Sign of the neutron-proton mass difference in an SU(2)×U(1) supersymmetric toy model: A possible scenario for solving the old puzzle

Bipin R. Desai and Guang-Hua Xu
Department of Physics, University of California, Riverside, California 92521
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Based on the idea that electromagnetism is responsible for mass differences within isotopic multiplets (e.g., pointlike neutron and proton or u and d quarks), we generalize an SU(2)×U(1) model in a toy field theory of vectors to a supersymmetric model and investigate the finite mass difference within the isotopic doublet. It is found that under soft-supersymmetry breaking, a positive n-p mass difference can be obtained under reasonable assumptions for the parameters involved.

I. INTRODUCTION

The question of the sign of the neutron-proton mass difference is an old puzzle.[1] Based on the idea that electromagnetism is responsible for the mass difference, calculations show that the proton is heavier than the neutron in contradiction with nature. In the standard model, which is couched in terms of the u and d quarks, the masses m_u and m_d are arbitrary parameters and one does not ask as to the origin of their mass difference. This view may well be correct but the fact that the u and d quarks are nearly degenerate in mass compared to the others in the quark families sets them apart and makes it important to explore the possibility that there exists a special relationship within this isotopic doublet.

In this paper we examine the finite mass differences within isotopic doublets in supersymmetric gauge field theories. The development of renormalizable gauge field theories within spontaneously broken gauge symmetry has yielded a different view of the origin of this approximate symmetry.[2-4] Three separate types of zeroth-order mass relations provide the mass differences in higher orders.[5] However, detailed calculations of the proton-neutron mass difference[6,7] in some illustrative models of the type-1 symmetry have shown that this mass difference $\Delta m|_{n-p}$ is manifestly negative. In all type-1 symmetry models, the mass differences can only be obtained after second order and they are zero at the tree level due to some symmetry (including discrete symmetry)[2,3] restrictions.

Since supersymmetry is a bigger symmetry, it is interesting to investigate whether or not the result can be improved in supersymmetric gauge field theories. In this paper based on the same idea as in gauge field theory,[3] we generalize the SU(2)×U(1) model of Freedman and Kummer[6] to a supersymmetric model to see whether the mass difference $\Delta m|_{n-p}$ can be improved in a second-order calculation. We find that the mass difference $\Delta m|_{n-p}$ is less negative in the exact-supersymmetry situation than the $\Delta m|_{n-p}$ obtained in Ref. 6. A positive $\Delta m|_{n-p}$, however, can be obtained under soft-supersymmetry breaking. In doing this, the mixing of scalar fermions and the mixing of neutral gauginos and Higgsinos play very important roles. To obtain a positive $\Delta m|_{n-p}$, the parameters we choose are consistent with the estimates of the supersymmetry (SUSY) phenomenology.

Our model ignores strong-interaction effects as contributing to the mass difference. In that sense the n-p and d-u mass differences are equivalent. The current estimates do indeed indicate that both mass differences are about the same (≈ 4 MeV).

The model we are considering cannot be incorporated in the standard electroweak model as it is not a chiral model and, unlike the standard model, it imposes strong isotopic symmetry for the fermion doublet. It is used here as a toy model to provide a scenario of how one might proceed if one were to continue, as in the past, to attribute the n-p mass difference to electromagnetism. Its applicability is likely limited to small mass differences. For the doublets (c,s) etc., the underlying cause of their typically large mass differences is very likely deeper than electroweak.

The e^--ν mass difference, however, can be understood within the context of this model as long as their zeroth-order masses do not vanish.

In Sec. II we generalize the simple model of type 1 with SU(2)×U(1) gauge symmetry and parity-conserving coupling to a model of supersymmetric SU(2)×U(1) gauge symmetry. In Sec. III we present the detailed calculation of $\Delta m|_{n-p}$ for the exact SUSY model. The improvements due to soft-SUSY breaking in this model are discussed in Sec. IV. In Sec. V we carry out explicit calculations of Δm for some special cases for the parameters in the soft-SUSY breaking terms.

II. SUPERSYMMETRIC GENERALIZATION OF THE SU(2)×U(1) MODEL

The original SU(2)×U(1) model[6] is a model of minimal algebraic complexity. It incorporates triplet and singlet gauge fields $\mathbf{V}_\mu$ and V'_μ and fermion doublet fields

$$\Psi = \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix} \tag{2.1}$$

as well as the scalar doublet

TABLE I. The fields in the SU(2)×U(1) model. Note: (1) The charge Q is obtained via $Q=T_3+Y/2$; (2) $\psi_i=(\psi_{iL}/\bar{\psi}_{iR})$; and (3) If we consider proton and neutron as point particles, and let $\psi=(p/n)$, then, we should have $y_L=1, y_R=-1$.

	Boson fields	Fermionic partners	$SU(2)_W$	Y
Gauge multiplets				
	V^a	λ^a	Triplet	0
	V'	λ'	Singlet	0
Matter multiplets				
Scalar fermions	$\tilde{\psi}_{jL}=(\tilde{\psi}_{1L},\tilde{\psi}_{2L})$ $=(\tilde{F}_{1L},\tilde{F}_{2L})$	$\psi_{jL}=(\psi_{1L},\psi_{2L})$ $=(F_{1L},F_{2L})$	Doublet	y_L
	$\tilde{\psi}_{jR}=(-\tilde{\psi}_{2R}^*,\tilde{\psi}_{1R}^*)$ $=(\tilde{F}_{1R},\tilde{F}_{2R})$	$\psi_{jR}=(-\psi_{2R},\psi_{1R})$ $=(-\psi_{2L}^c,\psi_{1L}^c)$ $=(F_{1R},F_{2R})$	Doublet	$y_R=-y_L$
Higgs bosons	H_1^j	$(\psi_{H_1}^1,\psi_{H_1}^2)$	Doublet	-1
	H_2^j	$(\psi_{H_2}^1,\psi_{H_2}^2)$	Doublet	1
	N	$\tilde{N}_L$	Singlet	0

