Isosinglet-quark decays to Higgs bosons in multi-Higgs-boson models

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We examine the decays of heavy $SU(2)_L$-singlet quarks to a Higgs boson and a light quark in a class of E_6 electroweak models with more than one Higgs multiplet. We calculate the branching fractions for several scenarios involving different Higgs sectors and quark mixing. We find that the enhanced couplings and decays to charged and pseudoscalar Higgs bosons can significantly increase the overall branching fraction to Higgs bosons. The increased Higgs-boson production rate improves the detection possibilities at hadron colliders of higher-mass Higgs bosons.

I. INTRODUCTION

In the minimal standard model[1] of the electroweak interactions, experimental verification of the Higgs symmetry-breaking mechanism has a high priority. However, if the mass of the neutral Higgs boson is comparable to the gauge-boson masses, this goal may not be accomplished until the next generation of hadron colliders such as the Superconducting Super Collider (SSC) become operational. On the other hand, new possibilities for Higgs-boson discovery are present in many extensions of the standard model.[2] An interesting possible new mechanism for producing the Higgs boson at a hadron collider has recently been pointed out by del Aguila, Kane, and Quiros,[3] in which heavy $SU(2)_L$-singlet charge $-\frac{1}{3}$ quarks, such as those present in E_6 models, decay to a Higgs boson and a light quark. In this particular model there is one Higgs doublet leading to a single neutral Higgs boson; subject to the value of the Higgs-boson mass, an appreciable fraction of isosinglet-quark decays may be into Higgs boson plus light quark.

In this paper we examine extensions of this scenario with more than one Higgs multiplet which could occur, for example, in E_6 superstring-inspired models with extra gauge bosons at the electroweak scale. The minimal number of Higgs multiplets in such models is generally three: two $SU(2)_L$ doublets to give masses to the u- and d-type quarks, and an $SU(2)_L$ singlet to give mass to the heavy isosinglet quarks (which we call D type). There may also exist a further Higgs doublet and singlet which can obtain vacuum expectation values (VEV's); see Table I. Thus, the coexistence of isosinglet quarks and extra Higgs multiplets is quite natural. We will examine two possibilities for the Higgs sector where the symmetry is broken by Higgs scalars in one **27** representation of E_6: when all five neutral Higgs bosons obtain a VEV [which we will call case (I)], and when only the neutral members of the minimal set of three Higgs multiplets obtain a VEV [case (II)].

The philosophy taken in this model is that the light-quark mass matrices are initially diagonal,[3,4] the nontrivial Cabibbo-Kobayashi-Maskawa (CKM) matrix then arises from mixing of the light d-type quarks with one or more heavy isosinglet D quarks. Before mixing, the down-quark mass is very small, but through this mixing it can become larger than the up-quark mass through a seesaw mechanism. Details of this quark mixing and the consequences for CKM phenomenology have been discussed in the literature for the case of one heavy D quark.[4] Mixing among light and heavy left-handed quarks is necessarily small to avoid large flavor-changing neutral currents. We will examine two possibilities for mixing between three generations of both light and heavy quarks: when the mixing among light and heavy right-handed quarks is small [case (a)], and when the mixing among right-handed quarks is large [case (b)].

With five Higgs multiplets the pattern of D-quark decays to Higgs bosons depends on the quark mixing scenario, so we will present separate results for the two possibilities (Ia) and (Ib). The pattern of decays in case (II) with three Higgs multiplets does not depend on the quark mixing scheme chosen. In all cases we find that enhanced quark couplings to Higgs bosons and new decay modes involving charged and pseudoscalar Higgs bosons may significantly increase the total Higgs-boson branching fraction of an isosinglet quark as compared to the model with a single Higgs doublet. In the limit where the lightest Higgs bosons are degenerate in mass or much lighter than the D quarks we find simple expressions for this enhancement factor which involve only ratios of the Higgs-field VEV's.

In Sec. II we present the relevant portions of the Lagrangian necessary for determining the heavy-quark couplings to Higgs bosons, quark mass matrices, and the spectrum of Higgs-boson states for each of our scenarios. We analyze the symmetry breaking to find the Higgs fields that may be produced in D decays. In Sec. III we use the results (Sec. II) to find expressions for the D-quark branching fractions in each scenario. In Sec. IV we discuss the implications of our results on the potential discovery of Higgs bosons at hadron colliders. In the Ap-

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TABLE I. Higgs-boson and quark quantum numbers in the **27** representation of E_6. The $U(1)_\eta$, $U(1)_\chi$, and $U(1)_\psi$ couplings are normalized so that their couplings are the same as the $U(1)_Y$ coupling.

	Q	T_{3L}	Y	$6Q_\eta$	$\sqrt{24}Q_\chi$	$\sqrt{72/5}Q_\psi$
$\Phi_1=(\phi_1^0,\phi_1^-)$	$(0,-1)$	$(\frac{1}{2},-\frac{1}{2})$	-1	1	-2	-2
$\Phi_2=(\phi_2^+,\phi_2^0)$	$(1,0)$	$(\frac{1}{2},-\frac{1}{2})$	1	4	2	-2
$\Phi_3=\phi_3^0$	0	0	0	-5	0	4
$\Phi_4=(\phi_4^0,\phi_4^-)$	$(0,-1)$	$(\frac{1}{2},-\frac{1}{2})$	-1	1	3	1
$\Phi_5=\phi_5^0$	0	0	0	-5	-5	1
(u,d)	$(\frac{2}{3},-\frac{1}{3})$	$(\frac{1}{2},-\frac{1}{2})$	$\frac{1}{3}$	-2	-1	1
u^c	$-\frac{2}{3}$	0	$-\frac{4}{3}$	-2	-1	1
d^c	$\frac{1}{3}$	0	$\frac{2}{3}$	1	3	1
D^c	$\frac{1}{3}$	0	$\frac{2}{3}$	1	-2	-2
D	$-\frac{1}{3}$	0	$-\frac{2}{3}$	4	2	-2

pendix we derive approximate expressions for the quark masses and mixing angles and discuss the implications for CKM mixing.

II. FORMALISM

A. Preliminaries

In general, electroweak models with n Higgs doublets have n-1 charged Higgs bosons after the gauge symmetry has been broken and one linear combination of charged Higgs bosons has been absorbed by the W boson. If there are in addition m neutral Higgs singlets, there will be $n+m-2$ pseudoscalars and $n+m$ scalars after the Z and Z' neutral gauge bosons become massive. Not all of these will remain light; some will be heavy with a mass on the order of the Z' mass.

