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# Dynamical symmetry breaking and the top-quark mass

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Dynamical symmetry breaking by fermion condensates is assumed to be the source of gauge-boson masses in the $SU(3)_C \times SU(2)_L \times U(1)$ model. For the minimal case of $t\bar{t}$ condensation alone, the predicted $m_t$ ranges from about 98 to 450 GeV as the scale of "new physics," responsible for $m_t, \Lambda$, goes from $\infty$ to $\sim 1$ TeV. The lighter values, $m_t \lesssim 200$ GeV, all correspond to "great desert" scenarios with $\Lambda \gtrsim m_{\text{Planck}} \simeq 2\times 10^{19}$ GeV. Including a fourth generation of fermion condensates can lower $m_t$. In the case of maximal $t$-$t'$ mixing, we find $m_t \simeq m_{t'} \simeq (\sqrt{3}/2)m_{b'}$ with $m_t$ ranging from about 90 to 250 GeV, and $m_t \simeq 140$ GeV for $\Lambda \simeq 10^{15}-10^{19}$ GeV. For smaller mixing, $m_t$ is lowered and $m_{t'}$ approaches $m_{b'}$. We speculate as to how, using strong-coupling unitarity conditions, one might have $m_t \lesssim 200$ GeV with $\Lambda$ far below $m_{\text{Planck}}$. Phenomenological constraints and consequences are also discussed.

Experimental searches have so far failed to discover the top quark. Recent results from CERN and Fermilab indicate $m_t \gtrsim 60$ GeV and that bound could soon rise to $\sim 90$ GeV when the Collider Detector Facility (CDF) at Fermilab analysis is complete. Why is the top quark so heavy relative to all other known fermions? Is nature telling us that there is something special about the top quark?

Recently, a number of authors[1-4] have speculated that the large top-quark mass may be a signal of dynamical symmetry breaking via $t\bar{t}$ condensation.[5] In such a scheme, some as yet unknown dynamics spontaneously breaks the $SU(2)_L \times U(1)$ symmetry by providing the top quark with mass. That mechanism gives rise to Goldstone bosons in the pseudoscalar $t\bar{b}$, $\bar{t}b$, and $t\bar{t}$ channels which become the longitudinal components of the $W^\pm$ and $Z$ bosons and endow them with mass. An attractive feature of such a scenario is that one can predict $m_t$ in terms of the known gauge-boson masses and couplings. The prediction is, however, sensitive to the scale of "new physics," $\Lambda$, responsible for $m_t$. As we shall see, $m_t$ ranges from about 98 GeV all the way to roughly 450 GeV (the perturbative unitarity bound)[6,7] as $\Lambda$ goes from $\infty$ to $\sim 1$ TeV. However, as recently emphasized by Bardeen, Hill, and Lindner,[4] there is a region of rather stable predictions[8] $m_t \simeq 220-230$ GeV for $\Lambda$ in the superstring-grand-unified-theory (GUT) regime $10^{19}-10^{15}$ GeV. So, determining $m_t$ may provide a window to the physics of very short distances. Of course, in such a minimal picture, there is an assumed "great desert," i.e., no new physics between $m_t$ and $\Lambda$ (modulo physics associated with the top quark), a bleak prospect for high-energy physics at accelerators. It implies that discovery of the top quark and study of its properties (including bound states) could mark the last hurrah or final frontier until we learn how to probe distances of $O(1/\Lambda)$.

If there is a "great desert" up to $10^{15}-10^{19}$ GeV and $m_t \simeq 220-230$ GeV, the top quark will not be directly discovered until the Superconducting Super Collider (SSC) turns on (more than a decade from now). In the meantime, we might discern $m_t$ from high-precision measurements of electroweak parameters. We, subsequently, discuss that possibility.

Before starting our analysis, let us comment on recent considerations of $t\bar{t}$ condensation as the source of dynamical symmetry breaking. Two groups, Miransky, Tanabashi, and Yamawaki[2] (MTY) and Bardeen, Hill, and Lindner[4] (BHL), have suggested a specific mechanism for self-consistent top-quark mass generation, a gauged Nambu–Jona-Jasinio model with four-Fermi interactions. The four-Fermi interactions are assumed to be effective manifestations of some "new" more fundamental interactions at short distances $1/\Lambda$. We, on the other hand, have kept our approach fairly general, i.e., not tied to a specific top-quark-mass-generating mechanism, and advocated use of the renormalization-group equations to circumvent unknown dynamics. We have, however, noted that the predicted $m_t$ is quite sensitive to $\Lambda$, the scale of "new physics" and given a prescription for estimating its effect. We later compare that prescription with the BHL compositeness condition which identifies $\Lambda$ with the scale where the effective Yukawa coupling of the top quark blows up. We remark that the Nambu–Jona-Lasinio model is a very interesting and instructive approach for sorting out the physics without specifying the dynamics. In that regard, it is similar to the ordinary Higgs mechanism, but in some respects has more predictive power. It shares, however, with the Higgs mechanism the problem of quadratic divergences (cut off by $\Lambda$). As such, a hierarchy problem exists when $\Lambda$ is very large and a fine-tuning is required. A possible way of reducing $\Lambda$ while keeping $m_t \lesssim 200$ GeV is to introduce a fourth generation of fermions. That possibility is demonstrated in this paper.

