Electromagnetic $N \rightarrow N^*(1535)$ transition in the relativistic constituent-quark model

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We study a light-cone model of the nucleon and the $S_{11}(1535)$ resonance which provides a relativistic generalization of the constituent-quark model in the nonperturbative low-Q^2 regime. The two parameters of the model, namely, the size parameter α and the constituent-quark mass $m_q \approx 300$ MeV, are fixed by the axial form factor. We find good agreement for the electromagnetic form factors of the nucleon up to $Q^2 \approx 1$–$2\ \mathrm{GeV}^2$. All static properties of the nucleon, except for the neutron charge radius, are reproduced within 10%. In addition, we show that the electromagnetic current of the nucleon is conserved. The $S_{11}(1535)$ transition amplitudes, where gauge invariance is achieved by including a three-body current, are calculated from a constraint-free form-factor set. They are in reasonable agreement with the available photo- and electroproduction data.

I. INTRODUCTION

Quantum chromodynamics (QCD) is now generally considered to be the gauge field theory of the strong interaction, although it remains unsolved at low momentum. Within its framework there is growing evidence[1] for the "constituent" light quark carrying an effective mass $m_u = m_d = m_q \approx m_N/3$, when the gluon fields are integrated out and meson-exchange dynamics emerges.[2] Skyrme models approximate the latter by nonlinear pion dynamics and are thought to be a long distance and large color $N_c \rightarrow \infty$ approximation of QCD. These aspects are practically ignored (except perhaps for m_q) in the successful nonrelativistic quark model (NQM). Since the mid 1970s it has been improved by ingredients from QCD, such as the color-magnetic part of an effective gluon exchange now known as the color hyperfine interaction.[3]

The NQM and its relativistic, chiral, or other generalizations are based on the dominance of valence quarks in the Fock-state expansion of hadrons. Relativistic effects are often large, e.g., the axial-vector coupling constant $g_A = \frac{5}{3}$ in the NQM changes to $\frac{5}{4}$ upon including them, and longitudinal-momentum distributions are broad and asymmetrical.[4] The p/m expansion of Foldy *et al.*[5] for light quarks allows including relativistic effects of low orders more consistently than in bag models.[6] However, the average quark momentum $\langle \mathbf{p}^2 \rangle^{1/2} \simeq m_q$ in the NQM. Hence the Gaussian wave functions of the underlying NQM also play the role of convergence factors so that relativistic effects become cutoff dependent.

The light-cone formalism provides a consistent relativistic theory for composite systems with a fixed number of constituents. Dirac's[7] front form has been implemented in light-cone time-ordered ($\tau = t + z$) perturbation theory in the asymptotic freedom phase of QCD.[8,9] This method converges even when relativistic wave functions are used that have only a power-law falloff at high momentum. On the light cone it is consistent to take particles on their mass shell, similar to the Schrödinger equation, but off-P^- shell in general. This feature allows using light-cone spinors for quarks in multiquark hadron wave functions rather than propagators in instant form. The linear dependence of $p^2 = p^+p^- - \mathbf{p}_\perp^2$ on p^- is crucial to satisfy the additive cluster decomposition property, a consistency requirement of relativistic many-body theory. A decisive advantage of the front form is the separation of the total momentum from the internal motion, which is connected to the transitivity of the seven kinematic (interaction-free) generators in momentum space.[10]

The construction of three-quark nucleon and resonance wave functions is guided by their nonrelativistic form in the NQM. The Melosh transformation to the light cone generates relativistic spin wave functions with approximately correct $\mathbf{J}^2$, which is difficult to achieve in general on the light cone because of the interaction dependence of $\mathbf{J}_\perp$.

Using the relativistic generalization of the harmonic $(1s)^3$ wave function allows comparing directly with the NQM to estimate relativistic effects. For the magnetic form factor of the nucleon such a calculation was done in Ref. 11. We show here in addition that, despite the phenomenological nucleon wave function, the nucleon current matrix elements satisfy gauge invariance. For the electromagnetic transition amplitudes to the nucleon resonance S_{11} with $J^\pi = \frac{1}{2}^-$, there are additional constraints from gauge invariance, which lead to the introduction of a three-body current. The slow q^2 falloff of the S_{11} helicity amplitude $A_{1/2}$ is of particular interest since it is hard to reconcile with the NQM.

Our main motivation has been to provide electromagnetic form factors from a relativistic quark model that interpolates between the static properties of bound three-quark states and their longitudinal distribution functions.[12] In the intermediate momentum region the data on electromagnetic N-N^* transitions is scarce and predictions are particularly useful for planning experiments at the Continuous Electron Beam Accelerator Facility (CEBAF), at the ELSA accelerator in Bonn, and at the MAMI accelerator in Mainz.

This paper is organized as follows. In Sec. II we give a brief review of the light-cone formalism with special emphasis on the three-quark kinematics. The construction of our $S_{11}(1535)$ and nucleon wave function can be found in Sec. III. The general form of transition current matrix elements and their evaluation on the light cone is discussed in Sec. IV. Finally, in Sec. V we consider current conservation and present our numerical results for the nucleon form factors and the $S_{11}(1535)$ transition amplitudes.

II. LIGHT-CONE FORMALISM

The light-cone formalism is known to be suitable to describe the relativistic many-body problem, since it decouples the total momentum (c.m.) motion from the internal motion. For a general review of this formalism the reader may be referred to Ref. 9. Here we just want to recall some basic facts. In light-cone dynamics one describes a system by its evolution in the "time" $\tau=x^0+x^3$ and by "coordinates" $(x^-,\mathbf{x}_\perp)=(x^0-x^3,x^1,x^2)$. For the ten generators of the Poincaré group there is, as Dirac[7] has shown, a maximum of seven generators free of interaction in the light-cone system. This subalgebra also includes the three boost operators, which take a particularly simple form, in contrast with the conventional instant form, where the boost operators are in general interaction dependent and not known for a composite system. On the other hand, rotational invariance is difficult to implement on the light cone for the interaction-dependent generators J^1 and J^2 (rotation around x and y axis). However, there is invariance under z rotations, where the z axis is an appropriate quantization axis.

In the light-cone approach all particles are on their mass shell; that is, a particle with mass m has the four-momentum

$$p^\mu=\left[p^+=p^0+p^3,p^-=\frac{m^2+\mathbf{p}_\perp^2}{p^+},\mathbf{p}_\perp\right] .$$

[Note that the scalar product in light-cone variables takes the form $a_\mu b^\mu=\frac{1}{2}(a^+b^-+a^-b^+)-\mathbf{a}_\perp\mathbf{b}_\perp$.]

A spin-$\frac{1}{2}$ fermion is described by a Dirac spinor with helicity $\pm\frac{1}{2}$:

$$u_{\uparrow,\downarrow}(p)=\frac{1}{\sqrt{2mp^+}}(p^++\beta m+\boldsymbol{\alpha}_\perp\mathbf{p}_\perp)\chi_{\uparrow,\downarrow} ,$$
$$\chi_\uparrow=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\1\\0\end{bmatrix}, \quad \chi_\downarrow=\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\0\\-1\end{bmatrix} \tag{1}$$

(with the metric convention of Ref. 13).

These spinors are solutions of the on-shell Dirac equation $(\not{p}-m)u_\lambda(p)=0$ and the $\uparrow$ and $\downarrow$ solutions refer to helicity eigenfunctions as measured from a system moving with $v\to c$ in $-z$ direction. Note that the light-cone spinors are identical to the conventional Dirac spinors for $\mathbf{p}=0$. However, they differ for $\mathbf{p}\neq 0$ since the light-cone boost generators differ from the conventional ones.

As suggested by deep-inelastic scattering, the n-body configuration (with $n=3$, e.g.) is conveniently described in terms of the longitudinal-momentum fractions (Bjorken-Feynman variables)

$$x_j=\frac{p_j^+}{P^+},\quad \sum_{j=1}^{n}x_j=1,\quad 0\le x_j\le 1 , \tag{2}$$

where $P^+=\sum_{j=1}^{n}p_j^+$ reflects the conservation of the total momentum P, and in terms of the relative momentum variables

$$q_3=\frac{x_2p_1-x_1p_2}{x_1+x_2} ,$$
$$Q_3=(x_1+x_2)p_3-x_3(p_1+p_2)=p_3-x_3P . \tag{3}$$

The crucial property of q_3 and Q_3 is their vanishing "invariant"+component: $Q_3^+=0=q_3^+$. Hence they are spacelike four-vectors $q_3^2=-\mathbf{q}_{3\perp}^2$, $Q_3^2=-\mathbf{Q}_{3\perp}^2$. It can be shown[9] that this property leads to a simple linear relation of the invariant mass $s=(\sum_{j=1}^{3}p_j)^2$ in terms of the relative variables and that in turn this feature leads to the so-called cluster decomposition property which is violated in most formulations of relativistic many-body dynamics.