The **27** representation of E_6 has five neutral fields which can obtain VEV's; three are members of $SU(2)_L$ doublets and two are singlets. The candidates for Higgs fields in the **27** representation are given in Table I. The Φ_1 and Φ_2 multiplets are identical to those found in the usual two-Higgs-doublet model, or in the minimal supersymmetric model.[5] The Φ_3 singlet is present in the minimal rank-5 E_6 model.[6] The Q_η, Q_χ, and Q_ψ quantum numbers come from different E_6 breakdown scenarios: the minimal rank-5 breakdown $E_6 \to SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_\eta$, and the rank-6 breakdown $E_6 \to SO(10)\times U(1)_\psi$, $SO(10)\to SU(5)\times U(1)_\chi$, $SU(5) \to SU(3)\times SU(2)_L \times U(1)_Y$. In principle, the rank-6 case can have any linear combination of $U(1)_\chi$ and $U(1)_\psi$ survive to form an effective rank-5 model at electroweak energies. However, if the symmetry is broken by Higgs-boson scalars in the **27** representation, only a subset of linear combinations is allowed.[7] In the table we also list the quantum numbers of the up, down, and isosinglet quarks for the left-handed fields q and q^c $(q=u,d,D)$.

B. Quark masses and Yukawa couplings

We will consider two symmetry-breaking possibilities: case (I), where the electroweak Lagrangian is invariant under the full rank-6 group and all five neutral scalars in the **27** achieve a VEV, and case (II), where the Lagrangian is invariant under the minimal rank-5 group and only the minimal rank-5 set of scalars (Φ_1, Φ_2, and Φ_3) break the symmetry.

In case (I) the allowed quark–antiquark–Higgs-boson Yukawa coupling terms (which may be deduced from examining the quantum numbers in Table I) are

$$L=\frac{\lambda_1^{jk}}{\sqrt{2}}(\phi_1-i\psi_1)\bar{d}_{Lj}d_{Rk}-\lambda_1^{jk}\phi_1^+\bar{u}_{Lj}d_{Rk}+\frac{\lambda_2^{jk}}{\sqrt{2}}(\phi_2-i\psi_2)\bar{u}_{Lj}u_{Rk}-\lambda_2^{jk}\phi_2^-\bar{d}_{Lj}u_{Rk}+\frac{\lambda_3^{jk}}{\sqrt{2}}(\phi_3-i\psi_3)\bar{D}_{Lj}D_{Rk}$$
$$+\frac{\lambda_4^{jk}}{\sqrt{2}}(\phi_4-i\psi_4)\bar{d}_{Lj}D_{Rk}-\lambda_4^{jk}\phi_4^+\bar{u}_{Lj}D_{Rk}+\frac{\lambda_5^{jk}}{\sqrt{2}}(\phi_5-i\psi_5)\bar{D}_{Lj}d_{Rk}+\text{H.c.}\ , \tag{1}$$

where summation over the generation indices j and k is assumed and ϕ_n^0 has been replaced by its real and imaginary parts:

$$\phi_n^0=(\phi_n+i\psi_n)/\sqrt{2}\ . \tag{2}$$

With Higgs-field VEV's,

$$\langle\phi_n\rangle=v_n,\quad \langle\psi_n\rangle=0\ , \tag{3}$$

the quark mass terms which arise from Eq. (1) are

$$L_{\text{mass}}=\bar{d}_L(M_d^0)d_R+\bar{u}_L(M_u^0)u_R+\bar{D}_L(M_D^0)D_R+\bar{d}_L(m')D_R+\bar{D}_L(m)d_R+\text{H.c.} \tag{4}$$

Here generation indices are suppressed and M_d^0, M_u^0, M_D^0, m', and m are 3×3 mass matrices given by (again with generation indices suppressed)

$$M_d^0=\frac{\lambda_1 v_1}{\sqrt{2}},\quad M_u^0=\frac{\lambda_2 v_2}{\sqrt{2}},\quad M_D^0=\frac{\lambda_3 v_3}{\sqrt{2}}\ ,$$
$$m'=\frac{\lambda_4 v_4}{\sqrt{2}},\quad m=\frac{\lambda_5 v_5}{\sqrt{2}}\ . \tag{5}$$

In Eq. (5) we have used the same general notation as Ref. 3 for the different sectors of the quark mass matrices. Note that if the full E_6 symmetry exists and the Φ_4 and Φ_5 do not obtain a VEV, then the d-D mixing necessary for the decays that we are studying will not occur.

For case (II), where the Lagrangian is invariant under the minimal rank-5 group, the Φ_4 and Φ_5 multiplets have identical quantum numbers to the Φ_1 and Φ_3 multiplets, respectively; i.e., they are redundant and need not obtain VEV's. Thus Φ_4 could be replaced by Φ_1 and Φ_5 by Φ_3 in Eq. (1). The v_4 and v_5 in the quark mass terms of Eq. (5) would then be replaced by v_1 and v_3, respectively. In principle, no degrees of freedom in the mass matrices are lost since the λ couplings are not related, but if we assume natural-sized couplings the scales of some of the quark mass terms would then be similar, i.e., $M_d^0 \sim m'$ and $M_D^0 \sim m$.

C. Quark mixing scenarios

The philosophy of the model is that the original light-quark mass matrices M_u^0 and M_d^0 are diagonal. Without loss of generality we can also assume that the heavy-quark mass matrix M_D^0 is diagonal, since if it were not we could diagonalize it without affecting M_d^0 (m and m' would change, but they are arbitrary anyway). Thus we have

$$M_u^0=\begin{bmatrix} m_u^0 & 0 & 0\\ 0 & m_c^0 & 0\\ 0 & 0 & m_t^0\end{bmatrix},$$
$$M_d^0=\begin{bmatrix} m_d^0 & 0 & 0\\ 0 & m_s^0 & 0\\ 0 & 0 & m_b^0\end{bmatrix}, \tag{6}$$
$$M_D^0=\begin{bmatrix} M_1^0 & 0 & 0\\ 0 & M_2^0 & 0\\ 0 & 0 & M_3^0\end{bmatrix}.$$