Our intention, then, is to first elaborate and extend our earlier study of dynamical symmetry breaking via renormalization-group equations, and apply that analysis to determine $m_t$ for different $\Lambda$. We compare our results with the approach of BHL and draw insight from their physical picture. For the case of a relatively small $\Lambda$, we propose a constraint for determining $\Lambda$ based on the requirement of perturbative unitarity.[7] Since, as emphasized by MTY and BHL, $m_t \simeq 220$–$230$ GeV is the natural scale for the minimal "great desert" model with $\Lambda \simeq 10^{15}$–$10^{19}$ GeV, we take this opportunity to comment on low-energy $m_t$ constraints and future precision electroweak measurements necessary to pinpoint $m_t$ (assuming no other "new physics") from loop effects. We also speculate that very large $m_t$ may not be ruled out, if "new physics" associated with $\Lambda$ partially screens the top contribution. Finally, anticipating that $m_t$ may turn out to be $<200$ GeV and preferring to reduce $\Lambda$ (if only because we find "great desert" scenarios up to near the Planck scale depressing), we examine the effect of a fourth generation of fermions. We show why $m_t$ is generally reduced for a given $\Lambda \leq m_{\rm Planck}$ and describe how $\Lambda$ may be reduced far below the Planck scale, perhaps even down to SSC energies for $m_t \lesssim 200$ GeV.

We begin by reviewing results from our previous work[3] relevant for the present discussion. There it was shown that the renormalization-group equation for

$$\kappa_t(\mu) \equiv \frac{\alpha_2(\mu)}{2} \frac{m_t^2(\mu)}{m_W^2(\mu)} \tag{1}$$

with $\alpha_2(\mu) \equiv g_2^2(\mu)/4\pi$ the running $SU(2)_L$ gauge coupling, provides a convenient means to study the relationship between $m_t$ and $m_W$ without specifying the underlying dynamics. It also avoids issues about gauge dependence. Two cases were considered. In the first case, effects of a Higgs scalar (or for this discussion, a very tightly bound pointlike $0^{++}$ $t\bar{t}$ state) were included in the evolution equation for $\kappa_t$, while in the second case, they were omitted. The two cases correspond to effective linear and nonlinear $\sigma$ models at scales $\ll \Lambda$. Ignoring light fermion masses and electroweak loops, one finds, for the effective $\sigma$ model in leading-log approximation,

$$\mu \frac{d}{d\mu} \kappa_t(\mu) = \frac{3}{4\pi} \kappa_t^2 - \kappa_t \left[ \frac{4}{\pi} \alpha_3 - \frac{3}{2\pi} \kappa_t \right] \quad (\text{pointlike } 0^{++}\ t\bar{t}) \tag{2}$$

with $\alpha_3$ the $SU(3)_C$ gauge coupling of QCD, while, in the nonlinear case,

$$\mu \frac{d}{d\mu} \kappa_t(\mu) = -\kappa_t \left[ \frac{4}{\pi} \alpha_3 - \frac{3}{2\pi} \kappa_t \right] \quad (\text{no pointlike } 0^{++}\ t\bar{t}) . \tag{3}$$

Which of these is relevant depends on the underlying dynamics. If $\Lambda$ is not very large in comparison with $m_t$, the $0^{++}$ Higgs-type bound state is less likely to be pointlike. Instead, it may be a broad resonance similar to the $\sigma$ of light-quark QCD with mass $O(\Lambda)$ and should not be necessarily included in the evolution of $\kappa_t(\mu)$. On the other hand, for $\Lambda$ very large, it is more likely that a very tightly bound $0^{++}$ $t\bar{t}$ state mimics the fundamental Higgs scalar and Eq. (2) is more relevant. Our reasoning is based on perturbative unitarity arguments[6,7] for $t_\pm t_\pm \to W_L^+ W_L^-$ or $Z_L Z_L$ scattering amplitudes (subscripts denote polarization). Perturbative unitarity in the zeroth partial-wave amplitude will be violated before $\sqrt{s} = \Lambda$ if