The six relative variables $x_1,x_2,\mathbf{q}_{3\perp},\mathbf{Q}_{3\perp}$ are translational invariant and invariant under the three light-cone boosts.[14] Thus they are convenient to decouple the internal dynamics from the c.m. motion: The free three-particle propagator G_{30} with

$$G_{30}^{-1}=P^2-s=P^+\left[P^--\sum_{j=1}^{3}p_j^-\right] \tag{4}$$

may be rearranged with the help of $\mathbf{P}_\perp=\sum_{j=1}^{3}\mathbf{p}_{j\perp}$ and $m^2=P^+P^--\mathbf{P}_\perp^2$ as

$$G_{30}^{-1}=m^2-\left[\sum_{j=1}^{3}\frac{m_j^2}{x_j}-q_3^2\frac{1-x_3}{x_1x_2}-Q_3^2\frac{1}{x_3(1-x_3)}\right]$$
$$=:m^2-M_3^2 . \tag{5}$$

Thus G_{30} is independent of the four-vector P_μ (except for the dependence on the Lorentz scalar $m^2=P_\mu P^\mu$).

Also the Weinberg equation of motion[15] written in terms of these variables

$$G_{30}^{-1}\psi_B(x,q_3,Q_3)=\int d\Gamma'\, V(x,q_3,Q_3;x',q_3',Q_3')\times\psi_B(x',q_3',Q_3') , \tag{6}$$

with the invariant phase-space volume element

$$d\Gamma'=\frac{1}{(16\pi^3)^2}d^2\mathbf{q}_{3\perp}'d^2\mathbf{Q}_{3\perp}'\delta\left[\sum_{i=1}^{3}x_i'-1\right]\prod_{i=1}^{3}\frac{dx_i'}{x_i'} \tag{7}$$

reflects the proper separation of internal motion and c.m. motion, since the internal baryon wave function $\psi_B(x,q_3,Q_3)$ does not change under boosts and translations.

III. WAVE-FUNCTION MODELS FOR THE NUCLEON AND THE S_{11} RESONANCE

For the nucleon a wave-function model exists[16,17] that represents the relativistic generalization on the light cone of the conventional three-quark spin-isospin wave function in conjunction with a Gaussian momentum wave function [see Eq. (18)]. This relativistic wave function is most convenient for estimating relativistic effects in a comparison with the nonrelativistic constituent-quark model (NQM).

The spin wave functions

$$|(\tfrac{1}{2}\tfrac{1}{2})S=0,\tfrac{1}{2};J=\tfrac{1}{2}m_J=\lambda\rangle = \frac{1}{\sqrt{2}}\sum_{\lambda_1,\lambda_2,\lambda_3}(\chi_1^\dagger i\sigma_2\chi_2^*)(\chi_3^\dagger\chi_\lambda)\chi_1\chi_2\chi_3 , \quad (8a)$$

$$|(\tfrac{1}{2}\tfrac{1}{2})S=1,\tfrac{1}{2};J=\tfrac{1}{2}m_J=\lambda\rangle = -\frac{1}{\sqrt{6}}\sum(\chi_1^\dagger\sigma i\sigma_2\chi_2^*)\cdot(\chi_3^\dagger\sigma\chi_\lambda)\chi_1\chi_2\chi_3 \quad (8b)$$

[with the spin S of quark pair (12)], and similar isospin wave functions lead to the nonstatic (i.e., momentum-dependent) Lorentz invariants

$$\begin{aligned} I_0 &= \bar{u}_1\gamma_5 G\bar{u}_2^T\bar{u}_3 u_N(P)+(23)1+(31)2 ,\\ I_1 &= \bar{u}_1\gamma^\mu\tau G\bar{u}_2^T\bar{u}_3\gamma_\mu\gamma_5\tau u_N(P)+(23)1+(31)2 , \end{aligned} \quad (9)$$

where $G=i\tau_2 C=G^T$ is the G parity with the charge-conjugation operator $C=i\gamma^2\gamma^0$ and $u_i=u(p_i)$, $i=1,2,3$ are the quark light-cone spinors. The summation over spin and isospin of the three quarks, as well as the totally antisymmetric color wave function, is understood but not written explicitly. With the conventional Dirac spinors the spin parts of I_0 and I_1 reduce in the nonrelativistic limit, where only the lowest order in p_j/m_q and P/m_N is retained, to Eqs. (8a) and (8b), respectively. However, we will use Eq. (9) with the light-cone spinors of Eq. (1).

Together with

$$I_2=\frac{1}{m_N}\bar{u}_1 \not{P}\gamma_5 G\bar{u}_2^T\bar{u}_3 u_N+(23)1+(31)2$$

these invariants form a complete basis for the nucleon ground state with totally symmetric momentum wave functions, since it can be shown[16] that all other invariants without explicit derivative terms depending on p_i (which correspond to P-, D-wave quarks in the nonrelativistic limit) are linearly dependent on I_0,I_1,I_2. More general invariants—beyond the totally symmetric momentum distribution—are discussed in Ref. 18.

In order to construct the spin invariants of the $N^*(S_{11})$ wave function we start with negative-parity invariants by substituting $u_N\to\gamma_5 u_N$ and adding an explicit p_j dependency. This yields

$$\begin{aligned} I_{*0} &= \bar{u}_1\gamma_5\tau G\bar{u}_2^T\bar{u}_3(\not{p}_1-\not{p}_2)\gamma_5\tau u_{N^*} +(23)1+(31)2 ,\\ I_{*1} &= \bar{u}_1\gamma^\mu\tau G\bar{u}_2^T\bar{u}_3(\not{p}_1-\not{p}_2)\gamma_\mu\tau u_{N^*} +(23)1+(31)2 ,\\ I_{*2} &= \frac{1}{m_*}\bar{u}_1\not{P}_*\gamma_5\tau G\bar{u}_2^T\bar{u}_3(\not{p}_1-\not{p}_2)\gamma_5\tau u_{N^*} +(23)1+(31)2 \end{aligned} \quad (10)$$

and leads to the following ansatz for the S_{11} wave function:

$$\psi_{N^*}(x,q_3,Q_3,\lambda)=N_{N^*}\sum_{n=0}^{2}\Phi_n(x,q_3,Q_3)I_{*n} \quad (11)$$

with totally symmetric momentum wave functions Φ_n.

In order to deal with the problem of angular momentum $\mathbf{J}^2$ in light-cone dynamics, where only J_z is well defined, we need some approximation to specify the N^* wave function completely. Therefore we set

$$\Phi_n(x,q_3,Q_3)=c_n\Phi_0(x,q_3,Q_3) , \quad (12)$$

where Φ_0 is a totally symmetric momentum distribution, and determine the constant coefficients c_n by comparison with the nonrelativistic $S_{11}(1535)$ wave function which is an eigenstate of total angular momentum $\frac{1}{2}$. This wave function may be written in momentum space as

$$\begin{aligned} |N_*,\lambda,m_T\rangle &= [|(\tfrac{1}{2}\tfrac{1}{2})S=0;\tfrac{1}{2}\rangle Y_1(\hat{\mathbf{p}}_\rho)]_\lambda^{1/2}\otimes|T;M_S\rangle+(23)1+(31)2\\ &= \frac{1}{\sqrt{8\pi}}\sum_{\lambda_1,\lambda_2,\lambda_3}(\chi_1^\dagger i\sigma_2\chi_2^*)(\chi_3^\dagger\boldsymbol{\sigma}\cdot\hat{\mathbf{p}}_\rho\chi_\lambda)\chi_1^I\chi_2^I\chi_3^I\otimes|T;M_S\rangle+(23)1+(31)2 , \end{aligned} \quad (13)$$

where $\mathbf{p}_\rho=(1/\sqrt{2})(\mathbf{p}_1-\mathbf{p}_2)$ is the relative momentum between quark 1 and 2 and $|T;M_s\rangle$ denotes the mixed symmetric isospin wave function

$$|(\tfrac{1}{2}\tfrac{1}{2})T=1;\tfrac{1}{2}m_T\rangle=-\frac{1}{\sqrt{6}}\sum_{t_1,t_2,t_3}(\Phi_1^\dagger\tau i\tau_2\Phi_2^*)(\Phi_3^\dagger\tau\Phi_{m_T})\Phi_1\Phi_2\Phi_3 , \quad (14)$$

which will be evaluated in the uds basis.[19] The superscript I in χ_i^I refers to the fact that the Pauli spinors are in the instant frame.

We now write the spin invariants I_{*i} of Eq. (10) explicit in the N^* rest frame where $P_*^\mu=(m_*,\mathbf{0})$ and only the upper component of u_{N^*} contributes. For eight different combinations of helicity components the results may be found in the Appendix, subsection 1 and Tables II and III. In the nonrelativistic limit we may approximate

$$p_2^+-p_3^+\approx(m_q+p_{2z})-(m_q+p_{3z})=p_{2z}-p_{3z} .$$

For the comparison with the nonrelativistic wave function we have to transform the instant spinors χ_i^I into the light-cone helicity eigenstates $|\lambda_i\rangle$. For free particles this transformation is known as the Melosh transformation[20]

$$\begin{aligned}\chi_{i\uparrow}^I &= w_i[(p_i^+ + m_i)|\uparrow_i\rangle - p_{iR}|\downarrow_i\rangle] ,\\ \chi_{i\downarrow}^I &= w_i[p_{iL}|\uparrow_i\rangle + (p_i^+ + m_i)|\downarrow_i\rangle]\end{aligned} \tag{15}$$

with $w_i = \{[2p_i^+(p_i^0+m_q)]^{1/2}\}^{-1} \to [(2m_q 2x_i P^+)^{1/2}]^{-1}$ in the nonrelativistic limit and $p_{iR,L} = p_i^1 \pm i p_i^2$. We use this transformation here, thus neglecting the quark binding effects but only for the determination of the coefficients c_n. A straightforward but lengthy calculation shows that agreement for all eight helicity components can only be achieved for the choice $c_0 = c_2 = 1$, $c_1 = 0$; thus

$$\psi_{N^*} = N_{N^*}\Phi_0(x, q_3, Q_3)(I_{*0} + I_{*2}) . \tag{16}$$

A similar analysis for the nucleon wave function leads to[21]

$$\psi_N = N_N \Phi_0(x, q_3, Q_{3_3})(I_0 + I_2) . \tag{17}$$

These results may also be obtained from a different starting point.[17] Using a separable scalar-quark–quark interaction the Weinberg equation (6) can be solved with the ansatz (11) and (12) and leads to $c_0 = c_2$, thus confirming (16) and (17).

