W^+W^- production in e^+e^- colliders: Test of a strongly interacting Higgs sector

F. Iddir,* A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal
Laboratoire de Physique Théorique et Hautes Energies, Université de Paris-Sud, Bâtiment 211, 91405 Orsay, France
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We study the production of W^+W^- pairs in e^+e^- collisions within the hypothesis of a strongly interacting Higgs sector. The $(Z^0+\gamma)$-exchange graphs and the ν-exchange one projected on the P wave and on $l>1$ waves are taken as Born approximation in a calculation of a final-state strong interaction for the longitudinally polarized W bosons. We adopt a Gounaris-Sakurai ansatz in the P wave with a vector ρ-like resonance at a mass between 1.5 and 2.0 TeV. We compare the number of expected events relative to the case in which the final-state strong interaction is ignored. We find a significant enhancement, mostly in the background hemisphere, that could be measured if the luminosity expectations are satisfied. We find an order-of-magnitude enhancement in the total number of WW pairs at the resonance. We also compute the effect of the strong interaction on the LR asymmetry.

I. INTRODUCTION

As is well known, if the Higgs boson were very massive, $m_H \geq 1$ TeV, this would imply a large Higgs-boson self-coupling λ, i.e., a strongly interacting Higgs sector.[1] In the standard model of electroweak interactions, the longitudinally polarized W degree of freedom comes from the Higgs field and hence if the Higgs boson has a strong interaction with itself, this implies a strong interaction between the longitudinally polarized W's. The purpose of this paper is to make a phenomenological study of the possibilities of finding experimental evidence of this strongly interacting sector in e^+e^- colliders. In a process such as $e^+e^- \to W^+W^-$, the S wave is forbidden because Z^0 and γ exchanges proceed in the P wave, as these are vector exchanges in the s channel, and ν exchange (Fig. 1) proceeds through waves $l \geq 1$ because of the helicity selection rules of the $V-A$ charged weak interactions (e^- must be left-handed, and e^+ right-handed resulting in a $J_Z=-1$ for the initial state along the e^+e^- axis). This process will therefore test a different partial wave than the celebrated scalar σ-like resonance in the $W_L W_L$ or $W_L Z_L$ channels that is allowed in gauge-boson bremsstrahlung processes, for example (Fig. 2).[2]

Longitudinal W's will behave like the π Goldstone bosons of chiral symmetry and in addition to wide 0^+ resonances we can expect a more easily identifiable 1^- ρ-like resonance. The process $e^+e^- \to W^+W^-$ precisely isolates to some extent this resonance. The exchanges of γ, Z^0 are P wave and ν exchange is dominated, as we will see below, by the P wave. However, because of the cancellations between the graphs of Fig. 1 that occur in the standard model to ensure s-channel unitarity, we cannot expect a large Born term and the final-state strong interaction (FSI) will appear as a correction to a cross section that will be small by itself. We want here to estimate quantitatively this effect to see if it can be measured in future colliders.

The paper is organized as follows. In Sec. II we write down the Born term and project on the desired partial wave. In Sec. III we discuss the numerical results, with an emphasis on the channel we are looking for: namely, W^+W^- pairs longitudinally polarized and in the P wave. In Sec. IV we introduce the final-state strong interaction. In Sec. V we show the numerical results and discuss the experimental feasibility at the light of the expected performances of future colliders.

II. BORN APPROXIMATION AND PARTIAL-WAVE ANALYSIS

A calculation of the amplitude $e^+e^- \to W^+W^-$ within the standard electroweak model in the Born approximation yields the results for ν exchange in the t channel and $Z^0+\gamma$ exchanges in the s channel (Fig. 1):

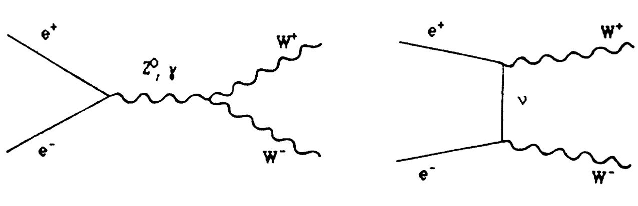


FIG. 1. Born approximation graphs for $e^+e^- \to W^+W^-$.

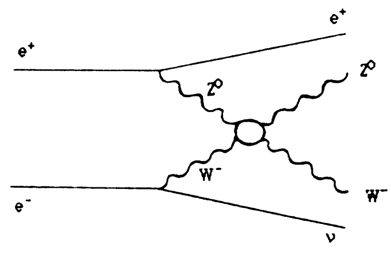


FIG. 2. Gauge-boson bremsstrahlung process.

$$A_\nu = -\frac{2ie^2}{\sin^2\theta_W}\frac{1}{q^2}v^+(1-\gamma_5)(\epsilon'^0+\boldsymbol{\epsilon}'\cdot\boldsymbol{\sigma})(q^0-\mathbf{q}\cdot\boldsymbol{\sigma})\times(\epsilon^0+\boldsymbol{\epsilon}\cdot\boldsymbol{\sigma})u \;, \quad (1)$$

$$A_{Z+\gamma} = -i\frac{e^2}{s(s-M_Z^2)}\bar{v}[(\not p-\not p')\epsilon'\cdot\epsilon+(p'\cdot\epsilon)\not\epsilon'-(p\cdot\epsilon')\not\epsilon+\not\epsilon'(p+p')\cdot\epsilon-\not\epsilon(p+p')\cdot\epsilon']u \times\begin{cases}\dfrac{s}{2\sin^2\theta_W}-M_Z^2 & (L)\;,\\ -M_Z^2 & (R)\;,\end{cases} \quad (2)$$

where $p(p'),\epsilon(\epsilon')$ are the momenta and polarizations of $W^-(W^+)$, $k(k'),u(v)$ are the momenta and spinors of $e^-(e^+)$, $q=k-p$ is the momentum transfer,

$$q^2 = M_W^2-\frac{s}{2}+\frac{s}{2}\beta\cos\theta \;, \quad (3)$$

where θ is the c.m.-system (c.m.s.) scattering angle, and $\beta=[1-(4M_W^2/s)]^{1/2}$.

In Eq. (2) $L(R)$ means the amplitude for the scattering of left-handed e^- on right-handed e^+ (right-handed e^- on left-handed e^+). Of course, the ν-exchange amplitude (1) is pure L, due to the helicity selection rules of charged-current weak interactions.

It is instructive to give the expressions of the polarization four-vectors of W^-,W^+ denoted by ϵ,ϵ'. Let us call $\epsilon^{(3)}(\epsilon'^{(3)})$ the longitudinal polarizations and $\epsilon^{(1)}$, $\epsilon^{(2)},(\epsilon'^{(1)},\epsilon'^{(2)})$ the two independent transverse polarizations:

$$\epsilon^{(3)}=\frac{1}{M_W}\left(\beta\frac{\sqrt{s}}{2},\frac{\sqrt{s}}{2}\hat{\mathbf{p}}\right),\quad \epsilon^{(2)}=\left(0,\frac{\mathbf{k}\wedge\mathbf{p}}{|\mathbf{k}\wedge\mathbf{p}|}\right),\quad \epsilon^{(1)}=\left(0,\frac{\boldsymbol{\epsilon}^{(2)}\wedge\mathbf{p}}{|\boldsymbol{\epsilon}^{(2)}\wedge\mathbf{p}|}\right), \quad (4)$$

$$\epsilon'^{(3)}=\frac{1}{M_W}\left(\beta\frac{\sqrt{s}}{2},-\frac{\sqrt{s}}{2}\hat{\mathbf{p}}\right),\quad \epsilon'^{(2)}=\left(0,\frac{\mathbf{k}\wedge\mathbf{p}'}{|\mathbf{k}\wedge\mathbf{p}'|}\right)=(0,-\boldsymbol{\epsilon}^{(2)}),\quad \epsilon'^{(1)}=\left(0,\frac{\boldsymbol{\epsilon}'^{(2)}\wedge\mathbf{p}'}{|\boldsymbol{\epsilon}'^{(2)}\wedge\mathbf{p}'|}\right)=(0,\boldsymbol{\epsilon}^{(1)}), \quad (5)$$

where $p(p')$ are the four-momenta of $W^-(W^+)$,

$$p=\left(\frac{\sqrt{s}}{2},\beta\frac{\sqrt{s}}{2}\mathbf{p}\right),\quad p'=\left(\frac{\sqrt{s}}{2},-\beta\frac{\sqrt{s}}{2}\mathbf{p}\right), \quad (6)$$

and $\mathbf{k}$ is the incident e^- three-momentum. The four-vectors (4) and (5) satisfy the constraints $\epsilon^2=-1$, $\epsilon\cdot p=0$. Let us call $A_{HMM'}$, H=L(R), the total amplitude ($Z^0+\gamma+\nu$ exchanges) for left-handed (right-handed) incident e^-, W^- polarization M (M=1,2,3), and W^+ polarization M' (M'=1,2,3). $A_{LMM'}$ will be given by the sum of (1) and the first piece of (2), and $A_{RMM'}$ by the second piece of (2). We will have each MM' component using the polarization vectors (4) and (5).