$$H=\begin{bmatrix} H_1 \\ H_2 \end{bmatrix}, \tag{2.2}$$

each with $T=\frac{1}{2}$ and $Y=1$. We consider the parity-conserving Lagrangian

$$\begin{aligned}\mathcal{L}=&-\tfrac{1}{4}(\partial_\mu \mathbf{V}_\nu-\partial_\nu \mathbf{V}_\mu+g\mathbf{V}_\mu\times\mathbf{V}_\nu)^2-\tfrac{1}{4}(\partial_\mu V'_\nu-\partial_\nu V'_\mu)^2\\ &+\bar{\Psi}(i\partial\!\!\!/-\tfrac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{V}-\tfrac{1}{2}g'V'-m)\Psi\\ &+|\partial_\mu H+i\tfrac{1}{2}g\boldsymbol{\tau}\cdot\mathbf{V}_\mu H+i\tfrac{1}{2}g'V'_\mu H|^2-\mu^2H^\dagger H\\ &+\lambda(H^\dagger H)^2\,,\end{aligned} \tag{2.3}$$

where we note that SU(2)×U(1) invariance permits a mass term with equal Ψ_1 and Ψ_2 masses but does not allow Yukawa couplings. This Lagrangian does not describe weak interactions as such but we are considering it as an illustrative model for the problem at hand.[6] In contrast with the standard model, where fermion masses are arbitrary parameters arising from the spontaneous gauge symmetry breaking, here we start with degenerate masses for the fermion isodoublets in order to answer the question about the sign and magnitude of their mass difference due to second-order loop corrections.

The supersymmetric generalization[8] of this model consists of the fields listed in Table I.

Note that we have additional scalar fields H_2 and N compared to Eqs. (2.1) and (2.2). H_2 is needed for generating masses for the supersymmetric partners of the gauge particles, and N and its supersymmetric fermionic partner are responsible for the existence of a unique ground state which breaks SU(2)×U(1) to U(1) at the tree level for this unbroken supersymmetric model, as we will see.

Following the same procedure as in Ref. 8, we can obtain the scalar potential[9] V as

$$\begin{aligned}V=&h^2(H_2^{i*}H_2^i+H_1^{i*}H_1^i)N^*N+|h\epsilon_{ij}H_1^iH_2^j+s|^2\\ &+\frac{g^2}{8}(\tilde{F}_{iL}^*\tilde{F}_{iL}\tilde{F}_{jL}^*\tilde{F}_{jL}+\tilde{F}_{iR}^*\tilde{F}_{iR}\tilde{F}_{jR}^*\tilde{F}_{jR}+H_1^{i*}H_1^iH_1^{j*}H_1^j+H_2^{i*}H_2^iH_2^{j*}H_2^j+4\tilde{F}_{iL}^*\tilde{F}_{iR}\tilde{F}_{jR}^*\tilde{F}_{jL}-2\tilde{F}_{iL}^*\tilde{F}_{iL}\tilde{F}_{jR}^*\tilde{F}_{jR}\\ &\quad+4\tilde{F}_{iL}^*H_1^iH_1^{j*}\tilde{F}_{jL}-2\tilde{F}_{iL}^*\tilde{F}_{iL}H_1^{j*}H_1^j+4\tilde{F}_{iL}^*H_2^iH_2^{j*}\tilde{F}_{jL}-2\tilde{F}_{iL}^*\tilde{F}_{iL}H_2^{j*}H_2^j+4\tilde{F}_{iR}^*H_1^iH_1^{j*}\tilde{F}_{jR}-2\tilde{F}_{iR}^*\tilde{F}_{iR}H_1^{j*}H_1^j\\ &\quad+4\tilde{F}_{iR}^*H_2^iH_2^{j*}\tilde{F}_{jR}-2\tilde{F}_{iR}^*\tilde{F}_{iR}H_2^{j*}H_2^j+4H_1^{i*}H_2^iH_2^{j*}H_1^j-2H_1^{i*}H_1^iH_2^{j*}H_2^j)\\ &+\frac{g'^2}{8}(y_L\tilde{F}_{iL}^*\tilde{F}_{iL}+y_R\tilde{F}_{iR}^*\tilde{F}_{iR}-H_1^{i*}H_2^i+H_2^{i*}H_2^i)^2\,.\end{aligned} \tag{2.4}$$

From Eq. (2.4), the only scalar fields which acquire nonzero vacuum expectation values are

$$\langle H_1\rangle=\frac{v}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix},\quad \langle H_2\rangle=\frac{v}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}, \tag{2.5}$$

which break SU(2)×U(1) down to $U(1)_{EM}$. The constant v is related to h and s of Eq. (2.4) by $\frac{1}{2}v^2h+s=0$. By inserting this solution back into Eq. (2.4), we note that at this minimum we have $V=0$ thus implying that the theory remains supersymmetric.

By considering the SUSY interactions between the gauge and matter multiplets and the self-interactions of the matter multiplets, after spontaneously gauge symmetry breaking, we can have the following mass eigenstates:

$$\tilde{\omega}_1^- = \begin{bmatrix} \Psi_{H_1}^2 \\ i\bar{\lambda}^+ \end{bmatrix}, \quad \tilde{\omega}_2^+ = \begin{bmatrix} \Psi_{H_2}^1 \\ i\bar{\lambda}^- \end{bmatrix}, \quad \text{with mass } m_v = \frac{gv}{\sqrt{2}} ,$$

$$\tilde{\xi} = \begin{bmatrix} \frac{1}{\sqrt{2}}(\Psi_{H_1}^1 - \Psi_{H_2}^2) \\ i\bar{\lambda}_Z \end{bmatrix} ,$$

$$\text{with mass } m_Z = \frac{1}{\sqrt{2}}(g^2+g'^2)^{1/2}v ,$$

$$\tilde{h} = \begin{bmatrix} \frac{1}{\sqrt{2}}(\Psi_{H_1}^1 + \Psi_{H_2}^2) \\ \overline{\Psi}_N \end{bmatrix}, \quad \text{with mass } m_h \equiv hv , \tag{2.6}$$

$$\tilde{\gamma} = \begin{bmatrix} -i\lambda_\gamma \\ i\bar{\lambda}_\gamma \end{bmatrix}, \quad \text{with mass } m_\gamma = 0 ,$$