For the totally symmetric momentum distribution we choose a Gaussian shape

$$\phi_0(x, q_3, Q_3) = \exp(-M_3^2/6\alpha^2) , \tag{18}$$

where M_3 is defined in Eq. (5). This anasatz will restrict our model to low-Q^2 values, since it does not exhibit the correct power-law falloff at high Q^2. But it should be emphasized that our model still leads well beyond the nonrelativistic quark model, because it incorporates the relativistic quark motion in the nonstatic spin wave function and a proper boost treatment.

There are only two free parameters in our model: the effective quark mass $m_q \approx m_N/3$ and the hadronic size parameter α, which will be fixed by static nucleon properties.

IV. CALCULATION OF NUCLEON FORM FACTORS AND ELECTROMAGNETIC S_{11} TRANSITION FORM FACTORS

The form factors of the nucleon are conventionally defined in terms of the electromagnetic current matrix element

$$\langle N\lambda'|J^\mu|\lambda\rangle = e\bar{u}_{\lambda'}(P')\left[\gamma^\mu F_1(q^2) + \frac{1}{4m_N} q_\nu[\gamma^\nu, \gamma^\mu] F_2(q^2)\right] u_\lambda(P) \tag{19}$$

with momentum transfer $q = P' - P$. It is known that this matrix element has an exact expression[22] in terms of the light-cone three-quark wave function $\psi_N(x_i, q_3, Q_3)$. The calculation on the light cone can be simplified considerably if one chooses the Drell-Yan frame[23] with the essential feature $q^+ = 0$:

$$\begin{aligned}P^\mu &= \left[P^+, \frac{m_N^2}{P^+}, 0_\perp\right] ,\\ q^\mu &= (0, q^-, \mathbf{q}_\perp) .\end{aligned} \tag{20}$$

Then the electromagnetic current matrix element $\langle N\lambda'|J^+|N\lambda\rangle$ is diagonal in the Fock-state basis. In particular, Fig. 1(b) does not contribute because on the light cone all fermions have $k^+, \bar{k}^+ > 0$ (Ref. 9) and the $+$ component is conserved at each vertex: $q^+ = \bar{k}^+ + k^+ > 0$. Furthermore, if we can restrict ourselves to the J^+-matrix element ($\sim\gamma^+$ upon neglecting anomalous magnetic moments of the constituent quarks), then we do not get contributions from the instantaneous fermion propagator parts $\sim\gamma^+/2k^+$ [Figs. 1(c) and 1(d)], since $\gamma^+\gamma^+ = 0$. Indeed a short calculation with the explicit light-cone spinors of the Appendix, subsection 1 in Eq. (19) shows that we can determine $F_1(q^2)$ in the Drell-Yan frame from the J^+ matrix element alone, i.e.,

$$\begin{aligned}eF_1(q^2) &= \frac{m_N}{P^+}\langle N(P')\uparrow|J^+|N(P)\uparrow\rangle ,\\ \frac{q_L}{2m_N} eF_2(q^2) &= -\frac{m_N}{P^+}\langle N(P')\uparrow|J^+|N(P)\downarrow\rangle .\end{aligned} \tag{21}$$

Hence only Fig. 1(a) contributes to the electromagnetic form factors of the nucleon and the matrix element for the three-quark nucleon wave function ψ_N of Eq. (17) reads

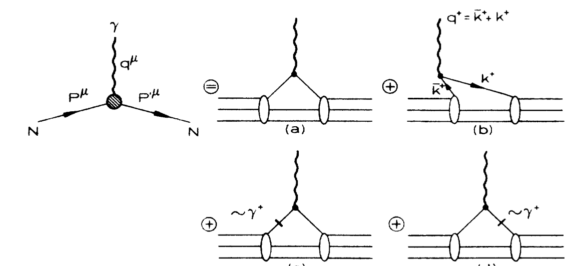

FIG. 1. Calculation of the electromagnetic current matrix element on the light cone. The matrix element is diagonal in the Fock-state basis, i.e., diagram (b) does not contribute, if we choose a frame with $q^+ = 0$. The diagrams (c) and (d) originating from the instantaneous part of the propagator do not contribute to the $+$ component of the current.

$$\left\langle N\lambda' \left| \frac{J^+}{P^+} \right| N\lambda \right\rangle = \sum_{j=1}^{3} \int d\Gamma\, \psi_N^\dagger(x_i',q_3',Q_3',\lambda') \frac{\bar{u}_{j'}}{\sqrt{p_j'^+}} e_j \gamma^+ \frac{u_j}{\sqrt{p_j^+}} \psi_N(x_i,q_3,Q_3,\lambda) , \tag{22}$$

where e_j is the charge of the struck quark and a sum over quark spins and isospins is implicit. Note that $x_i'=x_i$ since $q^+=0$.

If quark number 3 is active (i.e., $j=3$, $p_3'=p_3+q$, $p_1'=p_1$, $p_2'=p_2$ for the $+,\perp$ components) we find, from Eq. (3),

$$\mathbf{q}_{3\perp}'=\mathbf{q}_{3\perp},\quad \mathbf{Q}_{3\perp}'=\mathbf{p}_{3\perp}'-x_3'\mathbf{P}_\perp'=\mathbf{Q}_{3\perp}+(1-x_3)\mathbf{q}_\perp . \tag{23}$$

Note that $p_3'^- - p_3^- \neq q^-$ whereas $P'^- - P^- = q^-$. Details of the calculation may be found in the Appendix subsection 2.

The axial form factor $G_A(q^2)$ can be obtained in a very similar fashion from

$$\langle N(P')\lambda' | A_\alpha^+ | N(P)\lambda \rangle = \bar{u}'_N(P')\gamma_5\gamma^+ G_A(q^2)\tau_\alpha u_N(P)/2$$

and the analog of Eq. (22) in the light-cone model with the replacements $\gamma^+ \to \gamma_5\gamma^+$ and $e_j \to \tau_\alpha^j/2$.

For the S_{11} transition current some care is necessary to define a set of gauge-invariant transition form factors that are also constraint free, i.e., are independent of each other for all values of q^2. We follow the analysis of Devenish, Eisenschitz, and Körner[24] and start with the covariant, gauge-invariant, and constraint-free parametrization of the $(\frac{1}{2}^+ \to \frac{1}{2}^-)$-transition current as

$$\langle N^*(P_*)\lambda' | J^\mu | N(P)\lambda \rangle = e\bar{u}_{N^*}(P_*)[G_1(q^2)(q^2\gamma^\mu - \not{q}q^\mu)\gamma_5 + G_2(q^2)(p\cdot q\gamma^\mu - p^\mu \not{q})\gamma_5]u_N(P) . \tag{24}$$

With the help of the Gordon decomposition for the negative-parity case

$$(m_* - m_N)\bar{u}_{N^*}\gamma^\mu\gamma_5 u_N = \bar{u}_{N^*}[(P_* + P)^\mu + i\sigma^{\mu\nu}q_\nu]\gamma_5 u_N , \tag{25}$$

the transition current (24) may be rewritten in terms of the analog of Dirac and Pauli current

$$\langle N^*\lambda' | J^\mu | N\lambda \rangle = e\bar{u}_{N^*}(P_*)\left[F_{1*}(q^2)\gamma_5 \left[\gamma^\mu \frac{q^2}{m_* + m_N} + q^\mu \right] + F_{2*}(q^2) i\gamma_5\sigma^{\mu\nu}q_\nu \right] u_N(P) , \tag{24'}$$

where the form factors

$$F_{1*}(q^2) = -(m_* + m_N)[G_1(q^2) - \tfrac{1}{2}G_2(q^2)],\quad F_{2*}(q^2) = \tfrac{1}{2}(m_* + m_N)G_2(q^2) \tag{26}$$

are obviously still constraint-free, since they are obtained by linear, q^2-independent combinations of the G_i's.

