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The bounce and its negative eigenvalue: A new approach

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We derive an exact expression for the static bounce solution for $\lambda\phi^4$ theory with broken symmetry. We also propose a new approach to compute a nontrivial real contribution to the generating functional around a classical bounce-type solution avoiding problems due to the presence of a negative eigenvalue. We get this by using the arbitrariness in the mass scale of Wick ordering for the $\lambda\phi^4$ model in dimension $d=1+1$.

I. INTRODUCTION

The problem of the contribution of classical static solutions to the generating functional in field theories has been extensively studied.[1] In particular, the classical solutions associated to negative eigenvalues (bounce-type solutions) has been studied over the last two decades.

As bouncelike solutions are classically unstable (because of the presence of a negative eigenvalue), the usual treatments lead to a complex contribution to the generating functional.[2]

Another important point that has been discussed a lot in the literature is the convexity of the effective potential in the quantum field theory of the double-well potential.[3] The solution of this problem has been suggested by Callaway and Maloof,[4] through a Maxwell construction of the effective potential. The Maxwell construction was obtained by Fujimoto, O'Raifeartaigh, and Parravicini[5] and Bender and Cooper[6] by summing over the contributions from the degenerate minima of the potential in an incoherent way. This is well justified when the separation between the actual minima is sufficient to guarantee the independence of their contributions. However, when the overlapping between the two Gaussian approximations around the two minima are not small, this approach clearly fails and we might try to use nonconstant classical solutions which interpolate between the two minima. We are interested in looking for solutions that give information from one minimum and the other when we do not have an exact degeneracy of the potential. This is a way that one minimum can get information about the location of the other. The solution that gives this behavior is the bounce-type solution. This kind of solution has been considered as a small correction to the kink solution.[2,7] However, the fact that the bounce has a negative eigenvalue in the Schrödinger equivalent problem indicates that this is not the case.

In this paper we derive an exact expression for our bounce solution of the double-well potential of the model $\lambda\phi^4$ with negative squared mass, in the presence of a constant external current J. There is a critical current J_c such that for currents smaller than this one we have two minima and the bounce solution stays most of the time in the relative minimum, goes to the other side of the barrier and returns. We have been able to find an exact algebraic solution for $0<J<J_c$. Another point is to calculate the contribution around this classical solution and to show that the presence of the negative eigenvalue in the Schrödinger equivalent problem gives a nontrivial real contribution to the generating functional. This has been done already for the kink as well as for kink-antikink pair solutions.[8]

In Sec. II we present the model we consider in this paper and derive the exact algebraic expression for our bounce-type solution. We show that the bounce solution in this model can be written as a superposition of a kink and an antikink in a background constant field.

In Sec. III we point out that any contribution to the partition function must be real and this is assured by the $\lambda\phi^4$ term. To neglect the contributions from the cubic and quartic terms is only possible if the quadratic term has no negative eigenvalue. To solve this problem, we propose a new approach, by using the arbitrariness in the mass scale of Wick ordering. In Sec. III we also calculate the contribution from the bounce solution to the generating functional. The fact that this quantity is an intensive one allows one to show that only the contributions coming from the negative eigenvalue and the continuum spectrum of the Schrödinger equivalent problem are relevant.

In Sec. IV we present the conclusions of our work.

In the Appendix we study the asymptotic behavior of the wave functions of the Schrödinger equivalent problem.

II. AN EXACT CLASSICAL STATIC SOLUTION—THE BOUNCE

Let us consider a classical two-dimensional Euclidean field theory with a Lagrangian density of the form

$$\mathcal{L}[\phi,J]=-\tfrac{1}{2}(\partial_\mu\phi)^2-\vartheta(\phi)\ , \tag{2.1}$$

where

$$\vartheta(\phi)=\frac{g}{4}(\phi^2-a^2)^2-J\phi \tag{2.1a}$$

is a potential depending on the external constant current $J>0$.

We are looking for a static solution. The static solutions must satisfy the equation

$$\frac{d^2\phi(x)}{dx^2}=g\phi(x)[\phi^2(x)-a^2]-J=\vartheta'(\phi)\ , \tag{2.2}$$

which is equivalent to

$$\frac{1}{2}\left[\frac{d\phi(x)}{dx}\right]^2-\vartheta(\phi)=E\ , \tag{2.3}$$

where E is the an arbitrary constant.

The formal solution of (2.2) is given by

$$\frac{1}{\sqrt{2}}\int_{\phi_0}^{\phi(x)}\frac{d\phi'}{\sqrt{E+\vartheta(\phi')}}=x-x_0\ , \tag{2.4}$$

with the initial condition $\phi_0=\phi(x_0)$.

We see from (2.3) that this problem is formally equivalent to the one-dimensional motion of a particle of unit mass, with total energy E subject to a potential $-\vartheta(\phi)$, where the x coordinate plays the role of time and $\phi(x)$ the role of position.

There is a particular value $J=J_c$, see the discussion below, for which $J\in(-J_c,J_c)$, the potential $-\vartheta(\phi)$ has two maxima (see Fig. 1). From now on we are considering the case $0\le J<J_c$.

We are looking for a static solution having a finite energy such that starting in the remote past from the local (nonabsolute) maximum of $-\vartheta(\phi)$ it suffers reflection on the potential barrier associated with the other maximum and returns in the remote future to the original one. These characteristics will be made more clear below.

In fact, because of the particular form of the potential $-\vartheta(\phi)$ the problem has algebraic exact solutions, and the integral (2.4) can be calculated in a simple way at least in the three cases $E=-\vartheta(\phi_1)$, $E=-\vartheta(\phi_2)$, or $E=-\vartheta(\phi_3)$, when the total energy is one of the potential extrema.

