

Gluon fusion as a source for massive-quark polarization

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It is well known that single-polarization asymmetries are large in hyperon production in contrast with naive QCD predictions. We have explored the possibility of polarization of quarks through "gluon fusion," assuming that the quark mass can be significant at energies of interest. The fourth-order contribution to the single-spin asymmetry in $g + g \rightarrow s + \bar{s}$ is calculated. Properties of the polarization of the strange quark in the hadron center-of-mass frame are discussed. A fit to the hyperon polarization is presented that reproduces the unique and striking kinematic dependence of the data. This is evidence that gluon fusion can be taken as a serious candidate for the "seed" of polarization.

The polarization phenomena in strong interactions have been a field of considerable interest since the discovery of large transverse polarization in inclusive hyperon production at Fermilab.¹ During the last decade, experiments have explored these phenomena and the kinematic behavior has been extensively studied. The large polarization is a general property of all high-energy hyperon production by protons as well as other hadronic projectiles. The polarization depends on the incoming hadrons, grows in magnitude with the hyperon's transverse momentum, and is fairly insensitive to the energy of the beam. Remarkably simple and general systematic properties of the data² are striking and suggest something about the underlying dynamical mechanism for hyperon production by the strong interaction.

While the variety and amount of data has been increased, the progress in fitting these results into the standard QCD picture has been slow. In fact, QCD predicts zero polarization at high energies and transverse momenta (p_T). However, no trend toward zero polarization is detected at the highest available p_T . This has led to the widespread suspicion that either QCD is not the correct theory of strong interactions or that the assumptions made in perturbative QCD, neglecting confinement effects, are not appropriate. If confinement effects are dominant at available energies, more understanding of the nonlinear sector of the theory is necessary to make meaningful phenomenological calculations.

The existing models³⁻⁵ which have been constructed by incorporating some of the nonlinear character of the theory, explain the trend of the hyperon polarization data, but in a qualitative fashion. Each has its successful qualitative predictions and its severe limitations. One of the primary problems of all of these models is the classical or semiclassical nature of the descriptions and the corresponding arbitrariness in the application to real kinematics and data.

Assuming the more optimistic point of view, that the seed of polarization is hidden in QCD and can be uncovered through perturbative QCD which may be partially applicable at least at the highest available p_T , we have started to construct a model that is based on the actual one-loop-level QCD subprocesses involving massive

quarks. At the first step subprocesses involving strange quarks interacting with nonstrange quarks and gluons, "flavor-excitation" diagrams, were considered.⁶ The results were encouraging and had the correct sign, and energy and p_T dependence for hyperon polarization but the magnitude remained too small.

The next step is to look for the other important subprocesses. Specifically for the inclusively produced hyperons from protons, $p + p \rightarrow \Lambda + X$, in which the strange quark has to be created, the contribution from "gluon fusion," $g + g \rightarrow s + \bar{s}$, can be significant. In fact, this is the dominant contribution to the strange-quark produc-

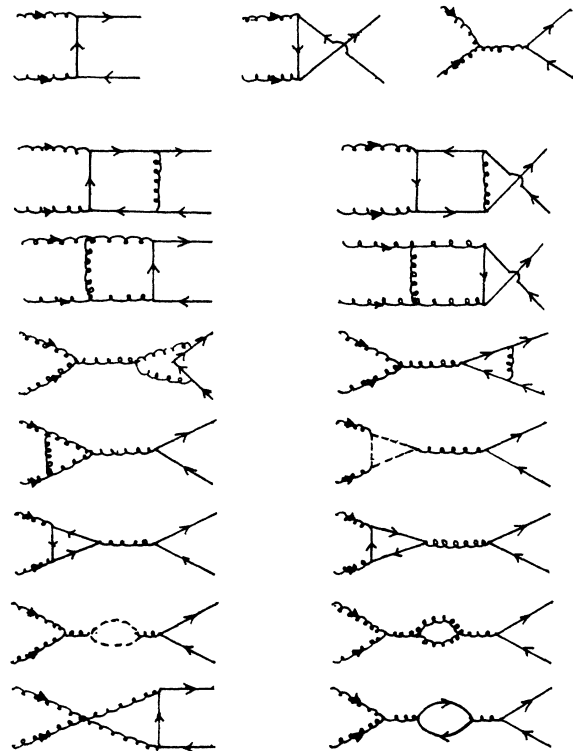


FIG. 1. Feynman diagrams for gluon fusion, $g + g \rightarrow s + \bar{s}$. In the second order, only the diagrams which contribute to the imaginary amplitude are shown.

tion at large p_T and small Feynman x . Furthermore, flavor excitation may be included in such gluon fusion subprocesses, in particular kinematic regions.⁷ In this paper we obtain the expression for the massive-quark polarization through gluon fusion. We explore the dependence of the polarization on various kinematic variables and on heavy-quark masses. The corresponding expression in the case of Abelian gauge theory or QED is extracted. Finally, a comparison with available data for hyperon polarization is made.