and

$$V_\mu^\mp = \frac{1}{\sqrt{2}}(V_\mu^1 \mp iV_\mu^2), \quad \text{with mass } m_v = \frac{gv}{\sqrt{2}} ,$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gV_\mu^3 - g'V_\mu') ,$$

$$\text{with mass } m_Z = \frac{1}{\sqrt{2}}(g^2+g'^2)^{1/2}v ,$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(gV_\mu^3 + g'V_\mu'), \quad \text{with mass } m_A = 0 ,$$

where

$$\lambda^\pm = \frac{1}{\sqrt{2}}(\lambda^1 \mp i\lambda^2), \quad \lambda_Z = \frac{g\lambda^3 - g'\lambda'}{\sqrt{g^2+g'^2}} ,$$

$$\lambda_\gamma = \frac{g'\lambda^3 + g\lambda'}{\sqrt{g^2+g'^2}} .$$

III. FINITE MASS DIFFERENCES

Without loss of generality, we consider the case $y_L = 1, y_R = 1$.

The interaction terms of the gauge and scalar-fermion multiplets, which can affect the self-energy of the fermions,[9] are given by

$$\begin{aligned}
\mathcal{L}_{\text{int}} = & -\frac{1}{2\sqrt{2}} g V_\mu^- \overline{\Psi}_1 \gamma^\mu \Psi_2 + \text{H.c.} - \frac{gg'}{(g^2+g'^2)^{1/2}} A_\mu \overline{\Psi}_1 \gamma^\mu \Psi_1 - \frac{g^2-g'^2}{2(g^2+g'^2)^{1/2}} Z_\mu \overline{\Psi}_1 \gamma^\mu \Psi_1 + \tfrac{1}{2}(g^2+g'^2)^{1/2} Z_\mu \overline{\Psi}_2 \gamma^\mu \Psi_2 \\
& - g(\bar{\tilde{\omega}}_1^- P_L \Psi_2 \tilde{\Psi}_{1L}^* + \text{H.c.} + \bar{\tilde{\omega}}_2^+ P_L \Psi_1 \tilde{\Psi}_{2L}^* + \text{H.c.}) + \left[\frac{g^2+g'^2}{2}\right]^{1/2} \bar{\tilde{\xi}} P_L \Psi_2 \tilde{\Psi}_{2L}^* + \text{H.c.} \\
& - \frac{g^2-g'^2}{[2(g^2+g'^2)]^{1/2}} \bar{\tilde{\xi}} P_L \Psi_1 \tilde{\Psi}_{1L}^* + \text{H.c.} - \frac{2gg'}{[2(g^2+g'^2)]^{1/2}} \bar{\tilde{\gamma}} P_L \Psi_1 \tilde{\Psi}_{1L}^* + \text{H.c.} \\
& - g(\tilde{\omega}_1^{-T} C^{-1} P_R \Psi_1 \Psi_{2R}^* + \text{H.c.} + \tilde{\omega}_2^{+T} C^{-1} P_R \Psi_2 \tilde{\Psi}_{1R}^* + \text{H.c.}) + \left[\frac{g^2+g'^2}{2}\right]^{1/2} \tilde{\xi}^T C^{-1} P_R \Psi_2 \tilde{\Psi}_{2R}^* + \text{H.c.} \\
& - \frac{g^2-g'^2}{[2(g^2+g'^2)]^{1/2}} \tilde{\xi}^T C^{-1} P_R \Psi_1 \tilde{\Psi}_{1R}^* + \text{H.c.} - \frac{2gg'}{[2(g^2+g'^2)]^{1/2}} \bar{\tilde{\gamma}} P_R \Psi_1 \tilde{\Psi}_{1R}^* + \text{H.c.}
\end{aligned} \tag{3.1}$$

The second-order contribution to Δm due to electroweak interaction can be readily calculated (see Fig. 1) following Freedman and Kummer.[6] Working in the U gauge, one finds that

$$\Delta m_1 = \frac{-mg^2g'^2}{8\pi^2(g^2+g'^2)} \int_0^1 dx(1+x)\ln\left[1+\frac{1-x}{x^2}\frac{m_Z^2}{m^2}\right] , \tag{3.2}$$

which is manifestly negative, where $\Delta m_i = \Delta m_i|_{2-1} = [m(\text{neutron}) - m(\text{proton})]$, and the subscript i indicates the figure number under consideration.

The contribution to Δm due to the SUSY particles can also be calculated (see Figs. 2 and 3). It is given by

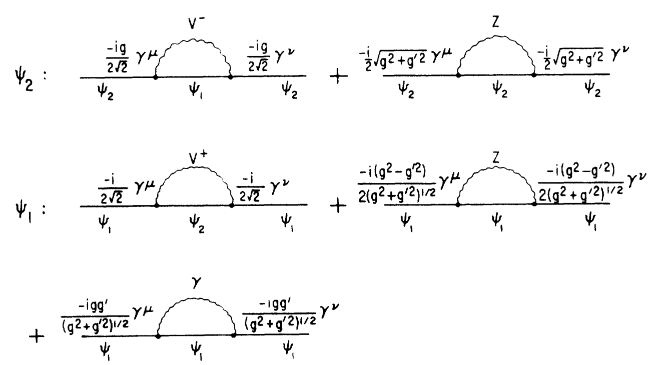

FIG. 1. The Feynman diagrams contributing to the fermion (ψ_1 and ψ_2) masses due to the interactions of the gauge particles and fermions.