$$\Lambda \leq \frac{4\pi\sqrt{2}}{3 G_F m_t} \tag{4}$$

($G_F = 1.166\,37 \times 10^{-5}$ GeV$^{-2}$) unless a Higgs-type $0^{++}$ state or some equivalent physics[6] cancels the bad large-$s$ behavior. So, if $\Lambda \gtrsim 5$ TeV, we expect the pointlike $0^{++}$ $t\bar{t}$ bound-state scenario described by Eq. (2) to be more plausible. Within the framework of the four-Fermi gauged Nambu–Jona-Lasinio model, BHL have (effectively) argued that the $0^{++}$ $t\bar{t}$ contribution in (2) should be included, and that up to $\simeq \Lambda$ it acts just like an ordinary fundamental Higgs scalar, with Yukawa coupling $\sim \sqrt{\kappa_t}$ to $t\bar{t}$. Given that we primarily consider very large $\Lambda$ and the persuasive nature of the BHL argument, we will assume throughout the rest of this paper that a tightly bound $0^{++}$ $t\bar{t}$ state does contribute as in (2). The effect of decoupling the $0^{++}$ is discussed in Ref. 3.

We can get a nice closed-form solution to (2) by employing

$$\mu \frac{d}{d\mu} \kappa_t = \left[ \mu \frac{d}{d\mu} \alpha_3 \right] \left[ \frac{d\kappa_t}{d\alpha_3} \right] , \tag{5a}$$

$$\mu \frac{d}{d\mu} \alpha_3 = -\frac{1}{2\pi} (11 - \tfrac{4}{3} n_g) \alpha_3^2 , \tag{5b}$$

where for $\mu > m_t$, $n_g = 3$, the number of generations. That converts Eq. (2) into a first-order homogeneous nonlinear differential equation,[9]

$$-\frac{7}{2\pi} \alpha_3^2 \frac{d\kappa_t}{d\alpha_3} = \frac{9}{4\pi} \kappa_t^2 - \frac{4}{\pi} \alpha_3 \kappa_t , \tag{6}$$

which admits the general family of solutions

$$\kappa_t(\mu) = \frac{2}{9} \frac{\alpha_3^{8/7}(\mu)}{\alpha_3^{1/7}(\mu) - C} , \tag{7a}$$

$$\alpha_3^{-1}(\mu) = \alpha_3^{-1}(m_t) + \frac{7}{2\pi} \ln(\mu/m_t) \tag{7b}$$

with $C$ an arbitrary constant. Specifying $C$ will determine $m_t$ via the value of $\kappa_t(m_t)$. In Ref. 3, no commitment to any "new physics" was made and the infrared stable solution $C = 0$ was advocated.[9] (It leads to $m_t \simeq 98$ GeV.) However, as stated there, it appears unlikely that QCD (or any other standard-model interaction) can cause symmetry breaking at the electroweak scale $\simeq 250$ GeV; so one seems forced to assume that some as yet unknown "new physics" gives rise to $m_t$ which in turn generates $W^\pm$ and $Z$ masses. To study the effect of "new physics" at scale $\Lambda$, the fermion loops responsible for $W^\pm$ and $Z$ mass generation were computed with a sharp cutoff at the "new physics" scale $\Lambda$. That prescription gave $C = \alpha_3^{1/7}(\Lambda)$ in (7a), i.e.,

$$\kappa_t(\mu) = \frac{2}{9} \frac{\alpha_3^{8/7}(\mu)}{\alpha_3^{1/7}(\mu) - \alpha_3^{1/7}(\Lambda)} . \tag{8}$$

It is easy to understand the zero in the denominator. The running $W$ mass is obtained from[3] (including the $0^{++}$ $t\bar{t}$ effect)

$$m_W^2(\mu) \simeq \frac{3}{8\pi}\int_{\mu^2} dp^2 \frac{\alpha_2(p)\Sigma_t^2(p^2)}{p^2+\Sigma_t^2(p^2)} , \tag{9}$$

where $\Sigma_t(p^2)$ is the top-quark self-energy. If we apply a sharp cutoff at $p^2=\Lambda^2$, then $m_W^2(\mu)$ must vanish at $\mu=\Lambda$ because there is no available phase space. That translates into $\kappa_t(\Lambda)$ blowing up and the form in (8).

In the limit $\Lambda\to\infty$, Eq. (8) goes into the $C=0$ solution, because asymptotic freedom requires $\alpha_3(\infty)=0$. However, the small exponent $\frac{1}{7}$ makes Eq. (8) quite sensitive to $\Lambda$. One should, therefore, view $C=0$, which corresponds to $m_t\simeq 98$ GeV as an approximate limiting case in which "new physics" either plays no role other than to generate $m_t$ or is effectively pushed to $\Lambda=\infty$. Since finite $\Lambda$ increases $m_t$, 98 GeV is an approximate minimum value above which consideration of dynamical symmetry breaking of the electroweak symmetry becomes interesting. Alternatively, one may view the range of $m_t$ corresponding to $\Lambda$ extending to $\infty$ as a measure of the uncertainty from "new physics" effects.