Again the F_{i*} may be obtained from the $+$ component of the current alone:

$$\begin{aligned} \frac{q^2}{m_* + m_N} eF_{1*}(q^2) &= -\frac{\sqrt{m_* m_N}}{P^+} \langle N^*(P_*)\uparrow | J^+ | N(P)\downarrow \rangle , \\ q_L eF_{2*}(q^2) &= \frac{\sqrt{m_* m_N}}{P^+} \langle N^*(P_*)\uparrow | J^+ | N(P)\downarrow \rangle . \end{aligned} \tag{27}$$

Since the $F_{i*}(q^2)$ are finite for all q^2 we can infer from Eq. (27) that $\langle N^*\uparrow | J^+(q^2=0) | N\downarrow \rangle$ must vanish linearly with q^2 and $\langle N^*\uparrow | J^+(q^2=0) | N\downarrow \rangle$ with q_L. However, there is still another set of form factors h_i which is of more physical importance. These form factors h_i enter diagonally into cross sections and correspond to definite helicity transitions

$$\begin{aligned} \langle N^*\lambda' | J^\mu | N\lambda \rangle &= \bar{u}_{N^*}(P_*)\left\{ h_1(q^2)\frac{1}{Q^-}(p\cdot q q^\mu - q^2 p^\mu)\gamma_5 + h_3(q^2)\frac{1}{Q^-} i\not{p}_* \epsilon^{\mu\alpha\beta\gamma} q_\alpha p_\beta \gamma_\gamma \right\} u_N(P) \\ &=: h_1(q^2) J^\mu_{\text{long}} + h_3(q^2) J^\mu_{\text{trans}} , \end{aligned} \tag{28}$$

where $Q^- := (m_* - m_N)^2 - q^2$ defines the pseudothreshold (at $Q^-=0$), and we use the convention $\epsilon_{0123} = -\epsilon^{0123} = 1$. The form factor $h_1(q^2)$ is called longitudinal since, for a transverse photon,

$$\epsilon_\mu^{\text{trans}} J^\mu_{\text{long}} = 0$$

[as is most easily seen in the laboratory system, $\mathbf{P}=0$, with $\mathbf{q}=q\hat{\mathbf{e}}_z$ and $(\epsilon^\mu)^{\text{trans}} = \mp(1/\sqrt{2})(0,1,\pm i,0)$], while $h_3(q^2)$ is called the transverse form factor because, for a longitudinal photon,

$$\epsilon_\mu^{\text{long}} J^\mu_{\text{trans}} = 0$$

[take, for instance, in the laboratory system

$$(\epsilon^\mu)^{\text{long}} = \frac{1}{\sqrt{-q^2}}(\sqrt{\nu^2 - q^2}, 0, 0, \nu)$$

with $\nu = p\cdot q/m_N$]. Using the identity

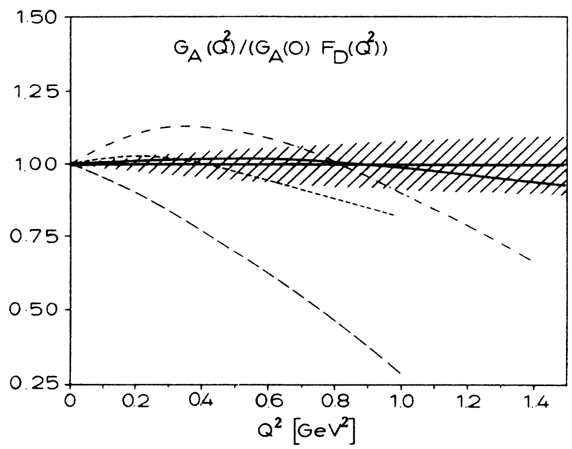

FIG. 2. Axial form factor of the nucleon normalized to the dipole fit (33) with $m_A=1.09$ GeV. ——: our calculation with $\alpha=0.33$ GeV, $m_q=0.38$ GeV; —·—·—·: recoil-corrected soliton bag model (Ref. 30); –··–: relativistic potential model (Ref. 27); — — —: static MIT bag model (Ref. 32). The experimental data of Ahrens *et al.* (Ref. 28) are displayed as the hatched area which corresponds to a dipole fit with $m_A=(1.09\pm0.05)$ GeV.

$$i\epsilon^{\mu\nu\rho\sigma}\gamma_\sigma=g^{\mu\nu}\gamma^\rho\gamma_5-g^{\mu\rho}\gamma^\nu\gamma_5+g^{\nu\rho}\gamma^\mu-\gamma^\mu\gamma^\nu\gamma^\rho\gamma_5$$

and the Gordon decomposition (25) we find

$$h_1(q^2)=2\left[\frac{m_*-m_N}{m_*+m_N}F_{1*}(q^2)-F_{2*}(q^2)\right],$$
$$h_3(q^2)=\frac{2}{m_*}\left[-\frac{q^2}{m_*+m_N}F_{1*}(q^2)+(m_*-m_N)F_{2*}(q^2)\right]. \tag{29}$$

From these equations we see that there is a constraint between h_1 and h_3 because

$$(m_*-m_N)h_1+m_*h_3=\frac{2}{m_*+m_N}Q^-F_{1*}$$

so that, at the pseudothreshold,

$$(m_*-m_N)h_1(q^2)|_{Q^-=0}+m_*h_3(q^2)|_{Q^-=0}=0\ . \tag{30}$$

Although the pseudothreshold $Q^-=0$ occurs for $q^2>0$ and is not accessible in electroproduction (where $-q^2>0$) a model for the transition form factors should obey the constraint (30).

The helicity amplitudes which enter into the radiative decay width $\Gamma(N^*\to N\gamma)$ of resonances are defined as[25]

$$A^N_{1/2}=\left[\frac{4\pi\alpha}{2K^*_w}\right]^{1/2}\langle N^*,J_z=\tfrac{1}{2}|-\mathcal{L}_{em}|N,J_z=-\tfrac{1}{2}\rangle\ ,$$
$$S^N_{1/2}=\left[\frac{4\pi\alpha}{2K^*_w}\right]^{1/2}\frac{|\mathbf{q}^*|}{\sqrt{-q^2}}\langle N^*,J_z=\tfrac{1}{2}|-\mathcal{L}_{em}|N,J_z=\tfrac{1}{2}\rangle\ , \tag{31}$$

where $\mathcal{L}_{em}=-\mathcal{H}_{em}=-\sum_{pol}\epsilon_\mu J^\mu$ is the electromagnetic interaction part of the Lagrangian. For $A^N_{1/2}$ ($S^N_{1/2}$) only transverse (longitudinal) photons with helicity $+1$ (0) contribute. In (31) K^*_w is the energy of an equivalent real photon and $|\mathbf{q}^*|$ the three-momentum transfer of the virtual photon, both in the isobar rest frame:

$$K^*_w=\frac{m_*^2-m_N^2}{2m_*}\ ,$$
$$\mathbf{q}^{*2}=\frac{Q^+Q^-}{4m_*^2}=\frac{(m_*^2-m_N^2+q^2)^2}{4m_*^2}-q^2\ ,$$
$$Q^\pm=(m_*\pm m_N)^2-q^2\ ,$$
$$\alpha=\frac{e^2}{4\pi}=\tfrac{1}{137}\ .$$

Evaluating (31) with the current defined in (28) yields finally

$$A^N_{1/2}=-m_*\left[\frac{4\pi\alpha Q^+}{8m_N(m_*^2-m_N^2)}\right]^{1/2}h^N_3(q^2)\ ,$$
$$S^N_{1/2}=-\frac{1}{2m_*}\left[\frac{4\pi\alpha Q^+(Q^+Q^-+2m_N^2q^2)}{8m_N(m_*^2-m_N^2)}\right]^{1/2}h^N_1(q^2)\ . \tag{32}$$

TABLE I. Static properties of the nucleon. The parameters α, m_q are given in GeV, the magnetic moments in Bohr magnetons μ_k and the rms radii in fm. (For the neutron charge radius we adopt the convention $\langle r^2\rangle=+6dG/dQ^2$.) Our results are compared with the recoil-corrected soliton bag model of Betz and Goldflam (Ref. 30) and the static MIT bag model. Experimental data are taken from Refs. 29 and 31.

	This calculation $\alpha=0.32$, $m_q=0.33$	This calculation $\alpha=0.32$, $m_q=0.36$	This calculation $\alpha=0.33$, $m_q=0.38$	Ref. 30	MIT	Experiment
g_A/g_V	1.20	1.23	1.23	0.91	1.09	1.255(6)
μ_p	2.80	2.73	2.67	2.63	2.57	2.793
μ_n	−1.73	−1.67	−1.62			−1.913
$\langle r^2_{E,p}\rangle^{1/2}$	0.83	0.81	0.79	0.96	0.97	0.862(12)
$\langle r^2_{M,p}\rangle^{1/2}$	0.78	0.76	0.74			0.858(56)
$\langle r^2_{E,n}\rangle^{1/2}$	0.15	0.16	0.16			0.342(42)
$\langle r^2_{M,n}\rangle^{1/2}$	0.79	0.76	0.73			0.876(70)

V. RESULTS AND DISCUSSION

A. Nucleon form factors

In our model there are two free parameters: namely, the constituent-quark mass $m_q \simeq m_N/3$ and the scale parameter α of the momentum distribution (18). It is well known that the pion cloud surrounding the three-quark core gives sizable contributions to the static electromagnetic properties of the nucleon (~10–20 %, see, e.g., Ref. 26). On the other hand, it can be shown[27] that the pion field, if introduced by a term $f_\pi \partial^\mu \Phi$ in the axial-vector current, does not contribute to the axial form factor $G_A(q^2)$. Thus the axial form factor has a special sensitivity to the quark core and is used as an input for our model.