The up-quark mass matrix M_u^0 remains diagonal, i.e., the physical charge $\frac{2}{3}$ quark masses are $m_q=m_q^0$ ($q=u,c,t$). The 6×6 charge $-\frac{1}{3}$ quark mass matrix is diagonalized as follows:

$$\begin{bmatrix} U_L & W_L\\ T_L & V_L\end{bmatrix}\begin{bmatrix} M_d^0 & m'\\ m & M_D^0\end{bmatrix}\begin{bmatrix} U_R^\dagger & T_R^\dagger\\ W_R^\dagger & V_R^\dagger\end{bmatrix}=\begin{bmatrix} M_d & 0\\ 0 & M_D\end{bmatrix}, \tag{7}$$

where M_d and M_D are diagonal and contain the physical light- and heavy-quark masses

$$M_d=\begin{bmatrix} m_d & 0 & 0\\ 0 & m_s & 0\\ 0 & 0 & m_b\end{bmatrix},\quad M_D=\begin{bmatrix} M_1 & 0 & 0\\ 0 & M_2 & 0\\ 0 & 0 & M_3\end{bmatrix}. \tag{8}$$

The 3×3 left-handed mixing matrices obey the unitarity relations

$$U_L U_L^\dagger+W_L W_L^\dagger=1,\quad U_L^\dagger U_L+T_L^\dagger T_L=1\ ,$$
$$U_L T_L^\dagger+W_L V_L^\dagger=0,\quad U_L^\dagger W_L+T_L^\dagger V_L=0\ , \tag{9}$$
$$T_L T_L^\dagger+V_L V_L^\dagger=1,\quad W_L^\dagger W_L+V_L^\dagger V_L=1\ .$$

The right-handed mixing matrices obey similar relations.

In this section we present only the general characteristics of the quark mixing. A more detailed analysis of these results is given in the Appendix. The mixing between light and heavy charge $-\frac{1}{3}$ quarks raises the d-quark mass via a seesaw mechanism. The origin of this mechanism can be understood by considering the one-generation equivalent of Eq. (7). If M_D^0 is large and $M_d^0 M_D^0 \ll mm'$, then the resulting light-quark mass is approximately $m'm/M_D$, where M_D is the physical heavy-quark mass.[8] For three generations $m_d^0 \ll m_s^0, m_b^0$, and only the scale of the d-quark mass is changed significantly. There are two possible scenarios depending on the scale of m.

Case (a). $m, m', M_d^0 \ll M_D^0$. This scenario leads to small mixing between light and heavy quarks. Thus the 3×3 mixing matrices W_L, T_L, W_R, and T_R are all small. The only mass in Eq. (8) that is shifted by the mixing from its original value in Eq. (6) is m_d, which obtains a value equal to an element in the matrix $m'(M_D^0)^{-1}m$ (for details, see the Appendix). If the heavy D-quark masses are of the order of 100 GeV, then the product of m and m' must be of order (1 GeV)^2 to give the proper scale for the d-quark mass.

Case (b). $m', M_d^0 \ll m \sim M_D^0$. This scenario has small mixing between light and heavy left-handed quarks (which is required by the absence of significant flavor-changing neutral currents), but large right-handed quark mixing. Therefore W_L and T_L are small, but all of the 3×3 mixing matrices in the right-handed sector are of order unity (in the absence of special conditions on the mixing terms). All of the charge $-\frac{1}{3}$ quark masses are shifted, but only the scale of m_d is changed significantly from its original value [by the same kind of seesaw mechanism that was operative in case (a)]. In this case, since the scale of both M_D^0 and m is 100 GeV, the scale of m' must be 10^{-2} GeV if the seesaw mechanism is to give the proper mass to the d quark.

Each of these quark mixing cases can be realized in the two Higgs sector cases (I) and (II) (although not always naturally). In the minimal standard model with no D quarks, the CKM mixing matrix is the product of up- and down-quark mixing matrices. In this model, because there is no mixing in the up-quark sector, the usual CKM matrix becomes the 3×3 mixing matrix among the light d quarks, which in our notation is $U_L^\dagger$. Because W_L and T_L are always small, we see from Eq. (9) that the CKM matrix $U_L^\dagger$ will be approximately unitary in all of our scenarios.

D. Higgs sector

We will now discuss the spectrum of Higgs bosons expected in E_6 models when the symmetry is broken by scalars in the **27** representation. Much work has been done in specific models on the Higgs-boson masses and mixings.[6] In this paper we do not confine ourselves to a particular model, but instead discuss the general properties of the Higgs-boson spectrum and Higgs-boson couplings to fermions. The gauge-covariant Higgs-boson Lagrangian is[9]

$$\begin{aligned} L = & \left| \partial\phi_1^- + i g_Z \left[\tfrac{1}{2}(1-2x_W)Z - \sqrt{x_W} Q_1' Z' + \frac{e}{g_Z} A \right] \phi_1^- - i\frac{g_L}{\sqrt{2}} W^- \phi_1^0 \right|^2 \\ & + \left| \partial\phi_1^0 - i g_Z (\tfrac{1}{2}Z + \sqrt{x_W} Q_1' Z')\phi_1^0 - i\frac{g_L}{\sqrt{2}} W^+ \phi_1^- \right|^2 \\ & + \left| \partial\phi_2^+ - i g_Z \left[\tfrac{1}{2}(1-2x_W)Z + \sqrt{x_W} Q_2' Z' + \frac{e}{g_Z} A \right] \phi_2^+ - i\frac{g_L}{\sqrt{2}} W^+ \phi_2^0 \right|^2 \\ & + \left| \partial\phi_2^0 - i g_Z (-\tfrac{1}{2}Z + \sqrt{x_W} Q_2' Z')\phi_2^0 - i\frac{g_L}{\sqrt{2}} W^- \phi_2^+ \right|^2 \\ & + \left| \partial\phi_3^0 - i g_Z \sqrt{x_W} Q_3' Z' \phi_3^0 \right|^2 + (\Phi_4 \text{ and } \Phi_5 \text{ terms, if present}) \,, \end{aligned} \tag{10}$$

where Q_j' are the Higgs-boson quantum numbers under the extra U(1)′ gauge group, $g_L = e/\sin\theta_W$, $g_Z = g_L/\cos\theta_W$, and $x_W = \sin^2\theta_W$. The Φ_4 and Φ_5 terms in Eq. (10), which will be present in case (I), are identical to the Φ_1 and Φ_3 terms, respectively, with Q_1' replaced by Q_4' and Q_3' by Q_5'.