At this point it is useful to compare our constraint on $\Lambda$ as embodied in Eq. (8) with the BHL condition.[4] Those authors define $\Lambda$ to be the scale at which $\kappa_t(\mu)$, their effective Yukawa coupling, blows up, the same as in (8). They arrive at that condition, however, in a much more formal manner by considering the effective potential of a gauged Nambu–Jona-Lasino model. The blow-up of $\kappa_t(\mu)$ indicates strong coupling which is generally an indication of a new-physics threshold.[6,7] Using perturbative $\beta$ functions to evolve couplings into a strong-coupling regime, however, can be dangerous and sometimes misleading. It has, therefore, been recently suggested[7] that one use perturbative unitarity bounds on couplings rather than their blow-up as an indicator of the onset of strong coupling and "new physics." For the case of $t\bar{t}\to t\bar{t}$ scattering, that constraint becomes

$$\kappa_t(\mu)\le \tfrac{2}{3} . \tag{10}$$

Requiring $\kappa_t(\Lambda)=\frac{2}{3}$ (i.e., saturate the unitarity bound) rather than $\infty$ should be more reliable in a perturbative approach; so we use it in our subsequent discussion. The difference is most important for relatively small $\Lambda$. To estimate $m_t$ as a function of $\Lambda$ we employ

$$\kappa(m_t)=\frac{2}{9}\frac{\alpha_3^{8/7}(m_t)}{\alpha_3^{1/7}(m_t)-C(\Lambda)} , \tag{11}$$

where $C(\Lambda)$ is determined by the unitarity bound

$$\kappa(\Lambda)=\frac{2}{3}=\frac{2}{9}\frac{\alpha_3^{8/7}(\Lambda)}{\alpha_3^{1/7}(\Lambda)-C(\Lambda)} . \tag{12}$$

As input, we take

$$\alpha_3(m_W)\simeq 0.107 ,\quad \alpha_2(m_W)\simeq 0.0340 ,\quad m_W\simeq 80 \text{ GeV} \tag{13}$$

and iterate using

$$\alpha_3^{-1}(m_t)=\alpha_3^{-1}(m_W)+\frac{23}{6\pi}\ln(m_t/m_W) ,$$
$$\alpha_2^{-1}(m_t)=\alpha_2^{-1}(m_W)+\frac{25}{12\pi}\ln(m_t/m_W) , \tag{14}$$
$$m_W^2(m_t)\simeq m_W^2[\alpha_3(m_t)/\alpha_3(m_W)]^{1/7} .$$

Also, to get the physical mass,[3] we finally multiply $m_t(m_t)$ by $1+\frac{4}{3}\alpha_3(m_t)/\pi$. That procedure misses the electroweak leading logarithms and, of course, neglects higher orders. We expect electroweak leading logarithms to increase[10] our $m_t$ values by about 4%. Using the above prescription leads to predictions for $m_t$ that range from 98 to about 450 GeV as $\Lambda$ goes from $\infty$ to $\sim 1$ TeV. For $\Lambda\simeq 10^{15}-10^{19}$ GeV, we find $m_t\simeq 214-202$ GeV. That result is about 17 GeV below the BHL result 231–220 GeV for the same range of $\Lambda$. At least part of the difference, about 6 GeV, stems from different input parameters and about 4 GeV from our use of the perturbative unitarity condition. The remaining 7-GeV difference is consistent with our neglect of electroweak leading logarithms and approximate iteration scheme. One might, therefore, increase our $m_t$ values by about 4% or fold that effect into the higher-order uncertainties.

Even stretching the uncertainties, it appears that $m_t$ cannot be brought much below 200 GeV for $\Lambda\lesssim 10^{19}$ GeV, as long as we assume a "great desert." (Higher-order effects are likely to increase $m_t$, but we have not undertaken a complete calculation.) Of course, the "new physics" beyond $\Lambda$ could always decrease $m_t$, but we need a complete theory to address that possibility. How does $m_t\gtrsim 200$ GeV compare with indirect phenomenological bounds on $m_t$ coming from loop-induced $m_t$-dependent radiative corrections? The tightest bound,[1] $m_t\lesssim 200$ GeV at 90% C.L., comes from global fits to weak neutral- and charged-current data along with gauge-boson masses. Those quantities exhibit $O(\alpha_2 m_t^2/m_W^2)$ corrections due to weak isospin breaking in the $W$ and $Z$ two-point function from the $t$-$b$ mass difference.[12] For very large $m_t\simeq 300-450$ GeV, it seems difficult to ignore that bound. A possible loophole is compensating loop effects that partially cancel or screen the large $m_t$-dependent effects. Such a scenario is plausible if additional low scale new physics enters or if $\Lambda$ is not very large such that "new physics" associated with it does not decouple.

