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Internal structure of black holes

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The gravitational effects associated with the radiative tail produced by a gravitational collapse with rotation are investigated. It is shown that the infinite blueshift of the tail's energy density occurring at the Cauchy horizon of the resulting black hole causes a classically unbounded inflation of the effective internal gravitational-mass parameter of the hole. Since this effect is causally disconnected from any external observer, the black-hole external mass remains bounded. The mass inflation phenomenon causes the spacetime curvature to grow to Planckian scales on a spacelike hypersurface in the vicinity of the Cauchy horizon, beyond which the classical laws of general relativity break down. A consequence is that an observer's trip to this hypersurface embraces all but the last Planck time of the entire black-hole classical history.

I. INTRODUCTION

Black-hole theory is without any doubt one of the major triumphs of classical general relativity. By a series of theorems[1] it has been established (subject to the plausible assumption of cosmic censorship[2]) that the external gravitational field of a black hole relaxes to a Kerr-Newman field described solely by three parameters: the hole's mass, charge, and angular momentum. This remarkable result is usually referred to as the no-hair theorem. The mechanism responsible for such a relaxation of the external field has been elucidated by Price[3] who showed that perturbations developing on the surface of a spherically collapsing star produce the emission of gravitational radiation which carries away all the initial characteristics of the star's gravitational field except the mass, charge, and angular momentum parameters. This radiation then interacts with the spacetime curvature: while some of it escapes to infinity, some is backscattered and absorbed by the resulting black hole. Most of this backscattering occurs soon after the emission: the amplitude of the infalling flux typically decays according to an inverse power law with advanced time.

Attempts have been made to extend these results to the black-hole internal gravitational field. For a Schwarzschild black hole, it has been demonstrated[4,5] that the asymptotic portion of spacetime near the singularity (corresponding to large values of advanced time v) is virtually free of aspherical perturbations propagated from the surface of the star since the gravitational radiation becomes infinitely diluted as it reaches the singularity. But this result breaks down as soon as the charge or angular momentum of the black hole is nonzero. The internal structures of the Reissner-Nordström and Kerr solutions differ drastically from that of the Schwarzschild solution: the singularity is now timelike and both of these spacetimes possess a Cauchy horizon, a null hypersurface beyond which predictability breaks down. Indeed, initial data specified (say) at the onset of the collapse are not sufficient to predict unambiguously what happens to the future of the Cauchy horizon.

In the analytically extended Kerr manifold, there is a relatively spacious region beyond the Cauchy horizon: it takes roughly the same proper time for a free-falling observer to travel from the event horizon to the Cauchy horizon as from the Cauchy horizon to the ring singularity. The presence of a region beyond the Cauchy horizon is an embarrassment: there is no way of predicting the course of events in this region and signals coming from the singularity could alter the physics in an unforeseeable manner. There is a further problem associated with the Cauchy horizon: it is a surface of infinite blueshift. If we recall that infalling gravitational radiation is expected to propagate inside the black hole with paths approaching that of the null generators of the Cauchy horizon, we realize that the energy density of this radiation will suffer an infinite blueshift as it approaches the Cauchy horizon. A free-falling observer would see the entire future history of the Universe flash before his eyes before encountering a wall of infinite density at the Cauchy horizon.

The first-order analysis of this phenomenon was undertaken by many people[6] and that perturbations diverge to linear order has been verified many times over. This result suggests that in a generic gravitational collapse, a singularity developing at the Cauchy horizon would seal off the "Kerr tunnel" that leads to other asymptotically flat universes, but so far, nobody has attempted to analyze the situation beyond linear order or to evaluate the effect on the black-hole geometry of such an infinite growth of the perturbations at the Cauchy horizon. The question of interest is to evaluate whether the perturbations, allowed to act as a source in Einstein's equations, can trigger the formation of a singularity of sufficient strength (e.g., stronger than that of a surface layer) to effectively stop the evolution of spacetime at the Cauchy horizon. This property would of course take care of the problems associated with the breakdown of predictability occurring there.

This question has been addressed by the present authors in two papers.[7] We have shown that the answer is yes: the combination of the outflux emitted by the collapsing star and its backscattered, blueshifted radiative tail propagating near the Cauchy horizon provoke a tremendous inflation of the black-hole internal mass parameter, which in fact becomes classically unbounded at the Cauchy horizon. Mass inflation then produces the inflation of curvature which too grows to infinity at the Cauchy horizon. The Cauchy horizon is the ultimate brick wall at which the evolution of spacetime is forced to stop.

The mechanism responsible for this catastrophic behavior is precisely the combination of the effects of the two radiation fluxes. At first sight, it might seem that only the blueshifted influx is the critical ingredient and that one could artificially turn off the outflux of radiation without altering the general conclusions. Indeed, one would expect the gravitational effects of this blueshifted energy to increase the black-hole gravitational mass parameter to arbitrarily large values. [It might seem that inside the black hole, the mass parameter has no direct operational meaning since one cannot go to infinity in order to measure it. But the mass parameter always possesses an important local meaning: it determines the "Coulomb" (Petrov type-D) component of the local curvature. An increase in the mass parameter would be manifested locally, for example, by an increase in the tidal forces felt by an (extended) observer falling radially inward near the Cauchy horizon, even though this observer would register the energy density due to the infalling radiative tail as almost unblueshifted and vanishingly small.]

The expectation of a mass growth near the Cauchy horizon due to the blueshifted radiative tail is not realized: the mass parameter remains almost unchanged as long as the outflux is not turned on. This apparent miracle can be explained as follows: while the Cauchy horizon is a surface of infinite blueshift for our own asymptotically flat universe, the inner apparent horizon of the hole is a surface of infinite redshift for future asymptotically flat universes. In the situation where only the influx is considered, the two horizons coincide and the blueshift and redshift cancel exactly thus preventing any spectacular increase in the mass parameter. When one turns on the outflux, however, the situation is radically different. The outgoing radiation escaping the surface of the collapsing star crosses the Cauchy horizon and focuses its null generators. But the apparent horizon now contracts faster (in fact, it deflates very rapidly) and the surfaces of infinite blueshift and redshift become distinct. The cancellation can therefore no longer occur and the mass parameter increases dramatically. One can think of the infinitely blueshifted influx as being the essential feature explaining mass inflation, but the phenomenon needs to be triggered by an arbitrarily small amount of outgoing radiation which produces the necessary separation between the Cauchy and apparent horizons.

In order to formulate the problem mathematically, it is necessary to make a few simplifying assumptions. The first is that we will use a spherical model where the black hole is charged and initially described by the Reissner-Nordström solution. This might appear to be a highly restrictive idealization, but we believe that this model can capture the essential physics, since the global structures of the Kerr and Reissner-Nordström black holes are very similar. Both manifolds possess a Cauchy horizon and according to our physical picture, the mass inflation phenomenon only depends on the two general features described above: the presence of a highly blueshifted influx and a separation between the Cauchy and inner apparent horizons. The spherical model should thus allow a qualitative understanding of the phenomenon without introducing too many difficulties in the mathematical analysis. The second assumption is that we will model the gravitational radiation emitted from the surface of the collapsing star and backscattered by the background curvature as two intersecting radial streams of infalling and outgoing lightlike particles following null geodesics. We furthermore assume that the streams do not interact with each other so that they are separately conserved. This too might appear as an important idealization, but since we are only interested in the highly blueshifted modes propagating near the Cauchy horizon, it is apparent that any further scattering should not be important and that Issacson's effective stress-energy description[8] for these high-frequency modes should be an adequate description. Indeed, since the radius of curvature of the background spacetime is always larger than the blueshifted wavelength of the infalling modes, the graviton geometric optics approximation should be an accurate approximation. As for the outgoing modes, we will see that their detailed description is not at all important for our conclusions: their only role is to irradiate the Cauchy and inner apparent horizons in order to produce a separation.

The mass inflation phenomenon completely changes our understanding of the black-hole internal structure. Instead of having the Cauchy horizon acting as a curtain beyond which a macroscopically large region of spacetime does not allow predictability from initial data specified earlier, we have near the Cauchy horizon a microscopic region of spacetime where the curvature is extremely high. If we imagine, as was speculated recently,[5,9] that curvature is naturally bounded at Planckian magnitude, we find that the effective mass parameter of the black-hole interior can reach the incredibly high value of $m_0^3/m_{\rm Pl}^2 \sim 10^{60}$ universe masses for a black hole of ten solar masses (m_0 is the external mass of the hole and $m_{\rm Pl}$ the Planck mass). In view of this fantastic increase of the mass, the charge and angular momentum of the black hole become totally irrelevant to the description of the geometry near the Cauchy horizon, which is then accurately described by the Schwarzschild metric with an enormous mass parameter. It is then easy to show that an observer's trip to the Cauchy horizon covers all but the last Planck time of the black hole's classical history. The spacetime region near the Cauchy horizon can then be imagined as a "fat cigar" (topology $S^2 \times R_+$) inside which the curvature is Planckian.

The paper is organized as follows: motivated by the discussion given above, we will consider as our model a background Reissner-Nordström spacetime irradiated by simultaneous infalling and outgoing radial fluxes. We will then derive in Sec. II the basic field equations for spherical spacetimes, expressed in a form covariant under arbitrary transformations of the coordinates of the radial two-spaces $(\theta,\phi)=\text{const}$. In Sec. III we will explore some exact solutions to the field equations: namely, the static Reissner-Nordström solution and the dynamic ingoing and outgoing generalized Vaidya solutions which describe a charged, spherical black hole irradiated by a pure in- or outflux. We will also derive the Dray–'t Hooft–Redmount (DTR) relation which describes the gravitational field before and after the collision of two spherical thin shells propagating at the speed of light, one expanding, the other contracting. The DTR relation is an exact solution to Einstein's equations for infalling and outgoing fluxes which can be described by δ-function pulses. We then proceed in Sec. IV with the formulation of the mass inflation phenomenon by formally integrating Einstein's equations with continuous infalling and outgoing radial fluxes. We show than it is possible to reach our conclusions making no additional assumption other than that given in our model. To evaluate the growth rate of the gravitational mass, however, we must make a formal perturbation expansion in terms of the product of the flux luminosities. Section V is finally devoted to quantum considerations: we speculate on the internal structure of the black hole should curvatures be bounded at Planckian magnitude. We summarize our conclusions in Sec. VI. Various technical details which might obscure the main line of thought are relegated to the Appendixes.

