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Scalar field in the Frolov-Markov-Mukhanov black-hole space-times

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The expectation values of the stress-energy tensor for a massless scalar field are investigated in a two-dimensional section of the Frolov-Markov-Mukhanov space-time representing a black hole with an interior de Sitter nucleus. The problem of regularity on the de Sitter Cauchy horizons is discussed.

I. INTRODUCTION

Singularities arising in general relativity are mostly regarded as an artifact of classical theory. They simply signal the breakdown of the classical Einstein equations and should not appear in a more correct framework given by a self-consistent quantum gravity theory. Unfortunately no such theory is available at the moment. This however has not refrained authors from speculating on how singularities arising in the classical treatment of gravitational collapse may be avoided. The mechanism proposed for this goal can be either vacuum polarization[1] or some more fundamental "new law of nature" which should prevent space-time curvature from growing behind an upper bound of Planckian magnitude.[2] In any case the picture which emerges is the following. As a result of the gravitational collapse of a massive body a black hole is formed. In its inner core when the curvature reaches order unity (in Planck units) quantum effects should produce a smooth transition towards a constant curvature (de Sitter) region which is indeed regular.

A crude model which mimics this behavior is that[3] of a black hole described by the Schwarzschild solution

$$ds^2=-\left[1-\frac{2M}{r}\right]dt^2+\left[1-\frac{2M}{r}\right]^{-1}dr^2+r^2(d\theta^2+\sin^2\theta\,d\phi^2) \tag{1.1}$$

down to a critical radius $r_0 \sim M^{1/3}$, $\lambda < r_0 < 2M$. Inside this radius the space-time is described by the de Sitter solution given in static coordinates by

$$ds^2=-\left[1-\frac{r^2}{\lambda^2}\right]dT^2+\left[1-\frac{r^2}{\lambda^2}\right]^{-1}dr^2+r^2(d\theta^2+\sin^2\theta\,d\phi^2)\,. \tag{1.2}$$

The transition region between the Schwarzschild and the de Sitter geometry is here supposed to be of short (timelike) extent and is modeled as a spacelike surface layer lying on a hypersurface of constant time r_0=const. Using Israel's junction conditions[4] one can show that such a static configuration is indeed allowed by the field equations and that it is stable against small variations of the physical parameters entering the model.[5] The intriguing global structure of such space-time is depicted in Fig. 1 where an asymptotically flat space-time is connected by a black-hole interior to a collapsing and then reexpanding de Sitter universe.

Physically more interesting models which take into account black-hole formation and Hawking evaporation are also discussed in Ref. 3.

In this paper we shall discuss in some detail within the context of the semiclassical approach to quantum gravity (i.e., quantum field theory in curved space-times) the behavior of a quantized massless scalar field $\hat{\phi}$ propagating in these black-hole space-times. Particular interest will be devoted to the behavior of the expectation values of the associated stress-energy tensor operator $\langle T_{ab}(\hat{\phi})\rangle$ near the de Sitter horizons. Consistency of the above models at the semiclassical level requires the fact that $\langle T_{ab}(\hat{\phi})\rangle$ be finite as $r \to \lambda$. A preliminary account of this investigation was given for the static configuration in Ref. 6 and the result found there (reviewed in Sec. III) was rather disappointing: $\langle T_{ab}\rangle$ is finite on the Cauchy horizons only if the transition region is located very close

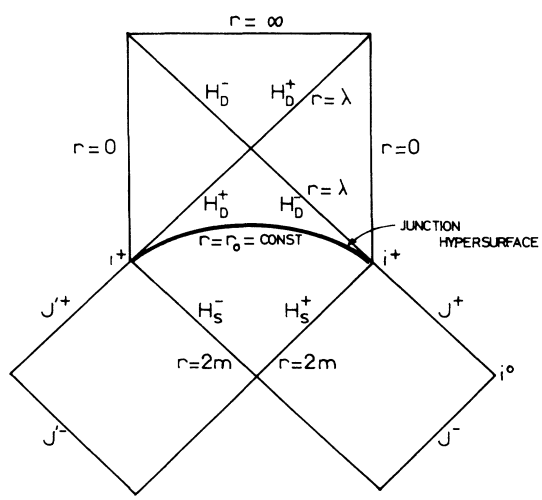

FIG. 1. Conformal diagram obtained by gluing the de Sitter space to the Schwarzschild one on the junction hypersurface of constant radial coordinate (r_0=const). $H_S^{\pm}$ ($r=2M$) are the event horizons of the black hole, while $H_D^{\pm}$ ($r=\lambda$) are the Cauchy horizons.

to the black-hole horizon $r=2M$. Here we treat more general (nonstatic) configurations in order to overcome this difficulty.