Let us now perform the partial-wave expansion, A_R is pure P wave as it comes only from $Z^0+\gamma$ exchanges, Eq. (2). A_L on the contrary will have all partial waves $l\geq 1$. The left amplitude corresponds to $m=J_Z=-1$ along the e^+e^- axis and therefore

$$A_L^l(\theta,\phi)=a_L^l Y_l^{-1}(\theta,\phi) \quad (7)$$

with

$$a_L^l=\int A_L(\theta,\phi)Y_l^{-1*}(\theta,\phi)d(\cos\theta)d\phi \;. \quad (8)$$

We want to isolate the P wave $l=1$ for which

$$a_L^l=\frac{3}{8\pi}\int A_L(\theta,\phi)\sin\theta e^{i\phi}d(\cos\theta)d\phi \;. \quad (9)$$

After some algebra, we find the amplitude $A_{HMM'}^l$, singularizing only the $l=1$ P wave. We make explicit the azimuthal-angle dependence $e^{im\phi}$ with $m=-1$ for L amplitudes and $m=+1$ for R amplitudes. We find, for W^+W^- both longitudinally polarized,

$$A_{L33}=i\frac{e^2\beta\sqrt{s}}{4\sin^2\theta_W}\sin\theta\, e^{-i\phi}\left[-\frac{2}{s\beta^2}\left(1+\frac{4M_W^2}{s}\right)+\frac{M_Z^2(1-2\sin^2\theta_W)}{s-M_Z^2}\left(\frac{1}{M_W^2}+\frac{2}{s}\right)-\frac{4M_W^2}{s\beta^2}\frac{1}{M_W^2-\frac{s}{2}+\frac{s}{2}\beta\cos\theta}\right],$$

$$A_{R33}=-\frac{ie^2\beta\sqrt{s}\,\sin\theta}{2}e^{i\phi}\left(\frac{1}{M_W^2}+\frac{2}{s}\right)\frac{M_Z^2}{s-M_Z^2} \;. \quad (10)$$

In A_{L33} we recognize the γ exchange (first term, in $1/s$), the Z exchange (second term), and ν exchange, behaving in $1/q^2$, where q^2 is given by (3). A_{R33} is pure P wave, as expected, and A_{L33} projects out into a P-wave piece and a piece with $l>1$:

$$A^{P}_{L33}=i\frac{e^2\beta\sqrt{s}\,\sin\theta e^{-i\phi}}{4\sin^2\theta_W}\left\{-\frac{2}{s\beta^2}\left[1+\frac{4M_W^2}{s}\right]+\frac{M_Z^2(1-2\sin^2\theta_W)}{s-M_Z^2}\left[\frac{1}{M_W^2}+\frac{2}{s}\right]-\frac{12M_W^2}{s^3\beta^4}\left[2M_W^2-s-\frac{2M_W^4}{s\beta}\ln\left[\frac{M_W^2-\frac{s}{2}(1-\beta)}{M_W^2-\frac{s}{2}(1+\beta)}\right]\right]\right\}, \tag{11}$$

$$A^{(l>1)}_{L33}=-i\frac{e^2\beta\sqrt{s}\,\sin\theta e^{-i\phi}}{4\sin^2\theta_W}\frac{4M_W^2}{s\beta^2}\left[\frac{1}{M_W^2-\frac{s}{2}+\frac{s}{2}\beta\cos\theta}-\frac{6M_W^2-3s}{s^2\beta^2}+\frac{6M_W^4}{s^3\beta^3}\ln\left[\frac{M_W^2-\frac{s}{2}(1-\beta)}{M_W^2-\frac{s}{2}(1+\beta)}\right]\right]. \tag{12}$$

These are the amplitudes, where we have two longitudinally polarized W bosons, that will be affected by the FSI, and A^P_{L33}, in particular, by the ρ-like resonance.

The amplitudes where both W^+W^- have transverse polarizations will be given by (we do not project on the P wave since the FSI does not affect these amplitudes)

$$A_{L11}=i\frac{e^2\beta\sqrt{s}\,\sin\theta e^{-i\phi}}{4\sin^2\theta_W}\left[\frac{\beta-2\cos\theta}{\left[M_W^2-\frac{s}{2}+\frac{s}{2}\beta\cos\theta\right]\beta}+\frac{2s-4M_Z^2\sin^2\theta_W}{s(s-M_Z^2)}\right],$$

$$A_{L22}=-i\frac{e^2\beta\sqrt{s}\,\sin\theta e^{-i\phi}}{4\sin^2\theta_W}\left[\frac{1}{M_W^2-\frac{s}{2}+\frac{s}{2}\beta\cos\theta}+\frac{2s-4M_Z^2\sin^2\theta_W}{s(s-M_Z^2)}\right], \tag{13}$$

$$A_{L12}=\frac{1}{4\sin^2\theta_W}\frac{e^2\sqrt{s}\,\sin\theta e^{-i\phi}}{M_W^2-\frac{s}{2}+\frac{s}{2}\beta\cos\theta},$$

$$A_{L21}=-A_{L12}$$

for the left amplitudes, and, for the right ones,

$$A_{R11}=-i\frac{e^2\beta\sqrt{s}M_Z^2}{s-M_Z^2}\sin\theta e^{i\phi},$$

$$A_{R22}=-A_{R11},\quad A_{R12}=0,\quad A_{R21}=0\,. \tag{14}$$

Finally we have the case where a longitudinal and a transverse W are produced:

$$A_{L13}=i\frac{e^2\beta e^{-i\phi}}{M_W}\left[\frac{\cos\theta}{s-M_Z^2}\left[-M_Z^2+\frac{s}{2\sin^2\theta_W}\right]+\frac{s[\sin^2\theta-(\beta-\cos\theta)^2]}{8\beta\sin^2\theta_W\left[M_W^2-\frac{s}{2}+\frac{s}{2}\beta\cos\theta\right]}\right],$$

$$A_{L23}=\frac{e^2\beta e^{-i\phi}}{M_W}\left[\frac{1}{s-M_Z^2}\left[-M_Z^2+\frac{s}{2\sin^2\theta_W}\right]-\frac{s(\beta^2\cos\theta-2\beta+\cos\theta)}{8\beta\sin^2\theta_W\left[M_W^2-\frac{s}{2}+\frac{s}{2}\beta\cos\theta\right]}\right], \tag{15}$$

$$A_{L31}=-A_{L13},\quad A_{L32}=A_{L23}\,,$$

for the left amplitudes, and, for the right ones,

$$A_{R13}=-i\frac{e^2\beta}{M_W}\frac{M_Z^2}{s-M_Z^2}\cos\theta e^{i\phi},$$

$$A_{R23}=\frac{e\beta^2}{M_W}\frac{M_Z^2}{s-M_Z^2}e^{i\phi}, \tag{16}$$

$$A_{R31}=-A_{R13},\quad A_{R32}=A_{R23}\,,$$

These results for the polarized amplitudes agree with the calculation of Gaemers and Gounaris,[3] except for the amplitude A_{L31}, probably due to a misprint in their paper. We agree with the results of Hagiwara *et al.*[3]

The expressions given above can allow us to compute, for example, the W^- density matrix for both $L\,e^-$ or $R\,e^-$:

$$\rho^{(L)}_{ij}=\sum_k A_{Lik}A^*_{Ljk} \tag{17}$$

and similarly for R. We sum over the W^+ polarizations (index k).