II. FIELD EQUATIONS FOR SPHERICAL SPACETIMES

We have given some justification in the Introduction for using a spherical model in which the black hole is initially described by the Reissner-Nordström solution. To describe the effects of the gravitational radiation propagating inside the hole, we will be interested in the field equations describing a charged, spherical black hole perturbed by cross-flowing radial fluxes of infalling and outgoing radiation. This idealized model should illustrate the basic physics behind the mass inflation phenomenon and is sufficiently simple to allow a detailed mathematical analysis. We now proceed to derive the field equations describing the above situation. In doing so, it is most convenient to use a coordinate system $x^\alpha=(x^a,\theta,\phi)$ $(a=1,2)$, where the coordinates x^a of the "radial" two-spaces $(\theta,\phi)=\text{const}$ are left unspecified. The field equations as thus written will be covariant under arbitrary transformations of the two-dimensional coordinates. The spacetime metric will then be written as

$$ds^2=g_{ab}dx^a dx^b+r^2 d\Omega^2 , \tag{2.1}$$

where g_{ab} is the metric on the radial two-spaces, $d\Omega^2$ the line element on the unit two-sphere, and r a function of x^a measuring the area of the two-spheres $x^a=\text{const}$. The various geometric quantities for such a metric are given in Appendix A, where it is shown that the Einstein tensor is given by

$$\begin{aligned} {}^4G_{ab}&=-[2rr_{;ab}+g_{ab}(1-r^{,a}r_{,a}-2r\Box r)]/r^2 , \\ G_{\theta\theta}&=\sin^2\theta G_{\phi\phi}=r\Box r-\tfrac{1}{2}r^2R . \end{aligned} \tag{2.2}$$

The semicolon denotes covariant differentiation with respect to the two-dimensional metric g_{ab} [we use the stroke (|) to denote the same with respect to the four-dimensional metric $g_{\alpha\beta}$], we write $\Box\psi\equiv g^{ab}\psi_{;ab}$ for any scalar field ψ and R denotes the two-dimensional Ricci scalar associated with the radial two-spaces (we use the superscript 4 to indicate four-dimensional quantities, when ambiguities might arise).

We write Einstein's field equations as

$$G_{\alpha\beta}=8\pi(E_{\alpha\beta}+T_{\alpha\beta}) , \tag{2.3}$$

where $E_{\alpha\beta}$ is the Maxwellian contribution to the stress-energy tensor representing the static electric field of a point charge of strength e located at the origin $r=0$:

$$E^\alpha{}_\beta=P_{\rm el}\text{diag}(-1,-1,1,1), \quad P_{\rm el}\equiv e^2/8\pi r^4 . \tag{2.4}$$

Expression (2.4) is valid in any system of coordinates (x^a,θ,ϕ) and follows from the hypothesis that the electromagnetic field (as seen by static observers) should be purely electric and radial. The non-Maxwellian contribution to the stress-energy tensor $T_{\alpha\beta}$ will later describe our cross flow of lightlike radiation. For the moment, we shall decompose it according to

$${}^4T^a{}_b=T^a{}_b, \quad T^\theta{}_\theta=T^\phi{}_\phi=P , \tag{2.5}$$

where P represents a tangential pressure; we will also denote the two-dimensional trace $T^a{}_a$ by the symbol T. The field equations then read

$$\begin{aligned} &2rr_{;ab}+(1-e^2/r^2-r^{,a}r_{,a}-2r\Box r)g_{ab}=-8\pi r^2T_{ab} , \\ &r\Box r-\tfrac{1}{2}r^2R-e^2/r^2=8\pi r^2P . \end{aligned} \tag{2.6}$$

At this point, it is useful to introduce the scalar fields $m(x^a)$, $f(x^a)$, and $\kappa(x^a)$ defined as

$$\begin{aligned} &g^{ab}r_{,a}r_{,b}\equiv f\equiv 1-2m/r+e^2/r^2 , \\ &\kappa\equiv-\tfrac{1}{2}\partial_r f=-(m-e^2/r)/r^2 . \end{aligned} \tag{2.7}$$

Substituting Eqs. (2.7) into (2.6), we obtain

$$\begin{aligned} &r_{;ab}-(\kappa+\Box r)g_{ab}=-4\pi rT_{ab} , \\ &\Box r-e^2/r^3-\tfrac{1}{2}rR=8\pi rP . \end{aligned} \tag{2.8}$$

Taking the trace of the first of Eqs. (2.8) yields $\Box r=-2\kappa+4\pi rT$ which can be substituted back to yield

$$r_{;ab}+\kappa g_{ab}=-4\pi r(T_{ab}-g_{ab}T) . \tag{2.9}$$

Substituting the same result into the second of Eqs. (2.8) then gives

$$R-2\partial_r\kappa=8\pi(T-2P) . \tag{2.10}$$

Equations (2.9) and (2.10) constitute our basic field equations. It is however useful to derive an additional equation by using Eqs. (2.7): we obtain $f_{,a}=-(2/r)m_{,a}-2\kappa r_{,a}$, which can be inverted to yield $m_{,a}$ in terms of $f_{,a}$ and $r_{,a}$. We can then use the field equation (2.9) using $f_{,a}=2g^{bc}r_{,b}r_{;ac}$. A little algebra then gives

$$m_{,a}=4\pi r^2(T_a{}^b-\delta_a{}^b T)r_{,b}\ . \tag{2.11}$$

We note that Eq. (2.11) follows directly from the conventional form of the field equations (obtained when one uses r and t as coordinates) and the requirement of two-dimensional covariance.

The conservation equations $(E^{\alpha\beta}+T^{\alpha\beta})_{|\beta}=0$ can be obtained by using the Christoffel symbols given in Appendix A. Since $E^{\alpha\beta}$ and $T^{\alpha\beta}$ are separately conserved, we find that

$$(r^2T^{ab})_{;b}=(r^2)^{;a}P\ . \tag{2.12}$$

The angular components of $T^{\alpha\beta}{}_{|\beta}=0$ are satisfied trivially, while the conservation equations for the electromagnetic stress-energy tensor confirm that $P_{\rm el}$ must be proportional to r^{-4} as was stated earlier.

In summary, we find that the field equations for spherical spacetimes for which the stress-energy tensor can be decomposed into two separately conserved Maxwellian and non-Maxwellian contributions are given by Eqs. (2.9) and (2.10) and that energy conservation is expressed by Eq. (2.12). Then Eq. (2.11) follows directly from the definition of m and Eq. (2.9). We note that similar equations were previously written down in Ref. 10.

We now specialize to the problem of interest. We want to construct a stress-energy tensor $T_{\alpha\beta}$ which would describe cross-flowing streams of infalling and outgoing lightlike particles following radial null geodesics. It is easy to see that it must take the form

$$^4T_{ab}=T_{ab}=\rho_{\rm in}l_al_b+\rho_{\rm out}n_an_b\ , \tag{2.13}$$

where l_a is a radial null vector pointing inwards and n_a is a radial null vector pointing outwards. The scalars $\rho_{\rm in}$ and $\rho_{\rm out}$ represent the energy density of the fluxes but do not have direct operational meaning since the null vectors can be arbitrarily normalized. Since we now have $P=T=0$, the field equations simplify to

$$\begin{aligned} &r_{;ab}+\kappa g_{ab}=-4\pi r T_{ab}\ ,\\ &R=2\partial_r\kappa=2(2m-3e^2/r)/r^3\ , \end{aligned} \tag{2.14}$$

$$m_{,a}=4\pi r^2T_a{}^br_{,b},\quad (r^2T^{ab})_{;b}=0\ . \tag{2.15}$$

From this system of equations, it is possible to derive three one-dimensional wave equations of the form $\Box\psi=\rho$ which are very useful since, as we will show in Sec. III, they can be formally integrated. First, taking the trace of the first of Eqs. (2.14) while noting that the trace of T_{ab} is zero yields

$$\Box r=-2\kappa\ . \tag{2.16}$$

Second, taking the derivative of the first of Eqs. (2.15) and using the conservation equation yields $\Box m=4\pi r^2T^{ab}r_{;ab}$. Equation (2.14) can then be used to give

$$\Box m=-(4\pi)^2r^3T^{ab}T_{ab}\ . \tag{2.17}$$

This equation will be very important in what follows. Note that $\Box m$ depends only on the product $\rho_{\rm in}\rho_{\rm out}$ and not on the individual linear contributions. The third wave equation requires a little more work. We want to find an expression for $\Box\ln f$ which will be used in Sec. III to derive the DTR relation. First note that, for any function ψ,

$$\Box\ln\psi=(\psi\Box\psi-\psi^{,a}\psi_{,a})/\psi^2\ . \tag{2.18}$$

Now, we can calculate $\Box f$ by taking the second derivative of the first of Eqs. (2.7) and get

$$\Box f=-2(\Box m/r-m^{,a}r_{,a}/r^2+\kappa\Box r+\kappa_{,a}r^{,a})\ , \tag{2.19}$$

which can be simplified further if we use Eqs. (2.16), and (2.14) in $\kappa_{,a}=\frac{1}{2}Rr_{,a}-m_{,a}/r^2$. A little algebra then yields

$$\Box f=-2\Box m/r+4m^{,a}r_{,a}/r^2+4\kappa^2-fR\ . \tag{2.20}$$

The product $f^{,a}f_{,a}$ involves the product $m^{,a}m_{,a}$ which can be recast in terms of $\Box m$ in the following way: $m^{,a}m_{,a}=-\frac{1}{2}rf\Box m$. Substituting this result into the expression for $f^{,a}f_{,a}$ yields

$$f^{,a}f_{,a}=-2f\Box m/r+8\kappa r^{,a}m_{,a}/r+4\kappa^2f\ . \tag{2.21}$$

Collecting the results finally gives

$$\Box\ln f=16\pi f^{-2}(f-2\kappa r)T^{ab}r_{,a}r_{,b}-R\ , \tag{2.22}$$

which is the result wanted. Note that $\Box\ln f$ is linear in T^{ab} and that $f-2\kappa r=1-e^2/r^2$. The two-dimensional Ricci scalar is given explicitly in Eq. (2.14).