The recent experimental data on $G_A(q^2)$, obtained from quasielastic antineutrino scattering[28] are well described by the dipole parametrization

$$\frac{G_A(Q^2)}{G_A(0)} = F_D(Q^2) := \left[1+\frac{Q^2}{m_A^2}\right]^{-2} \tag{33}$$

with $Q^2=-q^2>0$ and the axial regulator mass $m_A=(1.09\pm0.05)$ GeV, while the axial charge $G_A(0)=1.255(6)$ is obtained from neutron β decay.[29]

In Fig. 2 we show our result for the axial form factor normalized to the dipole fit and to $G_A(0)$, since not all other models reproduce the axial charge. However, in our model $G_A(0)$ is explicitly fixed and this puts the constraint $\alpha \simeq m_q$ on the parameters (Table I) just as in the NQM.[26] Our results for $G_A(0)$ and for the magnetic moments of the proton and the neutron (see below) agree with the equivalent calculation of Dziembowski.[11] The agreement with the data in Fig. 1 is quite impressive and shows the importance of a proper relativistic treatment of recoil and c.m. motion.

We turn now to the electromagnetic form factors of the nucleon. At this point it should be emphasized that gauge invariance holds for the electromagnetic current of

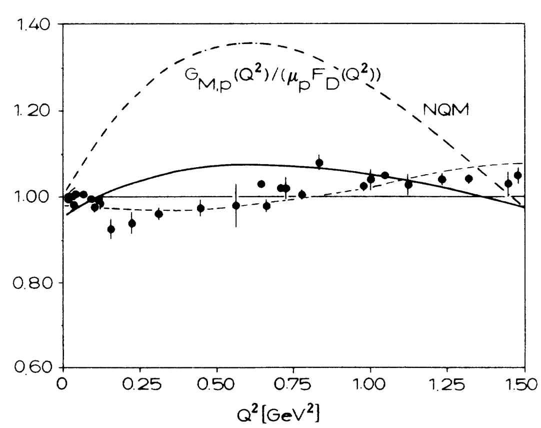

FIG. 3. Magnetic proton form factor normalized to the dipole fit (34). ——: this calculation with $\alpha=0.33$ GeV, $m_q=0.38$ GeV; —·—·—·: nonrelativistic quark model (NQM) with the same scale parameter α; — — —: relativized quark model of Warns *et al.* (Ref. 6). The experimental data are from Ref. 33.

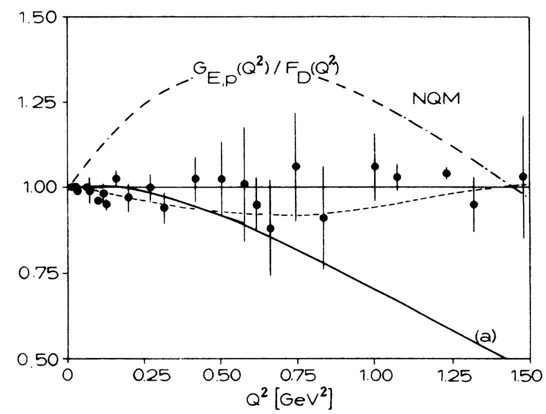

FIG. 4. Proton charge form factor normalized to the dipole fit (34). The curves are denoted as in Fig. 3.

the nucleon, i.e.,

$$\begin{aligned} 0 &= \frac{1}{2m_N} q_\mu \langle N'|J^\mu|N\rangle \\ &= \frac{1}{2m_N}\sum_{j=1}^{3} e_j \int d\Gamma\, \psi_N^\dagger \bar{u}'_j(\boldsymbol{P}'-\boldsymbol{P})u_j\psi_N \\ &= \sum_{j=1}^{3} e_j \int d\Gamma\, \psi_N^\dagger \bar{u}'_j((\boldsymbol{P}'+)-(\boldsymbol{P}+))u_j\psi_N\ , \end{aligned}$$

where we use the light-cone projection operators ($\boldsymbol{P}+$) of Eq. (A7). Thus we avoid the explicit calculation of the "bad" current components $\mathbf{J}_\perp$.

Our results for the electromagnetic form factors of the nucleon are displayed in Figs. 3–6. Since the magnetic form factors and the electric form factor of the proton are empirically well described by a dipole fit

$$\begin{aligned} G_E(Q^2) &= \frac{G_{M,p}(Q^2)}{\mu_p} = \frac{G_{M,n}(Q^2)}{\mu_n} \\ &= F_D := \left[1+\frac{Q^2}{m_D^2}\right]^{-2} \end{aligned} \tag{34}$$

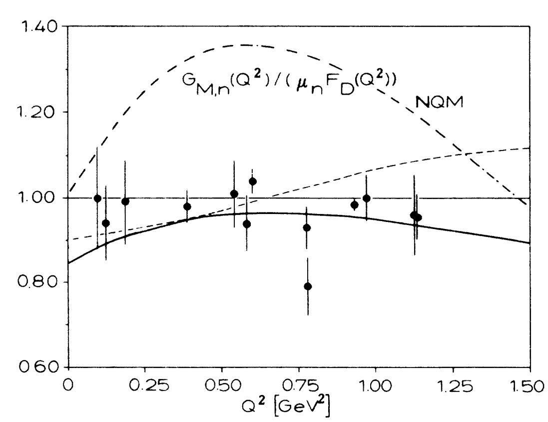

FIG. 5. Magnetic neutron form factor normalized to the dipole fit (34). The curves are denoted as in Fig. 3. The experimental data are from Ref. 34.

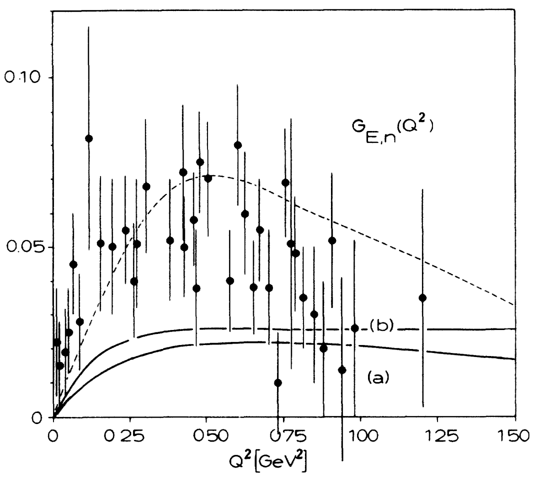

FIG. 6. Neutron charge form factor. The curves are denoted as in Fig. 3, (a) with G_M from light-cone calculation, (b) with G_M from dipole fit (see text).

with $m_D^2=0.71$ GeV^2, we display our results normalized to F_D. The magnetic form factors agree quite satisfactorily within 10–20% with the data, as expected without pionic corrections. It is interesting to note that pionic corrections to the magnetic moments as calculated by Cohen and Weber[26] raise their absolute values by $\sim 6\%$ (15%) for the proton (neutron). This correction, when applied to our three-quark core values of μ_p and μ_n (cf. Table I), would bring them remarkably close to the experimental values. However, it is not clear at this point to what extent pionic contributions are already incorporated in the constituent-quark mass m_q.

The dashed-dotted curve in Figs. 3–5 represents the results from the nonrelativistic quark model with the same parameter $\alpha=0.33$ GeV as in our calculation. Thus the ratio to the full line gives an estimate for the relativistic effects in the nucleon. For comparison we show in Figs. 3–6 also the results from the relativized quark model of Warns *et al.*[6] where relativistic effects are included via the p/m expansion of Foldy *et al.*[5]. However, since $\langle p^2\rangle^{1/2}\simeq m_q$ in the NQM it is not clear whether such an expansion will be convergent. Despite this conceptual difficulty their results agree quite well with the data, although pionic contributions are not included. But one should keep in mind that their model contains six parameters which are fitted especially to the electromagnetic properties of the nucleon.

The charge form factor of the proton falls off too fast compared to the dipole fit while the electric form factor of the neutron is about a factor of 2 too small in magnitude. However, at this stage of the model the result for $G_{E,n}$ should not be taken too seriously since it depends sensitively on the ratio of $F_{1,n}$ and $F_{2,n}$. If, for instance, we write

$$G_E=F_1+\frac{q^2}{4m_N^2}F_2=F_1+\frac{q^2}{4m_N^2}(G_M-F_1)$$

and take $G_M(Q^2)$ from the empirical dipole fit (34) instead of our model (which means only a 10% change in G_M), then $G_{E,n}$ changes by 50% to 80% [see curve (b) in Fig. 6].

B. Helicity amplitudes of the $S_{11}(1535)$

If we calculate the current matrix element $\langle N^*(P')\uparrow|J^+|N(P)\uparrow\rangle$ of Eq. (27) we find a nonvanishing contribution even at $q^2=0$, which would lead to an unphysical pole in the $S_{1/2}$ amplitude. This problem can be traced back to the nonorthogonality of the N and N^* wave function [cf. Eqs. (16) and (17)] in our simple model which in turn leads to a violation of gauge invariance. In order to restore current conservation we introduce a three-body convection current, thus replacing

$$J^\mu=e_q\gamma^\mu\rightarrow J^\mu=e_q\left[\gamma^\mu-\frac{\gamma\cdot q}{(P'+P)\cdot q}(P'+P)^\mu\right]. \tag{35}$$

which is necessary for a transition between states of different mass $m^2=P^2\neq P'^2=m_*^2$. The three-body current in (35) does not contribute in the nucleon case since there $\langle\gamma\cdot q\rangle=\langle \not P'-\not P\rangle=0$, as shown in Sec. V A.