As mentioned before, there exists a value of J, $J=J_c$ for which the minimum of $\vartheta(\phi)$ at $\phi=\phi_1$ becomes an inflection point. We shall call ϕ_1 and ϕ_2, respectively, the smaller and the greater of the two maxima of $-\vartheta(\phi)$ (see Fig. 1). For $0<J<J_c$ the two maxima are present and are given by[6]

$$\phi_1=\frac{2a}{\sqrt{3}}\cos\left[\frac{\theta+2\pi}{3}\right]$$

and (2.5)

$$\phi_2=\frac{2a}{\sqrt{3}}\cos\left[\frac{\theta}{3}\right]\ ,$$

where the "critical" value of the external current is

$$J_c=\frac{2}{3\sqrt{3}}ga^3 \tag{2.6}$$

and the angle θ is defined by

$$\tan\theta=\frac{1}{J}(J_c^2-J^2)^{1/2}\ . \tag{2.7}$$

Now, to obtain a solution of the field equation for any $0<J<J_c$, that is, to calculate the integral in (2.4) in the case we are interested in, we take the limiting case $E=-\vartheta(\phi_1)$, and we rewrite the potential $\vartheta(\phi)$ as a polynomial in $(\phi-\phi_1)$. After this is done, Eq. (2.4) takes the form

$$\frac{1}{\sqrt{2}}\int_{\phi_0}^{\phi(x)}\frac{d\phi'}{(\phi'-\phi_1)[A+B(\phi'-\phi_1)+C(\phi'-\phi_1)^2]^{1/2}}=x-x_0\ , \tag{2.8}$$

where

$$A=\frac{g}{2}(3\phi_1^2-a^2)=\tfrac{1}{2}\vartheta''(\phi_1),\quad B=g\phi_1\ , \tag{2.9}$$

and

$$C=\frac{g}{4}\ .$$

We remark that $A>0$ for any value of $J\in[0,J_c)$, since it is the second derivative of $\vartheta(\phi)$ respective to ϕ at $\phi=\phi_1$ and ϕ_1 is by definition a minimum of $\vartheta(\phi)$.

The integral in (2.8) is easily calculated,[9] and it gives, after some rearrangements,

$$\bar{\phi}(x)=\phi_1+\frac{4ANe^{-\sqrt{2A}(x-x_0)}}{(Ne^{-\sqrt{2A}(x-x_0)}-B)^2-4AC}\ , \tag{2.10}$$

where x is an arbitrary constant which reflects the

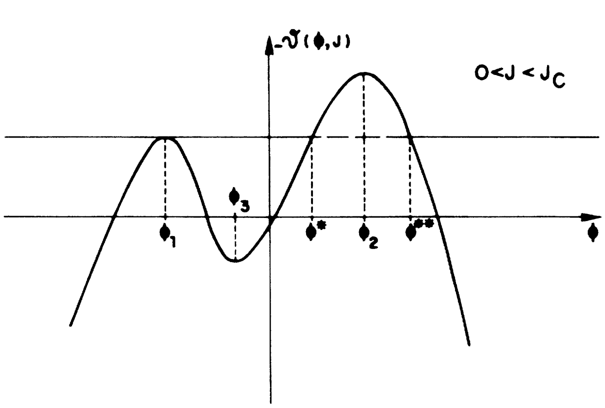

FIG. 1. The shape of the minus potential function $\vartheta(\phi)$ for $J\in(0,J_c)$. We draw the line for the energy $E=-\vartheta(\phi_1)$, and ϕ^* is the "position" where the unit particle reflects on the other side of the barrier. ϕ^{**} is the position of the unit particle when the motion is unbounded.

translational invariance of the solution and

$$N=\frac{1}{\phi_0-\phi_1}\{2\sqrt{A}\,[A+B(\phi_0-\phi_1)+C(\phi_0-\phi_1)^2]^{1/2}+B(\phi_0-\phi_1)+2A\}\ . \quad (2.10a)$$

We note that $\lim_{x\to\pm\infty}\bar{\phi}(x)=\phi_1$.

To see if there is any restriction to the value of N, we impose the condition $d\bar{\phi}(x)/dx=0$, and this gives the quadratic equation

$$Nz^2+4AC-B^2=0\ , \quad (2.11)$$

where $z=\exp[-\sqrt{2A}\,(x-x_0)]$, which for having a possible solution ($z\geq 0$) gives, after using (2.9), the condition

$$\phi_1^2\leq a^2\ , \quad (2.12)$$

which is always true.

So, we conclude that we have only one point x where the derivative $d\bar{\phi}(x)/dx$ vanishes, and this fact is *independent* of the chosen value of N. So, let us choose $d\bar{\phi}/dx=0$ for $x=x_0$, which corresponds to set $N=\sqrt{B^2-4AC}$. Then, defining a point $\bar{x}$ by the relation

$$\bar{x}=\frac{1}{\sqrt{2A}}\operatorname{arctanh}\left[\frac{\sqrt{4AC}}{-B}\right]\ , \quad (2.13)$$

it is possible after some algebraic manipulations to rewrite (2.10) in the more convenient form

$$\bar{\phi}(x)=\phi_1+\frac{1}{2}\left[\frac{A}{C}\right]^{1/2}\left\{\tanh\left[\left[\frac{A}{2}\right]^{1/2}(x+\bar{x}-x_0)\right]-\tanh\left[\left[\frac{A}{2}\right]^{1/2}(x-\bar{x}-x_0)\right]\right\}\ . \quad (2.14)$$

This is a solution of the bounce type,[10] which starts at $\phi=\phi_1$ for $x\to-\infty$, goes up to $\phi=\phi^*$ for $x=x_0$ [see Fig. 1 and remember that we are working in the case $E=-\vartheta(\phi_1)$], and goes backwards to $\phi=\phi_1$ for $x\to+\infty$. For this model, the bounce can be written exactly as a superposition of a constant and of a kink and an antikink centered, respectively, at $-\bar{x}+x_0$ and $\bar{x}+x_0$.