III. EXACT SOLUTIONS: REISSNER-NORDSTRÖM, VAIDYA, AND THE DTR RELATIONS

We will explore in this section a few exact solutions of the field equations derived in the previous section. The solution corresponding to a vanishing non-Maxwellian stress-energy tensor is the Reissner-Nordström solution and describes the geometry of a static, charged, spherical black hole. We shall explore this solution in Sec. III A. If we switch on either a pure influx, or pure outflux, of radiation (described either by $T_{ab}=\rho_{\rm in}l_al_b$ or $T_{ab}=\rho_{\rm out}n_an_b$), we obtain the charged Vaidya solution[11] (a generalization of the original Vaidya solution) which describes a charged black hole whose mass varies with either advanced time (in the case of pure inflow) or retarded time (in the case of pure outflow). We shall consider this solution in Sec. III B. An exact solution to the full field equations (inflow and outflow) exists in the specialized case where the fluxes can be modeled as thin shells moving at the speed of light. The energy densities $\rho_{\rm in}$ and $\rho_{\rm out}$ can then be expressed as δ functions and the field equations can be integrated in the generalized form of the Dray– 't Hooft–Redmount (DTR) relation.[12] This relation connects the masses in the different regions of spacetime separated by the shells. We shall explore this solution in Sec. III C.

A. The Reissner-Nordström solution

The Reissner-Nordström solution

$$ds^2=-f_0dt^2+f_0^{-1}dr^2+r^2d\Omega^2\ ,\qquad f_0=1-2m_0/r+e^2/r^2 \tag{3.1}$$

is the solution to Einstein's equations with vanishing $T_{\alpha\beta}$. The solution can be easily obtained by choosing $x^a=(t,r)$ and by writing $g_{ab}dx^adx^b=-e^{2\psi}fdt^2+f^{-1}dr^2$, where $\psi=\psi(r)$ and $f=1-2m(r)/r+e^2/r^2$. Field equation (2.15) yields that $m(r)=\text{const}\equiv m_0$ and the tt component of Eq. (2.14) yields that $\psi'=0$. The factor $e^{2\psi}$ is therefore a constant that can be absorbed by a rescaling of coordinate t to give Eq. (3.1).

For later use, we will now define other convenient coordinate systems to express the Reissner-Nordström metric: namely, the Eddington-Finkelstein null coordinates and the Kruskal null coordinates. So we define radial null coordinates u and v (the Eddington-Finkelstein coordinates) which we choose such that they both run forward inside the black hole's event horizon (see Fig. 1):

$$u=-t+r^*,\quad v=t+r^*\ , \tag{3.2}$$

where $r^*\equiv\int dr/f_0(r)$. Note that far from the black hole, $u\rightarrow -t+r$, $v\rightarrow t+r$ and that u is in fact an advanced time for observers in universe I_L. The metric (3.1) then takes the form

$$ds^2=f_0du\,dv+r^2d\Omega^2\ . \tag{3.3}$$

Coordinates u,v are singular on the black-hole horizons, but if one focuses attention on one horizon at a time, it is possible to rescale the coordinates such that the metric becomes manifestly regular on that horizon. Let us then focus attention on the inner horizon $r=r_0\equiv m_0-(m_0^2-e^2)^{1/2}$ and define the rescaled coordinates U,V (the Kruskal null coordinates) as

$$U=-e^{-\kappa_0 u},\quad V=-e^{-\kappa_0 v}\ , \tag{3.4}$$

where $\kappa_0\equiv -f_0'(r_0)/2=(m_0^2-e^2)^{1/2}/r_0^2$ is the surface gravity of the inner apparent horizon. That the metric is now regular at $r=r_0$ follows easily. We find that near the horizon $f_0\simeq -2UV$ and the metric element g_{UV} hence reads

$$g_{UV}=\tfrac{1}{2}f_0/\kappa_0^2UV\simeq -1/\kappa_0^2\ . \tag{3.5}$$

We will make use of the results derived here in Sec. IV.

B. The charged Vaidya solutions

The ingoing charged Vaidya solution[11]

$$ds^2=dv\,(2dr-f_{\text{in}}dv)+r^2d\Omega^2\ ,\qquad f_{\text{in}}=1-2m_{\text{in}}(v)/r+e^2/r^2 \tag{3.6}$$

is the solution to Einstein's equations with $T_{ab}=\rho_{\text{in}}l_al_b$ where the normalization of l_a is such that $l_a=-\partial_a v$. Coordinate v has here the same meaning as in Eq. (3.2): it reduces to the ordinary advanced time $t+r$ far from the black hole. It is easy to see that field equation (2.15) relates $m_{\text{in}}(v)$ and ρ_{in} as

$$dm_{\text{in}}(v)/dv=4\pi r^2\rho_{\text{in}}\ . \tag{3.7}$$

The ingoing Vaidya solution represents a spherical, charged black hole which is irradiated by an influx of radial lightlike radiation coming from $\mathcal{I}_R^-$ (Fig. 1) and following ingoing null geodesics. Similarly, the outgoing charged Vaidya solution

$$ds^2=du\,(2dr-f_{\text{out}}du)+r^2d\Omega^2\ ,\qquad f_{\text{out}}=1-2m_{\text{out}}(u)/r+e^2/r^2\ , \tag{3.8}$$

represents the black-hole interior being irradiated by an outflux of lightlike radiation coming from $\mathcal{I}_L^-$. Alternatively, we can imagine that the radiation really comes from the surface of the collapsing star (this is obviously the correct interpretation since the "parallel" universe to the left does not appear when a star is present) but since

$$dm_{\text{out}}(u)/du=4\pi r^2\rho_{\text{out}}\ , \tag{3.9}$$

for $T_{ab}=\rho_{\text{out}}n_an_b$, $n_a=-\partial_a u$, the mass parameter actually increases even though the star loses mass. This property corresponds to the fact that an observer near $\mathcal{I}_L^-$ would measure an increase of mass as radiation pours into the hole from his domain.

One could have chosen other null coordinates to express the Vaidya solutions (3.6) and (3.8). But the Eddington-Finkelstein coordinates are a good choice since they reduce to ordinary advanced and retarded times far from the black hole. Also, since the mass functions are objects one can measure at infinity (they have a direct operational meaning), the derivatives dm_{in}/dv and

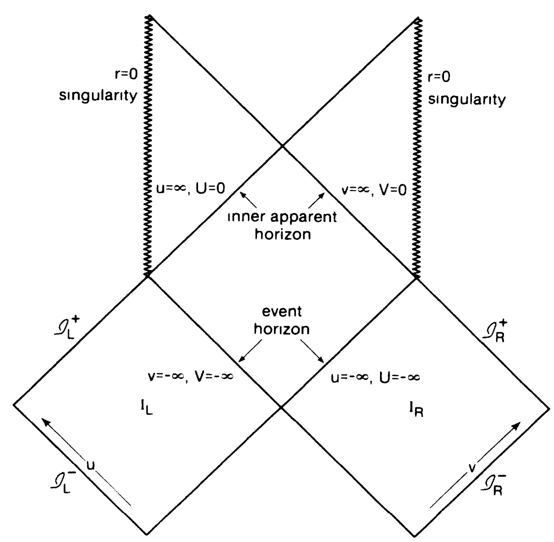

FIG. 1. A conformal diagram representing the Reissner-Nordström spacetime: we illustrate here the coordinates defined in the text. Coordinate v is defined to be an advanced time for observers in region I_R and u is defined to be an advanced time for observers in region I_L (note that coordinate t runs backward in this region).

$dm_{\rm out}/du$ also have direct operational meaning: they are the rates of variation of the black-hole gravitational mass as measured far from the black hole. It is through Eqs. (3.7) and (3.9) that the energy densities $\rho_{\rm in}$ and $\rho_{\rm out}$ acquire operational meaning.