By construction the current in (35) is conserved and from that we easily deduce the orthogonality of different states

$$0=q_\mu J^\mu=\tfrac{1}{2}(q^+J^-+q^-J^+-\mathbf{q}_\perp\mathbf{J}_\perp)$$
$$=\tfrac{1}{2}q^-J^+ \quad \text{for } q^2=0\ ,$$

where the last equality holds for a frame with $q^+=0$. Hence $\langle N^*|J^+(q^2=0)|N\rangle=0$, since

$$q^-=\frac{1}{p^+}(m_*^2-m_N^2-q^2)\neq 0 \quad \text{for } q^2=0\ ,$$

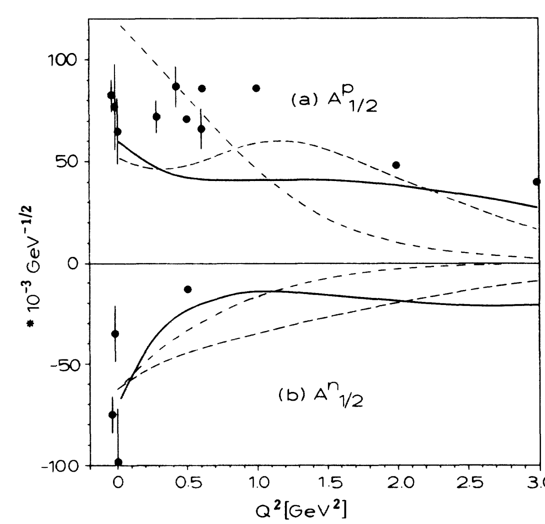

FIG. 7. Helicity transition amplitude $A_{1/2}$ of the $S_{11}(1535)$ resonance (a) from the proton, (b) from the neutron. ——: this calculation with $\alpha=0.33$ GeV, $m_q=0.38$ GeV; — — —: relativized quark model (Ref. 6); —·—·—·: nonrelativistic reduction. Data are taken from Refs. 36–40 (see also text). The points to the left of $Q^2=0$ correspond to three different photoproduction analyses.

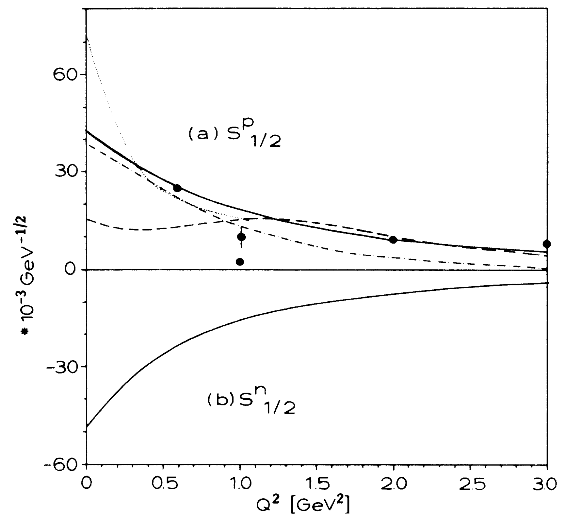

FIG. 8. Same as Fig. 7 for $S_{1/2}$. The dotted line is an analysis of η-electroproduction data (Ref. 41).

whereas $\langle N^*|J^+(q^2=0)|N^*\rangle$ may be different from zero. In fact we take $\langle N^*,\uparrow|J^+(q^2=0)|N^*,\uparrow\rangle=1$ as a normalization condition.

We calculate now the constraint-free form factors $F_{i*}(q^2)$ [cf. Eq. (27)] from our light-cone model, so that the pseudothreshold constraint (30) is trivially satisfied.

The results of the parameter-free calculation for the $S_{11}(1535)$ helicity amplitudes are displayed in Figs. 7 and 8 (solid lines). Data for these amplitudes are still scarce and are obtained by highly sophisticated fitting procedures to the observed cross sections so that they contain a lot of uncertainties, e.g., background and the use of fixed-t dispersion relations.[35] For the $A_{1/2}$ the three points to the left of $Q^2=0$ correspond to the last three analyses of pion photoproduction data in Refs. 36, 37, and 38, respectively. The electroproduction data are taken from a compilation of Foster and Hughes[39] and Krösen[40] and, for $S_{1/2}^P$ and $Q^2\leq 1$ GeV^2, from the η-electroproduction analysis of Gerhardt[41] (dotted line).

For comparison we show in Figs. 7 and 8 again the relativized quark model of Warns *et al.*[6] (dashed line) and the nonrelativistic result (dashed-dotted line). The striking feature of the $A_{1/2}^P$ data, namely, their slow falloff with Q^2 which cannot be reproduced in nonrelativisitic quark models, is seen qualitatively in both relativistic calculations. However, both results seem to be somewhat too low. For the $S_{1/2}^P$ the agreement with the analysis of Gerhardt is better than for the calculation of Ref. 6.

TABLE II. Light-cone matrix elements $2P^+2m_q\sqrt{x_2x_3}\bar{u}_2\Gamma\bar{u}_3^T$ (with $b_i,d_i=p_i^+\pm m_i$).

λ_2	λ_3	$\Gamma=\gamma_5 C$	$\Gamma=\gamma_0\gamma_5 C$
$\uparrow$	$\uparrow$	$2p_3^+p_{2L}-2p_2^+p_{3L}$	$2m_q(p_{2L}-p_{3L})$
$\uparrow$	$\downarrow$	$d_3d_2-b_3b_2$	$-(b_2b_3+d_2d_3+2p_{2L}p_{3R})$
$\downarrow$	$\uparrow$	$b_3b_2-d_3d_2$	$b_2b_3+d_2d_3+2p_{2R}p_{3L}$
$\downarrow$	$\downarrow$	$2p_3^+p_{2R}-2p_2^+p_{3L}$	$2m_q(p_{2R}-p_{3R})$

TABLE III. Light-cone matrix elements $(2P^+2m_q)^{1/2}\sqrt{x_3}\bar{u}_1\Gamma u_{N\uparrow}(P)$ for $\mathbf{P}=0$.

λ_1	$\Gamma=1$	$\Gamma=\gamma\cdot s_1\gamma_5$
$\uparrow$	b_1	$-b_1s_1^+-p_{1L}s_{1R}$
$\downarrow$	$-p_{1R}$	$p_{1R}s_1^+-b_1s_{1R}$

Better data of $S_{1/2}^P$ in the low-Q^2 region will be of great value, since they would allow a decisive distinction between various models.

VI. SUMMARY AND OUTLOOK

The main motivation for our calculation has been to study relativistic effects in the constituent-quark model for baryons. The appealing features of our light-cone approach are (i) the validity of the wave functions in any frame, (ii) the relativistic consistent treatment of quark spins and boosts, (iii) the proper decoupling of the c.m. motion. We considered a simple model for only the three-quark core of the baryons where the internal momentum distribution were taken to be of Gaussian shape.

With only two free parameters we got an excellent agreement with the experimental data for the axial form factor of the nucleon and good results for the static nucleon properties as well as the magnetic form factors up to $Q^2=1.5$ GeV^2. Relativistic effects are found to be sizable in comparison to the NQM. The proton charge form factor falls off too fast with Q^2 although $\langle r_{E,p}^2\rangle$ differs only by 10% from the experimental value. This discrepancy may be seen as the result of the imperfect momentum distribution of Gaussian shape. More realistic calculations should replace them by momentum distributions which are consistent with the underlying dynamics [see, e.g., Eq. (61) in Ref. 16] and with the power falloff at large Q^2.

The neutron charge form factor comes out too low by a factor of 2, because we did not consider configuration mixing of the nucleon ground state, which has been

TABLE IV. Light-cone matrix elements. For the definition of $(\not{p}_k)=(\not{p}_k+)$ see Eq. (A7).