III. THE CROSS SECTION FOR PRODUCING LONGITUDINAL POLARIZED W^+W^-

Let us now comment on the numerical results that we find for the cross section in the Born approximation and the different behavior of the contributions we have listed above, with emphasis in the channel we are interested: namely, W^+W^- both longitudinally polarized and in a P wave. The cross section for unpolarized beams will write

$$\frac{d\sigma}{d\Omega}=\frac{\beta}{256\pi^2}\sum_i |A_i|^2 , \tag{18}$$

where the sum over i runs over the indices $i=(L,R,W^+,W^-$ polarizations).

The cross section integrated over the angles decreases [Fig. 3(a)]:

$$\sqrt{s}=200\text{ GeV},\quad \sigma=1.7\times10^{-8}\text{ mb} ,$$

$$\sqrt{s}=1\text{ TeV},\quad \sigma=0.23\times10^{-8}\text{ mb} ,$$

$$\sqrt{s}=2\text{ TeV},\quad \sigma=0.6\times10^{-9}\text{ mb} ,$$

and the differential cross section is strongly peaked in the forward direction due to the ν exchange [Fig. 3(b) (Ref. 4)].

Let us now see which are the most favorable angular regions, i.e., where the production of two W longitudinally polarized $W_L W_L$ (the channel in which we are mostly interested) is not negligible. On general grounds, we must first say that at $\theta=0,\pi$, $W_L W_L$ and $W_T W_T$ are forbidden, and only $W_T W_L$ pairs are produced. Close to the forward direction, ν exchange strongly dominates and it produces almost essentially $W_T W_T$ (mostly pairs of the

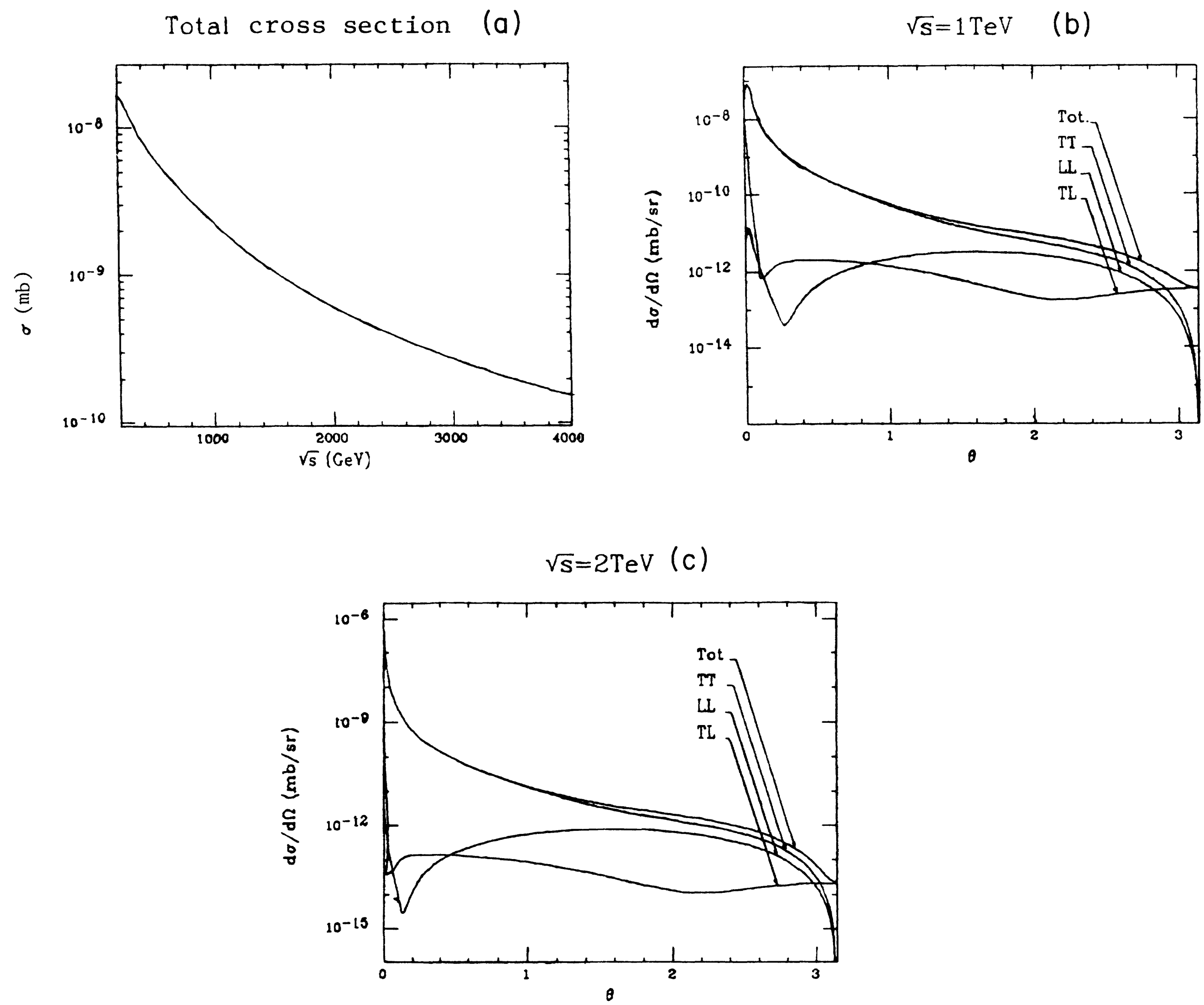


FIG. 3. (a) The integrated cross section $\sigma(e^+e^-\to W^+W^-)$ as a function of $\sqrt{s}$ and (b) and (c) the differential cross section $(d\sigma/d\Omega)(e^+e^-\to W^+W^-)$.

type $W_{T_1} W_{T_2}$, i.e., with orthogonal polarizations), that grow very quickly when θ increases, and also $W_T W_L$, that decrease very quickly with θ. As we will see, $W_L W_L$ pairs will only be produced in a sizable proportion in the backward hemisphere, at angles $\sim 7\pi/10$. The energy dependence is the following.

Close to threshold, $\sqrt{s} \simeq 200$ GeV, the WW production is maximal in the forward direction, but the proportion of $W_L W_L$ is small. Near the forward direction, ν exchange dominates and produces essentially $W_T W_L$ and $W_T W_T$ pairs. At $\theta=0$ we get zero $W_L W_L$, zero $W_T W_T$, and 100% $W_L W_T$. When θ grows the proportion changes: for example, at $\theta=0.314$ we get 9% of $W_L W_L$, 50% of $W_T W_T$, 41% of $W_T W_L$; at $\theta=0.628$ we get 10% of $W_L W_L$, 82% of $W_T W_T$, 8% of $W_L W_T$. The total production is in a maximum in this last zone. The proportion of $W_L W_L$ passes through two maxima (Fig. 4), at

$$\theta \simeq \frac{\pi}{6} \simeq 0.52, \quad \sim 11\% \text{ of } W_L W_L ,$$

$$\theta \simeq \frac{7\pi}{10} \simeq 2.20, \quad \sim 29\% \text{ of } W_L W_L .$$

At $\sqrt{s} \simeq 400-600$ *GeV,* $W_T W_L$ pair production dominates in the forward and backward directions, and, when θ grows, $W_T W_T$ production grows very quickly and remains always very important. For example, at $\theta=\pi/10$, we get 99% of $W_T W_T$, 0.5% of $W_L W_L$, 0.5% of $W_L W_T$. We find two maxima of production of $W_L W_L$ pairs with the most favorable zone in the backward hemisphere: $\sim 30\%$ $W_L W_L$ for $\theta \sim 3\pi/5 - 4\pi/5$.