II. QUANTUM STRESS TENSOR IN TWO-DIMENSIONAL SPACE-TIMES

Let us consider a quantized massless scalar field $\hat{\phi}$ propagating in a two-dimensional space-time whose metric can be written quite generally in the form

$$ds^2=-C(u,v)du\,dv \ , \tag{2.1}$$

where $u(v)$ is a retarded (advanced) null coordinate and C is the conformal factor.

The field equation for $\hat{\phi}$ assumes simply the flat-space form

$$\partial_u\partial_v\hat{\phi}=0 \ . \tag{2.2}$$

If u,v range from $-\infty$ to $+\infty$, the field operator $\hat{\phi}$ can be expanded in the normal modes:

$$(4\pi w)^{-1/2}e^{-iwu}, \quad (4\pi w)^{-1/2}e^{-iwv} \ . \tag{2.3}$$

This expansion defines a "vacuum" state $|u,v\rangle$. Now the expectation values of the renormalized stress-energy-tensor operator $T_{ab}(\hat{\phi})$ for the scalar field in the above state are given by the expression[7]

$$\langle u,v|T_{ab}|u,v\rangle=Q_{ab}-(48\pi)^{-1}Rg_{ab} \ , \tag{2.4}$$

where

$$Q_{uu}=-(12\pi)^{-1}C^{1/2}\partial_u^2C^{-1/2} \ , \tag{2.5a}$$

$$Q_{vv}=-(12\pi)^{-1}C^{1/2}\partial_v^2C^{-1/2} \ , \tag{2.5b}$$

$$Q_{uv}=Q_{vu}=0 \tag{2.5c}$$

and R is the two-dimensional Ricci scalar

$$R=4C^{-3}(C\partial_v\partial_uC-\partial_uC\partial_vC) \ . \tag{2.6}$$

The double null form of the metric (2.1) is preserved by reparametrizations of the form

$$u=A(\bar{u}), \quad v=B(\bar{v}) \quad \text{(conformal gauge)} \ . \tag{2.7}$$

In terms of the new null coordinates $\bar{u},\bar{v}$ the metric becomes

$$ds^2=-CA'B'd\bar{u}\,d\bar{v} \ , \tag{2.8}$$

where a prime indicates differentiation with respect to the relevant variable.

One can also expand $\hat{\phi}$ in terms of the set of normal modes:

$$(4\pi\bar{w})^{-1/2}e^{-i\bar{w}\bar{u}}, \quad (4\pi\bar{w})^{-1/2}e^{-i\bar{w}\bar{v}} \ . \tag{2.9}$$

This alternative expansion defines a new "vacuum" state, call it $|\bar{u},\bar{v}\rangle$, with respect to which the $\langle T_{ab}\rangle$ are given by

$$\langle\bar{u},\bar{v}|T_{ab}|\bar{u},\bar{v}\rangle=Q_{ab}-(48\pi)^{-1}Rg_{ab}+X_{ab} \ , \tag{2.10}$$

where $a,b=u,v$; Q_{ab} is given as before by Eqs. (2.5) and the state-dependent term X_{ab} of $\langle T_{ab}(\hat{\phi})\rangle$ is

$$X_{uu}=-(24\pi)^{-1}\left[\frac{3}{2}\left[\frac{A''}{A'}\right]^2-\frac{A'''}{A'}\right](A')^{-2} \ , \tag{2.11a}$$

$$X_{vv}=-(24\pi)^{-1}\left[\frac{3}{2}\left[\frac{B''}{B'}\right]^2-\frac{B'''}{B'}\right](B')^{-2} \ . \tag{2.11b}$$

$$X_{uv}=X_{vu}=0 \ . \tag{2.11c}$$

X_{ab} represents therefore a certain distribution of conserved massless radiation propagating along $\bar{u}=\text{const}$, $\bar{v}=\text{const}$ rays.