λ_i	Γ	$(\not{p}_k)\Gamma u_{\lambda_i}$	$\bar{u}_{\lambda_k}\Gamma u_{\lambda_i}$
$\uparrow$	1	$u_{k\uparrow}A_{ki}^+ + u_{k\downarrow}K_{ki}^R$	$\delta_{\lambda_k,\uparrow}A_{ki}^+ + \delta_{\lambda_k,\downarrow}K_{ki}^R$
$\downarrow$	1	$-u_{k\uparrow}K_{ki}^L + u_{k\downarrow}A_{ki}^+$	$-\delta_{\lambda_k,\uparrow}K_{ki}^L + \delta_{\lambda_k,\downarrow}A_{ki}^+$
$\uparrow$	γ_5	$-u_{k\uparrow}A_{ki}^- + u_{k\downarrow}K_{ki}^R$	$-\delta_{\lambda_k,\uparrow}A_{ki}^- + \delta_{\lambda_k,\downarrow}K_{ki}^R$
$\downarrow$	γ_5	$u_{k\uparrow}K_{ki}^L + u_{k\downarrow}A_{ki}^-$	$\delta_{\lambda_k,\uparrow}K_{ki}^L + \delta_{\lambda_k,\downarrow}A_{ki}^-$
$\uparrow,\downarrow$	γ^+	$u_{k\uparrow,\downarrow}X_{ki}$	$\delta_{\lambda_k,\lambda_i}X_{ki}$
$\uparrow$	γ^R	$u_{k\uparrow}\dfrac{x_kp_i^R}{m_{ki}}$	$\delta_{\lambda_k,\uparrow}\dfrac{x_kp_i^R}{m_{ki}}$
$\downarrow$	γ^R	$2u_{k\uparrow}A_{ki}^- + u_{k\downarrow}\dfrac{x_ip_k^R}{m_{ki}}$	$2\delta_{\lambda_k,\uparrow}A_{ki}^- + \delta_{\lambda_k,\downarrow}\dfrac{x_ip_k^R}{m_{ki}}$
$\uparrow$	γ^L	$u_{k\uparrow}\dfrac{x_ip_k^L}{m_{ki}} - 2u_{k\downarrow}A_{ki}^-$	$\delta_{\lambda_k,\uparrow}\dfrac{x_ip_k^L}{m_{ki}} - \delta_{\lambda_k,\downarrow}A_{ki}^-$
$\downarrow$	γ^L	$u_{k\downarrow}\dfrac{x_kp_i^L}{m_{ki}}$	$\delta_{\lambda_k,\downarrow}\dfrac{x_kp_i^L}{m_{ki}}$

found[42,6] to be more crucial for $G_{E,n}$ than relativistic effects. Future work in this direction should clearly include the color hyperfine interaction.

We have shown that for the elastic process $N\gamma \rightarrow N$ the electromagnetic current is conserved in the light-cone approach with only the diagram (a) of Fig. 1. We have constructed a negative-parity wave function for the $S_{11}(1535)$ resonance with the same prescriptions as for the nucleon to ensure that the S_{11} is approximately a $(J=\frac{1}{2})$ eigenstate on the light cone. In evaluating the transition amplitudes for the process $N\gamma \rightarrow S_{11}$ special care was taken to use a constraint-free form-factor set and to satisfy current conservation by adding a three-body convection current. As a result the available photo- and electroproduction data could be reproduced in sign and magnitude and the slow falloff for $A^{P}_{1/2}$ is seen as a relativistic effect. However, the anomalously high decay ratio of the S_{11} into the η channel ($\sim 50\%$) remains unexplained at this stage, as it is in other quark models. Future work in the light-cone approach should also include the $\Delta(1232)$ resonance, where more reliable data for $G^{M}_{\Delta}(Q^2)$ are available and the $D_{13}(1520)$ resonance, where the helicity asymmetry

$$A_1 = \frac{A^2_{1/2} - A^2_{3/2}}{A^2_{1/2} + A^2_{3/2}}$$

provides a more direct test of the photon quark dynamics than the individual amplitudes, since A_1 is insensitive to the specific form of the momentum distributions. For both resonances the spin wave functions can be constructed in a similar manner as for the neucleon and the S_{11}, respectively. Future experiments with electromagnetic probes at CEBAF and MAMI B are expected to provide more precise data so that we will hopefully get better insight into the photon-quark dynamics by testing various models.

ACKNOWLEDGMENTS

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APPENDIX

1. Helicity representation of the invariants I_i, I_{*i}

Following the metric convention of Bjorken and Drell[13] we can deduce from Eq. (1) the explicit form of the light-cone spinors

$$(u_\uparrow, u_\downarrow, v_\downarrow, v_\uparrow) = (\gamma_5 v_\downarrow, -\gamma_5 v_\uparrow, \gamma_5 u_\uparrow, -\gamma_5 u_\downarrow) = (-C\bar{v}^T_\uparrow, -C\bar{v}^T_\downarrow, -C\bar{u}^T_\downarrow, -C\bar{u}^T_\uparrow)$$
$$= \frac{1}{\sqrt{2p^+ 2m}} \begin{bmatrix} b & -p_L & d & -p_L \\ p_R & b & p_R & d \\ d & p_L & b & p_L \\ p_R & -d & p_R & -b \end{bmatrix}, \quad \text{(A1)}$$

$$\begin{bmatrix} \bar{u}_\uparrow \\ \bar{u}_\downarrow \\ \bar{v}_\downarrow \\ \bar{v}_\uparrow \end{bmatrix} = \begin{bmatrix} -\bar{v}_\downarrow\gamma_5 \\ \bar{v}_\uparrow\gamma_5 \\ -\bar{u}_\uparrow\gamma_5 \\ \bar{u}_\downarrow\gamma_5 \end{bmatrix} = \begin{bmatrix} +v^T_\uparrow C^T \\ +v^T_\downarrow C^T \\ +u^T_\downarrow C^T \\ +u^T_\uparrow C^T \end{bmatrix} = \frac{1}{\sqrt{2p^+ 2m}} \begin{bmatrix} b & p_L & -d & -p_L \\ -p_R & b & -p_R & d \\ d & p_L & -b & -p_L \\ -p_R & d & -p_R & b \end{bmatrix} \quad \text{(A2)}$$

with $p_{R,L} := p^1 \pm ip^2$, $b,d := p^+ \pm m$, and the charge-conjugation operator $C = i\gamma^2\gamma^0$ $(C^{-1} = C^\dagger = C^T = -C)$. The spinors are normalized so that $\bar{u}u = 1 = -\bar{v}v$.

In the *uds* basis[19] where the quarks are treated as distinguishable, we take quark 1 and 2 as up quarks and quark 3 as a down quark for the proton case. With the help of $\Phi^\dagger_u i\tau_2 \Phi^*_u \Phi^\dagger_d \Phi_u = 0$, $\Phi^\dagger_u i\tau_2 \Phi_d \Phi^\dagger_u \Phi_u = 1 = -\Phi^\dagger_d i\tau_2 \Phi_u \Phi^\dagger_u \Phi_u$ the invariant I_2 becomes, in the nucleon rest frame,

$$I_2(\mathbf{P}=0) = \left[\sum_{\lambda_i} (\bar{u}_2\gamma_0\gamma_5 C\bar{u}^T_3)(\bar{u}_1 u_p) - (\bar{u}_3\gamma_0\gamma_5 C\bar{u}^T_1)(\bar{u}_2 u_p) \right] |uud\rangle \quad \text{(A3)}$$

and, similarly for the $N^*(1535)$,

$$I_{*2}(\mathbf{P}_*=0) = \sum_{\lambda_i} [-2(\bar{u}_1\gamma_0\gamma_5 C\bar{u}^T_2)(\bar{u}_3 \not{s}_3 \gamma_5 u_{N^*}) + (\bar{u}_2\gamma_0\gamma_5 C\bar{u}^T_3)(\bar{u}_1 \not{s}_1 \gamma_5 u_{N^*}) + (\bar{u}_3\gamma_0\gamma_5 C\bar{u}^T_1)(\bar{u}_2 \not{s}_2 \gamma_5 u_{N^*})] |uud\rangle \quad \text{(A4)}$$

with $s_3 = p_1 - p_2$ and cyclic permutations. The invariants I_0 and I_{*0} can be obtained from I_2, I_{*2} by replacing $\gamma_0 \rightarrow 1$. The explicit form can be constructed with the matrix elements given in Tables II and III.

The corresponding invariants for the neutron and the $(m_T = -\frac{1}{2})$ channel of the S_{11} differ merely by an overall sign when written down in terms of $|ddu\rangle$.

The nonrelativistic $N^*(1535)$ wave function (13) can be written as

$$|N^*,\lambda=\uparrow,m_T=+\tfrac{1}{2}\rangle=[|\uparrow\uparrow\downarrow\rangle^I(s_{1z}-s_{2z})+|\downarrow\uparrow\uparrow\rangle^I(s_{2z}+2s_{3z})-|\uparrow\downarrow\uparrow\rangle^I(2s_{3z}+s_{1z})$$
$$-|\downarrow\downarrow\uparrow\rangle^I(s_{1R}-s_{2R})-|\uparrow\downarrow\downarrow\rangle^I(s_{2R}+2s_{3R})+|\uparrow\downarrow\uparrow\rangle^I(2s_{3R}+s_{1R})]|uud\rangle \tag{A5}$$

with $|\uparrow\uparrow\downarrow\rangle^I \equiv \chi^I_{1\uparrow}\chi^I_{2\uparrow}\chi^I_{3\downarrow}$, etc. Applying the Melosh transformation (15) to each $\chi^I_{\lambda_i}$ and collecting the terms,[43] one can compare the eight possible helicity amplitudes with the corresponding eight terms of $\sum_n c_n I_{*n}$. Since the Melosh transformation (15) does not involve $d_i=p_i-m_i$ the combination $\sum_n c_n I_{*n}$ should also be free of them. A glance at Table II shows that this is the case precisely for the combination $I_{*0}+I_{*2}$ and the explicit calculation confirms this for all eight amplitudes consistently.

