given by Chanowitz *et al.*[15] Since the γ_5's always occur in pairs in the amplitudes considered, this prescription guarantees the correct Ward identities. The calculation is straightforward, so we display only the final result in Appendix B. The diagrams evaluated are shown in Fig. 3.

We have ignored the mixing between generations for simplicity. There is a contribution from each generation, and the contribution to Δ from the top quark is the same as that for the up quark with the appropriate mass substitutions. The calculation of the diagrams involving squark loops is complicated by the mixing in the square sector.

We add soft supersymmetry-breaking terms to the scalar potential. The terms in the scalar potential involving squarks are[6]

$$V=V_F+V_D+V_{\rm soft} \ , \tag{4.1a}$$

where

$$\begin{aligned}V_F=&(\mu^* H_1^{i*}+f_2\tilde{Q}^{i*}\tilde{U}^*)(\mu H_1^i+f_2\tilde{Q}^i\tilde{U})\\&+(\mu^* H_2^{i*}+f_1\tilde{Q}^{i*}\tilde{D}^*)(\mu H_2^i+f_1\tilde{Q}^i\tilde{D})\\&+f_1^2|\epsilon_{ij}H_1^i\tilde{Q}^j|^2+f_2^2|\epsilon_{ij}H_2^i\tilde{Q}^j|^2\\&+(f_1H_1^{i*}\tilde{D}^*-f_2H_2^{i*}\tilde{U}^*)(f_1H_1^i\tilde{D}-f_2H_2^i\tilde{D}) \ ,\end{aligned} \tag{4.1b}$$

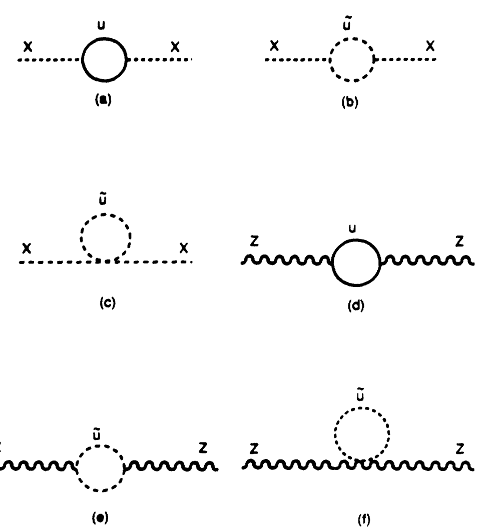

FIG. 3. The diagrams calculated in the MSE. There are the following number of nonvanishing diagrams of each type: (a) 4, (b) 12, (c) 8, (d) 1, (e) 3, (f) 2.

$$V_D=\tfrac{1}{8}g^2[4|H_1^{i*}\tilde{Q}^{\,i}|^2+4|H_2^{i*}\tilde{Q}^{\,i}|^2 -2(\tilde{Q}^{\,i*}\tilde{Q}^{\,i})(H_1^{i*}H_1^i+H_2^{i*}H_2^i)+(\tilde{Q}^{\,i*}\tilde{Q}^{\,i})^2] +\tfrac{1}{8}g'^2(H_2^{i*}H_2^i-H_1^{i*}H_1^i+y_q\tilde{Q}^{\,i*}\tilde{Q}^{\,i} +y_u\tilde{U}^*\tilde{U}+y_d\tilde{D}^*\tilde{D})^2\,, \tag{4.1c}$$

$$V_{\text{soft}}=\tilde{M}_Q^2\tilde{Q}^{\,i*}\tilde{Q}^{\,i}+\tilde{M}_u^2\tilde{U}^*\tilde{U}+\tilde{M}_d^2\tilde{D}^*\tilde{D} +m_6\epsilon^{ij}(f_1A_dH_1^i\tilde{Q}^{\,j}\tilde{D}-f_2A_uH_2^i\tilde{Q}^{\,j}\tilde{U}+\text{H.c.})\,. \tag{4.1d}$$

The conventional squark notation for the fields appearing in (2.1) and (4.1) is

$$\tilde{Q}^{\,i}=\begin{bmatrix}\tilde{u}_L\\ \tilde{d}_L\end{bmatrix},\quad \tilde{U}^*=\tilde{u}_R,\quad \tilde{D}^*=\tilde{d}_R\,. \tag{4.2}$$

The mass terms for the up squarks, for example, are

$$-(\tilde{u}_L^*\ \ \tilde{u}_R^*)\begin{bmatrix}\mathcal{A}_u & \mathcal{B}_u\\ \mathcal{B}_u & \mathcal{C}_u\end{bmatrix}\begin{bmatrix}\tilde{u}_L\\ \tilde{u}_R\end{bmatrix}, \tag{4.3a}$$

where

$$\mathcal{A}_u=\tilde{M}_Q^2+M_Z^2\cos2\beta(\tfrac{1}{2}-e_u\sin^2\theta_W)+m_u^2\,, \tag{4.3b}$$

$$\mathcal{B}_u=m_u(A_um_6+\mu\cot\beta)\,, \tag{4.3c}$$

$$\mathcal{C}_u=\tilde{M}_U^2+M_Z^2\cos2\beta(e_u\sin^2\theta_W)+m_u^2\,. \tag{4.3d}$$

A_um_6, $\tilde{M}_Q$, and $\tilde{M}_U$ are additional soft supersymmetric-breaking parameters that enter into the part of the scalar potential that involves squarks. The mass eigenstates can be defined as a mixture of these fields as

$$\begin{bmatrix}\tilde{q}_L\\ \tilde{q}_R\end{bmatrix}=O_{\theta_q}\begin{bmatrix}\tilde{q}_1\\ \tilde{q}_2\end{bmatrix}, \tag{4.4}$$

where O_{θ_q} are defined as in (3.8a). The mixing angles θ_q appear in the Feynman rules involving the squarks.

The coupling of the squarks to the Higgs bosons come from three places in the scalar potential. First the D terms contain contributions to the squark masses and to the squark–Higgs-boson coupling that are of $O(gM_Z)$. The F terms contain the Yukawa pieces that contribute a mass to the squarks equal to the quark mass (m_q), and terms of $O(gm_q)$ to the squark–Higgs-boson couplings. The F terms also contain the parameter μ which contributes to the off-diagonal entries in the mass matrix [see Eq. (4.3c)] as well as to the couplings. Finally the soft supersymmetry-breaking terms contribute the parameters A_qm_6 that contribute to the off-diagonal terms in the mass matrix and in the couplings. The soft-supersymmetry-breaking parameters $\tilde{M}_Q^2$ and $\tilde{M}_U^2$ above in (4.3) do not contribute to the couplings.