Let us now consider the case for which two-dimensional space-time is a two-dimensional section $\theta=\text{const},\phi=\text{const}$ of a de Sitter space-time given in static coordinates r,T by

$$ds^2=-\left[1-\frac{r^2}{\lambda^2}\right]du\,dv \ , \tag{2.12}$$

where now

$$u=T-r^* \ , \tag{2.13a}$$

$$v=T+r^* \ , \tag{2.13b}$$

and

$$r^*=\int\frac{dr}{1-\frac{r^2}{\lambda^2}} \ . \tag{2.14}$$

From Eqs. (2.5) we immediately obtain

$$Q_{uu}=Q_{vv}=-(48\pi\lambda^2)^{-1} \tag{2.15}$$

so if the field $\hat{\phi}$ is in the state $|u,v\rangle$ the expectation values of $T_{ab}(\hat{\phi})$ are

$$\langle u,v|T_{uu}|u,v\rangle=\langle u,v|T_{vv}|u,v\rangle =-(48\pi\lambda^2)^{-1} \ , \tag{2.16a}$$

$$\langle u,v|T_{uv}|u,v\rangle=\langle u,v|T_{vu}|u,v\rangle =-(24\pi\lambda^2)^{-1}\left[1-\frac{r^2}{\lambda^2}\right] \ . \tag{2.16b}$$

As is evident the tensor $\langle u,v|T_{ab}|u,v\rangle$ does not have the de Sitter-invariant form, i.e., $\langle T_{ab}\rangle\sim g_{ab}$, and even worse it behaves badly on the horizons $r=\lambda$.

In fact quite generally $\langle T_{ab}\rangle$ will be finite on the past horizon H_D^- in a coordinate system regular there if, as $r\to\lambda$,

$$|\langle T_{uu}\rangle|<\infty \ , \tag{2.17a}$$

$$(r^2-\lambda^2)^{-1}\langle T_{uv}\rangle|<\infty \ , \tag{2.17b}$$

$$(r^2-\lambda^2)^{-2}\langle T_{vv}\rangle|<\infty \ . \tag{2.17c}$$

Regularity on the future de Sitter horizon H_D^+ is expressed by similar inequalities with u and v interchanged. One can easily check that $\langle u,v|T_{ab}|u,v\rangle$ diverges on H_D^- as condition (2.17c) is not satisfied. Similarly one finds an analogue divergence on H_D^+.

A quantum state $|\bar{u},\bar{v}\rangle$ will therefore lead to physically acceptable values for $\langle T_{ab} \rangle$ on the horizons if, as $r \to \lambda$,

$$Q_{ab} + X_{ab} = 0 , \tag{2.18}$$

i.e., $X_{ab} \to (48\pi\lambda^2)^{-1}$ in a way that Eq. (2.17c) and the analogous equation for the (u,u) component are satisfied.

It is evident that if $X_{ab} = (48\pi\lambda^2)^{-1}$ identically the $\langle T_{ab} \rangle$ are not only regular throughout space-time but also of the de Sitter-invariant form. Such a state is the analogous of the Israel-Hartle-Hawking state in the Schwarzschild space-time and is constructed using Kruskal normal modes

$$(4\pi w)^{-1/2} e^{-iwU}, \quad (4\pi w)^{-1/2} e^{-iwV} , \tag{2.19}$$

where

$$U = e^{-u/\lambda}, \quad V = e^{v/\lambda} . \tag{2.20}$$

III. EXTERNAL BLACK HOLES

Let us consider following Ref. 3 the space-time described by the Penrose diagram in Fig. 1. It represents an eternal black hole whose inner core is constituted by a de Sitter region. The transition from the Schwarzschild to the de Sitter phase occurs along a spacelike hypersurface $r = r_0 = \text{const} < 2M$, where M is the mass of the hole. This surface has topology $R^1 + S^2$ (a cylinder) and represents an infinite (in the t direction) tube of radius r_0.

