PHYSICAL REVIEW D VOLUME 41, NUMBER 7 1 APRIL 1990

Remarks on the anomalous magnetic moment of the W boson

Cynthia L. Bilchak and John D. Stroughair
Physics Department, Shippensburg University, Shippensburg, Pennsylvania 17257
(Received 27 November 1989)

We comment on the experimental significance of limits on the electroweak-boson self-interactions derived from studies of the radiative corrections to currently well-measured quantities.

Now that copious numbers of Z's are available on both sides of the Atlantic, the next target for experimental investigation in the electroweak theory, failing a fortuitously easily accessible Higgs boson, is the self-interactions of the W and Z. The standard theory makes definite predictions for these interactions that so far are almost wholly unchecked by experiment. Given the success of the standard theory why should we expect its predictions for the self-interactions not to match experiment? The answer is that we do not expect the standard model to be a "final" theory of nature; it is too complicated and contains too many free parameters for that. At some point then the standard model must begin to fail and one of the few remaining unchecked places is the electroweak-boson self-interactions. Historically we can notice that one of the earliest hints that the proton and neutron were not fundamental particles was their anomalous magnetic moments.

The study of nonstandard electroweak-boson self-interactions has, by now, a long history.[1-4] Although the current situation is not completely clear and there has been a regrettable tendency for later contributions to reprise rather than clarify earlier work, three distinct approaches can be discerned: model making,[5-7] direct calculations of the effect of nonstandard couplings,[3,4,8-10] and indirect calculations (typically through radiative effects) of the effect of nonstandard couplings.[11-14] The latter two endeavors have been hampered by the fact that there does not yet exist any able contender to the standard model as a theory of the electroweak interactions. As a consequence, most calculations have tended to use a fairly *ad hoc* subset of the general interactions given by Gaemers and Gounaris.[2]

The lack of a viable alternative model has not been too great a problem for direct calculations: there the signal appears to be clear;[9,15] nonstandard couplings enhance the total cross section, particularly at large angles and higher energies, for two-boson production reactions, e.g., $e^+e^- \to W^+W^-$, $p\bar{p} \to W\gamma X$, etc. The situation with the indirect approach is less clear-cut; model dependencies can have a greater effect on results, witness the papers of Suzuki[12] and Neufeld, Stroughair, and Schildknecht[14] both of which attempt to limit κ (the W magnetic moment parameter) by considering corrections to the low-energy ρ parameter. Suzuki obtains bounds on $\Delta\kappa \equiv \kappa - 1$ at least one order of magnitude tighter than Neufeld, Stroughair, and Schildknecht; the difference is due entirely to the fact that Suzuki allows only κ to vary, whereas Neufeld, Stroughair, and Schildknecht, requiring a global SU(2) symmetry[7,16] for the interactions, must also vary other parameters and hence rediscover some of the cancellations that make the standard model renormalizable. We will refer to these two works again because they usefully illuminate certain points.

As well as the model-dependent uncertainties there are also problems with the physical meaning we can attach to these indirect results. It is this question that this note seeks to address. To do this we will have to first recap the methodology of the indirect studies. Typically a physical quantity is identified that has radiative corrections that depend on the electroweak-boson self-interactions, e.g., the muon anomalous magnetic moment[11] or the previously mentioned ρ parameter.[12,14] The next stage involves picking a set of deviations from the standard-model couplings and calculating the effect these deviations will have on the chosen physical quantity. The current experimental limits on this quantity then provide the bounds on the nonstandard couplings. The calculation itself is vexed by several nontrivial technical problems; once the boson self-interactions are changed, two of the standard model's most attractive features are lost: gauge invariance and renormalizability. This means that the R_ξ gauges are no longer available[12] and all calculations must be carried out in the physical unitary gauge, where the boson propagators take the canonical form of a massive spin-one particle. It is probably not supererogatory to stress that loss of gauge invariance is not a fatal flaw when one is no longer dealing with a gauge theory. The lack of renormalizability is dealt with by introducing an arbitrary cutoff Λ to control divergent integrals and by truncating the perturbation theory at one loop with a plausibility argument that the higher orders are "small" if Λ is not "too large." The use of a cutoff tends to lead to problems with U(1) electromagnetic gauge invariance, that unlike the weak gauge invariance is not negotiable and must be restored by hand using the Ward identities.