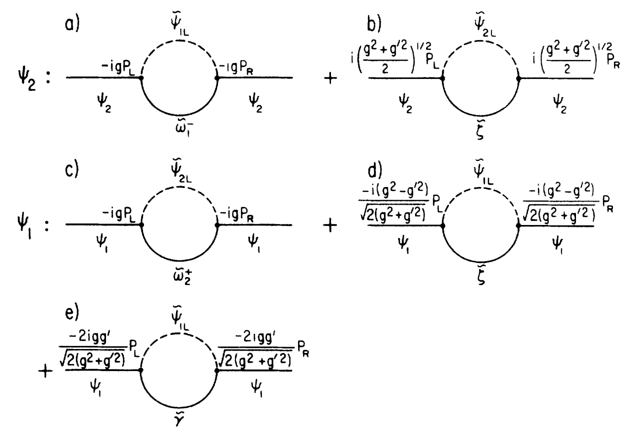

FIG. 2. A partial set of diagrams of the SUSY contributions to the fermion (ψ_1 and ψ_2) masses in the exact SUSY model.

$$\Delta m_2=\Delta m_3=\frac{mg^2g'^2}{16\pi^2(g^2+g'^2)}\int_0^1 dx\, x \ln\left[1+\frac{1-x}{x^2}\frac{m_Z^2}{m^2}\right]. \tag{3.3}$$

Thus the supersymmetric contribution is positive and, indeed, goes in the desired direction. However, the total Δm is still negative, given by

$$\Delta m=\Delta m_1+\Delta m_2+\Delta m_3$$
$$=\frac{-mg^2g'^2}{8\pi^2(g^2+g'^2)}\int_0^1 dx \ln\left[1+\frac{1-x}{x^2}\frac{m_Z^2}{m^2}\right]. \tag{3.4}$$

The fact that the supersymmetry part gives a positive contribution to Δm is encouraging. It is now interesting to consider what happens if this symmetry is broken which is expected to occur at low energies. In the next section, we will discuss the consequences due to soft-supersymmetry breaking.

IV. SOFT-SUPERSYMMETRY BREAKING

In the following, we consider soft-supersymmetry breaking in the Lagrangian given by Eq. (3.1) (Ref. 10).

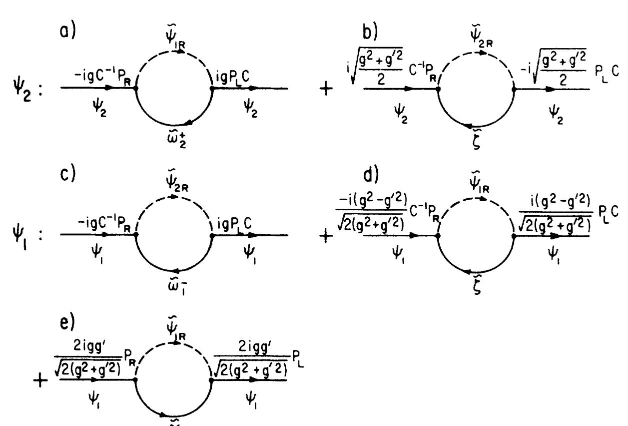

FIG. 3. A partial set of diagrams of the SUSY contributions to the fermion (ψ_1 and ψ_2) masses in the exact SUSY model.

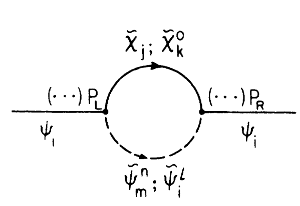

FIG. 4. A partial set of diagrams of the contributions to the fermion (ψ_1 and ψ_2) masses in the SU(2)×U(1) model with soft-SUSY breaking, where $i,j,l,m,n=1,2$; $i\neq m$; $\tilde{\chi}_k^0=\tilde{\chi}_2^0,\tilde{\chi}_3^0,\tilde{\gamma}$ as $i=1$; $\tilde{\chi}_k^0=\tilde{\chi}_2^0,\tilde{\chi}_3^0$ as $i=2$.

We will omit the SU(2)×U(1)-singlet fields (N,Ψ_N) to consider the minimal number of Higgs multiplets.

With soft-supersymmetry breaking, the vacuum expectation values of H_1 and H_2 will generally be replaced by

$$\langle H_1\rangle=\frac{v_1}{\sqrt{2}}\begin{bmatrix}1\\0\end{bmatrix},\quad \langle H_2\rangle=\frac{v_2}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix},\quad v_1\neq 2\,, \tag{4.1}$$

and the soft-SUSY-breaking terms will bring in mixing of scalar fermions, and of charged gauginos and Higgsinos, as well as of neutral gauginos and Higgsinos.[10]

(a) Mixing of the scalar fermions.

In exact supersymmetry, $\tilde{\Psi}_{iL}$ and $\tilde{\Psi}_{iR}$ are degenerate partners of Ψ_i. With soft-supersymmetry breaking, the following mass eigenstates can be written as

$$\tilde{\Psi}_i^1=\tilde{\Psi}_{iL}\cos\theta_i+\tilde{\Psi}_{iR}\sin\theta_i\,,$$
$$\tilde{\psi}_i^2=-\tilde{\psi}_{iL}\sin\theta_i+\tilde{\psi}_{iR}\cos\theta_i,\quad i=1,2\,, \tag{4.2}$$

with

$$m^2_{\tilde{\psi}_i^{1,2}}=m^2+\tfrac{1}{2}\{(L_i^2+R_i^2)\tilde{m}_i^2$$
$$\pm[(L_i^2-R_i^2)\tilde{m}_i^4+4A_i^2m^2\tilde{m}_i^2]^{1/2}\}\,, \tag{4.3}$$

where θ_i is determined by[10]

$$\tan 2\theta_i=\frac{2A_i m}{(L_i^2-R_i^2)\tilde{m}_i},\quad i=1,2\,, \tag{4.4}$$

and $L_i, R_i, A_i, \tilde{m}_i$ are free parameters with $m\neq 0$.

(b) Mixing of charged gauginos and Higgsinos.