The space-time of the model is therefore described by the metric

$$ds^2 = -\left[1 - \frac{2M}{r}\right] dt^2 + \left[1 - \frac{2M}{r}\right]^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) , \tag{3.1}$$

for $r > r_0$ and

$$ds^2 = -\left[\frac{r^2}{\lambda^2} - 1\right]^{-1} dr^2 + \left[\frac{r^2}{\lambda^2} - 1\right] dT^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2) \tag{3.2}$$

for $r < r_0$, with $\lambda < r_0 < 2M$.

According to the investigation of Ref. 5 this space-time represents a solution of Israel's matching equations and this solution is classically (meta)stable.

However, it is not at all granted that this resulting space-time is semiclassically regular. With this we mean simply that the expectation values of the stress tensor $\langle T_{ab} \rangle$ of some matter test field $\hat{\phi}$ propagating in this space-time are well behaved throughout the space-time. One might in fact expect dangerous divergences occurring along the Cauchy horizons $r = \lambda$ of the space-time of Fig. 1, leading to instabilities. We shall now show by a two-dimensional analysis of the problem how to make these horizons nonpathological by fine-tuning the location of the transition region so that Eqs. (2.17) are satisfied.

The static black hole can be considered to be in a thermal equilibrium with its own radiation at the Hawking temperature $T = (8\pi M)^{-1}$, so that the relevant quantum state in which evaluate $\langle T_{ab} \rangle$ for the test field $\hat{\phi}$ propagating in our space-time is the Israel-Hartle-Hawking vacuum.

The two-dimensional metric in the Schwarzschild region is given in double null form by

$$ds^2 = -\left[1 - \frac{2M}{r}\right] d\bar{u} \, d\bar{v} , \tag{3.3}$$

where

$$\bar{u} = t - r_s^*, \quad \bar{v} = t + r_s^* \tag{3.4}$$

and

$$r_s^* = \int \frac{dr}{1 - \frac{2M}{r}} . \tag{3.5}$$

The Israel-Hartle-Hawking state $|\bar{u},\bar{v}\rangle$ is defined by an expansion of $\hat{\phi}$ in terms of Kruskal modes

$$(4\pi w)^{-1/2} e^{-iw\bar{u}}, \quad (4\pi w)^{-1/2} e^{-iw\bar{v}} , \tag{3.6}$$

where now

$$\bar{u} = -4M \ln\left[-\frac{\bar{u}}{4M}\right], \quad \bar{v} = 4M \ln\left[\frac{\bar{v}}{4M}\right] . \tag{3.7}$$

Now using Eqs. (2.5) and (2.11) we can easily calculate Q_{ab} and the state-dependent part X_{ab}. The result reads

$$\langle \bar{u},\bar{v} | T_{\bar{u}\bar{u}} | \bar{u},\bar{v} \rangle = \langle \bar{u},\bar{v} | T_{\bar{v}\bar{v}} | \bar{u},\bar{v} \rangle = \left[1 - \frac{2M}{r}\right]^2 \left[1 + \frac{4M}{r} + \frac{12M^2}{r}\right] \times (768\pi M^2)^{-1} , \tag{3.8a}$$

$$\langle \bar{u},\bar{v} | T_{\bar{u}\bar{v}} | \bar{u},\bar{v} \rangle = \langle \bar{u},\bar{v} | T_{\bar{v}\bar{u}} | \bar{u},\bar{v} \rangle = (24\pi)^{-1} \left[1 - \frac{2M}{r}\right] \frac{M}{r^3} . \tag{3.8b}$$

One easily shows that $\langle \bar{u},\bar{v} | T_{ab} | \bar{u},\bar{v} \rangle$ is regular throughout the Schwarzschild region of space-time, in particular, on the black-hole horizons $r = 2M$.

The Kruskal modes Eq. (3.6) propagate across the boundary surface $r = r_0$ in the de Sitter region. The latter is described by the metric

$$ds^2 = \left[\frac{r^2}{\lambda^2} - 1\right] du \, dv , \tag{3.9}$$

where u and v are given by Eqs. (2.13).