The soft supersymmetry-breaking parameters $\tilde{M}_Q^2$, $\tilde{M}_U^2$, and A_um_6 are adjusted so that the squarks are sufficiently massive to have escaped detection while not so massive to destroy the stability of the electroweak scale to radiative corrections (i.e., the naturalness motivation for supersymmetry). The parameters $\tilde{M}_Q^2$ and $\tilde{M}_U^2$ show up in radiative corrections to Higgs-boson masses in diagrams such as that shown in Fig. 4. In the renormalization of the mass sum rule, the combination of these diagrams that arises is shown in Fig. 5. These diagrams sum exactly to zero. So while there are large corrections arising from $\tilde{M}_Q^2$ and $\tilde{M}_U^2$ to the mass of each Higgs boson, these contributions cancel in the sum rule. The sum rule is, therefore, insensitive to these parameters when they become large.

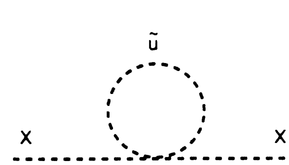

FIG. 4. Contributions to Higgs-boson masses that are quadratic in a scalar mass arise from these diagrams.

On the other hand, the supersymmetry-breaking parameter A_um_6 as well as the parameter μ contributes to the couplings of the squark to the Higgs bosons. If this parameter becomes large, substantial corrections can arise to the sum rule. These corrections occur only when there is substantial left-right mixing between the squarks.

The expression for Δ in Appendix B is composed of three parts: $\Delta=\Delta_4+\Delta_2+\Delta_0$. $\Delta_n\sim O(\alpha(\tilde{m}^{\,n}/M_W^2))$ where $\tilde{m}$ represents a mass parameter such as the up-quark mass or a parameter involving the squark sector such as A_um_6, μ, $m_{\tilde{u}_1}$, or $m_{\tilde{u}_2}$. We leave Δ in terms of the mixing angles α, β, and θ_q for convenience. The expressions for these angles in terms of physical masses are lengthy and not very illuminating. Expressions for α and β are given in Appendix A of Ref. 16.

The terms in Δ_4 give the largest contribution to Δ for large quark and squark masses. The terms involving the off-diagonal entries in the squark mass matrix (A_um_6 and μ) give large contributions provided the squark mixing angle θ_q is not small. Δ_2 contains terms that are $O(\alpha m_{\tilde{u}}^2)$, but these terms go to zero as the squark mass becomes large. This is a manifestation of the cancellation

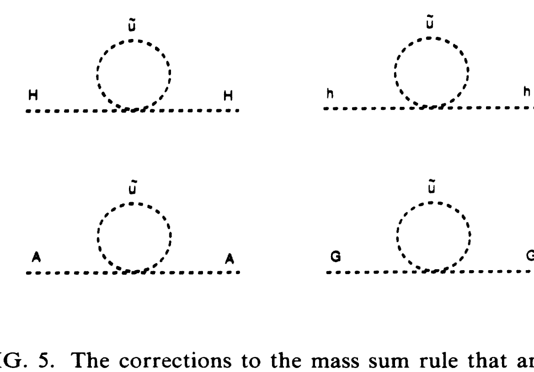

FIG. 5. The corrections to the mass sum rule that are quadratic in the squark mass cancel in the above diagrams. The restriction on naturalness from corrections to the Higgs-boson masses is, therefore, hidden in the sum rule.

of the diagrams in Fig. 5. The terms in Δ_2 of $O(\alpha m_{\tilde{u}}^2)$ come from the Z vacuum polarization only. Δ_0 is $O(\alpha M_Z^2)$ and is for our present purposes a negligible correction to the mass relation.

We will illustrate the result in Appendix B by considering the contribution from the top quark and top squark. Four parameters characterize the squark mass matrix in (4.3). We can take these to be $m_{\tilde{t}_1}^2$, $m_{\tilde{t}_2}^2$, θ_t, and μ. Then $A_t m_6$ is determined

$$A_t m_6 = \frac{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)\sin 2\theta_t}{2m_t} - \mu\cot\beta\ . \tag{4.5}$$

First consider the case in which there is no squark mixing, i.e., $\theta_t = 0$. This is expected to be approximately the case for all squark species except possibly the top squark. When $\theta_t = 0°$, the terms involving $A_t m_6$ and μ give only a small contribution to Δ. If the top quark and top squark are very massive $(m_t, m_{\tilde{t}} \gg M_W, M_H, M_A, M_h)$, we can neglect the other masses. Then we obtain

$$\Delta = \frac{g^2 m_t^4 N_c}{16\pi^2 M_W^2 \sin^2\beta} \ln\left[\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}\right]. \tag{4.6}$$

So we have large corrections to the mass relation just as there are large corrections $[O(m_t^4)]$ in the Higgs sector of the standard model.[17] We have plotted the correction Δ in Fig. 6. We have chosen the parameters $m_t = 100$ GeV, $\alpha = -48°$, $\beta = 30°$, and $\mu = 0$. For these parameters the tree-level Higgs-boson masses are $M_H = 140$ GeV, $M_h = 40$ GeV, and $M_A = 110$ GeV and $M_H^2 + M_h^2 = 2\times10^4$ GeV^2, so that each side of Eq. (1.1) is equal to 2×10^4 GeV^2 at the tree level. So for $\Delta = 200$ GeV^2, the correction is only one percent. We have plotted Δ for the case where $\theta_t = 0°$ in Fig. 6(a). The dependence on the squark masses is roughly logarithmic.

The expression in (4.6) diverges when $\sin^2\beta$ approaches zero. This reflects the fact that the Yukawa coupling giving the top quark a mass must diverge in this limit.

If there is significant mixing of the scalar quarks, large corrections can arise when there are large mass splittings between the squarks. In Fig. 6(b) we have taken $\theta_t = 20°$. Notice that the corrections are again small when $m_{\tilde{t}_1} \approx m_{\tilde{t}_2}$. If the squarks have significantly different masses, then there is a large negative Δ. These large corrections arise from large squark–Higgs-boson couplings that arise because $A_t m_6$ is very large.

The results displayed in Fig. 6 are typical. Other choices of the parameters m_t, α, β, and μ give similar results. If $\theta_t \approx 0$, then corrections tend to be small (i.e., the same order as the contribution of a t quark with mass m_t in the standard model). If θ_t is significant, then large negative contributions arise when $|m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2|$ become significant. Negative values for Δ imply that the sum of the scalar-Higgs-boson masses squared $M_H^2 + M_h^2$ is suppressed relative to the pseudoscalar-boson mass squared M_A^2.

Finally we note that large contributions to the mass sum rule are possible from a fourth generation as well, even when squark mixing is absent. As in the standard model the leading contribution for a heavy fermion $(m_f \gg M_W)$ goes like $\alpha m_f^4/m_W^2$ (Ref. 17). So *a priori* if a heavy fermion exists, we can expect large corrections to the masses in the Higgs sector just as in the standard model. The results given here, however, are valid for any fermion mass, and it is only if $m_f \gg M_W$ that Δ_4 can become very large. In Fig. 7 we have held the squark masses fixed and plotted Δ as a function of the top-quark mass. The values for α, β, and μ are the same as in Fig. 6.