With soft-supersymmetry breaking, it can be seen that

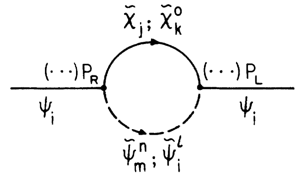

FIG. 5. A partial set of diagrams of the contributions to the fermion (ψ_1 and ψ_2) masses in the SU(2)×U(1) model with soft-SUSY breaking, where $i,j,l,m,n=1,2$; $i\neq m$; $\tilde{\chi}_k^0=\tilde{\chi}_2^0,\tilde{\chi}_3^0,\tilde{\gamma}$ as $i=1$; $\tilde{\chi}_k^0=\tilde{\chi}_2^0,\tilde{\chi}_3^0$ as $i=2$.

the possible mass terms in two-component notation are

$$\frac{ig}{\sqrt{2}}(v_1\lambda^+\psi_{H_1}^2+v_2\lambda^-\psi_{H_2}^1)+M\lambda^+\lambda^- -\mu\psi_{H_1}^2\psi_{H_2}^1+\text{H.c.} \tag{4.5}$$

Then, the mass eigenstates are given by

$$\tilde{\chi}_i=\begin{bmatrix}\chi_i^+\\ \bar{\chi}_i^-\end{bmatrix}, \tag{4.6}$$

with masses

$$M_\pm^2=\tfrac{1}{2}\{M^2+\mu^2+2m_W^2 \pm[(M^2-\mu^2)^2+4m_W^2\cos^2 2\theta_v +4m_W^2(M^2+\mu^2+2M\mu\sin 2\theta_v)]^{1/2}\}, \tag{4.7}$$

where

$$\chi_i^+=V_{i1}(-i\lambda^+)+V_{i2}\psi_{H_2}^1,$$

$$\chi_i^-=U_{i1}(-i\lambda^-)+U_{i2}\psi_{H_1}^2,$$

$$U=O_-,\quad V=O_+,\quad \det X\geq 0$$

$$V=\sigma_3 O_+,\det X<0 \text{ are unitary matrices},$$

where

$$O_\pm=\begin{bmatrix}\cos\phi_\pm & \sin\phi_\pm\\ -\sin\phi_\pm & \cos\phi_\pm\end{bmatrix},$$

$$X=\begin{bmatrix}M & m_W\sqrt{2}\cos\theta_v\\ m_W\sqrt{2}\cos\theta_v & \mu\end{bmatrix},$$

$$\tan 2\phi_-=\frac{2\sqrt{2}m_W(\mu\cos\theta_v+M\sin\theta_v)}{M^2-\mu^2+2m_W^2\cos 2\theta_W}, \tag{4.8}$$

$$\tan 2\phi_+=\frac{2\sqrt{2}m_W(\mu\sin\theta_v+M\cos\theta_v)}{M^2-\mu^2-2m_W^2\cos 2\theta_W},$$

$$\tan\theta_v\equiv\frac{v_1}{v_2}.$$

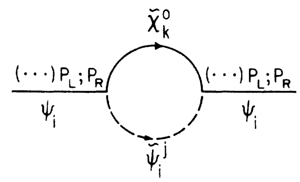

FIG. 6. A partial set of diagrams of the contributions to the fermion (ψ_1 and ψ_2) masses in the SU(2)×U(1) model with soft-SUSY breaking, where $i,j=1,2$; $\tilde{\chi}_k^0=\tilde{\chi}_2^0,\tilde{\chi}_3^0,\tilde{\gamma}$ as $i=1$; $\tilde{\chi}_k^0=\tilde{\chi}_2^0,\tilde{\chi}_3^0$ as $i=2$.

(c) Mixing of neutral gauginos and Higgsinos.

With soft-SUSY breaking, the possible mass terms are given by (in two-component notation)

$$\frac{i}{2}\sqrt{g^2+g'^2}\lambda_Z(v_1\psi_{H_1}^1-v_2\psi_{H_2}^2)+\tfrac{1}{2}(M\cos^2\theta_W+M'\sin^2\theta_W)\lambda_Z\lambda_Z +(M-M')\sin\theta_W\cos\theta_W\lambda_Z\lambda_\gamma+\tfrac{1}{2}(M'\cos^2\theta_W+M\sin^2\theta_W)\lambda_\gamma\lambda_\gamma+\mu\psi_{H_1}^1\psi_{H_2}^2+\text{H.c.}, \tag{4.9}$$

where M and μ are the same as the ones in Eq. (4.5). M, μ, and M' are independent free parameters in our case.[10]

We let $M=M'$. Then, the mass eigenstates with their corresponding masses are given by

$$\chi_1^0=-i\lambda_\gamma,\quad m_1=M,$$

$$\chi_2^0=(-i\lambda_Z)\cos\phi+\frac{v_1\psi_{H_1}^1-v_2\psi_{H_2}^2}{(v_1^2+v_2^2)^{1/2}}\sin\phi,$$

$$m_2=\left[\frac{1}{4}\left[\frac{\mu'}{2}-M\right]^2+\left[m_Z^2+\frac{M\mu'}{2}\right]\right]^{1/2}-\frac{1}{2}\left[\frac{\mu'}{2}-M\right],$$

$$-i\chi_3^0=\lambda_Z\sin\phi+\frac{v_1\psi_{H_1}^1-v_2\psi_{H_2}^2}{(v_1^2+v_2^2)^{1/2}}(-i\cos\phi), \tag{4.10}$$

$$m_3=\left[\frac{1}{4}\left[\frac{\mu'}{2}-M\right]^2+\left[m_Z^2+\frac{M\mu'}{2}\right]\right]^{1/2}+\frac{1}{2}\left[\frac{\mu'}{2}-M\right],$$

$$\chi_4^0=\frac{v_1\psi_{H_1}^1+v_2\psi_{H_2}^2}{(v_1^2+v_2^2)^{1/2}},\quad m_4=\frac{\mu'}{2},$$

where

$$\mu'=\frac{v_1^2+v_2^2}{v_1v_2}\mu,\quad \tan 2\phi=\frac{4m_Z}{2M+\mu'},$$

and we have inserted $(-i)$ in χ_3^0, so that m_3 can always be positive.

V. THE MASS DIFFERENCE, Δm, WITH SOFT-SUSY BREAKING

Substituting Eqs. (4.2), (4.6), (4.8), and (4.10) into Eq. (3.1), we obtain the interaction Lagrangian (see the Appendix) contributing to the mass differences. From this Lagrangian, we note that there are three types of one-loop Feynman diagrams, in the supersymmetric sectors, contributing to Δm. These are shown in Figs. 4–6. For the first two types, Figs. 4 and 5, the integral J appears in the form

$$P_L J P_R \quad \text{or} \quad P_R J P_L \tag{5.1}$$

in the calculation of Δm. These types of relations were already involved in Sec. III. For Fig. 6, however, the integral J appears in the form

$$P_L J P_L \quad \text{or} \quad P_R J P_R \ . \tag{5.2}$$

As we shall see, due to the free parameters in the soft-SUSY breaking terms, it will play an important role in obtaining positive mass differences.