From the relation between the two sets of null coordinates $\{u,v\}$ and $\{\bar{u},\bar{v}\}$ determined on the boundary

$$\frac{du}{d\bar{u}} \equiv A' = \frac{4M\alpha}{\bar{u}}, \quad \frac{dv}{d\bar{v}} \equiv B' = \frac{4M\alpha}{\bar{v}} , \tag{3.10}$$

where

$$\alpha = \left[\frac{\frac{2M}{r_0} - 1}{\frac{r_0^2}{\lambda^2} - 1}\right]^{1/2} \tag{3.11}$$

one can evaluate with the aid of Eqs. (2.11) the state-dependent part of $\langle \bar{u},\bar{v}|T_{ab}|\bar{u},\bar{v}\rangle$ in the de Sitter region

$$X_{uu}=X_{vv}=(768\pi M^2)^{-1}\alpha^{-2} . \tag{3.12}$$

On the other hand, from Eqs. (2.5) one has

$$Q_{uu}=Q_{vv}=-(48\pi\lambda^2)^{-1} . \tag{3.13}$$

Now the stress tensor will also be regular on the de Sitter part of space-time if Eq. (2.18) is satisfied. This can be rewritten in the form

$$r_0^3+(16M^2-\lambda^2)r_0-32M^3=0 . \tag{3.14}$$

One can easily see that in the allowed interval $\lambda<r_0<2M$ Eq. (3.14) admits only one solution for r_0 (Ref. 6).

Unfortunately this value for r_0 required by regularity is very close to $2M$, the black-hole horizon.

Physical considerations lead however to expect the transition region to be located in the very interior of the hole at nearly Planckian curvature,[1,3] i.e., $r_0\sim M^{1/3}$. So the classical (meta)stable model discussed in this section lacks physical consistency when quantum-field-theory arguments are included.

IV. PHYSICAL BLACK HOLE

In a realistic physical situation a black hole is supposed to be formed by collapsing matter. Furthermore if quantum effects are taken into account, one also expects the hole after formation to lose its mass by Hawking evaporation.

The space-time of the physical black hole then clearly differs from that depicted in Fig. 1.

Because the life of the hole is characterized by an evolution (formation, evaporation), one should not expect the transition region to occur necessarily along an $r=$const surface.

We shall see that by relaxing this hypothesis one can in some cases overcome the impasse encountered in the previous section.

Here we shall consider a more general situation than before in which the de Sitter interior metric of the hole is matched to an arbitrary exterior metric representing either the space-time of the collapsing matter or that of an evaporating black hole and examine under which conditions the regularity requirement Eq. (2.18) can be satisfied.

The relevant Penrose diagrams are given in Ref. 3. We reproduce in Fig. 2 one of them which will be relevant for our conclusion.

The space-time of the model therefore will be described by the de Sitter metric

$$ds^2=-\left[1-\frac{r^2}{\lambda^2}\right]du\,dv \tag{4.1}$$

for $r<R(\tau)$, where $R(\tau)$ is the boundary between the interior de Sitter metric and the exterior region. The latter is described by the metric

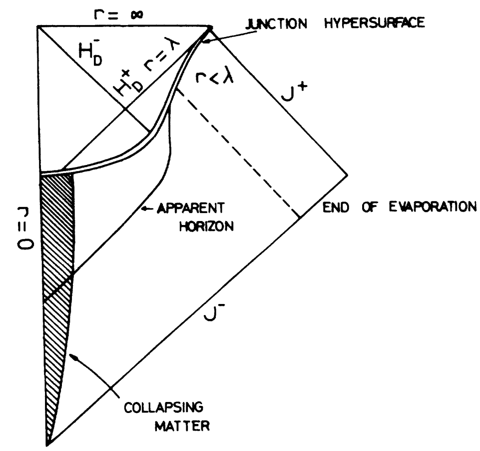

FIG. 2. Conformal diagram for an evaporating black hole formed by gravitational collapse with a de Sitter interior (see Ref. 3 for details).

$$ds^2=-f(\bar{u},\bar{v})d\bar{u}\,d\bar{v} . \tag{4.2}$$

Here f is an arbitrary (for the moment) function and $\{\bar{u},\bar{v}\}$ are a set of null coordinates:

$$\bar{u}=t-\rho , \tag{4.3a}$$

$$\bar{v}=t+\rho , \tag{4.3b}$$

where t is some exterior time and ρ a spacelike coordinate.