We do not wish to argue that the technical difficulties inherent in these calculations have been solved in a completely satisfactory manner and it is arguable that there are too many ambiguities to place much faith in them. They are though, for now, our only window onto these interesting interactions; if we accept the assumptions needed to carry out the calculations, what guidance can they give to the experimentalist hoping to measure the γWW and ZWW vertices at CERN LEP 200 or the Fermilab Tevatron?

First, note that experiments will be measuring an effective (summed to all orders) electroweak-boson interaction; whereas the studies of radiative effects have given us limits on parameters in the tree-level Lagrangian. This distinction has not been given much attention, probably because it is of no great practical significance in the standard model. This has been shown by the work of Bardeen, Gastmans, and Lautrup[17] and Bilchak, Gastmans, and van Proeyen[18] who considered the effect of radiative corrections on the parameter κ. κ is identically equal to unity at the tree level in the standard model; these authors have shown that κ picks up $O(\alpha/\pi)$ corrections at one loop. This level of change in κ is almost certainly unobservable at all currently planned machines.

The questions can then be stated as follows: given the existence of a Λ-dependent limit on, for definiteness, κ^0 (the tree-level value of κ), what can we then say about, for example, $\sigma^{1\ \mathrm{loop}}(e^+e^-\to W^+W^-;\Delta\kappa^0,\Lambda)$? In other words, we know that if $\kappa^0=1$, then $\Delta\kappa^{1\ \mathrm{loop}}=O(\alpha/\pi)$ and the one-loop corrections to $\sigma(e^+e^-\to W^+W^-)$ are small;[19,20] but if $\kappa^0\neq 1$ how sensitive is $\Delta\kappa^{1\ \mathrm{loop}}$ and hence $\sigma^{1\ \mathrm{loop}}$ to the fact that $\Delta\kappa^0$ is no longer zero?

We have tried to answer this question by calculating the one-loop corrections to κ for the case $\kappa^0\neq 1$. The relevant diagrams are given in Fig. 1. In this calculation we are faced with all the previously mentioned problems attendant with the lack of renormalizability and gauge invariance. We have not attempted to solve these in all generality, instead we have opted for consistency with previous calculations in the literature; if we are to take seriously a limit on $\Delta\kappa^0$ due to a one-loop calculation we should ask what would a $\Delta\kappa^0$ of this order imply for the observed value of κ, to one loop.

Suppressing all the nonstandard couplings except for κ and κ_Z, the analogue of κ for the ZWW interaction, we find, keeping only the dominant cutoff-dependent terms,

$$\kappa=\kappa^0+[\alpha\Lambda^2/(8\pi M_W^2)](2(\Delta\kappa^0)^2+(\Delta\kappa^0)^3-3\Delta\kappa^0/\sin^2\theta_W+\cot^2\theta_W\{\Delta\kappa^0(2\cos^2\theta_W+2)-\Delta\kappa_Z^0[2+\Delta\kappa_Z^0-\Delta\kappa^0(3+\Delta\kappa_Z^0)]\}) \tag{1}$$

with $\Delta\kappa^0\equiv\kappa^0-1$ and $\Delta\kappa_Z^0\equiv\kappa_Z^0-1$. Setting $\Delta\kappa_Z^0=0$, we arrive at

$$\kappa\simeq\kappa^0-0.045\Lambda^2(\mathrm{TeV})\{\Delta\kappa^0[1-2\Delta\kappa^0-(\Delta\kappa^0)^2]\}\ . \tag{2}$$

Rather unsurprisingly this tells us that if $\Delta\kappa^0$ is small then after the one-loop corrections κ will remain close to κ^0. As an example let us consider the previously mentioned result due to Suzuki,[12]

$$|\Delta\kappa^0|<7\times10^{-3}[\Lambda^4(\mathrm{TeV})]^{-1}\ , \tag{3}$$

and for numerical simplicity set $\Lambda=1.0$ TeV. Inserting this into Eq. (2) gives

$$0.9933\le\kappa\le1.0067\ . \tag{4}$$

So we have arrived at the reasonable result that if at the tree level κ deviates from its standard-model value by a small amount, then this deviation is stable at one loop, granted the assumptions inherent in this sort of calculation.