Next, we study the behavior of the solution (2.14) for small values of J, obtaining a divergent behavior for $\bar{x}$ in the limit $J\to 0^+$:

$$\bar{x}=-\frac{1}{2\sqrt{2g}}\frac{1}{a}\ln\left[\frac{J}{J_c}\right]+\text{const}\ . \quad (2.15)$$

This behavior points out that the bounce solution is not analytic in J and if we take the limit $J\to 0$ we cannot recover the kink or the antikink configuration. Actually the limit $J\to 0^+$ gives

$$\lim_{J\to 0^+}\bar{\phi}(x)=a\ .$$

This contradicts the usual belief that the bounce solution can be constructed by making small corrections to the kink configuration.[2,7]

From (2.15) we see that as J becomes smaller and smaller the separation between the kink and antikink components of the solution (2.14), $D=2\bar{x}$, grows indefinitely.

Next we calculate the classical action associated to the solution (2.14), considering the system enclosed in a rectangular box of sides L and T:

$$S=T\int_{-L/2}^{L/2}dx\left[\frac{1}{2}\left[\frac{d\bar{\phi}(x)}{dx}\right]^2+\vartheta(\bar{\phi}(x))\right]\ , \quad (2.16)$$

where the factor T comes from the integration over the t variable, since as $\bar{\phi}(x)$ is a static solution, the integrand of the action does not depend on t. We get

$$S=LT\vartheta(\phi_1)+\frac{TA^2}{8C}\int_{-L/2}^{L/2}dx\cosh^{-4}\left[\left[\frac{A}{2}\right]^{1/2}(x-x_0+\bar{x})\right]+\frac{TA^2}{8C}\int_{-L/2}^{L/2}dx\cosh^{-4}\left[\left[\frac{A}{2}\right]^{1/2}(x-x_0-\bar{x})\right]$$
$$-\frac{TA^2}{4C}\int_{-L/2}^{L/2}dx\cosh^{-2}\left[\left[\frac{A}{2}\right]^{1/2}(x-x_0+\bar{x})\right]\cosh^{-2}\left[\left[\frac{A}{2}\right]^{1/2}(x-x_0-\bar{x})\right]\ . \quad (2.17)$$

We see from (2.17) that, apart from a constant proportional to $\vartheta(\phi_1)$, the action is composed of two terms (the first two integrals) which give, respectively, the action of the kink and antikink components of (2.14), and a third integral in (2.17) which corresponds to the overlap of the kink and antikink components.

The first term has a volume type of divergence and it is exactly equal to the action of the constant configuration ϕ_1. The other terms in (2.17) have only a superficial divergence.

This volume divergence is responsible for the nonzero contribution to the free energy coming from a single bounce. Differently, in the case of a kink (antikink) we have to consider the approximation of a dilute gas of such configurations to get a nonzero contribution even at the classical level.

III. THE CONTRIBUTION OF THE BOUNCE TO THE GENERATING FUNCTIONAL—A NEW APPROACH

In this section we evaluate the one-loop contribution to the generating functional coming from the classical solution obtained in the preceding section and its neighborhood. The problem is treated in dimension $d=2(1+1)$. In this case, the superficial degree of divergence D for any graph is given by $D=2-2V$, where V is the number of vertices (remember that divergent graphs have $D\geq 0$). The only possible divergent graphs are those with $V=1$ (tad poles), which are excluded by Wick ordering of the products of fields operators, corresponding to the renormalization of the ground-state energy.[11]

The calculations will be done inside a two-dimensional box Λ of volume $L\times T$ with periodic boundary conditions and at the end we take the limit $L,T\to\infty$. Because of the use of the periodic boundary condition in a rectangular box centered at the origin of coordinates, we must set the value of the constant $x_0=0$ in the bounce solution $\bar{\phi}(x)$. Thus, the partition function in Euclidean space is (we are using the shorthand notation $dV=dx\,dt$ and $dV'=dy\,dt'$)

$$Z_\Lambda[J]=N^{-1}\int\mathcal{D}\phi\exp\left[-\int_\Lambda dV:\tfrac{1}{2}(\partial_\mu\phi)^2+\frac{g}{4}(\phi^2-a^2)^2-J\phi:_{m_0^2}\right], \quad (3.1)$$

with the normalization constant given by

$$N=\int\mathcal{D}\phi\exp\left[-\int_\Lambda dV:\tfrac{1}{2}(\partial_\mu\phi)^2:_{m_0^2}\right], \quad (3.1a)$$

where m_0 is an arbitrary mass scale introduced for Wick ordering of the products of operators, since the quadratic term in the field coming from the potential (2.1a) has a negative coefficient which cannot be interpreted as a squared mass parameter.

Our aim is to calculate the contribution coming from the classical solution (bounce), obtained in Sec. II, and its neighborhood to intensive quantities, such as the generating functional $F[J]=\lim_{\Lambda\to\infty}(1/\Lambda)\ln Z_\Lambda[J]$ for small $J>0$. We are interested in studying this quantity, once it gives information about symmetry breaking, and we are able to derive expressions in closed forms.

The usual way to treat the quantum correction from the bounce is to neglect the cubic and quartic terms in the potential $\vartheta(\phi)$ expanded around the bounce solution[2] [see Eq. (2.1a)]. In practice, this amounts to obtaining the spectrum of the operator $\mathcal{O}=-\frac{1}{2}[\partial^2-\vartheta''(\bar{\phi}(x))]$ which in the case of an expansion around the bounce solution has a negative eigenvalue in addition to the zero mode. The contribution from the zero eigenvalue is known in the literature.[1]

In our case, the operator $\mathcal{O}$ is given by

$$\mathcal{O}=-\tfrac{1}{2}\partial^2+V(x), \quad (3.2)$$

where

$$V(x)=A\left[1-\frac{3}{2}\left\{\operatorname{sech}^2\left[\left(\frac{A}{2}\right)^{1/2}(x-\bar{x})\right]+\operatorname{sech}^2\left[\left(\frac{A}{2}\right)^{1/2}(x+\bar{x})\right]\right\}\right],$$

with A and $\bar{x}$ given, respectively, by (2.9) and (2.13).