Case (I). For three doublets and two singlets, the physical Higgs sector after the symmetry is broken can be deduced from Eq. (10). Following the procedure of Ref. 9, we find that after some linear combinations of Higgs fields are absorbed by the $W^\pm$, Z, and Z', the original gauge eigenstates may be expressed as

$$\begin{aligned} &\phi_1^\pm \to \sin\theta_1 H_1^\pm + \sin\beta\cos\theta_1 H_2^\pm \,, \\ &\phi_2^\pm \to \cos\beta H_2^\pm \,, \\ &\phi_4^\pm \to -\cos\theta_1 H_1^\pm + \sin\beta\sin\theta_1 H_2^\pm \,, \end{aligned} \tag{11}$$

and

$$\begin{aligned} &\psi_1 \to \sin\theta_1 P_1^0 + \sin\beta\cos\theta_1 P_2^0 \,, \\ &\psi_2 \to \cos\beta P_2^0 \,, \\ &\psi_3 \to \sin\theta_3 P_3^0 \,, \\ &\psi_4 \to -\cos\theta_1 P_1^0 + \sin\beta\sin\theta_1 P_2^0 \,, \\ &\psi_5 \to -\cos\theta_3 P_3^0 \,, \end{aligned} \tag{12}$$

where $H_j^\pm$ and P_j^0 are charged and pseudoscalar Higgs bosons, respectively, that survive symmetry breaking, and the mixing angles are defined by

$$\begin{aligned} &\tan\beta = v_2/(v_1^2+v_4^2)^{1/2} \,, \\ &\tan\theta_1 = v_4/v_1, \quad \tan\theta_3 = v_5/v_3 \,. \end{aligned} \tag{13}$$

These states may undergo further mixing in the Higgs-boson potential to arrive at the actual mass eigenstates. In addition, there are five scalar fields ϕ_j, one from each multiplet, which can mix with each other. In one realization of the two-Higgs-doublet–one-Higgs-singlet model,[9] the field associated with the Higgs multiplet that has a large VEV (giving the Z boson and D quarks their heavy masses) is heavy, i.e., of the order of the Z' mass. Therefore, in our examples we correspondingly assume that ϕ_3 and P_3^0 are heavy in case (I), and in case (Ib), where m is large, we assume that ϕ_5 is also heavy since m is proportional to v_5.

Case (II). For two doublets and one singlet, there will be one charged, one pseudoscalar, and three scalar physical Higgs bosons. The original gauge eigenstates written in terms of these states are

$$\phi_1^\pm \to H^\pm \sin\beta, \quad \phi_2^\pm \to H^\pm \cos\beta \,, \tag{14}$$

and

$$\psi_1 \to P^0 \sin\beta, \quad \psi_2 \to P^0 \cos\beta, \quad \psi_3 \to 0 \,, \tag{15}$$

where $\tan\beta = v_2/v_1$ and $H^\pm$ and P^0 are mass eigenstates (for v_3 large, ψ_3 is absorbed by the Z'). There are also the three neutral scalars ϕ_j in this case which may mix with each other. Since ϕ_3 provides a large VEV, the potentially light neutral-scalar states are ϕ_1 and ϕ_2.

TABLE II. Higgs bosons which may be produced in D-quark decays.

Case	Higgs fields in D decays
Ia	$\phi_1, \phi_4, \phi_5, P_1^0, P_2^0, H_1^\pm, H_2^\pm$
Ib	$\phi_1, \phi_4, P_1^0, P_2^0, H_1^\pm, H_2^\pm$
II	$\phi_1, P^0, H^\pm$

Because ϕ_2 couples only to charge $\frac{2}{3}$ quarks, it will not be involved in D decays. A summary of the potentially light Higgs bosons in each scenario that could provide decay modes for the D quarks is given in Table II.

III. ISOSINGLET QUARK DECAYS

A. Lagrangian

We are now ready to study the isosinglet-quark decays to gauge and Higgs bosons. The Lagrangian for the charged-current couplings of the quarks is

$$L_{CC}=\frac{g_L}{\sqrt{2}}W^{\mu+}(\bar{u}_L\gamma_\mu T_L^\dagger D_L+\bar{u}_L\gamma_\mu U_L^\dagger d_L)+\text{H.c.}\ , \quad (16)$$

where generation indices are suppressed. The flavor-changing neutral-current interactions of the Z (we assume the Z' is too heavy to add significantly to D-quark decays) are

$$L_{FCNC}=-\tfrac{1}{2}g_Z Z^\mu(\bar{d}_L U_L T_L^\dagger \gamma_\mu D_L)+\text{H.c.} \quad (17)$$

The gauge couplings in Eqs. (16) and (17) are similar to those found in Ref. 3, but generalized here to three species of heavy D quarks.

The Lagrangian for D decays to Higgs bosons can be deduced by substituting the relations in Eqs. (11), (12), (14), and (15) into Eq. (1). After dropping terms which are small in all quark-mass scenarios, and writing the results in terms of the physical quark masses and mixing angles, we find, for case (I),

$$L_H=\frac{g_L}{2M_W}\left[\bar{d}_L(U_L T_L^\dagger M_D)D_R\frac{\cos\theta_1\phi_4+i\cos^2\theta_1 P_1^0-i\sin\beta\sin\theta_1\cos\theta_1 P_2^0}{\cos\beta\sin\theta_1\cos\theta_1}\right.$$
$$\left.+\bar{d}_L(M_d U_R T_R^\dagger)D_R\frac{-\cos\theta_1\phi_4+\sin\theta_1\phi_1-iP_1^0}{\cos\beta\sin\theta_1\cos\theta_1}\right]+\frac{g_L}{2M_W}\left[\bar{d}_R(U_R T_R^\dagger M_D^\dagger-M_d^\dagger U_L T_L^\dagger)D_L\frac{v}{v_5}\phi_5\right]$$
$$+\frac{g_L}{\sqrt{2}M_W}\left[\bar{u}_L(T_L^\dagger M_D)D_R\frac{\cos^2\theta_1 H_1^+-\sin\beta\sin\theta_1\cos\theta_1 H_2^+}{\cos\beta\sin\theta_1\cos\theta_1}+\bar{u}_L(U_L^\dagger M_d U_R T_R^\dagger)D_R\frac{-H_1^+}{\cos\beta\sin\theta_1\cos\theta_1}\right]+\text{H.c.}\ , \quad (18)$$

where $v^2=v_1^2+v_2^2+v_4^2$ is the usual standard-model VEV; for case (II) the result is

$$L_H=\frac{g_L}{2M_W}\bar{d}_L(U_L T_L^\dagger M_D)D_R\frac{\phi_1-i\sin\beta P^0}{\cos\beta}+\frac{g_L}{\sqrt{2}M_W}\bar{u}_L(T_L^\dagger M_D)D_R\frac{-H^+\sin\beta}{\cos\beta}+\text{H.c.} \quad (19)$$

The results of the model with just one neutral Higgs boson (Ref. 3) can be obtained from Eq. (19) by setting $\beta=0$.