C. The DTR relation

We are now ready to derive an exact solution to the Einstein equations when we have a crossflow of infalling and outgoing lightlike radiation which can be modeled as spherical thin shells moving with the speed of light, one expanding, the other contracting (see Fig. 2). The shells divide the radial two-space into four different sectors, each possessing a different mass and a different function f. The relationship between the masses is precisely the (generalized) DTR relation[12] which we shall now derive (note that the original DTR relation is only valid for a vanishing charge). In order to do so, and for later use, it is necessary to express the field equations (2.14)–(2.17) in radial double-null coordinates in which the two-dimensional metric assumes the form

$$g_{ab}dx^a dx^b = -2e^{2\sigma} dU\, dV \ , \tag{3.10}$$

where $\sigma=\sigma(U,V)$; for the time being we leave the null coordinates arbitrary: we do not necessarily assume that they are the Kruskalized coordinates of Eq. (3.4). We find that, for any scalar function ψ,

$$\Box\psi = -2e^{-2\sigma}\psi_{,UV} \ . \tag{3.11}$$

The Ricci scalar is calculated to be $R=-2\Box\sigma$ and this

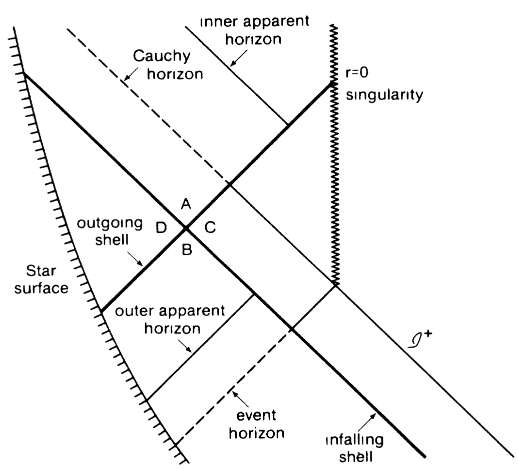

FIG. 2. Crossing null shells: two concentric, spherical thin shells propagating with the speed of light collide without interaction at event q, hence separating spacetime into four radial sectors $A,\ldots,D$. The energy content of the infalling shell is given by the difference $m_C - m_B$ and that of the outgoing shell (first expanding from the surface of the collapsing star but then forced to collapse to the singularity because of the gravitational pull) by $m_D - m_B$. The solution of the Einstein equations, the DTR relation, relates the mass m_A to the masses $m_B,\ldots,m_D$.

yields, when substituted into Eq. (2.14):

$$\Box\sigma = -\partial_r\kappa = -(2m-3e^2/r)/r^3 \ . \tag{3.12}$$

The stress-energy tensor takes the usual form (2.13) and we now choose

$$l_a = -\partial_a V, \quad n_a = -\partial_a U \ . \tag{3.13}$$

The conservation equations (2.15) express the fact that the product $r^2\rho_{\rm in}(r^2\rho_{\rm out})$ is independent of coordinate $U(V)$ so that we can write

$$\rho_{\rm in} = L_{\rm in}(V)/4\pi r^2, \quad \rho_{\rm out} = L_{\rm out}(U)/4\pi r^2 \ . \tag{3.14}$$

The luminosities $L_{\rm in}$ and $L_{\rm out}$ have no direct operational meaning since they depend on the definition of the coordinates U and V. Substituting these results into Eq. (2.17) yields

$$\Box m = -2(re^{4\sigma})^{-1}L_{\rm in}(V)L_{\rm out}(U) \tag{3.15}$$

and we note that $\Box\ln f$ is linear in $L_{\rm in}$ and $L_{\rm out}$ without any contribution from the bilinear $L_{\rm in}L_{\rm out}$. It is precisely this property which allows the integration of the field equations to obtain the DTR relation, as we will see presently.

Before we can do so, however, we need to derive a very useful result. We will show, as is well known, that any equation of the form $\Box\psi=\rho$ can be formally integrated. Suppose we are interested in the solution for ψ subject to boundary conditions on a characteristic sector Σ described by the equations $U=U_1$, $V=V_1$. It follows from a straightforward application of Green's identity that the solution at event (U,V) (assumed to be located to the future of sector Σ) is given by

$$\psi(U,V) = -\tfrac{1}{2}\int_{U_1}^{U}\int_{V_1}^{V} e^{2\sigma'}\rho' dU' dV' + \psi(U_1,V) + \psi(U,V_1) - \psi(U_1,V_1) \ , \tag{3.16}$$

where we denote $\sigma' \equiv \sigma(U',V')$, etc. The simplicity of this result comes from the fact that the Green's function for the operator $\Box$, when expressed in double-null coordinates, is given by the product of two Heaviside step functions. One can see that the solution at event (U,V) is given by a contribution from the boundary conditions specified on sector Σ, and by a surface integral over the past radial light cone of event (U,V). It is straightforward to differentiate Eq. (3.16) twice and to use Eq. (3.11) to verify that it is truly a solution of $\Box\psi=\rho$. This integral formula will be very useful in the following section.

The DTR relation now follows directly from Eq. (3.16) applied to $\psi\equiv\ln f$, by recalling that ρ is linear in $L_{\rm in}$ and $L_{\rm out}$ which are here given by δ functions. If we take our integration over an arbitrarily small lightlike rhombus around the point of collision, event q (Fig. 2), the absence of any bilinear term $L_{\rm in}L_{\rm out}$ in ρ ensures that the integral contribution will be arbitrarily small. We therefore get the generalized DTR relation

$$f_A(q)f_B(q) = f_C(q)f_D(q) \ , \tag{3.17}$$

which expresses the relationship between the four masses $m_A,\ldots,m_D$. It is remarkable that an exact solution to the complicated cross-flow field equations does exist and that it is so simple.

IV. MASS INFLATION

We now have everything in hand to formulate the mass inflation phenomenon. We consider a situation where an initially static Reissner-Nordström black hole with mass m_1 is perturbed by a crossflow of outgoing and infalling lightlike fluxes (see Fig. 3). More precisely, at advanced time V_1 we switch on the infalling flux, while at retarded time U_1 we switch on the outgoing flux. Therefore for $U < U_1$, $V < V_1$ spacetime geometry is described by the static Reissner-Nordström solution with mass m_1. For $V > V_1$ but $U < U_1$, the ingoing Vaidya solution takes over with a mass function $m_{\rm in}(V)$ which reduces to m_1 at $V = V_1$ and to the asymptotic limit m_0 on $\mathcal{J}_R^+$. Similarly, for $U > U_1$ but $V < V_1$, spacetime is described by the outgoing Vaidya solution with $m_{\rm out}(U_1) = m_1$. In the crossflow region $V > V_1$, $U > U_1$, spacetime will be described by the solution to the field equations (2.14) and (2.15) with the appropriate boundary conditions described above. We shall now attempt to find this solution.

Let us first recall the basic field equations. We shall continue to use our system of arbitrary radial null coordinates since they allow us to use the integral equation (3.16). The metric is then given by

$$ds^2 = -2e^{2\sigma} dU\, dV + r^2 d\Omega^2 , \tag{4.1}$$

while the non-Maxwellian contribution to the stress-energy tensor is given by

$$T_{ab} = [L_{\rm in}(V)/4\pi r^2] l_a l_b + [L_{\rm out}(U)/4\pi r^2] n_a n_b , \tag{4.2}$$

where $l_a = -\partial_a V$, $n_a = -\partial_a U$. The relation between the mass function and the metric element $e^{2\sigma}$ is

$$1 - 2m(U,V)/r + e^2/r^2 = f = -2e^{-2\sigma}(\partial_U r)(\partial_V r) \tag{4.3}$$

and the field equations for m are

$$\begin{aligned} \partial_U m &= -L_{\rm out}(U) e^{-2\sigma} \partial_V r , \\ \partial_V m &= -L_{\rm in}(V) e^{-2\sigma} \partial_U r . \end{aligned} \tag{4.4}$$

Its wave equation is given by Eq. (3.15).

The relation between $L_{\rm in}(V)$ and the measured quantity $dm_{\rm in}(v)/dv$ (v is, as always, the Eddington-Finkelstein advanced time) can be evaluated in the pure inflow region where $m = m_{\rm in}$, $f = f_{\rm in}$ and where the solution is known to be the ingoing charged Vaidya solution. By continuity, $L_{\rm in}(V)$ will assume the same form in the cross-flow region. By rewriting the second of Eqs. (4.4) in terms of v, and by using Eq. (3.6), it is easy to show that

$$L_{\rm in}(V) = \left[\frac{dv}{dV} \right]^2 \frac{dm_{\rm in}(v)}{dv} . \tag{4.5}$$

Similarly, we can show from comparing the first of Eqs. (4.4) to the outgoing Vaidya solution in the pure outflow region that

$$L_{\rm out}(U) = \left[\frac{du}{dU} \right]^2 \frac{dm_{\rm out}(u)}{du} . \tag{4.6}$$

To get the actual expression for $m_{\rm in}(v)$ we recall the analysis of Price[3] which showed that the amplitude of the backscattered gravitational radiation (modeled as a perturbing test field) varies as v^{-n} at late advanced times. If l is the multipole order of the perturbing field, n is given by $2l+2$. Now the energy density of the ingoing radiation, which is proportional to $dm_{\rm in}/dv$, will vary as v^{-2n}, so that the ingoing mass function will reach the asymptotic limit m_0 as $m_0 - m_{\rm in}(v) \sim v^{-(4l+3)}$. The dominant contribution to the influx will come from the quadrupole moment $l=2$ so that typically $m_0 - m_{\rm in} \sim v^{-11}$. So we can take

$$dm_{\rm in}(v)/dv \sim v^{-p} , \tag{4.7}$$

with $p \equiv 4(l+1) \geq 12$. As for $L_{\rm out}(U)$, we will see later that it is not necessary to write down a precise expression for it. We will only assume that it is a positive quantity: the radiation escaping the surface of the collapsing star should have a positive energy density.