Since the zero eigenvalue of this Schrödinger operator is given by the derivative of $\bar{\phi}(x)$, which has a node, then we have a negative eigenvalue E_{-1} (the ground state of operator $\mathcal{O}$). It is straightforward to compute E_{-1} in the lowest order in J (Ref. 12):

$$E_{-1}\simeq-\frac{36\sqrt{3}}{a}J. \quad (3.3)$$

Because of the existence of this negative eigenvalue, the quartic term of the exponent of (3.1) cannot be just thrown away, unless we conveniently modify the quadratic part, in order to assure that its spectrum is positive definite. We show in the following that this may be done by changing the mass scale in Wick ordering.

For this purpose let us recall some formulas for changing the scale m_0^2 to another one m^2 (Ref. 11):

$$:\phi^2(x):_{m_0^2}=:\phi^2(x):_{m^2}-\Delta C(m_0^2,m^2), \quad (3.4a)$$

$$:\phi^3(x):_{m_0^2}=:\phi^3(x):_{m^2}-3\Delta C(m_0^2,m^2)\phi(x), \quad (3.4b)$$

and

$$:\phi^4(x):_{m_0^2}=:\phi^4(x):_{m^2}-6\Delta C(m_0^2,m^2):\phi^2(x):_{m^2}+3[\Delta C(m_0^2,m^2)]^2, \quad (3.4c)$$

where $\Delta C(m_0^2,m^2)=\ln(m_0^2/m^2)/(4\pi)$.

To proceed further we expand $\phi(x)$ around $\bar{\phi}(x)$,

$$\phi(x,t)=\bar{\phi}(x)+\eta(x,t), \quad (3.5)$$

and the partition function (2.1) may be rewritten in the form

$$Z_\Lambda[J]=N^{-1}e^{-S[\bar{\phi}]}\int\mathcal{D}\eta\exp\left[-\int_\Lambda dV:\tfrac{1}{2}(\partial_\mu\eta)^2+\frac{g}{2}[3\bar{\phi}(x)^2-a^2]\eta^2+g\bar{\phi}(x)\eta^3+\frac{g}{4}\eta^4:_{m_0^2}\right], \quad (3.6)$$

and $S[\bar{\phi}(x)]$ is the classical action of the bounce solution $\bar{\phi}(x)$.

We can neglect the cubic and quartic terms that appear in expression (3.6) only if the quadratic term is positive definite. This can be done by exploiting the arbitrariness in the scale of Wick ordering. Therefore, we perform a change of mass scale $m_0^2\to m^2$, which gives, from (3.4) and (3.6),

$$Z[J]=N^{-1}e^{-S[\bar\phi(x)]}\exp\left[-\frac{3g}{(8\pi)^2}\ln^2\left[\frac{m^2}{m_0^2}\right]\Lambda\right]\exp\left[\int_\Lambda dV\frac{1}{4\pi}\ln\left[\frac{m^2}{m_0^2}\right]A(\bar\phi)\right]$$
$$\times\int\mathcal{D}\eta\exp\left\{-\int_\Lambda dV{:}\tfrac{1}{2}(\partial_\mu\eta)^2+\left[A(\bar\phi)-\frac{3g}{8\pi}\ln\left[\frac{m^2}{m_0^2}\right]\right]\eta^2-\frac{3g}{4\pi}\bar\phi(x)\ln\left[\frac{m^2}{m_0^2}\right]\eta+g\bar\phi(x)\eta^3+\frac{g}{4}\eta^4{:}_{m^2}\right\}, \quad (3.7)$$

where we have introduced, for any configuration $\phi(x,t)$, the notation

$$A(\phi)=\frac{g}{2}[3\phi^2(x,t)-a^2]\ ; \quad (3.8)$$

in particular, we have $A(\phi_1)=A$. Then, we may circumvent the negative eigenvalue problem by imposing that the operator

$$\mathcal{O}'\equiv-\partial^2+2A(\bar\phi)-\frac{3g}{4\pi}\ln\left[\frac{m^2}{m_0^2}\right] \quad (3.9)$$

has a positive spectrum. This may be accomplished by the condition

$$-\frac{3}{8\pi}\ln\left[\frac{m^2}{m_0^2}\right]\geq|E_{-1}|\ . \quad (3.10)$$

where E_{-1} is the negative eigenvalue given by formula (3.3) for small J. Thus, if we choose m such that

$$m^2\leq m_0^2\exp\left[\frac{8\pi}{3g}E_{-1}\right], \quad (3.11)$$

the operator (3.9) has no negative eigenvalue. If the strict equality is taken in (3.10) or (3.11), the operator $\mathcal{O}'$ will have a zero eigenvalue, but to avoid this we may take $m^2=m_0^2\exp[(8\pi/3g)(E_{-1}-\delta)]$, with $\delta>0$ and small, and put $\delta=0$ at the end of the computation. From now on, we take for simplicity the equality sign in (3.10) and (3.11).

Now we may neglect the cubic and quartic terms in the exponent of the functional integral (3.7), since the quadratic piece corresponding to the operator $\mathcal{O}'$ in (3.9) is well defined.