B. Partial widths

The Yukawa interactions involving a light quark q, heavy quark Q with mass M_Q, vector bosons W and Z, and scalar S with mass M_S, are

$$L=\frac{g_L}{2}C_W W_\mu^+\bar{q}\gamma^\mu\frac{1\pm\gamma_5}{2}Q+\frac{g_Z}{2}C_Z Z_\mu\bar{q}\gamma^\mu\frac{1\pm\gamma_5}{2}Q+\frac{g_L}{2M_W}C_S H\bar{q}\frac{1\pm\gamma_5}{2}M_Q Q+\text{H.c.}\ , \quad (20)$$

where the C_i are coupling-strength factors. The partial widths for Q decay (in the limit $m_q\to 0$) are[10]

$$\Gamma(Q\to qV)=C_V^2\frac{g_L^2}{128\pi}\frac{M_Q^3}{M_W^2}\left[1-\frac{M_V^2}{M_Q^2}\right]^2\left[1+2\frac{M_V^2}{M_Q^2}\right] \quad (V=W,Z)\ ,$$

$$\Gamma(Q\to qS)=C_S^2\frac{g_L^2}{128\pi}\frac{M_Q^3}{M_W^2}\left[1-\frac{M_S^2}{M_Q^2}\right]^2 . \quad (21)$$

From Eqs. (16) to (21) we see that the $W^\pm$ and charged-Higgs-boson partial widths are larger by a factor of 2 than the corresponding neutral-boson modes.[11]

We will now consider the decays of the heavy quark D_j into the kth generation of light quarks. In Eq. (18) the terms involving M_d are usually small. Of the remaining terms with the physical heavy-quark mass matrix M_D, all but the ϕ_5 term in case (Ia) have the same mixing factor T_{Ljk}^*. Since U_L is close to the identity matrix, the mixing factors of the charged and neutral decay modes of the D quarks will be identical. Thus, except for the ϕ_5 term in Eq. (18), the enhancement of the total Higgs-boson partial width in the multi-Higgs-boson models can be expressed as a function of just the angles β and θ_1, plus phase-space factors. In the limit that the light Higgs boson in Table II are much lighter than the D quarks, or in the limit that they are all degenerate in mass, the enhancement of the total Higgs-boson partial width in the multi-Higgs-boson model compared to the single-Higgs-boson model is

$$R_H=\frac{1+3\cos^2\theta_1+3\sin^2\beta\sin^2\theta_1}{\cos^2\beta\sin^2\theta_1}+R_5,\quad \text{Case(Ia)}\ ,$$
$$R_H=\frac{1+3\sin^2\theta_1+3\sin^2\beta\cos^2\theta_1}{\cos^2\beta\cos^2\theta_1},\quad \text{Case(Ib)}\ , \quad (22)$$
$$R_H=\frac{1+3\sin^2\beta}{\cos^2\beta},\quad \text{Case(II)}\ .$$

The R_5 term comes from $D_j \to q_k \phi_5$ and scales like $|(vm_{kj})/(v_5 m'_{jk})|^2$, which comes from the ratio of $|T_{Rjk}|^2$ to $|T_{Ljk}|^2$.

The sources of the enhancements in Eq. (22) are easy to understand. The denominator comes from the usual factors present in the Higgs-boson couplings to fermions in multi-Higgs-boson models, and the terms involving the factor of 3 in the numerator come from the additional modes available (1 from pseudoscalar-Higgs-boson and 2 from charged-Higgs-boson modes). Taking, for example, $\tan\beta = \tan\theta_1 = 1$ and ignoring R_5, we get $R_H = 13$ in cases (Ia) and (Ib), and $R_H = 5$ in case (II). Specific models tend to prefer $\tan\beta > 1$ (Ref. 12), since $v_2 > v_1$ is required to give a heavy top-quark mass; this would increase R_H even further. Thus, the Higgs-boson modes could easily dominate in D decays.

C. Higgs-boson branching fractions

The partial widths for the Higgs- and gauge-boson modes can be calculated from Eq. (21) using the couplings in Eqs. (16)–(19). From Eqs. (16)–(19) we see that except for $D \to q\phi_5$, the dominant terms in the decays $D \to qW$, $D \to qZ$, and $D \to qH$ are all proportional to $|T_{Ljk}|^2$. Therefore the total Higgs-boson branching fraction can be written as a function of the quantity R_H defined in Eq. (22). If all Higgs bosons participating in D decays are degenerate in mass and $M_W, M_Z, M_H < M_D$, then

$$B(D \to qH) = \frac{R_H \left[1 - \frac{M_H^2}{M_D^2}\right]^2}{R_H \left[1 - \frac{M_H^2}{M_D^2}\right]^2 + 2\left[1 - \frac{M_W^2}{M_D^2}\right]^2 \left[1 + 2\frac{M_W^2}{M_D^2}\right] + \left[1 - \frac{M_Z^2}{M_D^2}\right]^2 \left[1 + 2\frac{M_Z^2}{M_D^2}\right]} . \quad (23)$$

The result for the single-Higgs-boson model is recovered by setting $R_H = 1$.

To show the effects of phase-space suppression, we give an illustrative example. In Fig. 1 we show the total Higgs-boson branching fraction in D-quark decays versus the Higgs-boson mass for case (I), case (II), and the single-Higgs-boson model. In the figure we have assumed $\tan\beta = \tan\theta_1 = 1$, $R_5 = 0$ and that all of the light Higgs bosons available for the decays (listed in Table II) have the same mass. We have also ignored the effects of a non-negligible top-quark mass (which tend to enhance the Z- and Higgs-boson modes since $D \to tW$ is suppressed[3]). The results are shown for $M_D = 100$ GeV and $M_D = 150$ GeV.