To integrate Einstein's equations in the cross-flow region, we can make use of the integral formula (3.16) to find a formal expression for the mass function. Substituting Eq. (3.15) into Eq. (3.16) yields

$$m(U,V) = \int_{U_1}^{U} \int_{V_1}^{V} (r' e^{2\sigma'})^{-1} L_{\rm in}(V') L_{\rm out}(U') dU' dV' + m_{\rm in}(V) + m_{\rm out}(U) - m_1 , \tag{4.8}$$

with $L_{\rm in}(V)$ given by Eqs. (4.5) and (4.7), while $L_{\rm out}(U)$ is

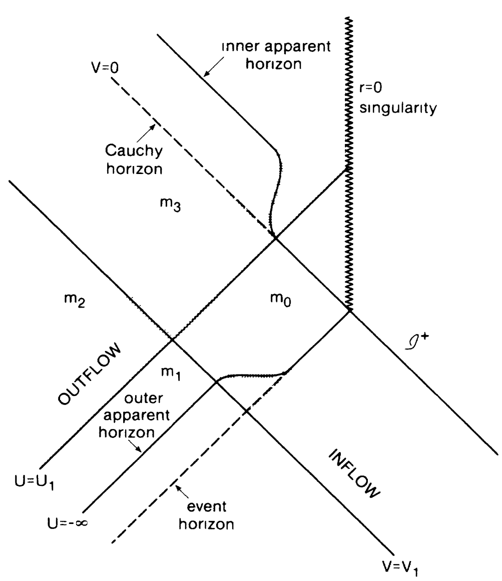

FIG. 3. Background Reissner-Nordström spacetime perturbed by cross-flowing streams of radial radiation: we turn on the influx at time $V = V_1$ and the outflux at $U = U_1$. If we suppose that the fluxes are later turned off, spacetime is characterized by four static regions with masses $m_0, \ldots, m_3$ and by inflow, outflow, and cross-flow regions. This figure represents the continuous counterpart of Fig. 2.

left arbitrary. So far, we have left the coordinates U and V unspecified; we shall now remedy this situation. Ingoing radiation is switched on at $V=V_1$, while the outflux is switched on at $U=U_1$. To make things conceptually clear, we can suppose that the fluxes are switched off at later times U_2 and V_2; we are of course interested in the situation where V_2 tends to the value of V on the Cauchy horizon. In this view, spacetime is characterized by four static regions with masses $m_0 \cdots m_3$ and by inflow, outflow, and cross-flow regions (Fig. 3). Each of the static regions can be described by a Reissner-Nordström solution with appropriate mass, and each static region possesses its own set of Eddington-Finkelstein null coordinates, defined as in Sec. III. We choose our coordinate v to be the Eddington-Finkelstein advanced time defined with respect to static region m_0. With this choice, the Cauchy horizon lies on the $v=\infty$ surface and v reduces to ordinary advanced time near $\mathcal{J}_R^-$. Similarly, we take u to be the Eddington-Finkelstein retarded time defined with respect to static region m_1. With this choice, u reduces to ordinary advanced time for observers near $\mathcal{J}_L^-$ and the outer horizon of region m_1 lies on the surface $u=-\infty$. We then choose our coordinate V to be the Kruskalized advanced time associated with the inner apparent horizon $r=r_0=m_0-(m_0^2-e^2)^{1/2}$:

$$V=-e^{-\kappa_0 v}\ , \tag{4.9}$$

where $\kappa_0=(m_0^2-e^2)^{1/2}/r_0^2$, such that $V=0$ on the Cauchy horizon. Similarly, we choose U to be the Kruskalized retarded time associated with the inner horizon $r=r_1=m_1-(m_1^2-e^2)^{1/2}$:

$$U=-e^{-\kappa_1 u}\ , \tag{4.10}$$

where $\kappa_1=(m_1^2-e^2)^{1/2}/r_1^2$, such that $U=-\infty$ on the outer horizon of region m_1. Note that both coordinates take negative values inside the black hole.

Now that we have chosen our system of coordinates, we can substitute Eqs. (4.7) and (4.9) into Eq. (4.5) to get

$$L_{\rm in}(V)\sim[-\ln(-V)]^{-p}/V^2\sim v^{-p}e^{2\kappa_0 v}\ . \tag{4.11}$$

If we note that, for $V\rightarrow 0$,

$$\int_{V_1}^{V} L_{\rm in}(V)dV\sim[-\ln(-V)]^{-p}/(-V)$$
$$\sim v^{-p}e^{\kappa_0 v}\ , \tag{4.12}$$

we conclude from Eq. (4.8) that $m(U,V)$ will diverge as $V\rightarrow 0$ unless the product $re^{2\sigma}$ goes to infinity quickly enough. Now, if $m(U,V)$ tends to become singular on the Cauchy horizon, we expect from Eq. (4.3) that $e^{2\sigma}$ will tend to go to zero, provided that the product of the derivatives $(\partial_U r)(\partial_V r)$ remains well behaved. We expect this to be the case since our coordinates U and V are well behaved in the vicinity of the Cauchy horizon and because we do not expect the Cauchy horizon to collapse too quickly due to the influence of the outflux of radiation. Indeed, the behavior of the Cauchy horizon [described by the function $r_{\rm CH}(U)\equiv r\ (U,V=0)$] is solely ruled by the amount of outgoing radiation crossing it and focusing its generators. The infinite blueshift plays no role in the contraction of the Cauchy horizon which then contracts only moderately. Of course, this is not so for the inner apparent horizon [described by the function $r_{\rm AH}(U,V)$ defined by $f(r_{\rm AH})=0$] which deflates catastrophically as the mass inflates to infinity. We therefore conclude, from our intuitive analysis, that $re^{2\sigma}$ will rather tend to go to zero in approaching the Cauchy horizon, hence feeding further mass inflation. The mass parameter is therefore bound to inflate to classically arbitrarily large values.

Although we cannot prove that $re^{2\sigma}$ actually goes to zero, we will now proceed with a formal proof that $re^{2\sigma}$ cannot go to infinity as we approach the Cauchy horizon. We construct the object $\psi=re^{2\sigma}$ and derive an expression for $\Box\ln\psi$ which can be formally integrated using Eq. (3.16). We will show that the solution for $\ln(re^{2\sigma})$ thus obtained, if not bounded, must nevertheless be bounded above. This shows that $re^{2\sigma}$ cannot go to infinity and verifies our conclusion that the mass parameter becomes unbounded at the Cauchy horizon. Using Eq. (2.18) we can calculate that $\Box\ln\psi=2\Box\sigma+\Box r/r-f/r^2$, which can be expressed in terms of r and e by using Eqs. (3.12), (2.16), and (2.7):

$$\Box\ln\psi=(3e^2-r^2)/r^4\ . \tag{4.13}$$

Substituting this in Eq. (3.16) yields

$$\ln\psi(U,V)=\ln[\psi_{\rm in}(U_1,V)\psi_{\rm out}(U,V_1)/\psi(U_1,V_1)]$$
$$-\tfrac{1}{2}\int_{U_1}^{U}\int_{V_1}^{V}\psi' r'^{-5}(3e^2-r'^2)dU'dV'\ , \tag{4.14}$$

where $\psi_{\rm in}(\psi_{\rm out})$ denotes the function $re^{2\sigma}$ which can be obtained from the known ingoing (outgoing) charged Vaidya solution. Now, the first term is finite since it can be shown that $\psi_{\rm in}(U_1,V)$ is actually finite on the Cauchy horizon. This nontrivial statement is proved in Appendix B: essentially, $\psi_{\rm in}$ is the radius of the Cauchy horizon r_0 times the metric element g_{UV} of the ingoing Vaidya solution evaluated near the Cauchy horizon; the Appendix shows that this metric element is indeed regular near the Cauchy horizon. This is a consequence of the fact that, when only influx is present, the surfaces of infinite blueshifts and redshifts coincide, thus preventing an unbounded increase of the mass parameter, as discussed in the Introduction. The second term is more serious, it might well be unbounded and what is crucial here is the sign of the contribution from the integral. If the integral is actually unbounded, the dominant contribution will come from values of the integrand near the Cauchy horizon. But it follows directly from $|e|<m_0$ (a necessary condition to the existence of the black hole) that $r_0<|e|$ so that in the vicinity of the Cauchy horizon, the contribution from the integral will be negative definite. This ensures that, if unbounded, $\ln\psi$ is necessarily not bounded below, but bounded above. This means that the product $re^{2\sigma}$ cannot go to infinity on the Cauchy horizon and hence cannot forbid the infinite increase of the mass parameter.

Some remarks are now in order: mathematically, the mass inflation phenomenon is expressed as the divergence of integral (4.8) as we approach the Cauchy horizon. We have shown that this integral must blow up, without actually having to evaluate it explicitly, which we cannot do exactly. We therefore know that mass actually inflates, but we do not know how fast. We have given a physical interpretation of this result in the Introduction: mass inflation can be explained by the combined effect of the infinitely blueshifted influx which piles up on the Cauchy horizon, and of the arbitrary outflux which produces the crucial separation of the Cauchy and inner horizons. While the Cauchy horizon contracts moderately, the inner horizon deflates much more rapidly. The mass inflation phenomenon can be understood further from an application of the DTR relation (3.17) (Fig. 2). If the two concentric null shells cross through each other near the inner horizon of sector B, f_B will be very small, but since the product $f_C f_D$ is finite and nonvanishing, the smallness of f_B must be balanced by the largeness of f_A. The closer to the inner horizon the shell propagates, the larger f_A will be. This can be interpreted as a violent increase of the mass parameter at the intersecting point. The DTR relation has been previously used by Blau[13] to elaborate Eardley's analysis[14] on the death of white holes by accretion and blueshift on the past horizon. However, the DTR relation does not offer any description of what happens at the interaction point, in contrast with our continuum analysis. In particular, the characteristic time scale over which mass inflation occurs is not given by the schematic DTR analysis.