To treat the linear term in the exponent of (3.7), we make the change of variables

$$\eta(x,t)=\xi(x,t)-\int_\Lambda dV'\mathcal{O}'^{-1}(x,t;y,t')F(y)\ , \quad (3.12)$$

with

$$F(y)\equiv-\frac{3g}{4\pi}\ln\left[\frac{m^2}{m_0^2}\right]\bar\phi(y)\ , \quad (3.12a)$$

and $\mathcal{O}'^{-1}$, the operator inverse of $\mathcal{O}'$, is given in terms of the energy eigenfunctions of operator $\mathcal{O}'$, that is,

$$\psi_{n,m}(x,t)=\frac{1}{\sqrt{T}}\chi_n(x)e^{i\omega_m t},\quad \omega_m=\frac{2\pi m}{T},\quad m\in\mathbb{Z}\ , \quad (3.13)$$

by the formula

$$\mathcal{O}'^{-1}(x,t;y,t')=\sum_{n,m}\frac{\psi_{nm}(x,t)\psi_n^*(y,t')}{E_{nm}}\ , \quad (3.14)$$

where we are summing the discrete and continuum part of the spectrum. E_{nm} are the energy eigenvalues, that is, $E_{nm}=\epsilon_n+(2\pi m/T)^2$, and ϵ_n being the eigenvalues satisfying the Schrödinger equation

$$\left[-\frac{d^2}{dx^2}+2A(\bar\phi)+2|E_{-1}|\right]\chi_n(x)=\epsilon_n\chi_n(x)\ . \quad (3.15)$$

For later convenience, let us introduce a function $\epsilon f(x)$ such that the Schrödinger equation associated with the operator $-\frac{1}{2}\partial^2+\epsilon f(x)$ has the same number of bound states as the operator $\mathcal{O}'$.

Then, inserting the change of variables (3.12) in (3.7), the partition function may be written as

$$Z_\Lambda[J]=\lim_{\substack{\Lambda\to\infty\\ \epsilon\to0}}N'^{-1}e^{-S[\bar\phi]}\exp\left[-\frac{3g}{(8\pi)^2}\ln^2\left[\frac{m^2}{m_0^2}\right]\Lambda\right]\exp\left[\frac{1}{4\pi}\ln\left[\frac{m^2}{m_0^2}\right]\int_\Lambda dV\,A(\bar\phi)\right]$$
$$\times\exp\left[\frac{1}{2}\int_{\Lambda,\Lambda'}dV\,dV'F(x)\mathcal{O}'^{-1}(x,t;y,t')F(y)\right]\int\mathcal{D}\xi(x,t)\exp\left[-\int_\Lambda dV{:}\tfrac{1}{2}\xi(x,t)\mathcal{O}'\xi(x,t){:}_{m^2}\right], \quad (3.16)$$

where

$$N'(\epsilon,f;\Lambda)=\int\mathcal{D}\xi\exp\left[-\int_\Lambda dV{:}[\tfrac{1}{2}(\partial_\mu\xi)^2+\epsilon f(x)\xi^2]{:}_{m^2}\right]. \quad (3.16a)$$

We proceed now to calculate the contribution from the bounce and its neighborhood to the generating functional:

$$F[J]=\lim_{\substack{L,T\to\infty\\ \epsilon\to0}}\frac{1}{LT}\ln Z_\Lambda[J]\ . \quad (3.17)$$

From (3.16), (3.16a), and (3.17) we obtain, by rewriting the Wick-ordered product in terms of simple products of fields,

$$F[J]=\lim_{\substack{L,T\to\infty\\ \epsilon\to 0}}\left[-\frac{1}{LT}S[\bar\phi]-\frac{3g}{(8\pi)^2}\ln^2\left[\frac{m^2}{m_0^2}\right]+\frac{1}{LT}\frac{1}{4\pi}\ln\left[\frac{m^2}{m_0^2}\right]\int_\Lambda dV\,A(\bar\phi)+\frac{1}{LT}\int_\Lambda dV[A(\bar\phi)+|E_{-1}|]C^0_{m^2}\right.$$
$$\left.+\frac{1}{LT}\frac{1}{2}\int_{\Lambda,\Lambda'}dV\,dV'F(x)\mathcal{O}'^{-1}(x,t;y,t')F(y)-\frac{1}{LT}\frac{1}{2}\ln\left[\frac{\det'(\mathcal{O}'/2)}{\det[-\frac{1}{2}\partial^2+\epsilon f(x)]}\right]\right], \tag{3.18}$$

where

$$C^0_{m^2}=\frac{1}{(2\pi)^2}\int\frac{d^2k}{k^2+m^2}\ . \tag{3.19}$$

Keeping only terms that give a nonvanishing contribution at the limit $L\to\infty, T\to\infty$, we find, after some manipulations,

$$F[J]=-\vartheta(\phi_1)-\frac{(E_{-1})^2}{2g}-\frac{2|E_{-1}|A}{g}$$
$$+\lim_{\substack{L,T\to\infty\\ \epsilon\to 0}}\frac{1}{LT}\left[\tfrac{1}{2}\int_{\Lambda,\Lambda'}dV\,dV'F(x)\mathcal{O}'^{-1}(x,t;y,t')F(y)\right.$$
$$\left.+\int_\Lambda dV[A(\bar\phi(x))+|E_{-1}|]C^0_{m^2}-\tfrac{1}{2}\ln\left[\frac{\det'(\mathcal{O}')}{\det[-\partial^2+2\epsilon f(x)]}\right]\right]. \tag{3.20}$$

Let us call, for short, I the last two terms between the large square brackets in Eq. (3.20). To calculate its contribution in the limit $\Lambda\to\infty$, some preliminaries are needed: we assume that the function $f(x)$ is such that $\lim_{|x|\to\infty}f(x)=\frac{1}{2}$, and moreover that the operator $-\partial^2+2\epsilon f(x)$ has the same number of bound states as the operator $\mathcal{O}'$, which will be denoted by λ_n, all of them being greater than zero. We use the finite-volume (discrete) version of (3.19):

$$C^0_{m^2}=\frac{1}{LT}\sum_{l=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}(k_n^2+\omega_l^2+m^2)^{-1}\ , \tag{3.21}$$

where

$$k_n=\frac{2\pi n}{L},\quad \omega_l=\frac{2\pi l}{T},\quad n,l=0,\pm1,\pm2,\ldots\ . \tag{3.22}$$