Polarization normal to the scattering plane arises as a result of the interference between nonflip and single-flip helicity amplitudes, which requires a complex amplitude. Since all amplitudes are real, even with massive quarks, in the lowest order in perturbation theory, only the imaginary parts of the one-loop-level amplitudes contribute to the single-spin asymmetry. Hence, the lowest-order Feynman diagrams and the second-order diagrams that contribute to the imaginary parts of amplitudes are considered in the calculation. The imaginary parts, the discontinuities across the unitarity cut in the physical s channel, are extracted by using the Cutkosky rules.⁸ The rest of the perturbative calculations are straightforward

and we will present the results below, leaving the details for a forthcoming publication.

The Feynman diagrams that contribute to the polarization through gluon fusion are shown in Fig. 1. Note that there are additional diagrams, both at the tree level and in the one-loop approximation, as compared with the pair annihilation, the QED analog. The existence of the triple-gluon coupling vertex, which is a result of the non-Abelian character of QCD, produces s -channel exchange in the lowest order. In one-loop level several new diagrams contribute to the complex amplitude that make the polarization distinctly different from QED. The imaginary parts of amplitudes of those loop diagrams are infrared divergent and are regulated by introducing an arbitrary small gluon mass. The delicate cancellation of the cutoff parameter, which must obtain in the polarization formula, is a good test of the correctness of such a calculation.

The resulting quark polarization for $g+g \rightarrow q+\bar{q}$ to order α_s , where α_s is the strong coupling constant which depends on the logarithm of the momentum transfer in the usual treatment of QCD, is given by the lengthy formula

$$\mathcal{P} = \alpha_s \frac{m(p^2 - k^2 \cos^2 \theta)}{24kD \sin \theta} \left[(N_1 + N_2)Y_+ + (N_1 - N_2)Y_- + N_3 \ln \frac{(p-k)}{(p+k)} + N_4 + 18k^3 \sin^2 \theta \cos \theta (\Sigma_1 + \Sigma_2) \right], \quad (1)$$

where

$$D = (9k^2 \cos^2 \theta + 7p^2)(k^4 \cos^4 \theta - 2k^2 m^2 \sin^2 \theta - p^4),$$

$$N_1 = 9k^2 \cos^3 \theta (p^2 + 2k^2) + 6kp \cos \theta (27p^2 + 11kp - 27k^2) + 27k^4 \cos \theta,$$

$$N_2 = kp \cos^2 \theta (11p^2 + 76k^2) - 162p^2 m^2 + 33k^3 p,$$

$$N_3 = p \cos \theta [243m^2 p \cos^4 \theta - \cos^2 \theta (324m^2 p - 54k^3) + 22kp^2 - 243m^2 p + 164k^3],$$

$$N_4 = -\frac{1}{4} k \sin^2 \theta \cos \theta [72 \cos^2 \theta (27p^3 - 18k^2 p + k^3) + 27(97k^2 p - 24p^3) - 8(22kp^2 + 45k^3)],$$

$$\Sigma_1 = \frac{2}{p^2} \sum_i \sqrt{p^2 - m_i^2} (2p^2 + m_i^2) \theta(p - m_i),$$

$$\Sigma_2 = \frac{1}{p^2} \sum_i \left[3m_i^2 p \ln \left[\frac{p - (p^2 - m_i^2)^{1/2}}{p + (p^2 - m_i^2)^{1/2}} \right] - 4(p^2 - m_i^2)^{3/2} \right] \theta(p - m_i),$$

$$Y_{\pm} = \ln \left[\frac{(p \pm k \cos \theta)^2}{m^2} \right].$$

The subprocess center-of-mass variables p , k , and θ are the gluon momentum, the quark momentum, and the scattering angle, respectively, and m is the mass of the final-state quark. The summation runs over all possible intermediate quark masses m_i . For strange-quark production, as an example, m_i takes up-, down-, and strange-quark masses at energies below the threshold for charm production. The axis of polarization is chosen in the direction given by $\mathbf{p}_a \times \mathbf{p}_c / |\mathbf{p}_a \times \mathbf{p}_c|$ where \mathbf{p}_a and \mathbf{p}_c are the center-of-mass momenta of the incoming gluon and the produced quark, respectively. These QCD polarization formulas have not been obtained before and have many interesting features that distinguish them from the analogous QED results and indicate some promise for producing the desired large polarization.

In the case of pair annihilation, the QED analog of gluon fusion, the corresponding expression for the polarization of the produced lepton can be obtained by considering the remaining diagrams of Fig. 1 after neglecting the gluon-gluon coupling. The resulting expression takes the form

$$\mathcal{P} = \alpha \frac{m(p^2 - k^2 \cos^2 \theta)}{4pD \sin \theta} \left[N_+ Y_+ + N_- Y_- + 2 \cos \theta \left[(p^2 + 5k^2) \ln \frac{p-k}{p+k} + 2kp \sin^2 \theta \right] \right] \quad (2)$$

where

$$D = k^4 \cos^4 \theta + 2k^2 p^2 \cos^2 \theta - 2k^4 \cos^2 \theta - p^4 - 2k^2 p^2 + 2k^4,$$

$$N_{\pm} = \pm [(p^2 + 2k^2) \cos^2 \theta + 3k^2 \pm 6kp \cos \theta].$$

The center-of-mass variables and Y_{\pm} are same as above. We have not found reference to such an expression in the recent literature. Apparently there is no great enthusiasm for performing such an obviously difficult experiment to test this order- α_{QED} effect. Also, the small QED coupling constant makes the polarization much smaller in the case of QCD.