A crude estimate of the mass parameter growth rate can be obtained if we formally expand the integral of Eq. (4.8) in powers of bilinear $L_{\text{in}}L_{\text{out}}$. If we keep only the first-order term, we can then take for $re^{2\sigma}$ their background values obtained from the static Reissner-Nordström solution with mass m_0 (to this order $m_0 \simeq m_1$). So, from Eq. (3.5), take $re^{2\sigma} \simeq r_0/\kappa_0^2$ and write, using Eq. (4.11),

$$L_{\text{in}}(V) \simeq \epsilon^2 (r_0/\kappa_0) \frac{[-\ln(-V)]^{-p}}{V^2} , \tag{4.15}$$

where ϵ^2 is an arbitrary dimensionless constant. Using Eq. (4.12) then yields

$$m(U,V) \simeq m_0 \epsilon^2 \Gamma(U) (\kappa_0 v)^{-p} e^{\kappa_0 v} , \tag{4.16}$$

where $\Gamma(U)$ is the fraction $\kappa_0 m_0^{-1} \int_{U_1}^{U} L_{\text{out}}(U') dU'$ of the star's mass radiated away. The dominant contribution, as noted above, comes from the quadrupole moment and so

$$m(U,V) \simeq m_0 \epsilon^2 \Gamma(U) (\kappa_0 v)^{-12} e^{\kappa_0 v} , \tag{4.17}$$

with ϵ representing a dimensionless quadrupole moment. Our crude estimate indicates that the mass parameter inflates exponentially with Eddington-Finkelstein advanced time with a characteristic time scale of $1/\kappa_0$. This is most certainly an underestimate since, as we mentioned earlier, the product $re^{2\sigma}$ does not actually remain constant but probably decreases to zero near the Cauchy horizon, hence precipitating further mass inflation. A numerical integration of the field equations would be necessary in order to say more.[15]

In conclusion, some comments on how the asphericities inevitably present in a realistic collapse might be expected to affect this idealized spherical picture. The broad aspects of mass inflation should be generic, since they depend only on the qualitative features of infinite blueshift at the Cauchy horizon and the separation of Cauchy and apparent horizons under transverse irradiation. Because angular momentum J is nearly conserved in the collapse (it would be conserved exactly for axial symmetry), the Kerr parameter $a = J/m$ should become negligibly small in comparison with m in the mass-inflated geometry near the Cauchy horizon. Thus the asymptotic field is expected to be very nearly Schwarzschild.

What about the effects of nonrotational (quadrupole) asymmetries? For a slightly aspherical, uncharged and nonrotating collapse it has been shown[4,7] that the "tail" (corresponding to large advanced time v) of the spacelike singularity formed at $r=0$ relaxes asymptotically to a stationary, exactly Schwarzschild-like form. This argument is easily adapted to the situation where a Cauchy horizon is present: a spacelike curve just outside the Cauchy horizon, where curvatures reach Planckian values, now assumes the role of the curve $r=0$. Asymmetries of the collapsing star can influence the (infinitely long) tail of this curve only through a very narrow window of the star's history, and they are exponentially redshifted. The geometry of the Cauchy horizon's tail is thus determined solely by the asymptotics of the external field as $v \to \infty$. But this is known to tend to a Kerr-Newman form, characterized by an inner-horizon surface gravity κ_0 which is a constant (independent of geographical location on the surface). Thus the exponential mass-inflation factor $e^{\kappa_0 v}$ is uniform, and nonuniformities $\Delta m/m \sim \Delta\theta$ of the mass aspect $m(\theta,\varphi,v)$ on angular scale $\Delta\theta$ will not grow exponentially, but should remain roughly constant during inflation. Near the Cauchy horizon these nonuniformities are negligible on length scales comparable with the local (near-Planckian) radius of curvature. On such length scales, the geometry should appear locally indistinguishable from a Schwarzschild solution of very large mass.

Thus, there appears to be no reason to expect our analysis and results to be significantly affected by the effects of rotation and asymmetries.[15a] Of course, a detailed analysis will be needed to confirm these tentative conclusions.

V. QUANTUM CONSIDERATIONS

How can quantum mechanics modify the classical picture described above? We shall attempt to find some clues to an answer in this section. It is now widely believed that singular behavior in general relativity signals a breakdown of the theory, and that it should not appear once a good theory of quantum gravity is formulated. Although there is hope from superstring theory, a quantum theory has not yet been discovered and much of the quantum effects associated with gravity are as yet uncertain. Nevertheless, it was speculated lately[5,9] that because of these unknown quantum effects, spacetime curvature should always be subject to an upper bound of Planckian magnitude. This corresponds to the most naive perception that one can have of a "nonsingular singularity": a region of spacetime where curvature is as extreme as we can imagine. As discussed in Ref. 5, one possible mechanism for slowing down the infinite rise of curvature is vacuum polarization associated with creation of virtual particles near the black-hole singularity. If the stress thus induced is a tension along the axes of the three-cylinders of constant time, spacetime tends to remain regular near the origin.

Let us imagine what could be the consequences of such an upper bound in curvature for the black-hole inner structure. We will come back later to the hard problem of justifying this assumption. Classically, the interaction of infalling and outgoing radiation inside the black hole produces an infinite rise in the hole's mass parameter. Quantum mechanically, we might expect an enormous increase, but not quite an infinite one. We recall that because of mass inflation and because the fluxes of radiation conserve (at least approximately) charge and angular momentum, the only relevant parameter describing the black-hole geometry near the Cauchy horizon is precisely the mass. We therefore infer that in this region, spacetime can be adequately described by the Schwarzschild metric

$$ds^2=-(1-2m/r)^{-1}dr^2+(1-2m/r)dt^2+r^2d\Omega^2 \ , \quad (5.1)$$

where m is the inflated final mass parameter. How large can it become? Spacetime curvature m/r^3 has become, according to our hypothesis, of order unity in Planck units. Since r is then of the order of the Cauchy horizon radius, itself of the order of the black-hole external mass m_0 (if we assume that $|e|\sim m_0$), we can conclude that

$$m\sim m_0^3/m_{\rm Pl}^2 \ , \quad (5.2)$$

where $m_{\rm Pl}$ is the Plank mass. For a black hole of ten solar masses, the internal mass parameter reaches the order of 10^{60} times the mass of the observable Universe.

If we suppose that we can accept the classical description up to the transition hypersurface Σ_Q on which curvature becomes Planckian as the mass parameter reaches its limiting value (5.2), we find that the transition region is spacelike since, classically, mass increases further to infinity on a null hypersurface occurring later. The separation between Σ_Q and the Cauchy horizon is of the order of the Planck time, as we will verify below. So, for all practical purposes, the Planckian phase, or end of the classical phase, appears to arise at a spacelike surface, as would be the case for the Schwarzschild singularity. It is interesting to note that the quantum evolution of the black-hole internal structure starts from a known and well-defined initial state on Σ_Q. This is to be contrasted with the situation in quantum cosmology where initial conditions have to be postulated.

Using metric (5.1) but assuming that $2m/r\gg 1$, it is easy to see that for an observer at fixed t, θ, and ϕ, the trip from the quantum radius $r_0\equiv m^{1/3}\sim r_{\rm CH}$ to the origin takes about one Planck time:

$$\tau=\int_{r_0}^{0}(r/2m)^{1/2}(-dr)\sim 1 \quad (5.3)$$

(the calculation is done in Planck units). This shows that an observer's trip to the vicinity of the Cauchy horizon covers all but the last Planck time of the black hole's entire classical history. This estimate is however modified if vacuum polarization allows curvature to remain bounded beyond the transition hypersurface Σ_Q. Let us assume specifically that metric (5.1) is valid at and near Σ_Q but that according to the analysis of Ref. 5, it would take the form

$$ds^2=-a^2d\tau^2+e^{-2\tau}(d\Omega^2+e^{2\psi(\tau)}dt^2) \quad (5.4)$$

beyond Σ_Q (a^2 is of order unity and is a measure of the curvature beyond Σ_Q, τ is the proper time for an observer at fixed t, θ, and ϕ, and ψ is an undetermined function depending on the equation of state of the quantum material). According to this new expression for the metric, the trip from Σ_Q to the origin would take an amount of time of the order of r_0. The possibility that vacuum polarization would increase this estimate even further is not excluded.

The picture emerging from these quantum considerations is that the region within Σ_Q is represented by a "fat cigar" (topology $S^2\times R_+$) of uncertain thickness enveloping the Cauchy horizon and inside which the evolution of spacetime is determined by unknown quantum effects which would, presumably, forbid curvature to grow beyond Planckian scales.

Let us now review some of the possibilities concerning the quantum evolution of spacetime geometry. Here, of course, we enter the realm of very uncertain speculation. One possibility would be to apply quantum cosmology in its current minisuperspace formulation where a wave function depending on the gravitational and external fields degrees of freedom is constructed. As mentioned earlier, our problem does not share quantum cosmology's difficulty concerning the formulation of boundary conditions. In view of metric (5.1), the simplest minisuperspace model for the black-hole interior would be the two-dimensional model

$$ds^2=-N^2(r)dr^2+a^2(r)dt^2+b^2(r)d\Omega^2 \ , \quad (5.5)$$

where a and b represent the gravitational degrees of freedom (N is not a dynamical variable). A quantum model of this kind, for pure gravity, has been considered by Nambu and Sasaki,[16] using the Einstein-Hilbert action. Classically, the trajectories representing the evolution of Schwarzschild spacetime must go toward the singularity. Should the wave function be peaked around some classically forbidden trajectory which escapes the singularity, one could interpret this as a quantum gravitational avoidance of singular behavior. Unfortunately, Nambu and Sasaki find that the wave function is exponentially damped for classically forbidden trajectories. This model therefore offers little hope for quantum gravitational justification of singularity avoidance. Further studies including matter fields[17] do not modify this pessimistic conclusion.