Keeping in mind that the relevant contribution from the coefficient of $C^0_{m^2}$ in (3.20) is the one that cancels the divergences in the quotient of the determinants and also that since $F[J]$ is an intensive quantity, only the continuum spectrum contributes. It is not difficult to get the following expression for I:

$$I=\lim_{\substack{L,T\to\infty\\ \epsilon\to 0}}\frac{1}{LT}\frac{1}{2}\left[M_0^2\sum_{l=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\frac{1}{\omega_n^2+k_l^2+m^2}-\sum_{n=0}^{N}\sum_{l=-\infty}^{\infty}\ln\left[\frac{\epsilon_n+\omega_l^2}{\lambda_n+\omega_l^2}\right]-\sum_{n=-\infty}^{\infty}\sum_{l=-\infty}^{\infty}\ln\left[\frac{M_0^2+k_n^2+\omega_l^2}{\epsilon+k_n^2+\omega_l^2}\right]\right],$$

where

$$M_0^2=A+|E_{-1}|=\tfrac{1}{2}\vartheta''(\phi_1)+|E_{-1}|\ . \tag{3.23a}$$

In the limit $\Lambda\to\infty$ (3.23) becomes, after some calculations and changes of variables,

$$I=-\frac{M_0^2}{8\pi}\left[1-\ln\left[\frac{M_0^2}{m^2}\right]\right]. \tag{3.24}$$

Coming back to the formula (3.20) let us evaluate the contribution

$$Q\equiv\lim_{L,T\to\infty}\frac{1}{LT}\frac{1}{2}\int_{\Lambda,\Lambda'}dV\,dV'F(x)\mathcal{O}'^{-1}(x,t;y,t')F(y)\ . \tag{3.25}$$

Again, in an analogous manner as for the preceding calculation of the quantity I, only the continuous part of the spectrum of the operator $\mathcal{O}'$ contributes. Nevertheless, it is important to point out that the presence of the term (3.25) in formula (3.20) is due to the existence of the *negative eigenvalue* of the operator $\mathcal{O}$ given by (3.2). Using (3.13) and (3.14) we may write (3.25) in the form

$$Q=\frac{1}{2}\lim_{L,T\to\infty}\frac{1}{LT}\sum_{n,m}\frac{\left|\int_\Lambda dV\,F(x)\psi_{nm}(x,t)\right|^2}{E_{nm}}\ , \tag{3.26}$$

where the summation goes over both discrete and continuous eigenstates, and where we have used the fact that the function $F(x)$ defined by (3.12a) is real.

Let us look at separately in (3.26) the contributions from the discrete and the continuum parts of the spectrum of the operator $-d^2/dx^2+2[A(\bar{\phi})+|E_{-1}|]$. Using (3.13), it is easy to see that the contribution of the discrete spectrum expression (3.26) is given by

$$Q=\frac{1}{2}\lim_{L\to\infty}\frac{1}{L}\sum_{n=1}^{N}\left|\int_{-L/2}^{L/2}dx\,F(x)\chi_n(x)\right|^2 , \qquad (3.27)$$

where N is the number of (bound) states. But, since $\chi_n(x)$ is a bound eigenstate and $F(x)$ goes to a constant as $|x|\to\infty$, the modulus of the integral in (3.27) is a finite quantity. Then, taking into account that N is finite, we conclude that (3.27) vanishes, which means that the discrete part of the spectrum of the operator $-d^2/dx^2+2[A(\bar{\phi})+|E_{-1}|]$ gives no contribution to Q.

In the Appendix we discuss the form of the wave functions associated with the continuous part of the operator $-d^2/dx^2+2[A(\bar{\phi})+|E_{-1}|]$. They may be chosen with definite parity, normalized, and have the form

$$\chi_n(x)=\frac{1}{\sqrt{L}}f_n(x)\cos[k_n(x)x-\delta_n^s] \qquad (3.28a)$$

or

$$\chi_n(x)=\frac{1}{\sqrt{L}}g_n(x)\sin[k_n(x)x-\delta_n^a] , \qquad (3.28b)$$

where if periodic boundary conditions are taken we have that, for k_n defined by $k_n=\lim_{|x|\to\infty}k_n(x)$,

$$k_n=\frac{2\pi n}{L}+\frac{2\delta^s}{L} \quad\text{or}\quad k_n=\frac{2\pi n}{L}+\frac{2\delta^a}{L} ,$$
$$n=0,\pm1,\pm2,\ldots , \qquad (3.29)$$

and $\delta^s(\delta^a)$ is the phase shift depending on n, $f_n(x)$ and $g_n(x)$ being bounded functions in the interval $x\in[-L/2,L/2]$.

Now, going back to the expression (3.26) for Q, we can calculate the contribution from the continuum part of the spectrum.

We restrict ourselves to the wave functions of even parity, since $F(x)$ is an even function. Let us take a fixed arbitrary M, such that $M\gg1$ and let $-\bar{x},+\bar{x}$ be the minima of the potential $A(\bar{\phi}(x))+|E_{-1}|$. For $x\gg M\bar{x}$, we get, from the Appendix,

$$\chi_n(x)\approx\frac{1}{\sqrt{L}}\cos(k_nx-\delta_n^s),\quad x\gg M\bar{x} . \qquad (3.30)$$

Inserting (3.30) in (3.26), using the fact that the function $F(x)$ tends exponentially to a constant F_0 as $|x|\to\infty$, we obtain, retaining only the terms that give nonvanishing contributions as $\Lambda\to\infty$,

$$Q=\frac{1}{2}\lim_{L\to\infty}\frac{1}{L}\sum\frac{1}{E_{n0}}\left\{F_0^2\left|\int_{-L/2}^{L/2}\chi_n(x)dx\right|^2+2\,\mathrm{Re}\left[F_0\left[\int_{-L/2}^{L/2}dx\,[F(x)-F_0]\chi_n(x)\right]\int_{-L/2}^{L/2}\chi_n(x)dx\right]\right\} , \qquad (3.31)$$