The contribution for a new top t' is given as in (4.6) while the new bottom b' will contribute (for $\theta_{b'} = 0$)

$$\Delta = \frac{g^2 m_{b'}^4 N_c}{16\pi^2 M_W^2 \cos^2\beta} \ln\left[\frac{m_{\tilde{b}_1'}^2 m_{\tilde{b}_2'}^2}{m_{b'}^4}\right]. \tag{4.7}$$

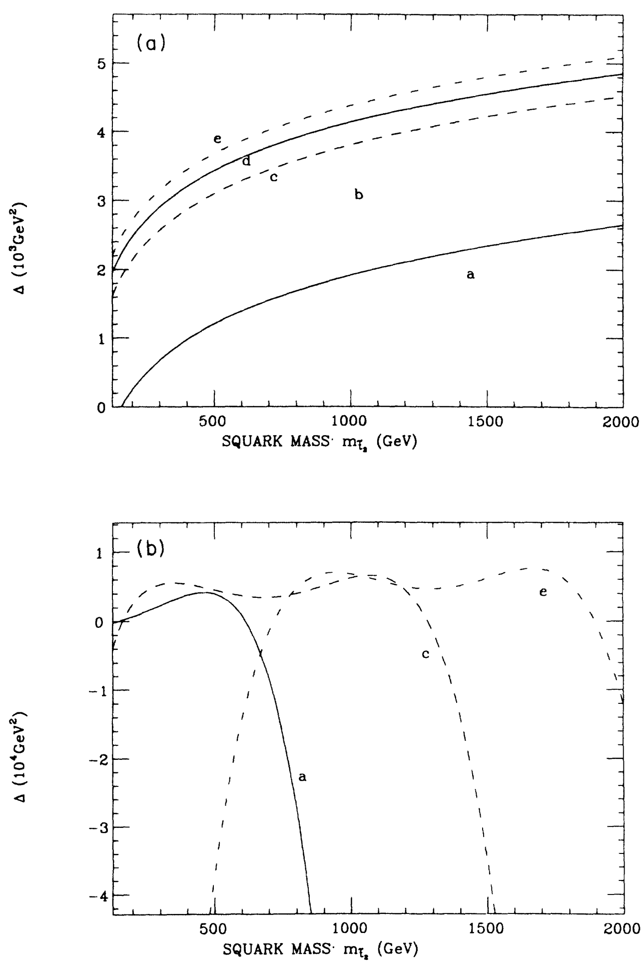

FIG. 6. We have plotted the correction Δ using the full expression given in Appendix B. The parameters used are given in the text. The squark mixing angle is $\theta_t = 0°$ and $20°$ in (a) and (b), respectively. The curves in the figures represent the following: *a*, $m_{\tilde{t}_1} = 100$ GeV; *b*, 400 GeV; *c*, 700 GeV; *d*, 1000 GeV; *e*, 1300 GeV. Large corrections occur when $\theta_t \neq 0$, and the squarks $\tilde{t}_1$ and $\tilde{t}_2$ have different masses.

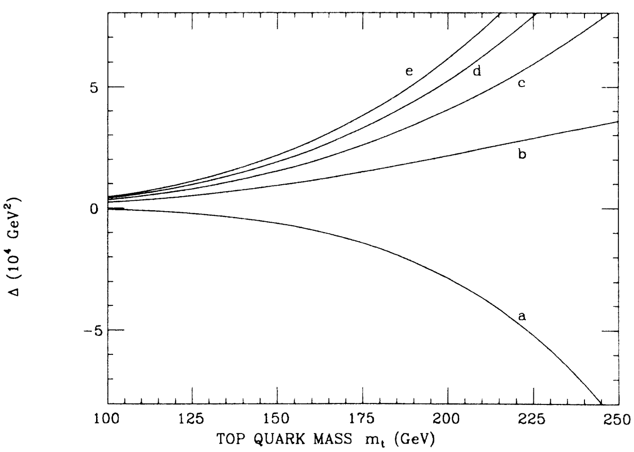

FIG. 7. We have plotted Δ as a function a, of the top-quark mass for five values of the squark masses: a, $m_{\tilde{t}_1} = m_{\tilde{t}_2} =$ 100 GeV; b, 400 GeV; c, 700 GeV; d, 1000 GeV; e, 1300 GeV.

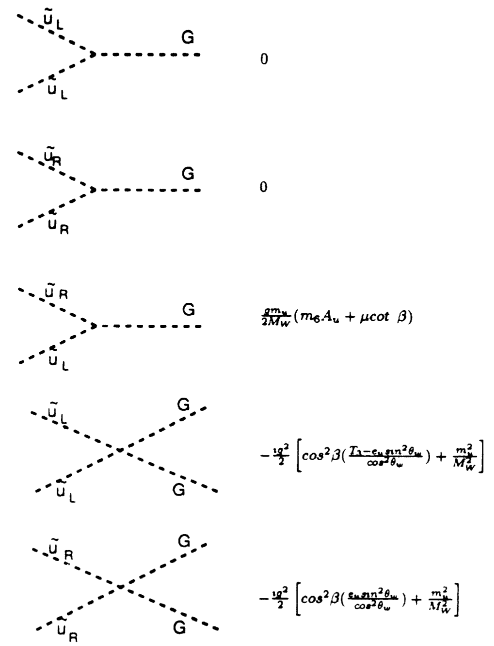

FIG. 8. Feynman rules involving Goldstone bosons and squarks. We have written these in the $\tilde{u}_L - \tilde{u}_R$ basis for simplicity.

These contributions have the same sign. This differs from the renormalization of the ρ parameter in that the ρ parameter is protected by a custodial symmetry which is not broken by equal-mass fermion doublets. The effects of a mass-degenerate heavy doublet has been discussed before in the context of the standard model.[18]

V. CONCLUSIONS

We have formulated the procedure for computing corrections to the Higgs-boson-mass relations in supersymmetric extensions to the standard model containing doublets. An explicit calculation in the case with just two doublets [the minimal supersymmetry extension (MSE)] was given. It was necessary to calculate self-energies of Higgs bosons and vacuum-polarization tensors as shown in (3.27) and (3.32). Coupling constant and wave-function renormalizations are not necessary. Tadpole contributions cancel exactly. The results in (3.27) and (3.32) are not destroyed in the presence of other Higgs representations (singlets, triplets, etc.) provided that no mixing between these fields and the Higgs doublets takes place. If mixing occurs, the tree-level mass relations (1.1) and (1.2) themselves will be destroyed. These results were generalized to the supersymmetric extensions to the standard model with more than two Higgs doublets (Appendix D).