At higher energies, this behavior becomes even sharper: $W_L W_T$ are produced at $\theta=0$, $\theta=\pi$ and decrease quickly elsewhere; $W_T W_T$ are forbidden at $\theta=0,\pi$ but are dominant elsewhere, rising very quickly for $\theta \neq 0,\pi$; $W_L W_L$ are forbidden at $\theta=0,\pi$ and rise slowly for $\theta \neq 0,\pi$, with a maximum of 30% in the region $\theta \sim 3\pi/5 - 4\pi/5$.

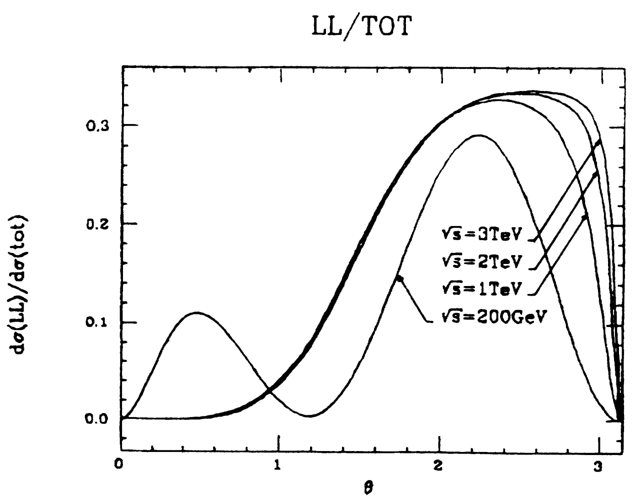


FIG. 4. The cross section for producing $W_L^+ W_L^-$ relative to the total number of produced W^+W^- for different values of $\sqrt{s}$.

It is useful to compute the number of $W_L W_L$ that can be produced per year in the favorable angular region, between $3\pi/5$ and $4\pi/5$, for a luminosity not unrealistic in future colliders, $\mathcal{L} \simeq 10^{33}$ cm^{-2}sec^{-1}.

At $\sqrt{s} \simeq 200$ *GeV,* we get a total number of 13 340 W^+W^- pairs with 27% $W_L W_L$, 45% $W_T W_T$, 28% $W_L W_T$.

When s grows, the proportion of $W_L W_L$ grows slowly, but the cross section drops steadily. Also, the proportion of $W_L W_T$ drops dramatically.

At $\sqrt{s}=1$ *TeV,* we have a total number of 220 W^+W^- pairs, with 31.2% $W_L W_L$, 65.9% $W_T W_T$, 2.9% $W_L W_T$.

At $\sqrt{s}=2$ *TeV,* we have only a total number of 50 W^+W^- pairs, with 31.5% $W_L W_L$, 67.9% $W_T W_T$, 0.6% $W_L W_T$.

We have discussed the production of $W_L W_L$, but we need to be more quantitative concerning the partial-wave analysis, as we are interested in the P wave, where the FSI is expected to be important due to a vector ρ-like resonance.

The W^+W^- pairs are produced in the P wave for $Z^0+\gamma$ exchanges and ν exchange produces both P and $l>1$ waves. Because of gauge theory cancellations, P-wave production from ν exchange and from $Z^0+\gamma$ exchange are of the same order of magnitude and opposite sign: their partial cancellation ensures s-channel unitarity,[4] the difference decreasing when s increases. This is important because it will decrease the number of events in the desired channel, P wave $W_L W_L$. On the other hand, the production cross section in the $l>1$ waves, that comes only from ν exchange, satisfies s-channel unitarity by itself.

The P-wave amplitude $A^P(Z+\gamma+\nu)$ compares in the following way to $A^{l>1}(\nu)$. Defining the ratio

$$r = \left| \frac{A^P}{A^{l>1}} \right| \tag{19}$$

we have the following behavior [Figs. 5(a) and 5(b)].

Near threshold ($\sqrt{s} \simeq 200$ GeV), $r<1$, mostly near the forward direction: $r \sim \frac{1}{25}$ for $\theta \sim 0$, and r increases as θ grows reaching an infinite value at $\theta \sim 1.35$ corresponding to the peak in the figure, and then decreases reaching a value 0.4–0.5 at large θ. This peak appears because the amplitude $|A^{l>1}|$ vanishes at a precise value of θ. For pure ν exchange, $|A^P(\nu)| > |A^{l>1}(\nu)|$ for all s and θ, but the gauge cancellations that occur in $A^P(\gamma+Z^0+\nu)$ can give $r<1$ in some domains of θ and s.

At $\sqrt{s} \simeq 400-800$ *GeV,* $r<1$ for $\theta \sim 0$, and $r \gg 1$ at large θ. When s grows, the region close to $\theta=0$ where $r<1$ becomes closer to the forward direction.

When $\sqrt{s} > 800$ *GeV,* $r>1$ for practically all θ, except $\theta \sim 0$ with r a function of θ. For example, *at* $\sqrt{s}=1$ *TeV,* $r \sim 10^{-3}$ at $\theta \sim 0$; $r \sim 10^2-10^3$ for large θ, decreasing to $r \sim 2-3$ for $\theta \sim \pi$.

At $\sqrt{s}=2$ *TeV,* $r \sim 10^{-2}$ at $\theta \sim 0$; $r \sim 2 \times 10^3$ for large $\theta(\sim 40°)$; $r \sim 10^2$ in the backward direction.

Notice also the peak of r for $\theta \sim 70°$, because of the vanishing of $A_{LL}^{l>1}$ (amplitude for producing $W_L W_L$ in the waves $l>1$). From (12) we can easily compute the value of θ for which $A_{LL}^{l>1}$ vanishes: this value decreases quickly from $\theta=90°$ at threshold to an asymptotic value $\theta\sim 70.53°$ for $\sqrt{s}>1$ TeV.

When introducing a phase shift in the P wave, we will be interested to be in a region where $A^P(W_L W_L)$ is big enough. Let us call $A_{LL}=A_{LL}^P+A_{LL}^{l>1}$ the total *amplitude for producing a pair of longitudinal W.* We have the following behavior.

Close to threshold ($\sqrt{s}\simeq 200$ GeV), the P wave does not dominate (especially close to the forward direction), and when A_{LL}^P is not negligible, there is a destructive interference, for example, at $\theta=1.26$, $|A_{LL}^P|^2>|A_{LL}|^2$.

At $\sqrt{s}\simeq 400$ *GeV,* we have, for example, for $\theta=0.314$, $|A_{LL}^P|^2<|A_{LL}|^2$, and for $\theta=0.6-1.0$, $|A_{LL}^P|^2 \gg |A_{LL}|^2$ and when $\theta>1$, $|A_{LL}^P|^2\sim|A_{LL}|^2$.