However, in a similar approach Neufeld, Stroughair, and Schildknecht[14] found that κ^0 can be an essentially free parameter if κ_Z^0 is related to κ^0 through

$$(\kappa^0)^2\sin^2\theta_W=1-(\kappa_Z^0)^2\cos^2\theta_W\ . \tag{5}$$

(Trivially satisfied by the standard model.) As we have mentioned earlier the only difference between the approach of Suzuki and that of Neufeld, Stroughair, and Schildknecht is that the latter authors retained some of the symmetry of the standard model. While Eq. (5) allows for large values of $\Delta\kappa^0$ [for example, $\kappa^0=-1$ and $\kappa_Z^0=-1$ are consistent with Eq. (5)], only for small values of $\Delta\kappa^0$ does Eq. (1) give sensible results. For example, if we set $\kappa^0=-1$ and impose the relation of Eq. (5), and take $\Lambda=1$ TeV, then Eq. (1) gives $\kappa=0.3$: a one-loop correction of the same order as the tree-level result. Obviously even our cutoff-dependent perturbation theory has broken down and must be discarded; therefore, the limits on κ^0, in this case, cannot be taken as a guide to the range of possible observed values of κ.

Our conclusions must perforce be a little negative. If a one-loop calculation of the effect of nonstandard couplings on a well-measured quantity leads to tight limits on the nonstandard couplings then our work indicates that these limits can be taken as a good guide to the size of the effects that might be expected in a direct experiment. Unfortunately these effects will also be small. On the other hand, if a particular model leads to loose limits on the self-interaction at the tree level, these limits cannot then be taken as a reliable guide to the possible effects to be seen in a direct experiment. For a definitive answer on the boson self-interactions we will have to wait for an experimental answer.

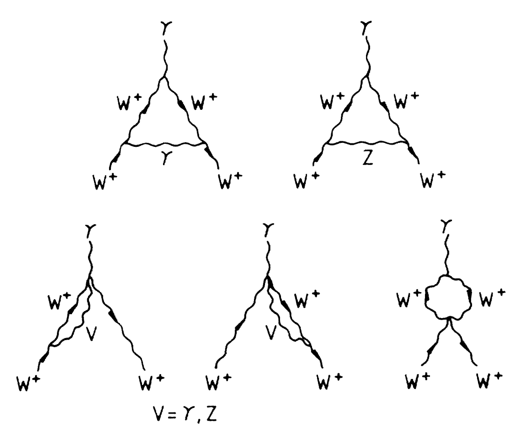

FIG. 1. Feynman diagrams contributing to κ at the one-loop level.

[1]P. Q. Hung and J. J. Sakurai, Nucl. Phys. **B143**, 81 (1978).

[2]K. J. F. Gaemers and G. J. Gounaris, Z. Phys. C **1**, 259 (1979).

[3]R. W. Brown and K. O. Mikaelian, Phys. Rev. D **19**, 922 (1979).

[4]R. W. Brown, D. Sahdev, and K. O. Mikaelian, Phys. Rev. D **20**, 1164 (1979).

[5]L. F. Abbott and E. Farhi, Phys. Lett. **101B**, 69 (1981).

[6]R. W. Robinett, Phys. Rev. D **28**, 1185 (1983); **28**, 1192 (1983).

[7]M. Kuroda, J. Maalampi, D. Schildknecht, and K. H. Schwarzer, Nucl. Phys. **B284**, 271 (1987).

[8]K. O. Mikaelian, M. A. Samuel, and D. Sahdev, Phys. Rev. Lett. **43**, 746 (1979).

[9]C. L. Bilchak and J. D. Stroughair, Phys. Rev. D **30**, 1881 (1984).

[10]M. Kuroda, J. Maalampi, D. Schildknecht, and K. H. Schwarzer, Phys. Lett B **190**, 217 (1987).

[11]F. Herzog, Phys. Lett. **148B**, 355 (1984).

[12]M. Suzuki, Phys. Lett. **153B**, 289 (1985).

[13]J. A. Grifols, S. Peris, and J. Sola, Report No. DESY 86-055, 1986 (unpublished).

[14]H. Neufeld, J. D. Stroughair, and D. Schildknecht, Phys. Lett. B **198**, 563 (1987).

[15]J. D. Stroughair and C. L. Bilchak, Z. Phys. C **26**, 415 (1984).

[16]R. Kögerler and D. Schildknecht, Report No. CERN TH 3231, 1982 (unpublished).

[17]W. A. Bardeen, R. Gastmans, and B. Lautrup, Nucl. Phys. **B146**, 319 (1972).

[18]C. L. Bilchak, R. Gastmans, and A. van Proeyen, Nucl. Phys. **B273**, 46 (1986).

[19]M. Lemoine and M. Veltman, Nucl. Phys. **B164**, 445 (1980).

[20]R. Phillipe, Phys. Rev. D **26**, 1588 (1982).