In the calculation of Δm, the following type of integral occurs:

$$F(m,\tilde{m};M_1,M_2) = \int_0^1 dt \ln \left[\frac{t\tilde{m}^2+(1-t)M_1^2-t(1-t)m^2}{t\tilde{m}^2+(1-t)M_2^2-t(1-t)m^2} \right] , \tag{5.3}$$

where m is the mass of the (degenerate) fermion isodoublet, $\tilde{m}$ is a gaugino mass and M_1, M_2 are scalar-fermion masses.

For small fermion masses (i.e., $m \to 0$), the above integration is particularly simple:

$$F(0,\tilde{m};M_1,M_2) = \left[\frac{M_1^2 \ln \left[\frac{M_1^2}{\tilde{m}^2} \right]}{M_1^2-\tilde{m}^2} - \frac{M_2^2 \ln \left[\frac{M_2^2}{\tilde{m}^2} \right]}{M_2^2-\tilde{m}^2} \right] . \tag{5.4}$$

If we make the further approximation that

$$\tilde{m} < M_1, M_2 , \tag{5.5}$$

which is quite consistent with the current phenomenological limits, one obtains, by expanding (5.4),

$$F(0,\tilde{m};M_1,M_2) = \left\{ \ln \left[\frac{M_1^2}{M_2^2} \right] + \tilde{m}^2 \left[\frac{1}{M_1^2} \ln \left[\frac{M_1^2}{\tilde{m}^2} \right] - \frac{1}{M_2^2} \ln \left[\frac{M_2^2}{\tilde{m}^2} \right] \right] \right\} . \tag{5.6}$$

We now consider the following two interesting cases for the choice of the parameters in the soft-SUSY-breaking terms.

First of all, we take M_{ij} to be the masses of the scalar fermions $\tilde{\psi}_i^j$ and choose

$$v_1 = v_2 (\Longrightarrow |V_{ij}| = |U_{ij}|) \ .$$

Case A. We take

$$\begin{aligned} &\theta_1=\theta_2=\theta \ , \\ &\mu'=2M[\Longrightarrow m_2=m_3=(m_Z^2+M^2)^{1/2}] \ , \\ &(1-t)m^2_{\text{gauginos}}+tm^2_{\text{s-fermions}}-t(1-t)m^2>0 \ , \\ &\qquad 0<t<1 \ . \end{aligned} \tag{5.7}$$

Then, the SUSY contribution to Δm from Fig. 6 will be

$$\begin{aligned} \Delta m_6 = \frac{M \sin 2\theta}{64\pi^2 (g^2+g'^2)} [&-(g^2+g'^2)^2 F(m,m_2;M_{21},M_{22}) \\ &+(g^2-g'^2)^2 F(m,m_3;M_{11},M_{12}) \\ &+4g^2 g'^2 F(m,M;M_{11},M_{12})] \ . \end{aligned} \tag{5.8}$$

One can easily verify that

$$\Delta m_6 > \frac{(\kappa-1)M(g^2+g'^2)}{64\pi^2} \sin 2\theta_1 F(m,m_2;M_{21},M_{22}) > 0 \tag{5.9}$$

provided

$$\frac{M_{11}}{M_{12}} > \frac{M_{21}}{M_{22}} , \tag{5.10}$$

where the quantity κ is defined by[11]

$$F(m,m_2;M_{11},M_{12}) = \kappa F(m,m_2;M_{21},M_{22}) \ . \tag{5.11}$$

It can be shown that $\kappa > 1$. Therefore, from (5.9), we have $\Delta m_6 \sim M$.

It can easily be verified (see the Appendix) that

$$|P_L J P_R| = |P_R J P_L| \sim m$$

and that

$$\Delta m_1 + \Delta m_4 + \Delta m_5 \sim m \ .$$

Hence, we have

$$\Delta m = \Delta m_1 + \Delta m_4 + \Delta m_5 + \Delta m_6 > 0$$

for M sufficiently large.

Indeed, under the limits

$$\begin{aligned} &m \ll M \ , \\ &m \ll M_{\text{s-fermion}} (m \neq 0) \ , \end{aligned} \tag{5.12}$$

and (5.5), one can write, using (5.6),

$$\Delta m = \frac{g^2+g'^2}{64\pi^2} M \sin 2\theta \ln \left[\frac{M_{11}}{M_{12}} \frac{M_{22}}{M_{21}} \right]^2 \tag{5.13}$$

which is positive if (5.10) is satisfied, where we have used the fact that the factors from the integrals do not affect the results in any substantial way as the SUSY particle masses appear inside the argument of the logarithm and are distributed in both numerators and denominators.

Case B. Here we take the scalar fermion masses M_{ij} to be

$$\begin{aligned} &M_{11}=M_{21}=M_1, \quad M_{12}=M_{22}=M_2 \ , \\ &\theta_1=\theta_2=\theta, \quad \mu'=0 \ . \end{aligned} \tag{5.14}$$

From (4.10), one then obtains

$$\begin{aligned} &m_2=(m_Z^2+\tfrac{1}{4}M^2)^{1/2}+\tfrac{1}{2}M \ , \\ &m_3=(m_Z^2+\tfrac{1}{4}M^2)^{1/2}-\tfrac{1}{2}M, \quad m_1=m_\gamma=M \ . \end{aligned} \tag{5.15}$$

The expression for Δm, which, as we stated earlier, comes entirely from Fig. 6, is

$$\Delta m_6 = \frac{\alpha \sin 2\theta}{\pi}[m_2 F(m,m_2;M_1,M_2)\cos^2\phi - m_3 F(m,m_3;M_1,M_2)\sin^2\phi - MF(m,M;M_1,M_2)] . \quad (5.16)$$

This can be further simplified using (5.6), under the limit (5.5) and small fermion masses, to

$$\Delta m = \frac{\alpha \sin 2\theta}{\pi} \frac{M\tilde{M}_Z^2}{\overline{M}^2} \ln \left| \frac{M_1^2}{M_2^2} \right| , \quad (5.17)$$

where we have expressed

$$m_2 + m_3 = 2\tilde{M}_Z \quad \text{and} \quad M_1 + M_2 = 2\overline{M} ,$$

and assumed that $\overline{M} > (M_1 - M_2)$.