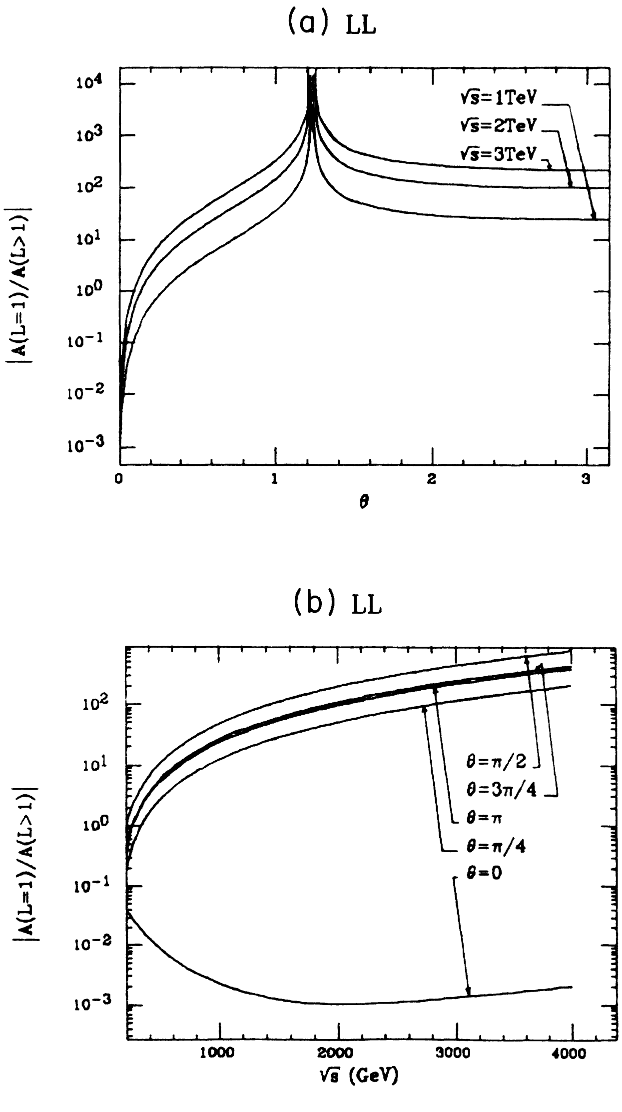


FIG. 5. (a) The ratio $r=|A^P/A^{l>1}|$ as a function of θ at fixed $\sqrt{s}$ and (b) as a function of $\sqrt{s}$ at fixed θ.

At higher energies, $|A_{LL}^P|^2\sim|A_{LL}|^2$, i.e., the P wave dominates, and this trend strengthens with the energy.

The interference between A_{LL}^P and $A_{LL}^{l>1}$ will be thus maximal close to $\theta=0$, but unfortunately this is not the region where $W_L W_L$ production is favored. In the zone where $W_L W_L$ production is maximal, ($\theta=3\pi/5-4\pi/5$), we have an increasing cross section in the P wave, $\sigma^P(WW)$, that represents, respectively, 66%, 84%, and 91% of the total cross section $\sigma(e^+e^-\to W_L^+W_L^-)$ for $\sqrt{s}=400$, 600, and 800 GeV.

To sum up, we are interested in the interference between $A^P(W_L W_L)$ and $A^{l>1}(W_L W_L)$ at high energy, for $\sqrt{s}$ between 1.5 and 2 TeV, the strong interaction among the $W_L W_L$ pairs taking place dominantly in the P wave, due to a ρ-like resonance in this region. At these energies, the $W_L W_L$ production is maximal in the region $\theta\simeq 3\pi/5-4\pi/5$ (about 30%), but the P wave dominates strongly and the interference will therefore be small. However, as we will see below, a sizable effect, characteristic of a strongly interacting $W_L W_L$ sector, will still remain, maybe detectable by the next generation of e^+e^- colliders, namely, the resonance enhancement of the absolute magnitude of the $W_L W_L$ P wave.

IV. FINAL-STATE INTERACTION IN THE P WAVE

The scattering of longitudinally polarized gauge bosons in the strongly coupled Higgs theory behaves like the scattering of Goldstone bosons. More precisely, there is the so-called equivalence theorem,[2] that relates the scattering amplitude among longitudinal gauge bosons and the physical Higgs field to the scattering amplitude for the Goldstone bosons that give the masses to the gauge bosons through the Higgs mechanism:

$$A(W_L,Z_L,H)\simeq A(w,z,H)+O(M_W/\sqrt{s})\ .$$

For reasons discussed below, we will not try to make calculations of strong-interaction effects starting from the Higgs Lagrangian. Rather, we will rely on an assumption of strict analogy between $W_L W_L$ and $\pi\pi$ scattering, which could be valid, for instance, in a technicolor scheme. In view of the equivalence theorem, the analogy is expected to hold for $s\gg M_W^2$ and results in a conversion factor for masses and widths

$$\frac{v}{F_\pi}\simeq 2674\ , \qquad (20)$$

where v is the Higgs vacuum expectation value $v=(\sqrt{2}G_F)^{-1/2}$ giving the W,Z masses, and F_π is the pion decay constant. The conversion factor is approximate because of the explicit breaking of chiral symmetry by the quark masses, which leads to a different scale, and because of gauge group algebraic factors.

Within such an assumption, the $W_L W_L\to W_L W_L$ P wave shall be described by a Gounaris-Sakurai model[5] with m_ρ,Γ_ρ scaled according to (20).

As to the electroweak process $e^+e^-\to W_L W_L$, pursuing the analogy, we will describe it by multiplying the Born amplitude obtained in the previous paragraph by the [scaled according to (20)] π form factor derived from the Gounaris-Sakurai model. Therefore we assume that the result is close to the Born result for small $s \ll m_V^2$, where m_V is the mass of a hypothetical vector resonance in the $W_L W_L$ channel. Of course, the left-hand singularities are quite different in the pion form-factor problem $e^+e^- \to \pi^+\pi^-$ and in $e^+e^- \to W_L W_L$, especially because of neutrino exchange, but we hope that the difference will not be too important for large s because in the limit $s \gg m_W^2, m_Z^2$, $A^P(W_L W_L)$ behaves indeed like in the case of Gounaris and Sakurai: $A(e^+e^- \to \pi^+\pi^-) \sim \sqrt{s}/s$.

In our case, the factor $\sqrt{s}$ comes from the longitudinal polarization of the W, Eq. (4), that behaves like a Goldstone boson. The W_L form factor will be given by

$$F_W(s) = \frac{m_V^2 + d m_V \Gamma_V}{(m_V^2 - s) + \Gamma_V \left[\frac{m_V^2}{k_V^3}\right] g(s) - i m_V \Gamma_V \left[\frac{k}{k_V}\right]^3 \frac{m_V}{\sqrt{s}}} , \tag{21}$$

where

$$k^2 = \tfrac{1}{4}(s - 4M_W^2) = \frac{s}{4}\beta^2 ,$$

$$k_V^2 = \tfrac{1}{4}(m_V^2 - 4M_W^2) , \tag{22}$$

$$d = \frac{3}{\pi}\frac{M_W^2}{k_V^2}\ln\left[\frac{m_V + 2k_V}{2M_W}\right] + \frac{m_V}{2\pi k_V} - \frac{M_W^2 m_V}{\pi k_V^3} ,$$

and

$$g(s) = k^2[h(s) - h(m_V^2)] + k_V^2 h'(m_V^2)(m_V^2 - s) \tag{23}$$

with $h(s)$ and $h'(m_\rho^2)[\beta_V = (1 - 4M_W^2/m_V^2)^{1/2}]$:

$$h(s) = \frac{2k}{\pi\sqrt{s}}\ln\left[\frac{\sqrt{s} + 2k}{2M_W}\right] = \frac{\beta}{\pi}\ln\left[\frac{(1+\beta)\sqrt{s}}{2M_W}\right] ,$$

$$h'(m_V^2) = \frac{2M_W^2}{\pi m_V^4 \beta_V}\ln\left[\frac{(1+\beta_V)m_V}{2M_W}\right] + \frac{\beta_V}{\pi}\left[\frac{2M_W^2}{m_V^4\beta_V(1+\beta_V)} + \frac{1}{2m_V^2}\right] . \tag{24}$$

Near the resonance, s close to m_V^2, $F_W(s)$ is reduced to the familiar resonance formula, as we can neglect the term proportional to $g(s)$ in the denominator:

$$F_W(s)\big|_{\text{near } s = m_V^2} = \frac{m_V^2\left[1 + d\frac{\Gamma_V}{m_V}\right]}{m_V^2 - s - i m_V \Gamma_V \left[\frac{k}{k_V}\right]^3 \left[\frac{m_V}{\sqrt{s}}\right]} . \tag{25}$$

The basis of the Gounaris-Sakurai formula is quite general. The unitarity condition requires that, *in the elastic region* $(4M_W^2 < s < 16M_W^2)$,

$$\mathrm{Im} t_{lI}(s) = \frac{k}{\sqrt{s}}|t_{lI}(s)|^2 , \tag{26}$$

where t_{lI} is the lth partial wave in the weak isospin state I [$I=0,2$ for $l=0$, $I=1$ for $l=1$ because of the Bose statistics and $I(W)=1$]. Unitarity, Eq. (26) implies

$$t_{lI} = \frac{\sqrt{s}}{k}e^{i\delta_{lI}}\sin\delta_{lI} , \tag{27}$$

$$t_{lI}^{-1} = -i\frac{k}{\sqrt{s}} + \frac{k}{\sqrt{s}}\cot\delta_{lI} . \tag{28}$$