This negative result is an illustration of Wheeler's principle of unanimity.[18] Roughly, it states that if the solutions to the classical equations of motion all exhibit singular behavior (except perhaps for a set of measure zero) then the quantum solutions should also be singular. According to this view, any model based on the Einstein-Hilbert action would not be freed of singularities by a quantum analysis. To avoid singular behavior, one would hence need to seek for alternative Lagrangians leading to nonsingular classical solutions. This is a rather natural step since it was realized long ago that the Einstein-Hilbert action should not be valid at extreme regimes.[19] Quantization of matter fields on a classical spacetime geometry typically induces quadratic terms in curvature in the effective gravitational Lagrangian and it is likely that higher-order terms would need to be included as well. A particular Lagrangian in ten dimensions is singled out by superstring theory. The Einstein-Hilbert+Gauss-Bonnet action[20]

$$\int d^{10}x\sqrt{-g}\,[R+\alpha(R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}-4R_{\alpha\beta}R^{\alpha\beta}+R^2)] \tag{5.6}$$

is the unique action principle leading to second-order differential equations for the ten-dimensional metric $g_{\alpha\beta}$ ($\alpha\sim l_{\rm Pl}^2$, the square of the Planck length). It would be interesting to attempt to find regular classical solutions to the field equations associated with this choice of Lagrangian for a generalized Kantowski-Sachs metric of the form

$$ds^2=-A(r)dr^2+B(r)dt^2+r^2d\Omega^2+\alpha\rho^2(r)g_{AB}dx^A dx^B\ , \tag{5.7}$$

where the nature of the six-dimensional line element $g_{AB}dx^A dx^B$ is rather uncertain. The behavior of the scale factor $\rho(r)$ should reflect the compactification of extra dimensions at the classical regime. Such an investigation is now underway.[21]

VI. CONCLUSION

It is likely that our views about the quantum evolutionary phase of spacetime inside black holes will change and that the many uncertainties surrounding it will not be resolved soon. But a definite classical picture appears to be emerging up to the formation of the "fat cigar singularity" near the Cauchy horizon and before the ambient curvature m/r^3 actually reaches Planck values, there does not seem to exist any mechanism capable of stopping the inflation phenomenon. For instance, scattering of the inflow by the now inflated curvature never becomes effective. Indeed, the radius of curvature, behaving like $m^{-1/2}\sim|V|^{1/2}$ always remains larger than the blueshifted wavelength of the ingoing modes, behaving like $|V|$. Our detailed mathematical analysis is based on a very idealized spherical model, but the basic physical mechanism (the combination of a highly focused and blueshifted shower of radiation propagating along the Cauchy horizon and the arbitrary irradiation of the latter by outgoing radiation emitted from the surface of the collapsing star) is independent of the model. We have therefore little reason to doubt that our conclusions should be valid for a generic, rotating black hole.[15a]

According to this classical picture (if we follow it to its most extreme consequences) the singularity arising in a generic black hole would be null rather than being spacelike as favored by strong cosmic censorship.[22] However, in a more pragmatic viewpoint where we consider curvatures of Planckian magnitude to be (classically) "singular," it makes little difference since the transition hypersurface on which curvature reaches unity in Planck units is a spacelike hypersurface. It is nevertheless an interesting question of principle to evaluate whether more realistic perturbing fields could succeed in producing an actual spacelike singularity. We will leave this question open for the time being.

The complicated field equations derived here do not allow us to find an explicit expression for the mass function $m(U,V)$. Equation (4.17) represents probably a very crude underestimate of its growth rate and to be able to say more, a numerical integration of the field equations should be performed. In the process, we would be able to gather more information about the relevant quantities of the problem, specifically, the behavior of the metric element $e^{2\sigma}$ and radius function r. Numerical integration of the equations is now in progress[15] and the results will be published elsewhere; for completeness, we derive in Appendix C the basic first-order system of differential equations necessary to write the numerical code.

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The mass inflation phenomenon, one of the most spectacular manifestations of the nonlinear aspects of general relativity, first dawned on us when we derived the generalized form of the Dray–'t Hooft–Redmount (DTR) relation presented in Sec. III. Shortly afterward, Steven Blau informed us of his very interesting and independent work[13] in which the original DTR relation is applied to show that white holes are typically engulfed by black holes soon after their formation. We are grateful to him for those enlightening discussions. Don Page's interest in our work was very stimulating and numerous discussions with him helped the elaboration of this paper. Our warmest thanks to him. This research was supported by the Canadian Institute for Advanced Research, Natural Sciences and Engineering Research Council of Canada, and Fonds pour la Formation de Chercheurs et l'Aide à la Recherche, Québec, Canada.

APPENDIX A: GEOMETRIC QUANTITIES FOR SPHERICAL SPACETIMES

We list in this appendix the relevant geometric quantities of a spherical spacetime geometry whose metric is expressed in the form

$$ds^2=g_{ab}dx^a dx^b+r^2d\Omega^2\ , \tag{A1}$$

where $x^a=(x^1,x^2)$ are arbitrary coordinates spanning the "radial" two-spaces $(\theta,\phi)=$const. Function $r(x^a)$ is defined in the usual way from the proper area of a two-sphere $x^a=$const: $\mathcal{A}=4\pi r^2$. The symbol $d\Omega^2$ denotes the two-dimensional line element of the unit two-sphere: $d\Omega^2=d\theta^2+\sin^2\theta\,d\phi^2$. It is most convenient to express the diverse four-dimensional quantities derived from the four-dimensional metric $g_{\alpha\beta}$ in terms of the two-dimensional quantities derived from g_{ab} and derivatives of r. We will denote by a semicolon the covariant derivative with respect to the two-metric, whereas a stroke ($|$) will denote the same with respect to the full four-dimensional metric. The d'Alembertian $\Box\psi$ of any scalar field ψ will be the two-dimensional quantity

$$\Box\psi\equiv g^{ab}\psi_{;ab} \; . \tag{A2}$$

We will express the four-dimensional quantities with the superscript 4, whereas we will leave the two-dimensional quantities free of additional indices. For example, the ab component of the four-dimensional Ricci tensor will be noted as ${}^4R_{ab}$ whereas the two-dimensional Ricci tensor will be written as R_{ab}. We use throughout the conventions of Misner, Thorne, and Wheeler.[23]

With the notation described above, the Christoffel symbols associated with the four-dimensional metric are

$$\begin{aligned} &{}^4\Gamma^a_{bc}=\Gamma^a_{bc}, \quad \Gamma^a_{\theta\theta}=\sin^{-2}\theta\Gamma^a_{\phi\phi}=-rr^{,a} \; , \\ &\Gamma^\theta_{a\theta}=\Gamma^\phi_{a\phi}=r_{,a}/r, \quad \Gamma^\theta_{\phi\phi}=-\sin\theta\cos\theta, \quad \Gamma^\phi_{\theta\phi}=\cot\theta \; . \end{aligned} \tag{A3}$$

The Riemann curvature tensor is

$$\begin{aligned} &{}^4R_{abcd}=R_{abcd}, \quad R_{a\theta b\theta}=\sin^{-2}\theta R_{a\phi b\phi}=-rr_{;ab} \; , \\ &R_{\theta\phi\theta\phi}=r^2\sin^2\theta(1-r^{,a}r_{,a}) \; . \end{aligned} \tag{A4}$$

Contracting over the first and third indices yields the Ricci tensor

$$\begin{aligned} &{}^4R_{ab}=R_{ab}-2r_{;ab}/r \; , \\ &R_{\theta\theta}=\sin^{-2}\theta R_{\phi\phi}=1-(r\Box r+r^{,a}r_{,a}) \; , \end{aligned} \tag{A5}$$

and the Ricci scalar is

$${}^4R=R+2(1-2r\Box r-r^{,a}r_{,a})/r^2 \; . \tag{A6}$$

We finally arrive at the Einstein tensor

$$\begin{aligned} &{}^4G_{ab}=-[2rr_{;ab}+g_{ab}(1-r^{,a}r_{,a}-2r\Box r)]/r^2 \; , \\ &G_{\theta\theta}=\sin^{-2}\theta G_{\phi\phi}=r\Box r-\tfrac{1}{2}r^2R \; . \end{aligned} \tag{A7}$$

These equations can be compared with the usual results obtained when one chooses a particular system of coordinates for x^a. For example, if one sets $g_{ab}dx^a dx^b=-2h\,du\,dv$, one recovers the results derived in Waugh and Lake.[24] Similarly, by choosing $g_{ab}dx^a dx^b=-e^{2\Phi}dt^2+e^{2\Lambda}dr^2$ one recovers the results given in Misner, Thorne, and Wheeler, page 844 of Ref. 23.

APPENDIX B: VAIDYA SOLUTION IN DOUBLE-NULL COORDINATES

A free-falling observer crossing the Cauchy horizon measures an infinite amount of energy density for the shower of infalling radiation propagating along it. We will verify this statement explicitly in this appendix, and we will show that, when expressed in regular coordinates, the metric is manifestly regular there. Interestingly enough, even though certain elements of the curvature tensor diverge, the invariant $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$ remains regular at the Cauchy horizon. This is a consequence of the fact that the singularity in the energy density is null.