We have performed an explicit computation of the radiative corrections to (1.1) from matter loops. We have found large corrections to the mass relation provided that the two complex squark fields mix. This results from large squark–Higgs-boson couplings. The potentially large contributions of $O(\alpha m_{\tilde{q}}^2)$ or $O(\alpha m_{\tilde{l}}^2)$ to Higgs-particle masses from a heavy squark and slepton sector in supersymmetric theories is hidden in the sum rule, i.e., cancels between the terms appearing in the sum rule. Provided that squark mixing is negligible, it is possible to imagine extremely large squark masses without inducing large radiative corrections to the sum rule.

ACKNOWLEDGMENTS

We wish to thank M. Chanowitz for suggesting this topic and for guidance, encouragement, and suggestions. We would also like to thank V. Jain for useful discussions. This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098 and in part by the National Science Foundation under Grant No. PHY85-15857.

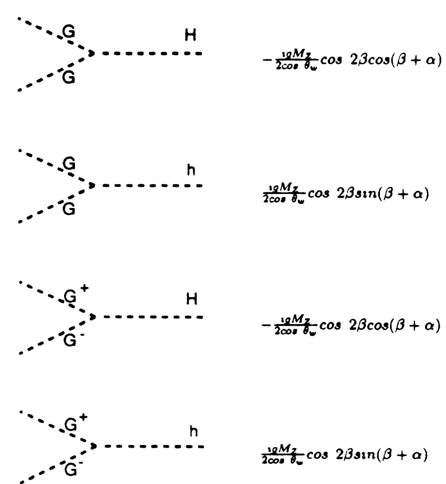

FIG. 9. Trilinear Higgs-boson couplings involving Goldstone bosons.

APPENDIX A: FEYNMAN RULES

In this appendix we display some Feynman rules that are needed in the calculation of Higgs-boson self-energies in the MSE. The Feynman rules we have used that are not included in the literature (to the best of our knowledge) are shown in Figs. 8 and 9. *CP* conservation demands that only an even number of pseudoscalars can emanate from a vertex.

APPENDIX B: THE CORRECTION TO THE MASS RELATION

The $O(\alpha)$ corrections Δ can be divided into pieces:

$$\Delta=\Delta_4+\Delta_2+\Delta_0 , \tag{B1}$$

where Δ_n is the part of Δ where the Nth power of the up-quark mass or parameters in the up-squark mass matrix (such as $A_u m_6$, μ, or the up-squark masses themselves) occur. The results of the calculation are as follows:

$$\begin{aligned}
\Delta_4=&\frac{g^2 m_u^4 N_c}{16\pi^2 M_W^2 \sin^2\beta}\{\sin^2\alpha[F(m_{\tilde u_1},m_{\tilde u_1},M_H)+F(m_{\tilde u_2},m_{\tilde u_2},M_H)-3F(m_u,m_u,M_H)]\\
&+\cos^2\alpha[F(m_{\tilde u_1},m_{\tilde u_1},M_h)+F(m_{\tilde u_2},m_{\tilde u_2},M_h)-3F(m_u,m_u,M_h)]\\
&+\cos^2\beta F(m_u,m_u,M_A)+\sin^2\beta F(m_u,m_u,0)\}\\
&+\frac{g^2 m_u^3 N_c(A_u m_6 \sin\alpha+\mu\cos\alpha)\sin2\theta_u}{16\pi^2 M_W^2\sin^2\beta}\{\sin\alpha[F(m_{\tilde u_1},m_{\tilde u_1},M_H)-F(m_{\tilde u_2},m_{\tilde u_2},M_H)]\}\\
&+\frac{g^2 m_u^3 N_c(A_u m_6 \cos\alpha-\mu\sin\alpha)\sin2\theta_u}{16\pi^2 M_W^2\sin^2\beta}\{\cos\alpha[F(m_{\tilde u_1},m_{\tilde u_1},M_h)-F(m_{\tilde u_2},m_{\tilde u_2},M_h)]\}\\
&-\frac{g^2 m_u^2 N_c}{64\pi^2 M_W^2\sin^2\beta}([2F(m_{\tilde u_1},m_{\tilde u_2},M_A)(A_u m_6\cos\beta-\mu\sin\beta)^2+2F(m_{\tilde u_1},m_{\tilde u_2},0)(A_u m_6\sin\beta+\mu\cos\beta)^2]\\
&-\sin^2 2\theta_u\{[F(m_{\tilde u_1},m_{\tilde u_1},M_H)+F(m_{\tilde u_2},m_{\tilde u_2},M_H)](A_\mu m_6\sin\alpha+\mu\cos\alpha)^2\\
&+[F(m_{\tilde u_1},m_{\tilde u_1},M_h)+F(m_{\tilde u_2},m_{\tilde u_2},M_h)](A_u m_6\cos\alpha-\mu\sin\alpha)^2\}\\
&-\cos^2 2\theta_u[2F(m_{\tilde u_1},m_{\tilde u_2},M_H)(A_u m_6\sin\alpha+\mu\cos\alpha)^2\\
&+2F(m_{\tilde u_1},m_{\tilde u_2},M_h)(A_u m_6\cos\alpha-\mu\sin\alpha)^2]) ,
\end{aligned}\tag{B2a}$$

$$\begin{aligned}
\Delta_2=&\frac{g^2 m_u^2 N_c}{8\pi^2\cos^2\theta_W\sin\beta}\{[\cos^2\theta_u(T_3-e_u\sin^2\theta_W)+\sin^2\theta_u(e_u\sin^2\theta_W)]\\
&\times[\sin\alpha\cos(\alpha+\beta)F(m_{\tilde u_1},m_{\tilde u_1},M_H)-\cos\alpha\sin(\alpha+\beta)F(m_{\tilde u_1},m_{\tilde u_1},M_h)]\\
&+[\sin^2\theta_u(T_3-e_u\sin^2\theta_W)+\cos^2\theta_u(e_u\sin^2\theta_W)]\\
&\times[\sin\alpha\cos(\alpha+\beta)F(m_{\tilde u_2},m_{\tilde u_2},M_H)-\cos\alpha\sin(\alpha+\beta)F(m_{\tilde u_2},m_{\tilde u_2},M_h)]\}\\
&+\frac{3g^2 m_u^2 N_c}{16\pi^2 M_W^2\sin^2\beta}[\sin^2\alpha M_H^2 G(m_u,m_u,M_H)+\cos^2\alpha M_h^2 G(m_u,m_u,M_h)-\cos^2\beta M_A^2 G(m_u,m_u,M_A)]\\
&+\frac{g^2 m_u^2 N_c}{32\pi^2\cos^2\theta_W}\ln\mu^2-\frac{g^2 m_u^2 N_c}{96\pi^2\cos^2\theta_W}+\frac{g^2 m_u N_c\cos(\alpha+\beta)}{16\pi^2\cos^2\theta_W\sin\beta}\sin2\theta_u(A_u m_6\sin\alpha+\mu\cos\alpha)\\
&\times\{[\cos^2\theta_u(T_3-e_u\sin^2\theta_W)+\sin^2\theta_u(e_u\sin^2\theta_W)]F(m_{\tilde u_1},m_{\tilde u_1},M_H)\\
&-[\sin^2\theta_u(T_3-e_u\sin^2\theta_W)+\cos^2\theta_u(e_u\sin^2\theta_W)]F(m_{\tilde u_2},m_{\tilde u_2},M_H)\\
&-\cos2\theta_u(T_3-2e_u\sin^2\theta_W)F(m_{\tilde u_1},m_{\tilde u_2},M_H)\}\\
&-\frac{g^2 m_u N_c\sin(\alpha+\beta)}{16\pi^2\cos^2\theta_W\sin\beta}\sin2\theta_u(A_u m_6\cos\alpha-\mu\sin\alpha)
\end{aligned}$$