Let us consider the $l=1$, $I=1$ channel. The approximation consists in writing an effective range formula for

$$\frac{k^3}{\sqrt{s}}\cot\delta_{11}(s) = k^2 h(s) + a_{11} + \frac{1}{2}r_{11}k^2 , \tag{29}$$

where $h(s)$ is given by (24) and has the role of canceling the spurious $1/\sqrt{s}$ singularity in $\mathrm{Im} t_{11}^{-1}$(28) and introducing no other singularities, a_{11} is the scattering length, and r_{11} the effective range. In the $l=1$, $I=1$ channel one assumes, in analogy with $\pi\pi$ scattering, the existence of a resonance V, and one can *define* its mass and width by

$$\cot\delta_{11}\big|_{s=m_V^2} = 0, \quad \frac{d\delta_{11}}{ds}\bigg|_{s=m_V^2} = \frac{1}{m_V\Gamma_V} . \tag{30}$$

Then, the Gounaris-Sakurai formula (21) follows from

$$F_W(s) = \frac{f(0)}{f(s)} , \tag{31}$$

where $f(s)$ is defined by

$$f(s) = k^2[t_{11}(s)]^{-1} \tag{32}$$

and $[t_{11}(s)]^{-1}$ is obtained from (28) with the approximation (29) together with (30) that allows us to express the unknown a_{11}, r_{11} in terms of m_V, Γ_V.

We will now write the amplitude taking into account the FSI in the P wave, and neglecting it the higher waves $l > 1$:

$$A^{\mathrm{FSI}} = A^P(W_L W_L)F_W + A^{l>1}(W_L W_L) . \tag{33}$$

The Born approximation amplitudes $A^P(W_L W_L)$, $A^{l>1}(W_L W_L)$, given by the last equation of (10), (11), and (12), can be written in the form, factorizing the azimuthal angle dependence,

$$A^P(W_L W_L) = i e^{-i\phi} A^P ,$$
$$A^{l>1}(W_L W_L) = i e^{-i\phi} A^{l>1} \tag{34}$$

with A^P, $A^{l>1}$ real, and the form factor $F_W(s)$ can be parametrized:

$$F_W(s) = \frac{X}{Y - iZ} . \tag{35}$$

If we write A^{FSI} in the form

$$A^{\mathrm{FSI}} = A e^{i\delta} \tag{36}$$

with $A = i e^{-i\phi} A'$, A' real, we find

$$\tan\delta = \frac{XZA^P}{XYA^P + (Y^2+Z^2)A^{l>1}} . \quad (37)$$

We have still to determine the parameters a_{11}, r_{11} in (29). This can be done by estimating m_V and Γ_V. In the technicolor model with $SU(4)_{TC}$ as the strong-interaction gauge group, the appropriate rescaling gives[6]

$$m_V = \left[\frac{3}{N}\right]^{1/2} \frac{v}{F_\pi} m_\rho ,$$
$$\Gamma(V \to W_L W_L) = \left[\frac{3}{N}\right]^{3/2} \frac{v}{F_\pi} \frac{\Gamma(\rho\to\pi\pi)}{\left[1-\frac{4m_\pi^2}{m_\rho^2}\right]^{3/2}} \left[1-\frac{4M_W^2}{m_V^2}\right]^{3/2} , \quad (38)$$

where $N=4$ is the number of colors for $SU(4)_{TC}$. From (20) and $F_\pi = 92$ MeV we obtain

$$m_V = 1.783 \text{ TeV}, \quad \Gamma_V = 0.325 \text{ TeV} . \quad (39)$$

These values give for the parameters entering the Gounaris-Sakurai formula: $d=0.339$, $k_V=0.888$ TeV, $\beta_V=0.996$, $h(m_V^2)=0.976$, $h'(m_V^2)=0.533\times10^{-7}$.

The parameter a_{11} could also be deduced,[7,8] as done previously by Brown and Goble[8] from the low-energy theorems of $SU(2)\times SU(2)$, which are valid for $W_L W_L$ scattering if $s \gg M_W^2$ (then the condition is, on the whole, $M_W^2 \ll s \ll m_H^2$).

The low-energy behavior of the amplitude for the process involving Goldstone bosons (longitudinally polarized W bosons),

$$w_i w_j \to w_k w_l , \quad (40)$$

$(i,j,k,l = 1,2,3$, weak-isospin components of $W^\pm, Z)$ will be, in close analogy with $\pi\pi\to\pi\pi$ scattering,[7,8]

$$T_{ijkl}(s,t) \sim \frac{1}{v}(\delta_{ij}\delta_{kl}s + \delta_{ik}\delta_{jl}t + \delta_{il}\delta_{jk}u) \quad (41)$$

(up to $M_W/\sqrt{s}$ terms). Decomposing the amplitude into partial waves $t_{lI}(s)$, the current-algebra constraint (41) implies, in particular for the amplitude we are interested in,

$$t_{11} = \frac{1}{12\pi v^2}k^2 \quad (M_W^2 \ll s \ll m_H^2) , \quad (42)$$

i.e., the same formula as for $t_{11}(\pi\pi\to\pi\pi)$ with $v \to F_\pi$, $M_W \to m_\pi$.

The estimate of a_{11} is

$$a_{11} = 12\pi v^2 \quad (43)$$

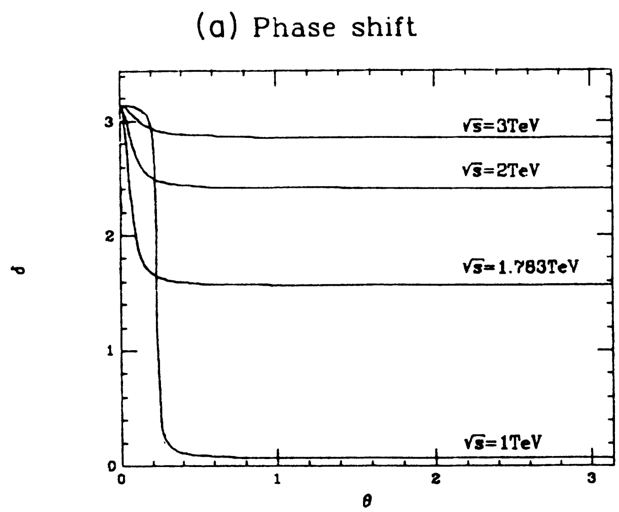
and would lead to comparable results as above, but would only give a relation between m_V and Γ_V, and not the absolute magnitudes. The relation is

$$\Gamma_V \simeq \frac{m_V^3}{96\pi v^2} \quad (44)$$

yielding $\Gamma_V \simeq 450$ GeV for $m_V \simeq 2$ TeV.

Let us now briefly discuss how general it is to expect the presence of a vector resonance in the W_L-W_L P-wave scattering channel, and what can be learned about its parameters. The current-algebra predictions are in principle valid and would lead to (43). But, as noticed before, this is not sufficient to answer the question. This question was discussed several years ago by Peskin;[9] he has stressed the increasing effect of the techni-rho to be expected in technicolor, and also considered the possibility of a nonresonant enhancement in elementary Higgs models. We would like to reconsider it in the light of our present knowledge.