Matching the interior and exterior metric across the separation surface one gets

$$\dot{T}=NC^{-1} , \tag{4.4}$$

where

$$N^2=f(1-\dot{\rho}^2)C+\dot{R}^2 , \tag{4.5}$$

$$C=1-\frac{R^2}{\lambda^2} \tag{4.6}$$

and an overdot means derivation with respect to the exterior time. So $\dot{R}\equiv(dR/d\tau)/(d\tau/dt)$; τ is a parameter along the matching surface.

Now consider a "vacuum" state $|\bar{u},\bar{v}\rangle$ defined by expanding the field operator $\hat{\phi}$ in terms of normal modes constructed with the null coordinates $\bar{u},\bar{v}$:

$$(4\pi\omega)^{1/2}e^{-i\omega\bar{u}},\quad (4\pi\omega)^{-1/2}e^{-i\omega\bar{v}} . \tag{4.7}$$

In the model considered here J^- is a Cauchy surface (see Fig. 2) and the outgoing modes are simply the reflection throughout the (nonsingular) origin $r=0$ of the ingoing modes. These modes coming from the exterior region enter and propagate in the interior de Sitter domain and naturally define the appropriate vacuum in this region.

As we have seen previously for practical use we need only know how to identify an ingoing null geodesic $\bar{v}=$const ($\bar{u}=$const) in the exterior region with the corresponding null geodesic $v=$const ($u=$const) in the de Sitter region to construct the state-dependent part of

$\langle T_{ab} \rangle$.

Concentrating ourselves only on the ingoing modes we have from Eqs. (2.13b) and (4.3b) that at the boundary

$$B' \equiv \frac{dv}{d\bar{v}} = \frac{dv}{dt}\left[\frac{d\bar{v}}{dt}\right]^{-1} = C^{-1}(N+\dot{R})(1+\dot{\rho})^{-1} . \quad (4.8)$$

This is the desired expression which by further derivation will allow us to compute the state-dependent part X_{ab} of $\langle \bar{u},\bar{v}|T_{ab}|\bar{u},\bar{v}\rangle$ in the de Sitter region.

Since we are interested primarily in the regularity conditions Eqs. (2.17), we shall give here only the limiting behavior of X_{ab} as the de Sitter horizon is approached. The X_{vv} term describes massless radiation propagating along v_{const} rays. Let us suppose that as the generator of H_D^- is approached $C \to 0$, i.e., $R(\tau) \to \lambda$, the matching surface crosses the past de Sitter horizon as in Fig. 2. Furthermore let the functions describing the exterior metric be well behaved in this limit. So for $C \to 0$ we obtain, from Eq. (4.8) by subsequent differentiation,

$$\frac{3}{2}\frac{(B'')^2}{(B')^4} = \frac{3}{2}\frac{(C')^2}{4\dot{R}^2}(1+\dot{\rho})^2 + O(C) , \quad (4.9a)$$

$$\frac{B'''}{(B')^3} = \frac{2(C')^2}{4\dot{R}^2}(1+\dot{\rho})^2 + O(C) , \quad (4.9b)$$

$$\frac{d}{d\bar{v}} = \frac{1}{\dot{\rho}+1}\frac{d}{dt} ; \quad (4.10)$$

as usual all functions appearing should be regarded as evaluated at the boundary $r = R(\tau)$.

We can now construct the state-dependent part of $\langle \bar{u},\bar{v}|T_{ab}|\bar{u},\bar{v}\rangle$, namely, X_{vv}, see Eq. (2.11b), in the above limit:

$$\lim_{R\to\lambda} X_{vv} = (48\pi)^{-1}\left[\frac{\partial C}{\partial R}\right]^2\Bigg|_{R=\lambda} = (48\pi\lambda^2)^{-1} . \quad (4.11)$$

But this value is precisely $-Q_{vv}$ so

$$\lim_{R\to\lambda} Q_{vv} + X_{vv} = 0 \quad (4.12)$$

which is exactly what condition (2.18) requires.