$$\times\{[\cos^2\theta_u(T_3-e_u\sin^2\theta_W)+\sin^2\theta_u(e_u\sin^2\theta_W)]F(m_{\tilde u_1},m_{\tilde u_1},M_h)$$
$$-[\sin^2\theta_u(T_3-e_u\sin^2\theta_W)+\cos^2\theta_u(e_u\sin^2\theta_W)]F(m_{\tilde u_2},m_{\tilde u_2},M_h)$$
$$-\cos 2\theta_u(T_3-2e_u\sin^2\theta_W)F(m_{\tilde u_1},m_{\tilde u_2},M_h)\}$$
$$-\frac{g^2m_{\tilde u_1}^2N_c}{8\pi^2\cos^2\theta_W}\{[\cos^2\theta_u(-T_3+e_u\sin^2\theta_W)^2+\sin^2\theta_u(e_u\sin^2\theta_W)^2]F(m_{\tilde u_1},m_{\tilde u_1},0)$$
$$-[\cos^2\theta_u(-T_3+e_u\sin^2\theta_W)+\sin^2\theta_u(e_u\sin^2\theta_W)]^2F(m_{\tilde u_1},m_{\tilde u_1},M_Z)$$
$$-\tfrac{1}{2}\sin^2\theta_u\cos^2\theta_u H(m_{\tilde u_1},m_{\tilde u_2},M_Z)\}$$
$$-\frac{g^2m_{\tilde u_2}^2N_c}{8\pi^2\cos^2\theta_W}\{[\sin^2\theta_u(-T_3+e_u\sin^2\theta_W)^2+\cos^2\theta_u(e_u\sin^2\theta_W)^2]F(m_{\tilde u_2},m_{\tilde u_2},0)$$
$$-[\sin^2\theta_u(-T_3+e_u\sin^2\theta_W)+\cos^2\theta_u(e_u\sin^2\theta_W)]^2F(m_{\tilde u_2},m_{\tilde u_2},M_Z)$$
$$-\tfrac{1}{2}\sin^2\theta_u\cos^2\theta_u H(m_{\tilde u_2},m_{\tilde u_1},M_Z)\}\ , \tag{B2b}$$

$$\Delta_0=\frac{g^2M_Z^2N_c}{16\pi^2\cos^2\theta_W}\{[\cos^2\theta_u(T_3-e_u\sin^2\theta_W)+\sin^2\theta_u(e_u\sin^2\theta_W)]^2$$
$$\times[\cos^2(\alpha+\beta)F(m_{\tilde u_1},m_{\tilde u_1},M_H)+\sin^2(\alpha+\beta)F(m_{\tilde u_1},m_{\tilde u_1},M_h)]$$
$$+[\sin^2\theta_u(T_3-e_u\sin^2\theta_W)+\cos^2\theta_u(e_u\sin^2\theta_W)]^2$$
$$\times[\cos^2(\alpha+\beta)F(m_{\tilde u_2},m_{\tilde u_2},M_H)+\sin^2(\alpha+\beta)F(m_{\tilde u_2},m_{\tilde u_2},M_h)]$$
$$+\tfrac{1}{2}\sin^2 2\theta_u(T_3-2e_u\sin^2\theta_W)^2[\cos^2(\alpha+\beta)F(m_{\tilde u_1},m_{\tilde u_2},M_H)+\sin^2(\alpha+\beta)F(m_{\tilde u_1},m_{\tilde u_2},M_h)]\}$$
$$-\frac{g^2M_Z^2N_c}{16\pi^2\cos^2\theta_W}\{[\cos^2\theta_u(-T_3+e_u\sin^2\theta_W)+\sin^2\theta_u(e_u\sin^2\theta_W)]^2 2G(m_{\tilde u_1},m_{\tilde u_1},M_Z)$$
$$+[\sin^2\theta_u(-T_3+e_u\sin^2\theta_W)+\cos^2\theta_u(e_u\sin^2\theta_W)]^2 2G(m_{\tilde u_2},m_{\tilde u_2},M_Z)$$
$$+\sin^2\theta_u\cos^2\theta_u G(m_{\tilde u_1},m_{\tilde u_2},M_Z)$$
$$+4[(-T_3+e_u\sin^2\theta_W)^2+(e_u\sin^2\theta_W)^2]G(m_u,m_u,M_Z)\}\ , \tag{B2c}$$

where $N_c=3$ colors and

$$F(m_1,m_2,m_3)=\int_0^1 dx\,\ln\left[\frac{xm_1^2+(1-x)m_2^2-x(1-x)m_3^2}{\mu^2}\right]\ , \tag{B3}$$

$$G(m_1,m_2,m_3)=\int_0^1 dx\,x(1-x)\ln\left[\frac{xm_1^2+(1-x)m_2^2-x(1-x)m_3^2}{\mu^2}\right]\ , \tag{B4}$$

$$H(m_1,m_2,m_3)=\int_0^1 dx\,x\,\ln\left[\frac{xm_1^2+(1-x)m_2^2-x(1-x)m_3^2}{\mu^2}\right]\ . \tag{B5}$$

T_3 is the weak isospin (which is $+\frac{1}{2}$ for up-type quarks).

The expression for Δ in (B2) should be independent of the renormalization point μ. We have checked that this is indeed the case in both the analytic expression and in our computer program for calculating Δ, which provides a partial check of our answer (equivalent to the cancellation of divergences). We have also verified that (3.28) is satisfied which is a check on the value of the Goldstone self-energy that enters in (3.27).