We note that

$$\Delta m > 0$$

provided

$$M_1 > M_2 . \quad (5.18)$$

Finally, we observe that, due to the small mass difference between e^- and ν, the same approach as above can be used to calculate the e^--ν mass difference as long as the zeroth-order masses do not vanish. For the case when the zeroth-order fermion masses vanish, one can easily check that Δm will vanish identically.

VI. CONCLUSIONS

We have found that in order to obtain a positive Δm it is extremely crucial, first of all, to incorporate supersymmetry and, second, to have soft-supersymmetry breaking so that mixing occurs for the gauginos and for the scalar fermions.

This breaking must be complete as is evident from the formulas (5.13) and (5.17) for the cases considered. In other words, the sign of Δm is remarkably sensitive to the fact that the scalar fermions mix (i.e., $\theta \neq 0$) with stringent conditions on the mass ratios [see (5.10) and (5.18)] and, of course, that photino acquires large enough mass. In the absence of any one of these conditions, Δm reverts to the negative sign.

The cases we have considered for the supersymmetry parameters cover values that have certain restrictions but are reasonable according to current estimates. The future discovery of these particles will naturally shed further light on this subject.

In terms of the magnitude of Δm, the experimental value is about 3–4 MeV for the (n-p) doublet and for the (u-d) doublet. If the masses of the supersymmetric particles are ~ 100 GeV, and the values of the logarithms [see (5.13) and (5.17)] involving scalar-fermion mass ratios are in the range 10^{-1} to 1 for small mixing angle $\theta \sim 10^{-1}$, then Δm of the above order is quite possible.

In the absence of supersymmetry, on the other hand,

$$\Delta m \sim -\alpha m ,$$

so that for $m \sim 3$–4 MeV, not only is Δm of the wrong sign but almost a hundred times smaller than the expected value.

To obtain the correct mass difference it is not essential to involve the full machinery of supersymmetry. In our model, we notice by examining case A considered earlier, for example, that the essential items are that the photino gets large enough mass, as well as the mixing of the scalar fermions such that (5.10) can be satisfied.

Our model should be applicable for those cases which involve typically small mass differences with electromagnetism as their principal underlying cause. The large mass differences within the doublets $(c,s),(t,b)$ (neglecting the mixings among these quarks), etc., however, have, very likely, a totally different origin. Furthermore, as stated in the Introduction, our SU(2)×U(1) isodoublet model is not consistent with the standard electroweak model. It is rather a toy model simplified to include only the vectors. It may, however, suggest a possible scenario if one were to follow the historic path of attributing the n-p mass difference to electromagnetism. If, indeed, one succeeds in this task then the neutron-proton mass difference will no longer remain a puzzle.[12]

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APPENDIX

Substituting (4.2), (4.8), and (4.10) into the interaction Lagrangian (3.1), we have

$$\mathcal{L}_{\text{int}} = \frac{-1}{2\sqrt{2}} g V_\mu^- \bar{\psi}_1 \gamma^\mu \psi_2 + \text{H.c.} - \frac{gg'}{(g^2+g'^2)^{1/2}} A_\mu \bar{\psi}_1 \gamma^\mu \psi_1 - \frac{g^2 - g'^2}{2(g^2+g'^2)^{1/2}} Z_\mu \bar{\psi}_1 \gamma^\mu \psi_1 + \tfrac{1}{2}(g^2+g'^2)^{1/2} Z_\mu \bar{\psi}_2 \gamma^\mu \psi_2$$

$$- g \left[\frac{1}{\det V}(V_{22}\overline{\tilde{\chi}}{}_1^c - V_{12}\overline{\tilde{\chi}}{}_2^c) P_L \psi_2 (\tilde{\psi}_1^{1*}\cos\theta_1 - \tilde{\psi}_1^{2*}\sin\theta_1) + \frac{1}{(\det V)^*}(V_{22}^* \bar{\psi}_2 P_R \tilde{\chi}_1^c - V_{12}^* \bar{\psi}_2 P_R \tilde{\chi}_2^c)(\tilde{\psi}_1^1 \cos\theta_1 - \tilde{\psi}_1^2 \sin\theta_1) \right.$$

$$+ \frac{1}{\det U}(U_{22}\bar{\tilde{\chi}}_1 - U_{12}\bar{\tilde{\chi}}_2) P_L \psi_1 (\tilde{\psi}_2^{1*}\cos\theta_2 - \tilde{\psi}_2^{2*}\sin\theta_2)$$