For very large $\Lambda$, one expects the usual perturbative loop corrections to carry over to the dynamical-symmetry-breaking case. (Unless some other additional physics $\ll\Lambda$ comes into play.) A specific realization of that point has been given by BHL. They showed how within the framework of the gauged Nambu–Jona-Lasino model with very large $\Lambda$, top-quark mass corrections to the $\rho$ parameter[12] were equivalent to those computed with a standard Higgs mechanism, i.e., $\Delta\rho=(3G_F/8\sqrt{2}\pi^2)m_t^2$. So, for $\Lambda$ very large one expects the usual bounds on $m_t$ to apply. Of course, as emphasized by BHL, $m_t<200$ GeV is only a 90%-C.L. bound which extends to about 230 GeV at 95% C.L.; so, it is much too premature to dismiss $m_t$ values in the 200–250 GeV range. In fact, this new "great desert" scenario with $\Lambda \gtrsim 10^{15}$ GeV should provide a strong incentive to push low-energy weak parameter measurements such as $\sin^2\theta_W$, $\rho$, $m_W$, and $m_Z$ to the highest precision possible to uncover a hint of $m_t \gtrsim 200$ GeV. Indeed, if that idea is correct, it implies lean years ahead for high-energy physics. The SSC is the only approved project capable of finding and studying a top quark with $m_t \gtrsim 200$ GeV and it is at least a decade away. In the meantime, top-quark physics could be indirectly studied (inferred) from small loop corrections to $m_Z$, $m_W$, $\sin^2\theta_W$, and $\rho$. Already $m_Z$ measurements have reached high precision,[13]

$$m_Z = 91.11 \pm 0.23 \text{ GeV Mark II} ,$$
$$m_Z = 90.9 \pm 0.3 \pm 0.2 \text{ GeV CDF} , \tag{15}$$

and the errors will drop much further when CERN LEP gets data. What is now needed is a measurement of a second parameter with precision at least comparable to (15) which can be compared with $m_Z$. For $m_t \simeq 200$ GeV, one needs a measurement of $m_W$ to about $\pm 100$ MeV or $\sin^2\theta_W$ to 1% to infer the value of $m_t$ to about $\pm 15$ GeV. Such measurements are possible, but they must await new initiatives such as the DØ detector at Fermilab, LCD at LAMPF, polarization asymmetry measurements at the SLAC Linear Collider (SLC), etc. One could also directly try to measure $\rho$ by mounting a new round of high-precision deep-inelastic $\overset{(-)}{\nu}_\mu N$ scattering experiments. To determine $m_t$ to 15 GeV would require about a 0.2% determination of $\rho$. We might comment that for $m_t \simeq 230$ GeV one expects[14] [based on (15)] $m_W \simeq 80.8$ GeV and $\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2 = 0.213$. The latter value is in slight conflict with the present world average from deep-inelastic $\nu_\mu N$ scattering,[11]

$$\sin^2\theta_W = 0.231 \pm 0.003 \pm 0.005 \quad (\nu_\mu N \text{ data}) , \tag{16}$$

which when combined with (15) favors $m_t \simeq 100$–$120$ GeV (with large uncertainties). Of course, as emphasized by BHL, it is premature to rule out $m_t \simeq 230$ GeV on the basis of such indirect, less-than-overwhelming evidence. If $\sin^2\theta_W = 0.231$ as suggested by (16), one expects $m_W = 79.9$ GeV. It will be interesting to see whether the ongoing CDF determination of $m_W$ favors 80.8 or 79.9 GeV. At present, their central value is 80.0 GeV, but the large error $\approx \pm 0.7$ GeV makes the result inconclusive.

Information about $m_t$ can also be inferred[15] from loop-induced flavor-changing neutral-current processes such as $K_L \to \mu^+\mu^-$, $K^+ \to \pi^+\nu\bar{\nu}$, etc. Unfortunately, uncertainties in the CKM quark mixing matrix and in some cases hadronic effects make a precise determination of $m_t$ difficult. Nevertheless, in a few years, global studies of rare decays and $CP$ violation in the kaon system should be able to discern a heavy $m_t$ effect, particularly if $m_t \gtrsim 200$ GeV (barring screening from new physics). Similar comments apply to $B$ decays and oscillations which may ultimately be our best indirect probe of top-quark physics.