The contribution for down squarks and down quarks is easily obtained from this result. The substitutions are shown below:

$$\theta_u \to \theta_d \ , \tag{B6a}$$

$$m_u \to m_d \ , \tag{B6b}$$

$$m_{\tilde{u}_{1,2}} \to m_{\tilde{d}_{1,2}} \ , \tag{B6c}$$

$$e_u \to e_d \ , \tag{B6d}$$

$$T_3 = \tfrac{1}{2} \to -\tfrac{1}{2} \ , \tag{B6e}$$

$$\sin\beta \to \cos\beta \ , \tag{B6f}$$

$$\cos\beta \to \sin\beta \ , \tag{B6g}$$

$$\cos\alpha \to \sin\alpha \ , \tag{B6h}$$

$$\sin\alpha \to \cos\alpha \ . \tag{B6i}$$

The last four equations imply $\sin(\alpha+\beta) \to \sin(\alpha+\beta)$ and $\cos(\alpha+\beta) \to -\cos(\alpha+\beta)$. To obtain the proper result requires the *further* substitutions

$$\sin(\alpha+\beta) \to -\sin(\alpha+\beta) \ , \tag{B6j}$$

$$\cos(\alpha+\beta) \to -\cos(\alpha+\beta) \ . \tag{B6k}$$

For example, the first two terms in Δ_2 for the down quark and squarks should be

$$\begin{aligned} \frac{g^2 m_d^2 N_c}{8\pi^2 \cos^2\theta_W \cos\beta} & [\cos^2\theta_d (T_3 - e_d \sin^2\theta_W) + \sin^2\theta_d (e_d \sin^2\theta_W)] \\ & \times [\cos\alpha \cos(\alpha+\beta) F(m_{\tilde{d}_1}, m_{\tilde{d}_1}, M_H) + \sin\alpha \sin(\alpha+\beta) F(m_{\tilde{d}_1}, m_{\tilde{d}_1}, M_h)] \\ & + [\sin^2\theta_d (T_3 - e_d \sin^2\theta_W) + \cos^2\theta_d (e_d \sin^2\theta_W)] \\ & \times [\cos\alpha \cos(\alpha+\beta) F(m_{\tilde{d}_2}, m_{\tilde{d}_2}, M_H) + \sin\alpha \sin(\alpha+\beta) F(m_{\tilde{d}_2}, m_{\tilde{d}_2}, M_h)] \ . \end{aligned} \tag{B7}$$

The contributions for the lepton and slepton loops are given in terms of the contributions for the up- and down-quark loops. The electron and selectron contribution is obtained from the expression for the down quarks with the appropriate mass and SU(2)×U(1) quantum-number replacements. Similarly the contributions from the neutrino and the sneutrino are given by an expression similar to that for the up quark with the appropriate mass and SU(2)×U(1) quantum-number substitutions.

APPENDIX C: TADPOLE CONTRIBUTIONS

In this appendix we demonstrate explicitly that the tadpole contribution to Δ in (3.27) and to $\tilde{\Delta}$ in (3.32) vanish in the minimal supersymmetry extension (MSE). The result can be seen by examining the Feynman rules that are present in the MSE. We display the vertices that are needed for the calculation of the tadpole diagrams in Fig. 10. The contribution to the sum in (3.27) from the tadpole diagrams in Fig. 11 is now easily seen to vanish using the couplings in Fig. 10.

We also display the vertices needed for the tadpole diagrams contributing to (3.32) in Fig. 12. The combination of tadpole diagrams in Fig. 13 vanishes.

These results generalize to the $2N$ Higgs-doublet models discussed in Appendix D. The Π's in (D16) and (D18), therefore, include all contributions to Higgs-boson self-energies besides tadpole diagrams. Similarly, tad-

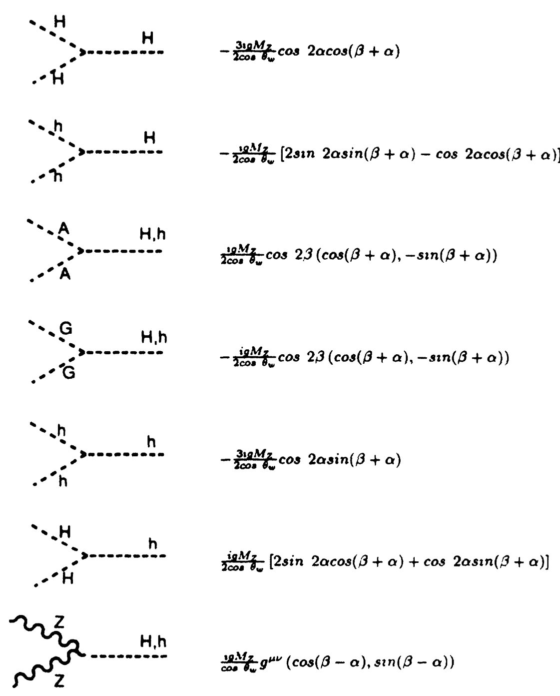

FIG. 10. Trilinear couplings relevant to tadpole contributions to (3.27).

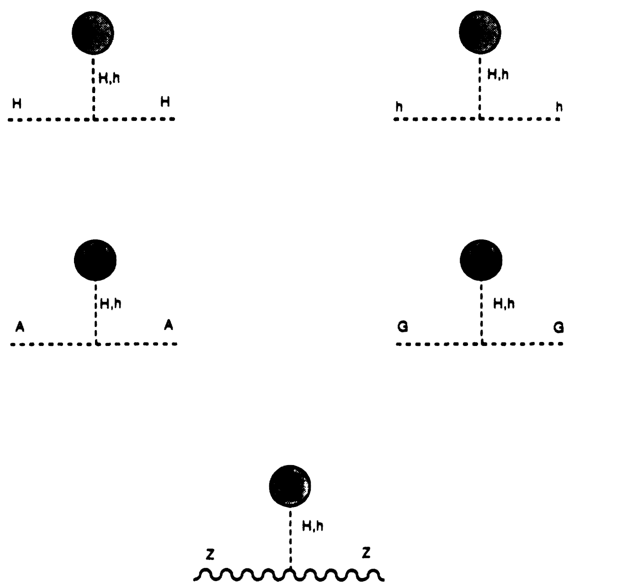

FIG. 11. These diagrams contribute to the sum in (3.27). The couplings in Fig. 10 show that this contribution is zero.

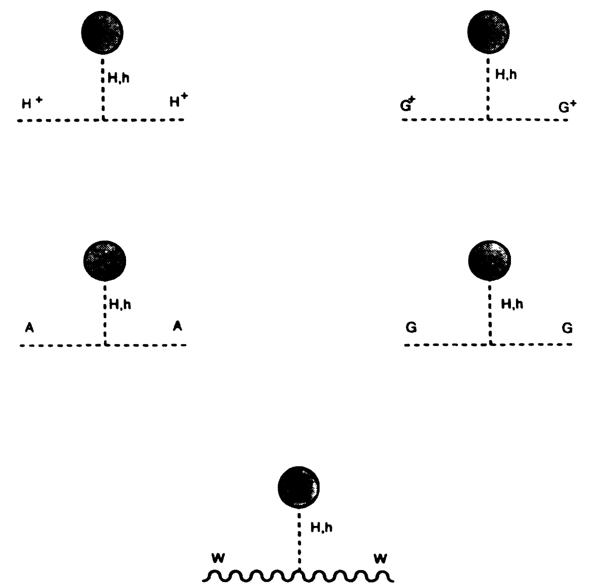

FIG. 13. These diagrams contribute to the sum in (3.32). The couplings in Fig. 12 show that this contribution is zero.

poles are not to be included in the contributions from the vacuum-polarization tensor either.