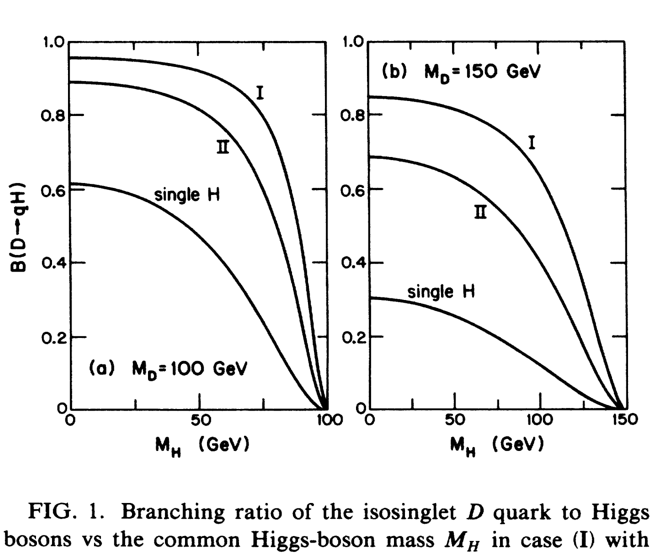

FIG. 1. Branching ratio of the isosinglet D quark to Higgs bosons vs the common Higgs-boson mass M_H in case (I) with $R_H = 13$, case (II) with $R_H = 5$, and the single-Higgs-boson model ($R_H = 1$), for (a) $M_D = 100$ GeV and (b) $M_D = 150$ GeV.

IV. DETECTION AT HADRON COLLIDERS

The detection of the minimal standard-model Higgs boson in hadron colliders can be problematical because of backgrounds from other standard-model processes.[2] The merit of isosinglet quark decays to Higgs bosons is an increased production rate with a distinctive signature. A discussion of Higgs-boson production and detection via isosinglet quark decays is given in Ref. 3. We review the highlights here and discuss the effects of the multi-Higgs-boson scenarios on the possibilities for detection.

The main production mechanism for isosinglet quarks in hadron colliders is through gluon fusion, $g + g \to D + \bar{D}$. The D quarks subsequently decay to a light quark plus boson ($W^{\pm}$, Z, or Higgs boson). The Higgs boson generally decays into the heaviest available fermion pair. For $M_H > 10$ GeV most of the neutral-Higgs-boson decays will be to $b\bar{b}$ pairs, which will be difficult to distinguish from the QCD background. A possible signature is when one D quark decays to a gauge boson and the other decays to a Higgs boson. If the gauge boson subsequently decays leptonically, the signature will be a hard electron or muon plus missing transverse momentum plus two to four jets (when $W^{\pm} \to e\nu$ or $\mu\nu$) or two hard electrons or muons plus two to four jets (when $Z \to ee$ or $\mu\mu$). The number of jets depends on whether the light quarks in the primary D decay have sufficient transverse momentum to be seen. In order to distinguish the two Higgs-boson decay b jets from ordinary jets, vertex detectors will be necessary. The Higgs-boson mass can be reconstructed from the two-jet mass peak in these events. Identifying the charged Higgs-boson events [which constitute 46% of the signal in case (I) and 40% in case (II) in our illustrative examples] will be difficult since the dominant decay is $H^+ \to c\bar{s}$.

Well above the decay thresholds, the ratio of the Z to $W^{\pm}$ branching fractions in D decays is

$$r=\frac{\Gamma(D\to Zq)}{\Gamma(D\to Wq)}$$
$$=\frac{(M_D^2-M_Z^2)^2(M_D^2+2M_Z^2)}{2(M_D^2-M_W^2)^2(M_D^2+2M_W^2)}\quad (M_D>M_Z)\ ,$$
$$=0\quad (M_D\le M_Z)\ . \tag{24}$$

Assuming that Higgs-boson modes make up the remainder of the D decays, the fraction of $D+\bar{D}$ pairs which will give the $(W,Z)+H$ signature will be

$$F=B_H(1-B_H)\frac{0.44+0.12r}{1+r}\ , \tag{25}$$

where B_H is the D-quark branching fraction to all Higgs-boson modes and we have taken the gauge-boson branching fractions to leptons to be $B(W\to\nu e)=0.11$ and $B(Z\to ee)=0.03$. The quantity r depends only on the D-quark mass, while B_H varies with the Higgs-boson mass and the enhancement factor R_H. We see that if the Higgs-boson modes totally dominate D decays ($B_H\to 1$), we lose the signature since there are not enough events with $D\to qW$ or $D\to qZ$. For a given value of M_D, the signal will have its maximum value at $B_H=\frac{1}{2}$:

$$F_{\max}=\frac{0.11+0.03r}{1+r}\ . \tag{26}$$

The function $F_{\max}$ varies from 0.11 when $M_D=M_Z$ to 0.083 when M_D is very large, so we can say that no more than about one-tenth of the $D+\bar{D}$ events can provide a distinctive signature, irrespective of the Higgs-boson scenario.

In Fig. 2 we show F versus Higgs-boson mass in case (I), case (II), and the single-Higgs-boson model, for $M_D=100$ GeV and $M_D=150$ GeV. As in Fig. 1, we take $\tan\beta=\tan\theta_1=1$, $R_5=0$ and assume a common mass for all of the light Higgs bosons. In the single-Higgs-boson model F approaches its maximum value at low M_H where the phase-space suppression of the Higgs-boson mode is not severe. At higher M_H, $B(D\to qH)$ goes down and the signal decreases. On the other hand, in the multi-Higgs-boson models the enhancement factor R_H causes the $D\to qH$ decays dominate at low Higgs-boson mass, thereby suppressing the gauge-boson decays which are essential to the signal. The signal reaches its peak at higher Higgs-boson mass, where the enhancement factor compensates for the phase-space suppression of the Higgs-boson modes.

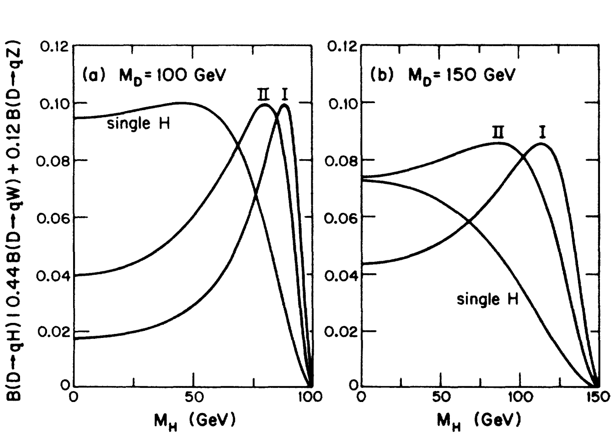

FIG. 2. Fraction of $D+\bar{D}$ pairs which decay into $(W,Z)+H$ with subsequent gauge-boson decay to leptons ($W\to e\nu$ or $\mu\nu$, $Z\to e\bar{e}$ or $\mu\bar{\mu}$), shown vs the common Higgs-boson mass in the three cases described in Fig. 1 for (a) $M_D=100$ GeV and (b) $M_D=150$ GeV.