Let us examine the integral $I_n\equiv\int_{-1/2}^{1/2}\chi_n(x)dx$. We have, since $\chi_n(x)$ is an even function,

$$I_n=2\int_0^{L/2}\chi_n(x)dx$$
$$=2\left[\int_0^{M\bar{x}}\chi_n(x)dx+\int_{M\bar{x}}^{L/2}\chi_n(x)dx\right] , \qquad (3.32)$$

where M and $\bar{x}$ have been defined previously. Estimates for the integrals that appear in (3.32) are

$$\int_0^{M\bar{x}}\chi_n(x)dx\le\frac{1}{\sqrt{L}}\mathrm{Sup}[f(x)](M\bar{x})=O(L^{-1/2}) \qquad (3.33)$$

and, using the boundary condition $\sin(k_nL/2-\delta_n^s)=0$, we have

$$\int_{M\bar{x}}^{L/2}\chi_n(x)dx=O(L^{-1/2})\quad\text{for}\quad n\neq O\ (k_n\neq0) . \qquad (3.34)$$

For $n=0$ ($k_0=0$), we have

$$\int_{M\bar{x}}^{L/2}\chi_n(x)dx=\frac{\sqrt{L}}{2}\cos\delta_0^s . \qquad (3.35)$$

Inserting (3.35), (3.34), and (3.33) in (3.32) and in (3.31), we obtain

$$Q=\frac{1}{2}\frac{F_0^2}{E_{00}}\cos^2\delta_0^s , \qquad (3.36)$$

or, from (3.12a) and (3.23a), and using the fact that $\delta_0^s=0$ is an acceptable solution for the phase shift problem (cf. the Appendix), we have then that

$$Q=\frac{2|E_{-1}|^2}{M_0^2}\phi_1^2 . \qquad (3.37)$$

Replacing (3.37) and (3.24) in (3.20) and using (3.10) with the equality sign, we finally get

$$F[J]=-\frac{|E_{-1}|^2}{6g}-\vartheta(\phi_1)+\frac{2|E_{-1}|^2\phi_1^2}{\vartheta''(\phi_1)+2|E_{-1}|}-\frac{\vartheta''(\phi_1)+2|E_{-1}|}{8\pi}\times\left[1-\ln\left[\frac{\vartheta''(\phi_1)+2|E_{-1}|}{m_0^2}\right]\right] . \qquad (3.38)$$

This expression is valid for $J\in(0,J_c)$.

IV. CONCLUSIONS

We have shown that despite the fact that the bounce solution is unstable at the classical level it gives a real contribution to the generating functional for the model we are considering. Of course, we can imagine other possibilities for handling the negative eigenvalue in the path integral, which possibly will lead to a better approximation scheme to the complete integral.

To make contact with other ways of computing approximations to the generating functional we can compute the vacuum expectation value of the field $\langle\phi\rangle$ at zero external current $J=0$. This is obtained by

$$\langle \phi \rangle_{0^+} = \lim_{J \to 0^+} \frac{dF[J]}{dJ} ,$$

which gives, using (3.40) and (3.3),

$$\langle \phi \rangle_{0^+} = -a + \frac{72\sqrt{3}-3}{8\pi a} \ln \frac{2ga^2}{m_0^2} . \tag{4.1}$$

Of course, using $J<0$ instead of a positive J will lead to $\langle \phi \rangle_{0^-} = -\langle \phi \rangle_{0^+}$.

If we perform this computation by using an expansion around the minima ϕ_1 or ϕ_2 of the classical potential (2.1a) we obtain

$$\langle \phi \rangle_{0^+} = -a - \frac{3}{8\pi a} \ln \frac{2ga^2}{m_0^2} \tag{4.2}$$

for the relative minimum ϕ_1 and

$$\langle \phi \rangle_{0^+} = a + \frac{3}{8\pi a} \ln \frac{2ga^2}{m_0^2} \tag{4.3}$$

for the absolute minimum ϕ_2.

Comparing (4.1) with (4.2) and (4.3) we see that our result contains an additional term

$$\frac{72\sqrt{3}}{8\pi a} \ln \frac{2ga^2}{m_0^2} , \tag{4.4}$$

which is a quantum correction to the classical value $-a$ and it is originated by the negative eigenvalue associated to the bounce solution. The term (4.4) has an opposite sign relative to the usual quantum correction

$$-\frac{3}{8\pi a} \ln \frac{2ga^2}{m_0^2}$$

and for small a Eq. (4.4) tends to drag the vacuum expectation closer to a zero value.

Of course, in oder for this to be effective we must have a ratio $2ga/m_0 > 1$ which is natural if we interpret the Lagrangian density (2.1) as a zero mass theory with a quartic and a negative quadratic interaction terms. Therefore, the Wick-ordering scale m_0 should be consistently a small parameter [it cannot be exactly zero because this would lead to a (infrared) divergence].

Hence, we have a possible mechanism for symmetry restoration which results from the fact that we are beginning with a tunneling classical field configuration between the two classical minima.