APPENDIX D: GENERALIZATION TO 2N HIGGS DOUBLETS

Models with more than two Higgs doublets have mass relations analogous to (1.1) and (1.2). In an extension of

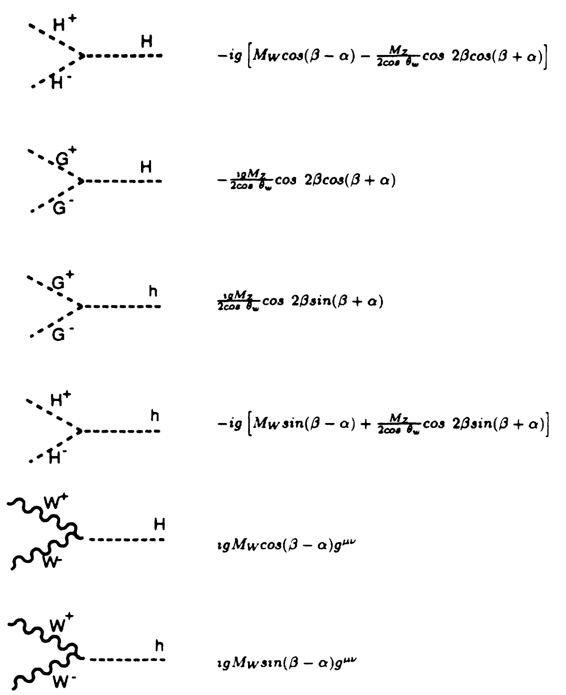

FIG. 12. Trilinear couplings relevant to tadpole contributions to (3.32).

the standard model with $2N$ Higgs doublets, there are $8N$ Higgs degrees of freedom. After spontaneous symmetry breaking three of these are Goldstone bosons, leaving $4N-2$ charged Higgs bosons $H_i^{\pm}$ and $4N-1$ neutral Higgs bosons. We shall denote the neutral Higgs scalar by H_i and the neutral Higgs pseudoscalar by A_i. In the supersymmetric version of the $2N$ doublet model, the couplings and masses in the Higgs sector are again constrained. The mass relations that arise are[3]

$$\sum_{i=1}^{2N} M_{H_i}^2 = \sum_{i=1}^{2N-1} M_{A_i}^2 + M_Z^2 , \tag{D1}$$

$$\sum_{i=1}^{2N-1} M_{H_i^{\pm}}^2 = \sum_{i=1}^{2N-1} M_{A_i}^2 + M_W^2 \tag{D2}$$

which generalize (1.1) and (1.2).

The Higgs potential for the model in the extension with $2N$ doublets is[3]

$$\begin{aligned} V = {} & \sum_{i=1}^{2N} m_i^2 \phi_i^\dagger \phi_i - \sum_{j<i}^{2N} m_{ij}^2 (\phi_i^\dagger \phi_j + \phi_j^\dagger \phi_i) \\ & + \tfrac{1}{8} g'^2 \sum_{i=1}^{2N} |(-1)^{i+1} \phi_i \phi_i|^2 \\ & + \tfrac{1}{8} g^2 \sum_{a=1}^{3} \left| \sum_{i=1}^{2N} (-1)^{i+1} \phi_i \sigma^a \phi_i \right|^2 . \end{aligned} \tag{D3}$$

This equation is the $2N$ doublet analog of (2.4) where arbitrary soft supersymmetry-breaking terms have been included.

There is now a vacuum-expectation value v_i for each of the $2N$ doublets ϕ_i. We can eliminate the m_i in favor of the VEV's v_i. The neutral scalar and neutral pseudoscalar mass matrices are $2N \times 2N$ matrices. The neutral scalar mass matrix M^2 is given by

$$M_{ii}^2=\sum_{j\neq i} m_{ij}^2\frac{v_j}{v_i}+\tfrac{1}{2}(g^2+g'^2)v_i^2 \quad \text{(no sum on } i\text{)}\ , \tag{D4a}$$

$$M_{ij}^2(i\neq j)=-m_{ij}+(-1)^{j-i}\tfrac{1}{2}(g^2+g'^2)v_iv_j \tag{D4b}$$

while the neutral pseudoscalar mass matrix M'^2 is given by

$$M_{ii}'^2=\sum_{j\neq i} m_{ij}^2\frac{v_j}{v_i}\ , \tag{D5a}$$

$$M_{ij}'^2(i\neq j)=m_{ij}\ . \tag{D5b}$$

M'^2 has a zero eigenvalue corresponding to a neutral Goldstone boson. Since both M^2 and M'^2 are real and symmetric, they can be diagonalized by orthogonal transformations that preserve their traces, i.e., $\sum_i M_{H_i}^2=\sum_i M_{ii}^2$ and $\sum_i M_{A_i}^2=\sum_i M_{ii}'^2$. Using (D4) and (D5), one can obtain (D1) and (D2).

The renormalization of the mass relations in (D1) and (D2) is a generalization of the arguments in Sec. III. The wave-function renormalization matrices $Z_S^{1/2}$ and $Z_P^{1/2}$ become $2N\times 2N$ matrices. The mass matrices (D4) and (D5) are symmetric and are diagonalized by

$$(M_S^2)_D=O_S^{-1}M^2O_S,\quad (M_P^2)_D=O_P^{-1}M'^2O_P\ , \tag{D6}$$

where O_S and O_P are orthogonal matrices. $(M_S^2)_D$ and $(M_P^2)_D$ are diagonal matrices whose nonzero entries are the masses $M_{H_i}^2$ and $M_{A_i}^2$, respectively. We shift parameters as in (3.5):

$$(m_{ij}^2)_b=m_{ij}^2+\delta m_{ij}^2 \quad (i\neq j)\ , \tag{D7a}$$

$$(v_i)_b=v_i+\delta v_i\ , \tag{D7b}$$

$$(M_Z^2)_b=M_Z^2+\delta M_Z^2\ . \tag{D7c}$$

The unrenormalized propagators are given by formulas analogous to (3.12):

$$i\Gamma_S(p^2)=(Z_S^{1/2})^TZ_S^{1/2}p^2-(Z_S^{1/2})^T(M_S^2)_DZ_S^{1/2}-\delta M_S\ , \tag{D8}$$

$$i\Gamma_P(p^2)=(Z_P^{1/2})^TZ_P^{1/2}p^2-(Z_P^{1/2})^T(M_P^2)_DZ_P^{1/2}-\delta M_P\ , \tag{D9}$$