The actual discovery limit for the Higgs boson in the $(W,Z)+H$ mode will depend on the production cross section for $D+\bar{D}$ pairs as well as the value of F. We take as an example the Fermilab Tevatron and assume the present integrated luminosity of 4.7 pb^{-1} at $\sqrt{s}=1.8$ TeV. The cross section for producing three generations of $D+\bar{D}$ pairs is about 300 pb for $M_D=100$ GeV and 30 pb for $M_D=150$ GeV (Ref. 13). If we assume that ten events (before efficiency factors are included) are necessary to confirm the signal, then the minimum value of F needed to find the Higgs-boson signal is $F_{\min}=0.0067$ for $M_D=100$ GeV and $F_{\min}=0.067$ for $M_D=150$ GeV. From Fig. 2 we see that for $M_D=100$ GeV Higgs bosons below about 90 GeV may be detectable in all Higgs-boson scenarios, even though F is somewhat suppressed at low Higgs-boson mass in the multi-Higgs-boson models. The upper discovery limit on M_H is a little higher in multi-Higgs-boson models than for the single-Higgs-boson model, but in all cases it is close to M_D.

The situation at higher D mass is more interesting. For example, at $M_D=150$ GeV (where $F_{\min}=0.067$) the signal may be detectable in the single-Higgs-boson model for a Higgs-boson mass up to about 45 GeV. In the multi-Higgs-boson models it may be observable for Higgs-boson masses as high as 130 GeV for case (I) and 115 GeV for case (II) for the parameters used in Fig. 2, although in case (I) a Higgs boson lighter than about 80 GeV will not be seen because the Higgs-boson branching fraction is then high enough to choke off the W and Z decay modes. Thus, for higher D masses, the discovery limit for Higgs bosons is higher in the multi-Higgs-boson models as compared to the single-Higgs-boson models, although sensitivity to lighter Higgs-boson masses may sometimes be lost.

In Fig. 3 we show the discovery region in M_D vs M_H at the Tevatron for integrated luminosities of 4.7 pb^{-1} and 100 pb^{-1} using the criteria discussed above. We use the results of Ref. 13 for the production cross section of $D+\bar{D}$ pairs. We see that the potential limit for the D mass is similar for the multi-Higgs-boson and single-Higgs-boson models, but that the potential search limit on the Higgs-boson mass is significantly higher in the multi-Higgs-boson model. With 100 pb^{-1} at the Tevatron, a D quark with mass up to 250 GeV may be detectable with this signal, while the realizable search limits for Higgs bosons are about 200 GeV in the multi-Higgs-boson models (for $R_H=5$ to 13) and 150 GeV in the single-Higgs-boson model ($R_H=1$). In the limit of very large R_H, the search limit on the Higgs-boson mass approaches that of the D-quark mass, about 250 GeV for integrated luminosity of 100 pb^{-1}. If the charged-Higgs-boson modes cannot be distinguished from background,

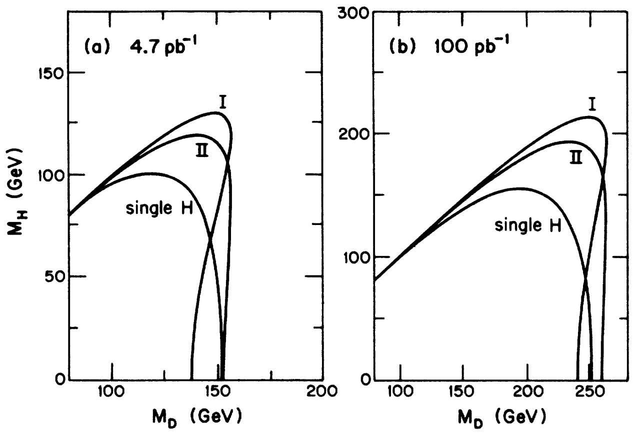

FIG. 3. Possible search region in M_D vs M_H for the three cases described in Fig. 1 at a proton-antiproton collider with $\sqrt{s}=1.8$ TeV and integrated luminosity of (a) 4.7 pb^{-1} and (b) 100 pb^{-1}.

then the D quark and Higgs-boson search limits are reduced by about 20 GeV in these examples.

The Higgs-boson branching-ratio formula in Eq. (23) is accurate only when M_W, M_Z, and M_H are all less than M_D. If $M_W, M_Z < M_D < M_H$ the gauge-boson modes dominate, and there is no signal. Similarly, if $M_H < M_D < M_W, M_Z$ the Higgs-boson modes dominate, and although Higgs bosons are copiously produced in D decays, the necessary leptonic signature of an accompanying gauge-boson decay is lost. If $M_D < M_W, M_Z, M_H$, then both the gauge and Higgs bosons in D decays are virtual; however, because the Higgs-boson couplings to fermions is suppressed by the fermion–to–gauge-boson mass ratio, the Higgs-boson branching fraction will be small. Thus the region shown in Fig. 3 is the most likely one for detection of the Higgs boson through isosinglet quark decays.

The D-quark diagonal couplings to Higgs bosons are suppressed (in some cases by d-D mixing and in others by a factor proportional to $M_Z/M_{Z'}$) so that gluon fusion through a D-quark loop will not be a significant Higgs-boson production mechanism in this model.

After this paper was submitted we received a paper by F. del Aguila, Ll. Ametller, G. L. Kane, and J. Vidal, Michigan Report No. UM-TH-89-17A, which extends the work in Ref. 3.

ACKNOWLEDGMENTS

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APPENDIX

In this appendix we will analyze in more detail the two quark mixing scenarios discussed in the main text. Our main goal here is to show (i) how m_d achieves its mass from the quark mixing, (ii) what determines the scale of the mixing between light and heavy quarks, and (iii) what the implications for the CKM matrix are in each case.

Case (a). $m, m', M_d^0 \ll M_D^0$. The eigenvalues in Eq. (8) can written as an expansion in inverse powers of the M_j^0. After some lengthy algebra, we find that the first two terms for the light-quark masses are

$$m_d = m_d^0 - \mu_{11}, \quad m_s = m_s^0 - \mu_{22}, \quad m_b = m_b^0 - \mu_{33} , \tag{A1}$$

where we have defined the 3×3 matrix

$$\mu = m'(M_D^0)^{-1} m . \tag{A2}$$

Since m_d^0 is supposed to be very small in this model, the physical d-quark mass m_d must be generated by μ_{11} through an extended seesaw mechanism. If the M_k^0 are of order 100 GeV, then the product of m and m' must be of order (1 GeV)^2. Since m_s^0 and m_b^0 are much bigger than the scale of μ, these masses are nearly unaffected. The eigenvalues for the heavy D quarks are almost unchanged, i.e., $M_j^0 \approx M_j$.