$$
\begin{aligned}
&\qquad \left. + \frac{1}{(\det U)^*} (U^*_{22} \bar{\psi}_1 P_R \tilde{\chi}_1 - U^*_{12} \bar{\psi}_1 P_R \tilde{\chi}_2)(\tilde{\psi}^1_2 \cos\theta_2 - \tilde{\psi}^2_2 \sin\theta_2) \right] \\
&+ \left[\frac{g^2+g'^2}{2} \right]^{1/2} (\bar{\bar{\chi}}{}^0_2 \cos\phi - i \bar{\bar{\chi}}{}^0_3 \sin\phi) P_L \psi_2 (\tilde{\psi}^{1*}_2 \cos\theta_2 - \tilde{\psi}^{2*}_2 \sin\theta_2) + \text{H.c.} \\
&- \frac{g^2-g'^2}{\sqrt{2(g^2+g'^2)}} (\bar{\bar{\chi}}{}^0_2 \cos\phi - i \bar{\bar{\chi}}{}^0_3 \sin\phi) P_L \psi_1 (\tilde{\psi}^{1*}_1 \cos\theta_1 - \tilde{\psi}^{2*}_1 \sin\theta_1) + \text{H.c.} \\
&- \frac{2gg'}{\sqrt{2(g^2+g'^2)}} \bar{\tilde{\gamma}} P_L \psi_1 (\tilde{\psi}^{1*}_1 \cos\theta_1 - \tilde{\psi}^{2*}_1 \sin\theta_1) + \text{H.c.} \\
&+ g \left[\frac{1}{\det V} \bar{\psi}_1 P_L (V_{22} \tilde{\chi}_1 - V_{12} \tilde{\chi}_2)(\tilde{\psi}^1_2 \sin\theta_2 + \tilde{\psi}^2_2 \cos\theta_2) + \text{H.c.} + \frac{1}{\det U} \bar{\psi}_2 P_L (U_{22} \tilde{\chi}^c_1 - U_{12} \tilde{\chi}^c_2)(\tilde{\psi}^1_1 \sin\theta_1 + \tilde{\psi}^2_1 \cos\theta_1) + \text{H.c.} \right] \\
&- \left[\frac{g^2+g'^2}{2} \right]^{1/2} \bar{\psi}_2 P_L (\tilde{\chi}^0_2 \cos\phi - i \tilde{\chi}^0_3 \sin\phi)(\tilde{\psi}^1_2 \sin\theta_2 + \tilde{\psi}^2_2 \cos\theta_2) + \text{H.c.} \\
&+ \frac{g^2-g'^2}{\sqrt{2(g^2+g'^2)}} \bar{\psi}_1 P_L (\tilde{\chi}^0_2 \cos\phi - i \bar{\bar{\chi}}{}^0_3 \sin\phi)(\tilde{\psi}^1_1 \sin\theta_1 + \tilde{\psi}^2_1 \cos\theta_1) + \text{H.c.} + \frac{2gg'}{\sqrt{2(g^2+g'^2)}} \bar{\psi}_1 P_L \tilde{\gamma} (\tilde{\psi}^1_1 \sin\theta_1 + \tilde{\psi}^2_1 \cos\theta_1) + \text{H.c.}
\end{aligned}
\tag{A1}
$$

If we choose the parameters as (5.7), then, the SUSY contribution to $\Delta m|_{2-1}$ from Figs. 1, 4, and 5 will be

$$
\begin{aligned}
\Delta m_1|_{2-1} + \Delta m_5|_{2-1} + \Delta m_6|_{2-1} = {} & \frac{-mg^2g'^2}{8\pi^2(g^2+g'^2)} \int_0^1 dt (1+t) \ln \left[1 + \frac{1-t}{t^2} \frac{m_Z^2}{m^2} \right] \\
& + \frac{m(g^2-g'^2)^2}{64\pi^2(g^2+g'^2)} \int_0^1 dt\, t \ln \left| \frac{(1-t)m_2^2 + tM_{21}^2 - t(1-t)m^2}{(1-t)m_2^2 + tM_{11}^2 - t(1-t)m^2} \right| \\
& + \frac{m(g^2-g'^2)^2}{64\pi^2(g^2+g'^2)} \int_0^1 dt\, t \ln \left| \frac{(1-t)m_2^2 + tM_{22}^2 - t(1-t)m^2}{(1-t)m_2^2 + tM_{12}^2 - t(1-t)m^2} \right| \\
& + \frac{4mg^2g'^2}{64\pi^2(g^2+g'^2)} \int_0^1 dt\, t \ln \left| \frac{(1-t)m_2^2 + tM_{21}^2 - t(1-t)m^2}{(1-t)m_\gamma^2 + tM_{11}^2 - t(1-t)m^2} \right| \\
& + \frac{4mg^2g'^2}{64\pi^2(g^2+g'^2)} \int_0^1 dt\, t \ln \left| \frac{(1-t)m_2^2 + tM_{22}^2 - t(1-t)m^2}{(1-t)m_\gamma^2 + tM_{12}^2 - t(1-t)m^2} \right| ,
\end{aligned}
\tag{A2}
$$

where $m_\gamma = M$ and m_2, m_3 are given by (4.10).

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[9]For a theory which can describe weak interactions, gauge invariance does not permit an $f\epsilon_{ij}\tilde{F}_{iL}\tilde{F}_j$ term. In the gauge group we are using, if we use the Higgs fields as shown in Table I, this term, however, is allowed. Then in the scalar potential V, we will have the term $|h\epsilon_{ij}H_{1i}H_{2j} + f\epsilon_{ij}\tilde{F}_{iL}\tilde{F}_{jR} + s|^2$. For our purposes, we can choose the vacuum expectation values as

$$\langle H_1 \rangle = \frac{v}{\sqrt{2}} \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \langle H_2 \rangle = \frac{v}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$\langle \tilde{F}_L \rangle = \langle \tilde{F}_R \rangle = 0 .$$

Then, the scalar potential, however, will not change significantly. The $f\epsilon_{ij}\tilde{F}_{iL}\tilde{F}_{jR}N$ term can contribute a Yukawa coupling term $N\bar{\psi}\psi$. But this term does not contribute to the mass differences within isotopic doublets at least in second order and does not produce fermion masses after spontaneously gauge symmetry breaking because $\langle N \rangle = 0$. Finally, we do

not use the scalar field N in discussing soft-SUSY breaking to obtain mass differences. Therefore, we neglect the term $f\epsilon_{ij}\tilde{F}_{iL}\tilde{F}_{jR}$ as it is irrelevant for our purposes.

[10]To consider soft-SUSY breaking we follow the paper by Haber and Kane (Ref. 8). We note that the soft-SUSY breaking terms are the same as the ones in SUSY standard model. This is not surprising because the two models break down to the same symmetry group after spontaneously gauge symmetry breaking.

[11]We have made use of

$$\frac{a+c}{b+c} > \frac{a+d}{b+d} ,$$

if $a > b, d > c > 0, b+d > b+c > 0$ and (4.3), (4.10), and (5.3).

[12]A possible larger group in which the standard model can be embedded and which may be capable of reproducing some of our results is being currently investigated by us.