We have used the homology with $\pi\pi$ scattering because we had in mind technicolorlike models where this analogy is well grounded, since the Goldstone bosons as well as the Higgs boson are fermion-antifermion bound states in a confining gauge theory similar to QCD. In such models, the presence of a rho seems unavoidable



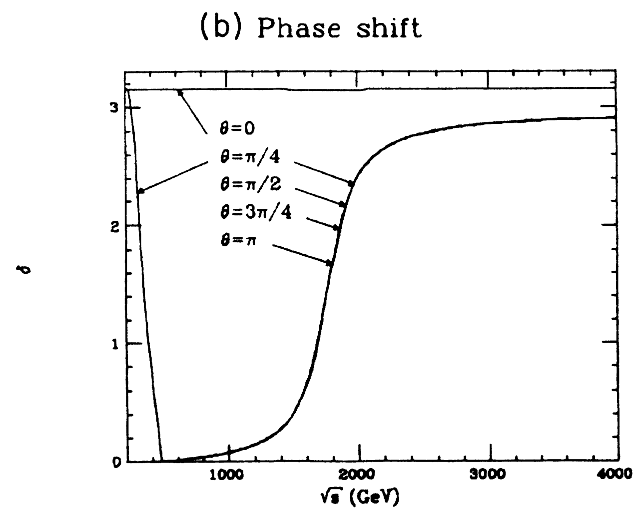


FIG. 6. (a) The phase shift δ in the $l=1$, $I=1$ channel as a function of θ at fixed $\sqrt{s}$ and (b) as a function of $\sqrt{s}$ at fixed θ.

and its parameters are reasonably approximated by the scaling principle.

Such a close analogy is not really to be expected in elementary Higgs models such as the linear sigma model. As an example, Basdevant and Lee found a vector resonance, but much narrower than the actual rho.[10] Anyhow, such theories are now believed to be trivial or, if treated with a cutoff, to lead to a small effective coupling constant, i.e., to no strongly interacting sector at all.[11]

Thus our present exploration of models seems to imply the presence of a vector resonance if there is a strongly interacting sector, and does not offer an alternative to technicolorlike models (fortunately such models are still alive with the possibility of walking technicolor[12]). Truly, little is known about the Weinberg-Salam symmetry-breaking mechanism, and one would like to know for instance what spectrum will result from the lattice Weinberg-Salam theory without Higgs bosons.[13]

As a side remark it is worth mentioning that loop corrections due to very heavy fermions can also raise the $e^+e^- \to W^+W^-$ cross section.[14]

V. RESULTS AND DISCUSSION

For our numerical discussion we adopt the values (39) for m_V, Γ_V. The phase shift δ that parametrizes the FSI through Eqs. (33)–(36), is plotted in Fig. 6(a) at $\sqrt{s}$ fixed as a function of θ and in Fig. 6(b) at θ fixed as a function

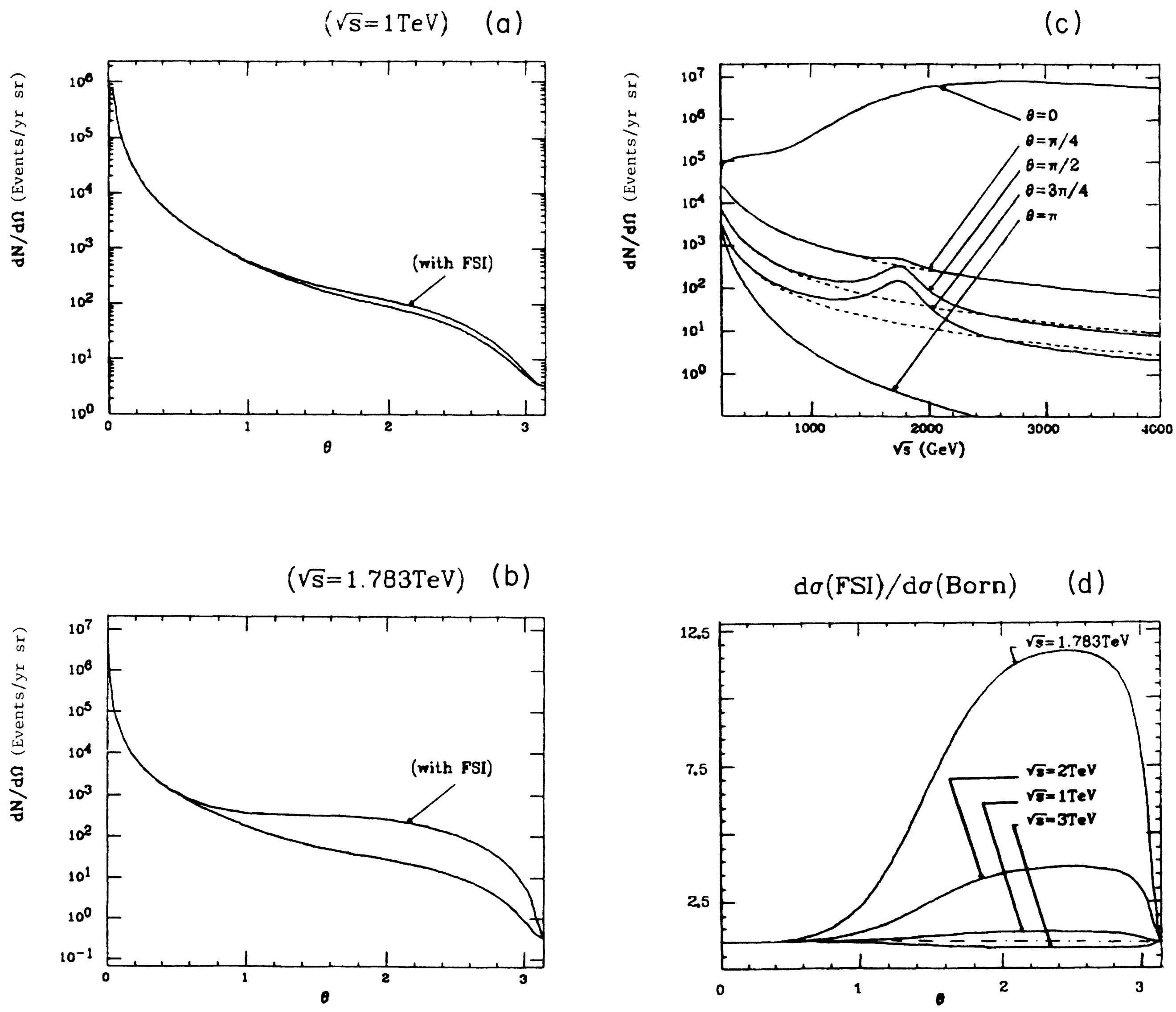


FIG. 7. (a) and (b) the number of events/yr sr without and with FSI as a function of θ at fixed $\sqrt{s}$ and (c) as a function of $\sqrt{s}$ at fixed θ. The luminosity assumed is $\mathcal{L} = 10^{33}$ cm^{-2}sec^{-1}. (d) The ratio $d\sigma(\text{FSI})/d\sigma(\text{Born})$ as a function of θ at fixed $\sqrt{s}$.

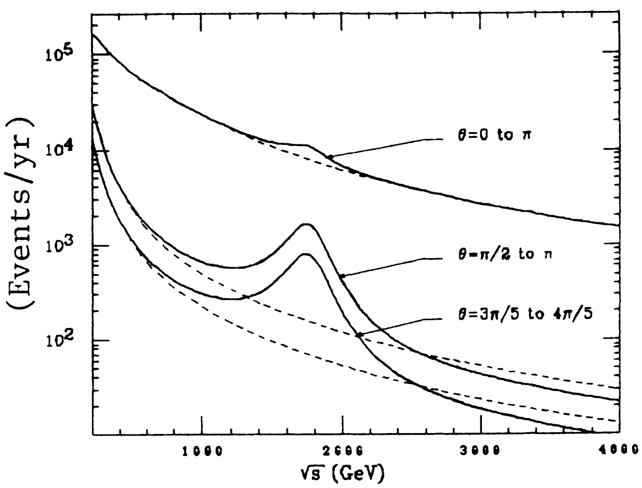


FIG. 8. The number of events/year integrated over the most favorable region $0.6\pi \le \theta \le 0.8\pi$, over the whole backward hemisphere, and over the total angular space (solid line, with FSI; dashed line, without FSI).

of the energy. At fixed θ we observe the typical resonance behavior at $\sqrt{s} = m_V$, as we see from Figs. 6(a) and 6(b). As a function of $\sqrt{s}$ we see a wide peak with $\delta \sim \pi$ at small θ and when θ grows δ falls sharply. When s grows, the peak at $\delta \sim \pi$ becomes narrower, and the phase shift grows on average, $\tan\delta$ being maximum at $\sqrt{s} = m_V$.