What if $m_t$ is found to lie in the range 90–180 GeV, the approximate discovery potential of Fermilab's $p\bar{p}$ collider? Within the dynamical-symmetry-breaking picture, that range could be accommodated in many ways. One could effectively push $\Lambda \gg 10^{19}$ GeV perhaps by arguing that the "new physics" does not introduce a sharp cutoff or suggest larger uncertainties in the calculation (if $m_t$ is not too far below 200 GeV). Another possibility is to introduce additional low-energy physics, i.e., eliminate the "great desert" hypothesis. There are, of course, many ways to populate the desert. The simplest, albeit least imaginative possibility, is to introduce a fourth generation of fermions with condensates which also contribute to $W^\pm$ and $Z$ mass generation. In general, one might expect that the more condensates there are, the less their individual contribution need be and one can get away with smaller fermion masses. There are a number of interesting new-physics possibilities suggested by such a scenario. The remainder of this paper will discuss some generic features of a fourth generation and its possible role in dynamical symmetry breaking.

One could use the fourth-generation condensates $t'\bar{t}'$ and $b'\bar{b}'$ (along with perhaps the charged lepton $L\bar{L}$) alone to produce $W^\pm$ and $Z$ masses. Solving the coupled renormalization-group equations for $\kappa_{t'}$ and $\kappa_{b'}$, one finds (neglecting electroweak corrections and all other fermion masses and mixing)

$$\kappa_{t'}(\mu) \simeq \kappa_{b'}(\mu) \simeq \frac{7}{18} \frac{\alpha_3^{24/17}(\mu)}{\alpha_3^{7/17}(\mu) - \alpha_3^{7/17}(\Lambda)} , \tag{17}$$

where, for $\mu > m_{t'}$,

$$\alpha_3^{-1}(\mu) = \alpha_3^{-1}(m_{t'}) + \frac{17}{6\pi} \ln\mu/m_{t'} \tag{18}$$

and the perturbative unitarity constraint is now[6,7] $\kappa_{t'} \leq \frac{2}{5}$ from a coupled-channel analysis.

Naively, for $\Lambda \to \infty$, one expects by comparing (18) with (7a) an increase by $\sqrt{7/4}$ over our previous $m_t$ analysis. However, for finite $\Lambda$ that is modified because of the slower running of $\alpha_3$ and the larger exponent $\frac{7}{17}$ in (17) compared with our previous $\frac{1}{7}$. In other words, the prediction is less sensitive to $\Lambda$. For $\Lambda = \infty$, one expects $m_{t'} \simeq m_{b'} \simeq 128$ GeV. That value increases to about 190 GeV for $\Lambda \simeq 10^{15}$ GeV (Ref. 16). For $\Lambda$ as low as 1 TeV, one finds $m_{t'} \simeq m_{b'} \simeq 350$ GeV, the unitarity bound.[6] However, because the $t'$-$b'$ mass difference is small, they do not upset low-energy phenomenology (unless they have significant mixing with the other generations) even for very large masses: $\sim 350$ GeV.

Given that, for a wide range of $\Lambda$, $m_{t'}$ and $m_{b'}$ are predicted to be not much larger than our expectations about $m_t$ (from experiment), it is unrealistic to leave the top quark out of the analysis. Instead, we consider the coupled renormalization-group equations for $\kappa_t$, $\kappa_{t'}$, $\kappa_{b'}$, and $\kappa_L$. (We subsequently neglect $\kappa_L$.) Again ignoring electroweak effects and light fermion masses, one finds

$$\mu\frac{d}{d\mu}\kappa_t=\frac{3}{4\pi}\kappa_t\left[\frac{\kappa_t}{c_1}-\frac{\kappa_{b'}}{c_2}\sin^2\theta\right]-\kappa_t\left[\frac{4}{\pi}\alpha_3-\frac{1}{2\pi}(3\kappa_t+3\kappa_{t'}+3\kappa_{b'}+\kappa_L)\right],$$
$$\mu\frac{d}{d\mu}\kappa_{t'}=\frac{3}{4\pi}\kappa_{t'}\left[\frac{\kappa_{t'}}{c_2}-\frac{\kappa_{b'}}{c_2}\cos^2\theta\right]-\kappa_{t'}\left[\frac{4}{\pi}\alpha_3-\frac{1}{2\pi}(3\kappa_t+3\kappa_{t'}+3\kappa_{b'}+\kappa_L)\right],\tag{19}$$

$$\mu\frac{d}{d\mu}\kappa_{b'}=\frac{3}{4\pi}\kappa_{b'}\left[\frac{\kappa_{b'}}{c_2}-\frac{\kappa_{t'}}{c_2}\cos^2\theta-\frac{\kappa_t}{c_1}\sin^2\theta\right]-\kappa_{b'}\left[\frac{4}{\pi}\alpha_3-\frac{1}{2\pi}(3\kappa_t+3\kappa_{t'}+3\kappa_{b'}+\kappa_L)\right],$$
$$\mu\frac{d}{d\mu}\kappa_L=\frac{3}{4\pi}\frac{\kappa_L^2}{c_3}+\kappa_L\frac{1}{2\pi}(3\kappa_t+3\kappa_{t'}+3\kappa_{b'}+\kappa_L),$$