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APPENDIX: WAVE FUNCTIONS OF THE CONTINUOUS SPECTRUM OF THE OPERATOR: $-d^2/dx^2+2[A(\bar{\phi})+|E_{-1}|]$

We must investigate the Schrödinger equation

$$\left[-\frac{d^2}{dx^2} + 2[A(\bar{\phi}) + |E_{-1}|] \right] \chi_n(x) = \epsilon_n \chi_n(x) , \tag{A1}$$

from (2.14) and (3.8) we have the explicit expression for the potential:

$$V(x) = A(\bar{\phi}) + |E_{-1}| = A \left[1 - \frac{3}{2} \left\{ \operatorname{sech}^2 \left[\left[\frac{A}{2} \right]^{1/2} (x+\bar{x}) \right] + \operatorname{sech}^2 \left[\left[\frac{A}{2} \right]^{1/2} (x-\bar{x}) \right] \right\} \right] + |E_{-1}| , \tag{A2}$$

where A and $\bar{x}$ are given by (2.9) and (2.13). The points $x = \pm \bar{x}$ are the minima of $V(x)$ and $A + |E_{-1}|$ its asymptotic value.

We are interested in studying the wave function in the asymptotic region, for $\epsilon_n \geq M_0^2$ [see Eq. (3.25a)] and for $|x| \gg M\bar{x}$, $M \gg 1$. We make a change of variables,

$$z = \sqrt{2A}\, x, \quad \lambda_n = \frac{2}{A}(\epsilon_n - M_0^2) , \tag{A3}$$

and write the hyperbolic functions in its exponential form. Using (A2) and (A3), Equation (A1) becomes

$$\frac{d^2 \chi_n(z)}{dz^2} - 6 \left[\frac{e^{-(z+\bar{z})}}{(1+e^{-(z+\bar{z})})^2} + \frac{e^{-(z-\bar{z})}}{(1+e^{-(z-\bar{z})})^2} \right] \chi_n(z) = \frac{\lambda_n}{4} \chi_n(z) . \tag{A4}$$

In the region $x > \bar{x}$, we expand (A4) in power series, rewrite the wave function as

$$\chi_n(z) = e^{ikz} \sum_{n=0} a_{mn} e^{-z(n+s)} , \tag{A5}$$

and define a new variable $\omega = e^{-z}$; by equating coefficients we obtain the condition

$$s(s-1) - (2ik-1)s + \frac{\lambda_m}{4} - k^z = 0 , \tag{A6}$$

which yields

$$s = ik \pm i \left[\frac{\lambda_m}{4} \right]^{1/2} . \tag{A7}$$

Thus, going back to the variable x, the wave functions may be written in the form

$$\chi_m(x) \approx e^{\pm i\sqrt{k_m}x} \sum_{n=0}^{\infty} a_{mn} \exp(-2n\sqrt{A/2}x), \quad x > \bar{x} , \tag{A8}$$

where $k_m=\epsilon_m-M_0^2$.

We have an analogous formula for $x<-\bar{x}$:

$$\chi_m(x)\approx e^{\mp i\sqrt{k_m}x}\sum_{n=0}^{\infty}\bar{a}_{mn}\exp(-2n\sqrt{A/2}x),\quad x<-\bar{x}\ . \tag{A8a}$$

The potential in the Schrödinger operator $-d^2/dx^2+2V(x)$ is even. Therefore, we may choose the eigenfunctions $\chi_n(x)$ with a definite parity. There are two possible expansions of the type (A5) with the coefficients noted a_{mn}^+ and a_{mn}^-, respectively, which give two possible sets of eigenfunctions:

$$\chi_m^{\pm}(x)\approx e^{\mp i\sqrt{k_m}x}\sum_{n=0}^{\infty}a_{mn}^{\pm}\exp(-2n\sqrt{A/2}x),\quad x>\bar{x} \tag{A9}$$

and analogously for $x<-\bar{x}$. We need only the even wave functions. These can be written as a particular superposition of $\chi_m^+(x)$ and $\chi_m^-(x)$ in the regions $x<-\bar{x}$ and $x>\bar{x}$; that is, (i) for $x>\bar{x}$,

$$\chi_m^{\text{even}}(x)=\cos(\sqrt{k_m}x)\sum_{n=0}^{\infty}\left[\frac{a_{mn}^{+}+a_{mn}^{-}}{2}\right]\exp(-2n\sqrt{A/2}x)+\sin(\sqrt{k_m}x)\sum_{n=0}^{\infty}\left[\frac{a_{mn}^{-}-a_{mn}^{+}}{2}\right]\exp(-2n\sqrt{A/2}x)\ ; \tag{A9a}$$

(ii) for $x<-\bar{x}$,

$$\chi_m^{\text{even}}(x)=\cos(\sqrt{k_m}x)\sum_{n=0}^{\infty}\left[\frac{\bar{a}_{mn}^{+}+\bar{a}_{mn}^{-}}{2}\right]\exp(2n\sqrt{A/2}x)+\sin(\sqrt{k_m}x)\sum_{n=0}^{\infty}\left[\frac{\bar{a}_{mn}^{+}-\bar{a}_{mn}^{-}}{2}\right]\exp(2n\sqrt{A/2}x)\ , \tag{A9b}$$

since the coefficients satisfy the conditions $\bar{a}_{mn}^{+}=a_{mn}^{+}$ and $\bar{a}_{mn}^{-}=a_{mn}^{-}$.

In order to obtain the asymptotic behavior of the wave functions, we keep only the oscillatory terms

$$\chi_m^{\text{even}}(x)\approx\cos(\sqrt{k_m}x-\delta_m),\quad x\gg M\bar{x} \tag{A10}$$

and

$$\chi_m^{\text{even}}(x)\approx\cos(\sqrt{k_m}x+\delta_m),\quad x\ll -M\bar{x}\ , \tag{A10a}$$

where

$$\delta_m=\arccos\left[\frac{a_{m0}+a_{m0}}{2}\right]. \tag{A11}$$

Using the periodic boundary conditions, we obtain

$$k_m=\left[\frac{2\pi m}{L}+\frac{2\delta m}{L}\right]^2,\quad n\in\mathbb{Z}\ . \tag{A12}$$

Finally, we remark that using the expressions of $\chi_m^{\text{even}}(x)$ [cf. (A9a) and (A9b)], $\delta_0=0$ is an acceptable solution.

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