We will assume a luminosity of $\mathcal{L} = 10^{33}$ $\mathrm{cm^{-2}sec^{-1}}$ and plot *the number of expected* $e^+e^- \rightarrow W^+W^-$ *events per year and per sr.* In Figs. 7(a) and 7(b) we plot the number of events without FSI and with FSI as a function of θ for different fixed values of $\sqrt{s}$. As we see, the effect becomes sizable in the backward hemisphere, being maximum at the resonance. In Fig. 7(c) we plot the number of events per year as a function of $\sqrt{s}$ at different values of θ. In Fig. 8 we plot the total number of events/year as a function of $\sqrt{s}$ integrated over the most favorable interval $0.6\pi \le \theta \le 0.8\pi$ and the same quantity for the more easily experimentally accessible backward hemisphere $\pi/2 \le \theta \le \pi$, and for the whole range $0 \le \theta \le \pi$. The effect

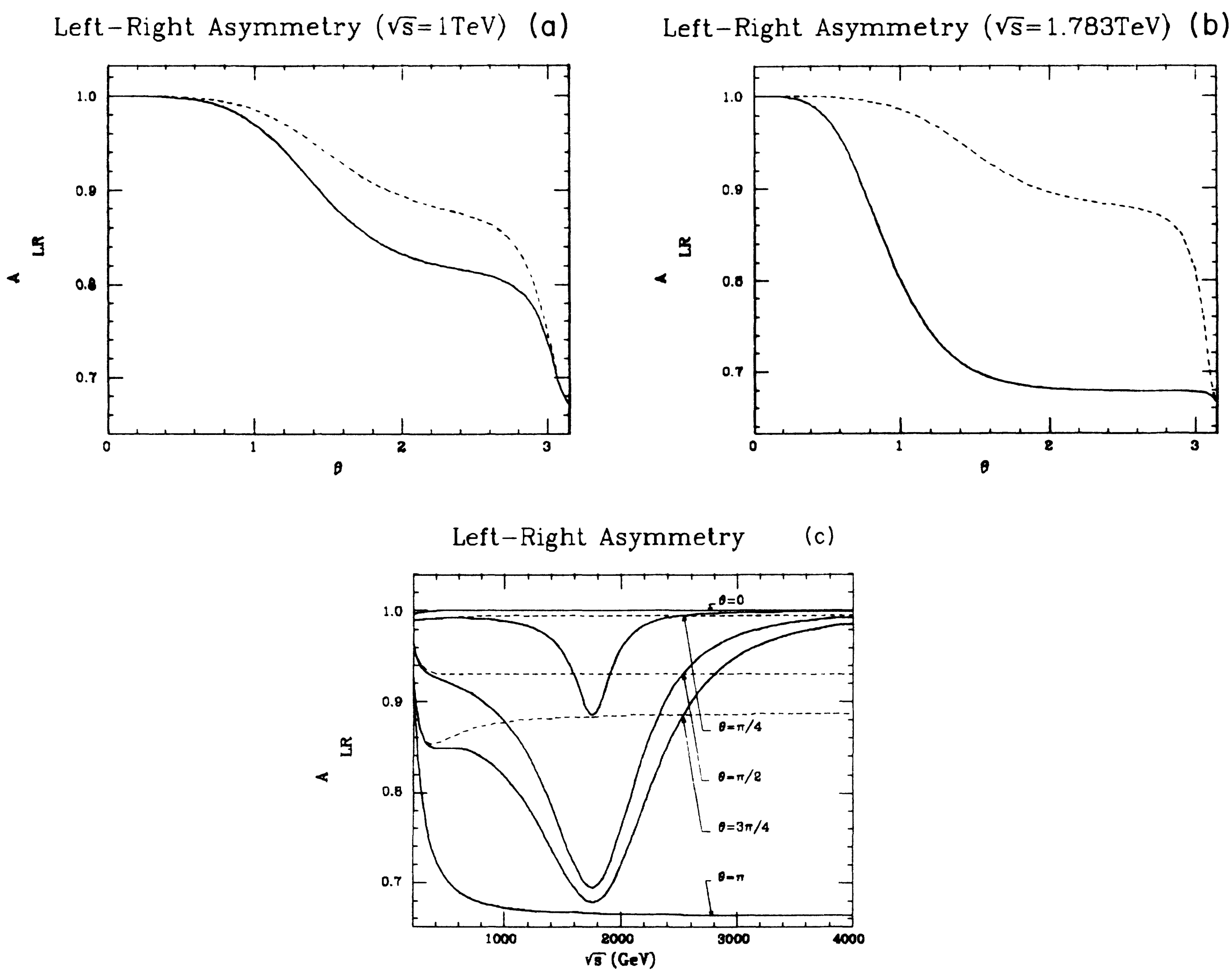


FIG. 9. (a) and (b) the left-right asymmetry A_{LR} as a function of θ at fixed values of $\sqrt{s}$, and (c) as a function of $\sqrt{s}$ for fixed values of θ (solid line, with FSI; dashed line, without FSI).

could be quite visible as we find an effect of one order of magnitude at the resonance. We must emphasize however that at these energies the total number of events is rather small: in the Born approximation, at 1 TeV we have 220 W^+W^- per year, and 50 W^+W^- at 2 TeV with about 30% of $W_L W_L$. At the resonance we will have a huge increase of the total number of WW pairs due to the FSI among the longitudinal W bosons. We find indeed an increase of one order of magnitude of the total number of events, relative to the Born approximation, in this region [Fig. 7(d)].

For completeness, let us consider the L-R asymmetry and the effect on it of a FSI, recently considered by Chiapetta and Feruglio[15] in a different theoretical scheme than ours: namely, a nonlinear realization of the symmetry-breaking sector of the standard model. The point in common with our work is that they consider the possibility of a vector resonance, although with rather low values for its mass, unrealistic in our opinion. We consider the asymmetry

$$A_{LR}(s,\theta)=\frac{\sigma_L(s,\theta)-\sigma_R(s,\theta)}{\sigma_L(s,\theta)+\sigma_R(s,\theta)}\ , \tag{45}$$

where $\sigma_L(\sigma_R)$ is obtained from the $A_L(A_R)$ amplitudes, Eqs. (10), (13), and (14). In Figs. 9(a) and 9(b) we plot A_{LR} as a function of θ for a few values of $\sqrt{s}$, with and without a FSI in the P wave. Finally, in Fig. 9(c) we plot A_{LR} as a function of $\sqrt{s}$ for a few values of θ. Our results for the left-right asymmetry agree qualitatively with the authors of Ref. 15.

To conclude, we have studied the effect of a strongly interacting Higgs sector on the reaction $e^+e^-\rightarrow W^+W^-$. As the lowest partial wave is the P wave, we have assumed the existence of a vector resonance in the channel of production of two longitudinally polarized W bosons, at a mass $m_V\sim 1.78$ TeV, as suggested by a technicolor scheme. In the Born approximation, longitudinally polarized $W_L W_L$ pairs are mainly produced in the backward hemisphere. Using a Gounaris-Sakurai ansatz for the FSI in the P wave we find, at the resonance, an order of magnitude increase of the $\sigma(e^+e^-\rightarrow W^+W^-)$ in the angular region where $W_L W_L$ pairs are mainly produced, $0.6\pi\le\theta\le 0.8\pi$. With a foreseen luminosity of $\mathcal{L}=10^{33}$ cm^{-2}sec^{-1}, we predict in the favorable region $0.6\pi\le\theta\le 0.8\pi$ a total number of 772 W^+W^- events per year while in the Born approximation one would expect about 67 events. In the whole backward hemisphere we expect 1586 events with the FSI and 151 in the Born approximation.

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*Permanent address: Laboratoire de Physique Theorique, Institut des Sciences Exactes, Université d'Oran-es-Sénia, Oran, Algérie.

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