where $\delta M_S^2=O_S^{-1}\delta M^2O_S$ and $\delta M_P^2=O_P^{-1}\delta M'^2O_P$. δM^2 and $\delta M'^2$ are analogous to the matrices constructed in the two-Higgs-doublet case. Since the trace of the matrices is invariant under orthogonal transformations we have

$$\mathrm{Tr}\delta M_S^2=\mathrm{Tr}\delta M^2\ , \tag{D10}$$

$$\mathrm{Tr}\delta M_P^2=\mathrm{Tr}\delta M'^2\ . \tag{D11}$$

From the expressions for the mass relations in (D4) and (D5) we have

$$\mathrm{Tr}\delta M^2=\mathrm{Tr}\delta M'^2+\delta M_Z^2 \tag{D12}$$

so that

$$\mathrm{Tr}\delta M_S^2=\mathrm{Tr}\delta M_P^2+\delta M_Z^2\ . \tag{D13}$$

The renormalization conditions analogous to those in (3.20) are[10]

$$i\tilde{\Gamma}_{H_iH_i}(M_{H_i}^2)=0 \quad \text{(no sum)}\ , \tag{D14a}$$

$$i\tilde{\Gamma}_{H_iH_j}(M_{H_i}^2)=i\tilde{\Gamma}_{H_iH_j}(M_{H_j}^2)=0 \quad \text{(no sum)}\ , \tag{D14b}$$

$$i\tilde{\Gamma}'_{H_iH_i}(M_{H_i}^2)=1 \quad \text{(no sum)}\ . \tag{D14c}$$

If we define the radiative corrections to (D1) as

$$\sum_{i=1}^{2N}M_{H_i}^2=\sum_{i=1}^{2N-1}M_{A_i}^2+M_Z^2+\Delta \tag{D15}$$

we obtain the result

$$\Delta=-\sum_{i=1}^{2N}\Pi_{H_iH_i}(M_{H_i}^2)+\sum_{j=1}^{2N}\Pi_{A_jA_j}(M_{A_j}^2)-A_{ZZ}(M_Z^2)\ , \tag{D16}$$

where the sum over the pseudoscalar Higgs-boson A_j self-energies includes the neutral Goldstone-boson self-energy $\Pi_{GG}(0)$. It can be shown that the tadpoles cancel just as in the MSE. Similarly it can be shown that the correction $\tilde{\Delta}$ to (D2) defined as

$$\sum_{i=1}^{2N-1}M_{H_i^\pm}^2=\sum_{i=1}^{2N-1}M_{A_i}^2+M_W^2+\tilde{\Delta} \tag{D17}$$

is given by

$$\tilde{\Delta}=-\sum_{i=1}^{2N}\Pi_{H_i^+H_i^\pm}(M_{H_i^\pm}^2)+\sum_{j=1}^{2N}\Pi_{A_jA_j}(M_{A_j}^2)-A_{WW}(M_W^2)\ , \tag{D18}$$

where the sum over the pseudoscalar Higgs-boson A_j self-energies includes the neutral Goldstone-boson energy $\Pi_{GG}(0)$, and the sum over the charged Higgs-bosons $H_i^\pm$ self-energies includes the charged Goldstone-boson self-energy $\Pi_{G^\pm G^\pm}(0)$.

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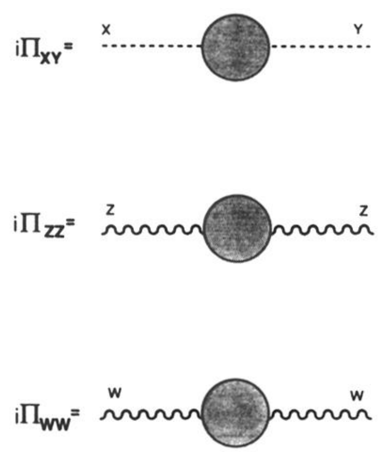

FIG. 1. The self-energy diagrams are defined as shown with the external legs amputated. $X, Y = H, h, A, G$.

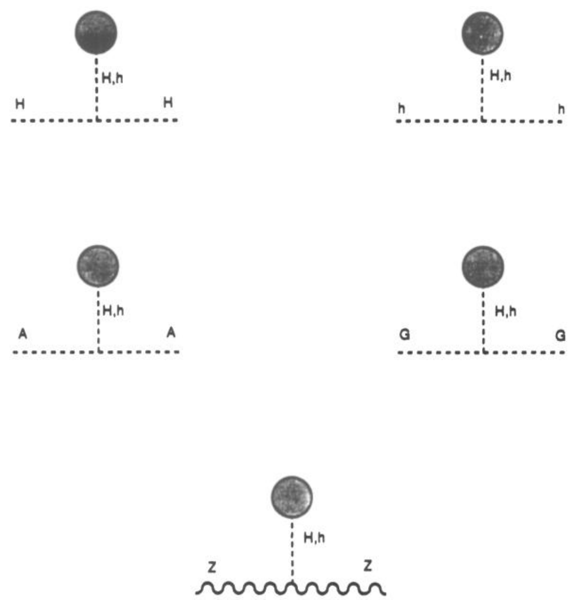

FIG. 11. These diagrams contribute to the sum in (3.27). The couplings in Fig. 10 show that this contribution is zero.

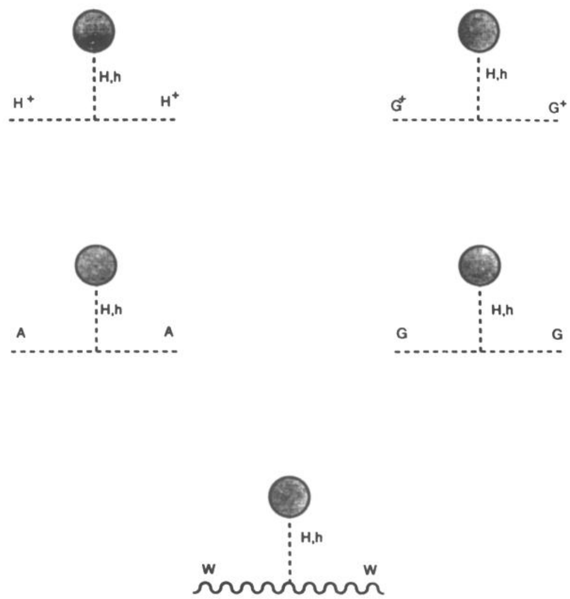

FIG. 13. These diagrams contribute to the sum in (3.32). The couplings in Fig. 12 show that this contribution is zero.

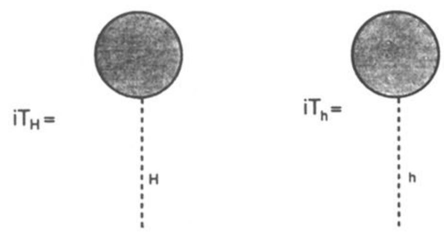

FIG. 2. The two kinds of tadpoles.