2. Calculation of current matrix elements

As an example we calculate the current $\langle N\lambda'|J^\mu|N\lambda\rangle$ of Eq. (22) for the proton in the *uds* basis $|uud\rangle$:

$$\frac{1}{p^+}\langle N\lambda'|J^\mu|N\lambda\rangle = N_N^2\int d\Gamma\,\Phi^*(M_3'^2)\Phi(M_3^2)\sum_{j=1}^3 (I_0'+I_2')^+(\lambda')\frac{\bar u_j'}{\sqrt{p_j'^+}}e_j\gamma^\mu\frac{u_j}{\sqrt{p_j^+}}(I_0+I_2)(\lambda)$$
$$=\frac{1}{m_N^2}N_N^2\int d\Gamma\,\Phi^*(M_3'^2)\Phi(M_3^2)\sum_{j=1}^3[\bar u_N' u_1' u_3'^T C^{-1}\gamma_5(\mathbf{P}'+m_N)u_2' - 2(13)]$$
$$\times\frac{\bar u_j'}{\sqrt{p_j'^+}}e_j\gamma^\mu\frac{u_j}{\sqrt{p_j^+}}[\bar u_2(\mathbf{P}+m_N)\gamma_5 C\bar u_3^T\bar u_1 u_N-(31)2]$$
$$=:\frac{1}{P^+}\sum_{j=1}^3\langle N\lambda'|J^\mu_{(j)}|N\lambda\rangle\ . \tag{A6}$$

For each term in the sum over j we may separately permute the quark indices so that quark number 3 becomes the active one (but keep e_j fixed), i.e., $p_3'=p_3+q$, and use $u_i'=u_i$ (since $p_i'=p_i$) for the spectator quarks $i=1,2$. After some spinor algebra, with the use of the light-cone projection operators

$$(\not p_i\pm):=\left[\frac{\pm\not p_i+m_i}{2m_i}\right]=\begin{cases}\sum_{\lambda_i}u_i\bar u_i\ ,\\ -\sum_{\lambda_i}v_i\bar v_i=\sum_{\lambda_i}(C\bar u_i^T)(u_i^T C^{-1})\end{cases} \tag{A7}$$

and the identities

$$C^\dagger=-C=C^{-1}=C^T,\quad C\gamma_\mu^T C^{-1}=C^{-1}\gamma_\mu^T C=-\gamma_\mu,\quad [C,\gamma_5]=0\ ,$$

we obtain, for example,

$$\langle N\lambda'|J^\mu_{(1)}|N\lambda\rangle=\tfrac{2}{3}eN_N^2\int d\Gamma\,\Phi^*(M_3'^2)\Phi(M_3^2)\frac{1}{x_3}$$
$$\times 4\bar u_N'\{(\not p_3')\gamma^\mu(\not p_3)\,\mathrm{Tr}[(\not p_1)(\not P)(\not p_2)(\not P')]+(\not p_3')\gamma^\mu(\not p_3)(\not P)(\not p_2)(\not P')(\not p_1)$$
$$+(\not p_1)(\not P)(\not p_2)(\not P')(\not p_3')\gamma^\mu(\not p_3)+(\not p_1)\,\mathrm{Tr}[(\not p_2)(\not P)(\not p_3)\gamma^\mu(\not p_3')(\not P')]\}u_N\ , \tag{A8}$$

where $(\not p_i)\equiv(\not p_i+)$ is used as a shorthand.

The matrix element $\langle N\lambda'|J^\mu|N\lambda\rangle$ can be further simplified by using the fact that each term is invariant under index exchange $1\leftrightarrow 2$ and that in a trace of γ matrices their order may be reversed anticyclicly:

$$\mathrm{Tr}(\not a_1\not a_2\cdots\not a_{2n})=\mathrm{Tr}(\not a_{2n}\cdots\not a_2\not a_1)\quad\text{if } [a_i,a_j]=0\ .$$

A very similar result is obtained for the neutron. Altogether then

$$\langle N\lambda'|J^+|N\lambda\rangle = \tfrac{4}{3}eN_N^2 \int d\Gamma\, \Phi^*(M_3'^2)\Phi(M_3^2)\frac{1}{x_3}\bar{u}'_N\{(1+3\tau_0)(\not p'_3)\gamma^+(\not p_3)\,\mathrm{Tr}[(\not p_1)(\not P)(\not p_2)(\not P')]$$
$$+(1+3\tau_0)(\not p'_3)\gamma^+(\not p_3)(\not P)(\not p_2)(\not P')(\not p_1)$$
$$+(1+3\tau_0)(\not p_1)(\not P)(\not p_2)(\not P')(\not p'_3)\gamma^+(\not p_3)$$
$$+(1-3\tau_0)(\not p_1)(\not P)(\not p_3)\gamma^+(\not p'_3)(\not P')(\not p_2)\}u_N\ , \qquad \text{(A9)}$$

where N denotes proton or neutron, respectively. The normalization constant $N_N = N_p$ is determined from

$$1 = F_1^p(q^2=0) = \frac{1}{e}\frac{m_N}{P^+}\langle p\uparrow|J^+(q^2=0)|p\uparrow\rangle\ . \qquad \text{(A10)}$$

To evaluate the six-dimensional integral $\int d\Gamma$ we proceed as follows. We define

$$m_{ik} = \sqrt{x_i m_i x_k m_k},\quad A_{ik}^{\pm} = \frac{1}{2m_{ik}}(x_i m_k \pm x_k m_i)\ ,$$
$$K_{ik}^{R,L} = \frac{1}{2m_{ik}}(x_i p_k^{R,L} - x_k p_i^{R,L}),\quad X_{ik} = \frac{2p^+}{2m_{ik}}x_i x_k \qquad \text{(A11)}$$

for $i,k = 1,2,3,N,N'$ with $x_N = x_{N'} = 1$, $\mathbf{p}_{N\perp} = \mathbf{P}_\perp$, $\mathbf{p}_{N'\perp} = \mathbf{P}_\perp + \mathbf{q}_\perp$, $p^{R,L} = p_1 \pm ip_2$. If the momenta

$$\mathbf{p}_{1\perp} = \mathbf{q}_{3\perp} - \frac{x_3}{1-x_3}\mathbf{Q}_{3\perp} + x_1\mathbf{P}_\perp,\quad \mathbf{p}_{2\perp} = -\mathbf{q}_{3\perp} - \frac{x_2}{1-x_3}\mathbf{Q}_{3\perp} + x_2\mathbf{P}_\perp,\quad \mathbf{p}_{3\perp} = \mathbf{Q}_{3\perp} + x_3\mathbf{P}_\perp$$

[from the inverse of Eq. (3)] are used and if we substitute the light-cone spin sums of Eq. (A7), then Eq. (A9) can be expressed in terms of $x_i, \mathbf{q}_{3\perp}, \mathbf{Q}_{3\perp}$. (As a check, the result must not depend on $\mathbf{P}_\perp$.) Useful relations are given in Table IV.

This rather lengthy and tedious but straighforward computation was facilitated by using the symbolic formula manipulation program REDUCE. The $\mathbf{q}_{3\perp}$ and $\mathbf{Q}_{3\perp}$ integrations can be done analytically, while the remaining two-dimensional integrations over x_1 and x_2 were performed numerically.

For completeness we list here also the axial-vector current for the nucleon and the transition current to the S_{11} resonance:

$$\langle N(P')\lambda'|A_\alpha^+|N(P)\lambda\rangle = 8N_N^2 \int d\Gamma\, \Phi^*(M_3'^2)\Phi(M_3^2)\frac{1}{x_3}\bar{u}_{N'}\tau_\alpha\{(\not p'_3)\gamma_5\gamma^+(\not p_3)\,\mathrm{Tr}[(\not p_1)(\not P)(\not p_2)(\not P')]$$
$$+(\not p'_3)\gamma_5\gamma^+(\not p_3)(\not P)(\not p_2)(\not P')(\not p_1)$$
$$+(\not p_1)(\not P)(\not p_2)(\not P')(\not p'_3)\gamma_5\gamma^+(\not p_3)$$
$$+(\not p_1)(\not P)(\not p_3)\gamma_5\gamma^+(\not p'_3)(\not P')(\not p_2)\}u_N\ ,$$

$$\langle N^*(P_*)\lambda'|J^+|N(P)\lambda\rangle = 8N_{N^*}N_N \int d\Gamma\, \Phi^*(M_3'^2)\Phi(M_3^2)\frac{1}{x_3}\bar{u}_{N^*}\{-(1+3\tau_0)\not s'_1\gamma_5(\not p_1)(\not P)(\not p_2)(\not P_*)(\not p'_3)\gamma^+(\not p_3)$$
$$+(1+\tau_0)\not s'_1\gamma_5(\not p_1)(\not P)(\not p_3)\gamma^+(\not p'_3)(\not P_*)(\not p_2)$$
$$-2\tau_0\not s'_1\gamma_5(\not p_1)\,\mathrm{Tr}[(\not p_2)(\not P)(\not p_3)\gamma^+(\not p'_3)(\not P_*)]$$
$$+(1+\tau_0)\not s'_3\gamma_5(\not p'_3)\gamma^+(\not p_3)(\not P)(\not p_2)(\not P_*)(\not p_1)\}u_N\ .$$

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