where $\theta$ is the mixing angle between the third and fourth generations. (It may have its own renormalization-group equation.) The $c_i$, $i=1,2,3$ are the relative weights of different scalar and pseudoscalar contributions (including Goldstone bosons) with $c_1+c_2+c_3=1$. In general, the $c_i$-dependent terms in (19) may be much more complicated; however, to ascertain the structure of those terms more precisely requires knowledge about the fermion-mass-generating dynamics. We do not address that issue. A symmetric scenario, $c_1=c_2=\frac{1}{2}$ and $\sin^2\theta\simeq\frac{1}{2}$ seems natural. That situation is analogous to a two-Higgs-doublet model with equal vacuum expectation values.[17] However, depending on the effective potential, there could be even more (or less) scalars and low-energy pseudoscalars and a more complicated pattern of couplings to fermions. For simplicity, we will take $c_1=c_2=\frac{1}{2}$, $c_3=0$ and assume the form in (19). The general features of our results are not terribly sensitive to that assumption, except that it naturally leads to large $t$-$t'$ mixing.

To simplify the analysis, we neglect $\kappa_L$. That seems like a valid approximation, since $L$ carries no color and is thus only about $1/\sqrt{3}$ as effective as the quarks for providing $W^\pm$ and $Z$ masses. Also, if we use the BHL condition that the underlying dynamics of mass generation at $\Lambda$ requires all large Yukawa couplings to become strong at $\Lambda$, then $\kappa_L$ at low energies should be relatively small since it has no QCD interactions (only QFD) to keep it from growing. (Including $\kappa_L$ would reduce our subsequent predictions for quark masses but not significantly.)

If $\kappa_t$, $\kappa_{t'}$, and $\kappa_{b'}$ are all to become strong (diverge) at the same $\Lambda$, independent of $\Lambda$, then a consistent solution to (19) is given by (for $c_1=c_2=\frac{1}{2}$)

$$\kappa_t\simeq\kappa_{t'}\simeq\tfrac{3}{4}\kappa_{b'}\quad\text{with }\sin^2\theta=\tfrac{1}{2},$$

or

$$m_t\simeq m_{t'}\simeq 0.87 m_{b'}\tag{20}$$

with

$$-\frac{17}{6\pi}\alpha_3^2\frac{d\kappa_t}{d\alpha_3}=\frac{1}{2\pi}\kappa_t^2-\kappa_t\left[\frac{4}{\pi}\alpha_3-\frac{5}{\pi}\kappa_t\right].\tag{21}$$

(For a more complicated effective scalar sector $\sin^2\theta=\frac{1}{2}$ will not in general be required.)

Following our above prescription, $m_t$ is found from the general solution

$$\kappa_t(\mu)=\frac{7}{33}\frac{\alpha_3^{24/17}(\mu)}{\alpha_3^{7/17}(\mu)-C(\Lambda)}.\tag{22}$$

If we require that $\kappa_t(\mu)$ blow up at $\Lambda$, then $C(\Lambda)=\alpha_3^{7/17}(\Lambda)$. If instead we use the requirement of partial wave unitarity in $t\bar{t}\to t\bar{t}$ to determine at what strength strong couplings begins, we find[6,7] (approximately)

$$\kappa_t\le\tfrac{1}{5}\quad\text{(unitarity bound)}.\tag{23}$$

One can understand an extra factor of $\frac{1}{2}$ in (23) by thinking of a two-Higgs-doublet model with equal vacuum expectation values $v$. Since both give mass to the $W^\pm$ and $Z$, each $v$ is $1/\sqrt{2}$ times the one-Higgs-doublet case. To compensate, a fermion of given mass which couples to only one doublet must have a larger Yukawa coupling by $\sqrt{2}$. So, Yukawa couplings in multi-Higgs-doublet models are larger and unitary bounds on fermion masses thus more constraining. The extra factor of $\frac{1}{2}$ can be important for relatively small $\Lambda$ considerations. As an illustration, we take $c_1=c_2=\frac{1}{2}$ and fix $C(\Lambda)$ by saturating the unitarity bound at $\Lambda$ in (23):

$$\frac{\alpha_3^{24/17}(\Lambda)}{\alpha_3^{7/17}(\Lambda)-C(\Lambda)}=\frac{33}{35}.\tag{24}$$

Using that condition, one finds that $m_t$ ranges from about 95 to 250 GeV as $\Lambda$ goes from $\infty$ to $\sim 1$ TeV. The scale of "new physics" could be well below the GUT scale, and we can easily get $m_t\lesssim 200$ GeV. For enlarged more general effective scalar sectors, and a more constraining coupled-channel unitarity bound, one might even bring $\Lambda$ down farther and avoid even a "small desert." Perhaps $\Lambda$ could even be reduced to a scale accessible at the SSC.