We look at the ingoing charged Vaidya solution

$$\begin{aligned} &ds^2=dv\,(2dr-f dv)+r^2d\Omega^2 \; , \\ &f=1-2m(v)/r+e^2/r^2 \; , \end{aligned} \tag{B1}$$

where the mass function is given by

$$m(v)=m_0-\mu(v) \; , \tag{B2}$$

where $\mu(v)\sim v^{-(p-1)}$. The slow increase of the mass parameter is produced by an influx of lightlike particles described by the stress-energy tensor

$$T_{\alpha\beta}=\rho l_\alpha l_\beta \; , \tag{B3}$$

where $l_\alpha=-\partial_\alpha v$ and $4\pi r^2\rho=dm/dv$. We recall that because of the large blueshifts occurring near the Cauchy horizon (which we shall verify below) the effective stress-energy tensor description (B3) should be an accurate description for the backscattered gravitational radiation falling into the black hole at late advanced times. We will first show that the energy density measured by a free-falling observer with four-velocity u^α,

$$\rho_{\text{obs}}\equiv T_{\alpha\beta}u^\alpha u^\beta=\rho(l_\alpha u^\alpha)^2 \; , \tag{B4}$$

becomes infinite at the Cauchy horizon $v=\infty$. It is only necessary to compute the v component of the four-velocity; if we choose a radial observer, the normalization condition $u^\alpha u_\alpha=-1$ yields (the overdot denotes differentiation with respect to the observer's proper time)

$$\dot v=[\dot r-(\dot r^2+f)^{1/2}]/f\simeq 2\dot r/f \; , \tag{B5}$$

where the approximation holds near the Cauchy horizon. Therefore

$$(u^\alpha l_\alpha)^2=\dot v^2\sim 1/f^2 \; , \tag{B6}$$

which is infinite at the Cauchy horizon. It is possible to integrate (B5) explicitly if we approximate f by its asymptotic expression $f_0=1-2m_0/r+e^2/r^2$. This is in fact a good approximation since $m(v)$ is slowly varying near the Cauchy horizon. We find that on the path of the free-falling observer

$$f\simeq -2e^{-\kappa_0 v} \; , \tag{B7}$$

where $\kappa_0\equiv(m_0^2-e^2)^{1/2}/r_{\text{CH}}^2$ such that combining Eqs. (B2), (B4), and (B7), we arrive at

$$\rho_{\text{obs}}\sim v^{-p}e^{2\kappa_0 v} \; , \tag{B8}$$

which manifestly blows up at the Cauchy horizon. Note that not only ρ_{obs} blows up, but also its integrated value over the path of the observer.

It is not at first sight obvious that metric (B1) is regular at the Cauchy horizon since the coordinates themselves become singular there: both r and v are constant on the Cauchy horizon. Even a rescaling of v of the form

$V=-e^{-\kappa_0 v}$ does not remove this difficulty. But because $m(v)$ is a slowly varying function, it is possible to approximately transform metric (B1) into a double-null form which is manifestly regular at the Cauchy horizon. (Such a transformation is not possible in general.[24]) We now proceed with this transformation. We suppose that the influx of radiation is switched on at advanced time v_1 and that $m(v_1)=m_0$, not only do we suppose that m_0 is the asymptotic limit of the mass function, we also suppose that it is its initial value. This choice has the important advantage of allowing us to define the various quantities of interest (e.g., coordinate u, function κ_s below) with respect to m_0 rather than another value of the mass parameter. The difference $m_0-m(v)\equiv\mu(v)$ will therefore be assumed to be zero for $v<v_1$, and will be assumed to be non-negative thereafter until it decreases asymptotically to zero as v goes to infinity. Only this asymptotic behavior will be of interest to us and we will not be concerned with the initial decrease of the mass function. Although we are interested in the situation where the asymptotic behavior of $\mu(v)$ is that of an inverse power law, the following calculation only assumes that $\mu(v)/m_0$ is small enough so that we can ignore second-order corrections.

We start by expanding $f[m(v),r]$ around the initial value of the mass m_0 and around function $r_s(u,v)$, the solution of

$$dr_s=\tfrac{1}{2}f_s(du+dv)\ , \tag{B9}$$

where $f_s\equiv f[m_0,r_s]$ (the subscript stands for "static"). We thus obtain

$$f\simeq f_s+2\mu/r_s-2\kappa_s(r-r_s)\ , \tag{B10}$$

where

$$\kappa_s=-\tfrac{1}{2}\partial_r f|_s=(m_0^2-e^2)^{1/2}/r_s^2 \tag{B11}$$

is a function of r_s. Function $r(u,v)$ is related to the coordinates by

$$dr=\tfrac{1}{2}f(e^{\phi}du+dv)\ , \tag{B12}$$

where e^{ϕ} plays the role of an integrating factor. We therefore find that the metric element which we want to evaluate is

$$g_{uv}\equiv h=\tfrac{1}{2}fe^{\phi}=\partial r/\partial u\ . \tag{B13}$$

To find $r(u,v)$, we use Eqs. (B9), (B10), and (B12) to obtain a differential equation for the difference $r-r_s$:

$$\partial(r-r_s)/\partial v+\kappa_s(r-r_s)=\mu/r_s\ , \tag{B14}$$

which can be integrated to yield

$$r-r_s=f_s\int_{v_1}^{v}dv'\mu(v')/r_s'f_s'\ , \tag{B15}$$

where we suppose that $v>v_1$. For $v<v_1$, $\mu(v)=0$, and $r(u,v)$ reduces to $r_s(u,v)$. [We use the notation $r_s'\equiv r_s(u,v')$, etc.] It is now straightforward to show that differentiation of Eq. (B15) with respect to u and use of Eq. (B13) yields

$$h=\tfrac{1}{2}f_s\left[1-2\int_{v_1}^{v}dv'\frac{\mu(v')}{r_s'}\frac{\kappa_s-\kappa_s'}{f_s'}-\int_{v_1}^{v}dv'\frac{\mu(v')}{r_s'^2}\right]\ , \tag{B16}$$

where we recognize in front of the large parentheses the static expression associated with the initial mass m_0. It is manifest that the correction terms are bounded at $v\to\infty$ provided that $\int_{v_1}^{\infty}\mu(v')dv'<\infty$. This property is clearly satisfied by our choice for $\mu(v)$.

It is now easy to transform to coordinates U and V in which the metric is well behaved. This has the effect of changing the coefficient $\tfrac{1}{2}f_s$ in Eq. (B16) to the expression (3.5). This finally shows that metric (B1), when expressed in well-behaved coordinates, is perfectly regular at and near the Cauchy horizon. Note however that second derivatives in V will produce a divergent curvature of the order of ρ_{obs}. The curvature invariant

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}=48m^2(v)/r^6\ , \tag{B17}$$

however, remains regular. The reason lies in the fact that the singularity is null: any contraction of the kind $l^{\alpha}l_{\alpha}$ will give a contribution of zero.

APPENDIX C: FIRST-ORDER EQUATIONS FOR NUMERICAL INTEGRATION

For completeness, we include here the system of first-order differential equations which would enable a numerical analyst to integrate Einstein's equations numerically. All we will do is recast the field equations (4.1)–(4.4) in a more convenient form. We shall choose for our null coordinates the usual Schwarzschild retarded and advanced times.

Define first additional variables x and y according to

$$e^{-x}\equiv\partial_u r/\tfrac{1}{2}f,\quad e^{-y}\equiv\partial_v r/\tfrac{1}{2}f\ . \tag{C1}$$

Substituting this in Eq. (4.3) yields

$$e^{2\sigma}=-\tfrac{1}{2}fe^{-(x+y)} \tag{C2}$$

and then into Eqs. (4.4) gives

$$\begin{aligned}\partial_u m&=e^{x}dm_{\text{out}}(u)/du\ ,\\ \partial_v m&=e^{y}dm_{\text{in}}(v)/dv\ .\end{aligned} \tag{C3}$$

It is then straightforward to derive additional differential equations for x and y starting from Eqs. (C1). We find

$$\begin{aligned}\partial_u y&=-(2e^{x}/rf)dm_{\text{out}}(u)/du\ ,\\ \partial_v x&=-(2e^{y}/rf)dm_{\text{in}}(v)/dv\ .\end{aligned} \tag{C4}$$

Since x is determined up to an arbitrary rescaling of coordinate u, it is not necessary to write down an expression for $\partial_u x$. The same is true for $\partial_v y$.

In the initial static Reissner-Nordström region ($u<u_1$, $v<v_1$), we have $x=y=0$, $m(u,v)=m_1$, and $r(u,v)$ is given implicitly by $u+v=2r^*$. In the pure inflow region ($u<u_1$, $v>v_1$), we have $\partial_v x$ given by Eq. (C4), $y=0$,

$m(u,v)=m_{\rm in}(v)$, and $\partial_v r = f/2$. An expression for $\partial_u r$ is not needed since by continuity, $r(u_1,v)$ is known. Finally, in the pure outflow region ($u>u_1$, $v<v_1$) we have $x=0$, $\partial_u y$ given by Eq. (C4), $m(u,v)=m_{\rm out}(u)$ and $\partial_u r = f/2$. As before, an equation for $\partial_v r$ is not needed. All the above quantities can then be obtained in the cross-flow region by numerical integration of Eqs. (C1)–(C4) with the boundary conditions given above.

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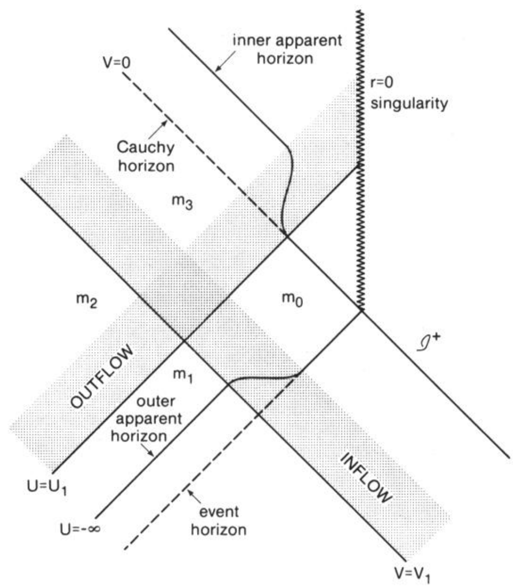

FIG. 3. Background Reissner-Nordström spacetime perturbed by cross-flowing streams of radial radiation: we turn on the influx at time $V=V_1$ and the outflux at $U=U_1$. If we suppose that the fluxes are later turned off, spacetime is characterized by four static regions with masses $m_0, \ldots, m_3$ and by inflow, outflow, and cross-flow regions. This figure represents the continuous counterpart of Fig. 2.