Physically this means that the positive-energy radiation coming from the moving boundary (X_{vv}) cancels exactly the infinite negative-energy vacuum polarization (Q_{vv}) on the horizon. Similarly under the same hypothesis one can show that $\langle \bar{u},\bar{v}|T_{ab}|\bar{u},\bar{v}\rangle$ is regular on the future de Sitter horizon. Let us finish by considering two examples where our hypothesis is not satisfied and the regularity condition is not satisfied.

The first is represented by taking the Schwarzschild metric in the (r,T) coordinates as exterior one and matching it along a $r=$const surface to the interior de Sitter metric. For this case $C=$const and $B'=$const so that $X_{vv}=0$ identically.

The second example is obtained by considering the Vaidya metric as representing the exterior evaporating black-hole space-time:

$$ds^2 = -\left[1-\frac{2M(\bar{v})}{r}\right]d\bar{v}^2 + 2d\bar{v}\,dr . \quad (4.13)$$

Let us further suppose as argued in Ref. 3 that the matching surface $r = R(\bar{v})$ evolves according to

$$R(\bar{v}) = \left[\frac{2M(\bar{v})}{\lambda}\right]^{1/3}\lambda^2 . \quad (4.14)$$

So that in this case $C \neq$const and $C=0$ as the surface crosses the horizon.

It is easy to see that matching the Vaidya metric (4.13) to the de Sitter one along the surface (4.14) one gets

$$\left[1-\frac{R^2}{\lambda^2}\right]\left[\frac{dv}{d\bar{v}}\right]^2 - 2R'\frac{dv}{d\bar{v}} + 2R' - \left[1-\frac{R^2}{\lambda^2}\right] = 0 \quad (4.15)$$

which admits as a solution $B' \equiv dv/d\bar{v} = 1$.

Even in this case $X_{vv}=0$ and the interior de Sitter space-time cannot be semiclassically regular.

Here we make a final comment on the limits of the applicability of the semiclassical approach we used to reach our conclusions.

As we said at the beginning, the transition from the Schwarzschild-like to the de Sitter region is supposed to occur at some $r_0 \sim M^{1/3}$ where the curvature of black-hole space-time grows to order unity. For a solar-mass black hole $r_0 \sim 10^{13}$ in Planck units or 10^{-20} cm. In this regime one might expect the space-time geometry to remain effectively classical and be governed by the semiclassical Einstein equations, the source being the expectation values of the quantum stress tensor $\langle T_{ab}\rangle$. Consistency clearly requires $\langle T_{ab}\rangle$ to be finite.

The Cauchy horizon, on the other hand, is located at some $r = \lambda < r_0$ (probably $\lambda \ll r_0$). Its actual value is presently unknown but one might expect it to depend on the various fields entering the theory.[1] In order that our semiclassical approach be trustworthy even at the Cauchy horizon one should require this value to be greater than order 1 (10^{-33}cm); otherwise still unknown quantum-gravitational effects need to be taken into account.

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[1]E. Poisson and W. Israel, Class. Quantum Grav. **5**, L201 (1988).

[2]M. A. Markov, Pis'ma Zh. Eksp. Teor. Fiz. **36**, 214 (1982) [JETP Lett. **36**, 265 (1982)].

[3]V. P. Frolov, M. A. Markov, and V. F. Mukhanov, Phys. Lett. B **216**, 272 (1989); Phys. Rev. D **41**, 383 (1990).

[4]W. Israel, Nuovo Cimento **44B**, 1 (1966); **48B**, 463 (1967).

[5]R. Balbinot and E. Poisson, Phys. Rev. D **41**, 395 (1990).

[6]R. Balbinot, Phys. Lett. B **227**, 30 (1989).

[7]See, for example, N. D. Birrell and P. C. W. Davies, *Quantum Field Theory in Curved Spaces* (Cambridge University Press, Cambridge, England, 1982).