Assuming that the GUT-superstring domain $10^{15}-10^{19}$ GeV is the scale $\Lambda$ of "new physics," we find $m_t\simeq 140$ GeV, an interesting region. For large $\Lambda$, that prediction is rather stable with respect to changes in the effective Higgs sector. With regard to the heavy charged lepton, we venture a "guess" based on the form of the renormalization-group equations at large scales that $\kappa_L\lesssim\kappa_t/3\simeq\kappa_{b'}/4$.

Inclusion of $\kappa_L$ in the renormalization-group equations then lowers $m_t$ by less than 4%. Therefore, in the case $\Lambda\simeq 10^{15}-10^{19}$ GeV, we expect, for maximal mixing, $\sin^2\theta\sim\frac{1}{2}$,

$$m_t\simeq m_{t'}\simeq 140\text{ GeV},\quad m_{b'}\simeq 160\text{ GeV},\qquad m_L\lesssim 80\text{ GeV}.\tag{25}$$

If $\sin^2\theta<\frac{1}{2}$, we expect $m_{t'}$ and $m_{b'}$ to move closer together while $m_t$ decreases. For no mixing, $m_{t'}\simeq m_{b'}$. Of course, the specific masses in (25) are not as important as the distinct pattern and range of values.

Even for near maximal mixing, one expects some splitting (at least several GeV) between $t$ and $t'$. It would be fun trying to untangle the simultaneous appearance of two new charge-$\frac{2}{3}$ quarks with nearly the same masses and mixing. The signal would be "top" production at a hadron collider with twice the expected cross section via glue-glue scattering.

In the case of low-energy phenomenology, the near degeneracy of $t$ and $t'$ means (for large $\Lambda$) they act together like top quarks at $\simeq 140$ GeV with no mixing. So, the $m_t \lesssim 200$ GeV constraint is easily satisfied. Loop-induced rare $K$ and $B$ decays are also essentially the same as for the top quarks alone at $\simeq 140$ GeV; however, there might be interesting consequences for $CP$ violation.

We find the idea of large (near maximal) mixing quite appealing in that it may help explain why $m_b \ll m_t$. The large mixing may stem from the $b$-$b'$ sector where mixing interactions could increase $m_{b'}$, and send $m_b$ toward zero. Note also, that large mixing could find its way into the first and second generations. For example, one might find $V_{ub'} \gtrsim V_{ub}$ or $V_{cb'} \gtrsim V_{cb}$ in the $4\times4$ CKM matrix, a rich source of new phenomenology. Of course, lack of knowledge of the effective scalar sector prevents any real prediction for $\sin^2\theta$.

A clear positive signal of the fourth generation would be discovery of a fourth neutrino either in precision $Z$ width measurements or in $e^+e^- \rightarrow \nu\bar{\nu}\gamma$. Also, since we expect $m_L < m_t$, the heavy charged lepton may be found at LEP II. In the above scheme, for large $\Lambda$, there should also be several charged and neutral tightly bound spin-0 states that act much like physical Higgs scalars (pseudoscalars).

In conclusion we have described two scenarios for dynamical symmetry breaking. The first minimal version employs $t\bar{t}$ condensation alone to break the symmetry. It suggests that the top quark is likely to be heavy, $\gtrsim 200$ GeV, for a "new physics" cutoff $\lesssim 10^{19}$ GeV. If the top quark is in the 200–230 GeV region, that scheme implies a "great desert" lies beyond the discovery of top and its associated Higgs-type $t\bar{t}\,0^{++}$ state; a depressing prospect. Our second scenario includes a fourth generation with charged lepton and heavy quarks near the top-quark mass. It exhibits potentially marvelous properties such as possible (near) maximal mixing and is rich in new phenomenology without upsetting low-energy constraints on $m_t$. In that scenario, a "great desert" is also possible, but only after the fourth generation along with its physics is untangled. Of course, the minimal model requires only heavy top quarks (and some unspecified dynamics at very short distances). We are almost certain that the top quark exists and is heavy; so that scheme has much in its favor. On the other hand, at present there is no hint of a fourth generation.

*Note added in proof.* Recent precision measurements of the $Z$ width at SLAC and CERN appear to rule out the possibility of a light fourth neutrino. A fourth generation with a heavy neutrino is still a viable possibility. In the scenario described in this paper, one would then expect $m_{\nu_L} \simeq m_L$ with both in the $\sim 70$ GeV range.

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