Using the eigenvalues given above, the eigenvectors can be determined. To leading order in powers of M_j^0, the 3×3 matrix U_L introduced in Eq. (7) is

$$U_L = \begin{bmatrix} 1 & \mu_{12}/m_s & \mu_{13}/m_b \\ -\mu_{12}^*/m_s & 1 & \dfrac{m_s\mu_{32}^* + m_b\mu_{23}}{m_b^2 - m_s^2} \\ -\mu_{13}^*/m_b & -\dfrac{m_s\mu_{32} + m_b\mu_{23}^*}{m_b^2 - m_s^2} & 1 \end{bmatrix} \quad [\text{Case(a)}] . \tag{A3}$$

Equation (A3) is a straightforward generalization of the single heavy-quark scenario discussed in Refs. 3 and 4. It is evident that the off-diagonal elements of U_L are all small as required for CKM mixing. Since U_L is close to the identity matrix, the unitarity of the full 6×6 left-handed mixing matrix implies that V_L is approximately unitary and the matrices W_L and T_L are both small in magnitude. From Eqs. (7) and (9) we find that the approximate forms for W_L and T_L are

$$\begin{aligned} W_L &= -U_L(M_d^0 m^\dagger + m' M_D^{0\dagger})(M_D^0)^{-2} , \\ T_L &= V_L(M_D^0)^{-2}(m M_d^{0\dagger} + M_D^0 m'^\dagger) \quad [\text{Case(a)}] . \end{aligned} \tag{A4}$$

The form of the matrix V_L which determines the mixing among the heavy quarks depends upon the original heavy masses M_j^0. If they are nondegenerate, then V_L is approximately equal to the identity matrix (with corrections of order M_j^{0-2}) and T_L in Eq. (A4) simplifies to just $-W_L^\dagger$. If the heavy quarks are degenerate ($M_1^0=M_2^0=M_3^0$), then by squaring Eq. (7) and keeping only the leading terms, one can show that V_L is such that $V_L(m^\dagger m)^T V_L^\dagger$ is diagonal. Thus, although remaining approximately unitary, V_L can have large off-diagonal terms. Regardless of the form of V_L, both W_L and T_L are small in magnitude.

The analysis and results for the right-handed sector 3×3 matrices (U_R, W_R, T_R, V_R) are similar to those for the left-handed sector. The only difference is that one must make the substitutions $m \leftrightarrow m'^\dagger$, $M_d^0 \rightarrow M_d^{0\dagger}$, and $M_D^0 \rightarrow M_D^{0\dagger}$ to derive the right-handed results from the left-handed ones. Thus, U_R is close to the identity matrix, V_R is nearly unitary (and close to the identity if the heavy quarks are not degenerate), and W_R and T_R are small in magnitude.

Case (b). $m', M_d^0 \ll m \sim M_D^0$. If m_d^0 is very small (once again, assuming the down-quark must get its mass from mixing), then the product of the six masses of the charge $-\frac{1}{3}$ quarks is approximately

$$m_d m_s m_b M_1 M_2 M_3 \approx -\mu_{11} m_s^0 m_b^0 M_1^0 M_2^0 M_3^0 \ , \qquad \text{(A5)}$$

which is in fact the same as for case (a). However, because m has the same scale as M_D^0, the heavy masses are shifted away from their original values, i.e., $M_j \neq M_j^0$. This also causes shifts in the strange- and bottom-quark masses, i.e., $m_s \neq m_s^0$ and $m_b \neq m_b^0$ (these shifts will be discussed in a little more detail below). Nevertheless, the scales of these masses will remain the same. The down quark will still get its mass by an extended seesaw mechanism, and that mass will have the same scale as μ_{11}. Thus, since $m \sim M_D^0 \sim 100$ GeV, we must have $m' \sim 10^{-2}$ GeV. For the down-quark mass seesaw mechanism to work then imposes the further constraint $m' \ll M_d^0$.

Because both M_D^0 and m are large, there is large d_R-D_R mixing. Therefore none of the 3×3 mixing matrices U_R, W_R, T_R, and V_R are in general small, i.e., all are of order unity, and they obey unitarity relations analogous to those in Eq. (9). This will not affect the standard electroweak phenomenology because the d_R and D_R charge and weak isospin are the same, and there is a generalized Glashow-Iliopoulos-Maiani (GIM) mechanism[14] in effect. Therefore we do not give the details of the right-handed mixing.

In the left-handed sector, one can derive from Eqs. (7) and (9) the approximate expressions

$$\begin{aligned} W_L &= -U_L M_d^0 m^\dagger (mm^\dagger + M_D^{02})^{-1} \ , \\ T_L &= V_L (mm^\dagger + M_D^{02})^{-1} m M_d^{0\dagger} \quad [\text{Case(b)}] \ . \end{aligned} \qquad \text{(A6)}$$

From Eq. (A6) we see that W_L and T_L are small and therefore U_L and V_L must be approximately unitary. By squaring Eq. (7), using Eq. (A6), and keeping only the leading terms one can show that

$$\begin{aligned} U_L M_d^0 [1 - m^\dagger (mm^\dagger + M_D^{02})^{-1} m] M_d^{0\dagger} U_L^\dagger &= M_d^2 \ , \\ V_L [mm^\dagger + M_D^{02}] V_L^\dagger &= M_D^2 \quad [\text{Case(b)}] \ . \end{aligned} \qquad \text{(A7)}$$

Since m is in general the same scale as M_D^0, Eq. (A7) shows that all of the eigenvalues are shifted, and that U_L and V_L diagonalize the left-handed light- and heavy-quark sectors, respectively. Although we do not give the details here, the b-d and s-d mixing are small in the limit that m_d^0 is very small, and b-s mixing is proportional to m_s^0/m_b^0. Thus $U_L^\dagger$ (which is the CKM matrix) will have small off-diagonal terms in this case, and the CKM mixing can agree with observed phenomenology. The mixing in the heavy-quark sector, determined by V_L, could be large unless there are further constraints on the matrix m.

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