To estimate the polarization from the full QCD formula at the subprocess level one needs quark masses and α_s . What quark mass should be inserted—current, constituent, or perhaps a larger effective mass incorporating confinement effects—at energies of interest where the perturbative QCD is partially valid? It is an interesting point to mention that Eq. (1) can be written as a function of α_s , θ , and m_i/p , eliminating k by using the kinematic relation $p^2 = k^2 + m^2$. This shows that the properties of Eq. (1) can be discussed for any given quark mass and for a higher mass the same properties will appear at a higher momentum. Hence, for simplicity, we use constituent quark masses in our estimation. One can argue that the coupling constant at these energies may differ from that of usual perturbative QCD. We use a “reasonable,” constant value for α_s which is an overall factor in the expression.

For $\alpha_s = 0.4$ and constituent quark masses, 0.3, 0.5, 1.5, and 4.5 GeV/c^2 for up (or down), strange, charm, and bottom quarks, respectively, the polarization of the produced strange quark as a function of scattering angle (θ) for various values of gluon momenta (p) is shown in Fig. 2. The polarization vanishes at $\pi/2$ and the sign changes leaving the magnitude the same under $\theta \rightarrow \pi - \theta$ as one expects from constraints on helicity amplitudes under identical particle interchange. More interesting is the momentum structure of the polarization. For small momenta, polarization increases rapidly with p until the peak value is reached and then decreases slowly, a much slower rate than one would get from the leading-logarithmic approximation in m/E of Eq. (1). Also, the properties shown in Fig. 2 are distinctly different if one takes the

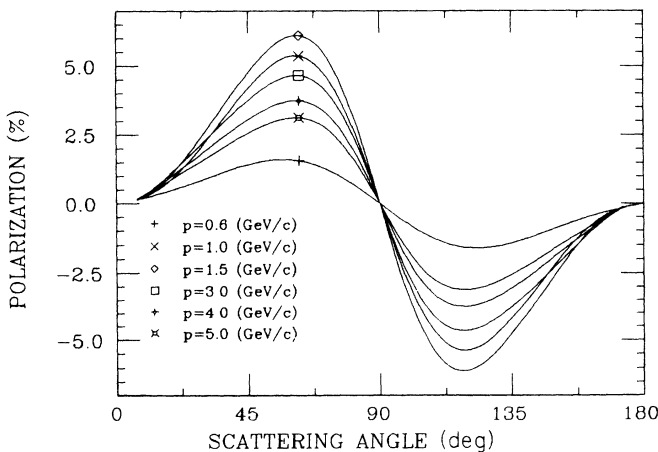


FIG. 2. Polarization of outgoing s quark in the subprocess center-of-mass frame as a function of the scattering angle with $\alpha_s = 0.4$, $m_u = m_d = 0.3 \text{ GeV}/c^2$, $m_s = 0.5 \text{ GeV}/c^2$, $m_c = 1.5 \text{ GeV}/c^2$, $m_b = 4.5 \text{ GeV}/c^2$.

leading-logarithmic approximation.

The properties discussed above for strange quark polarization remain the same for any flavor creation. A heavier quark will not necessarily give a larger polarization, but if the center-of-mass energy is well above the threshold for pair creation then the heaviest quark acquires the largest polarization. As an example, at $p = 13 \text{ GeV}/c$, which is the required momentum for largest b -quark polarization, the polarization increases with mass as shown in Fig. 3. Note that the magnitude of the b -quark polarization at the peak is approximately the same as that of s -quark polarization shown in Fig. 2. Hence, for heavier quarks the polarization is roughly the same except for the fact that the peak appears at a higher momentum as mentioned earlier.

It is an interesting point to mention, even though an individual Feynman diagram is not a gauge-invariant entity, that the dominant contribution to the polarization comes from one particular diagram, namely from the radiative corrections to the gluon propagator—“vacuum polarization.” This appears in the s channel and contributes to the imaginary parts of amplitudes in gluon fusion, in contrast to the gluon-propagator corrections in the t channel for flavor-excitation subprocesses^{6,7} with no contribution to the polarization. The existence of the diagram with an s -channel intermediate gluon, for the subprocess considered, is a result of the non-Abelian character of the theory which makes the polarization large and distinct from the analog QED expression [Eq. (2)].

Having obtained the subprocess polarization, the next step is to make a comparison with hyperon polarization. That needs the convolution with the initial-state hadron structure functions and a mechanism to produce hyperons from polarized strange quarks. Convolution is performed for initial-state protons by using the simplest (Q^2 independent) version of gluon distribution functions.⁹

The result obtained from Monte Carlo calculations for the polarization of strange quarks in the center-of-mass frame of protons, $P_{\text{c.m.}} = 14.0 \text{ GeV}/c$, equivalent to a beam of $400 \text{ GeV}/c$, is shown in Fig. 4. Note that the polariza-

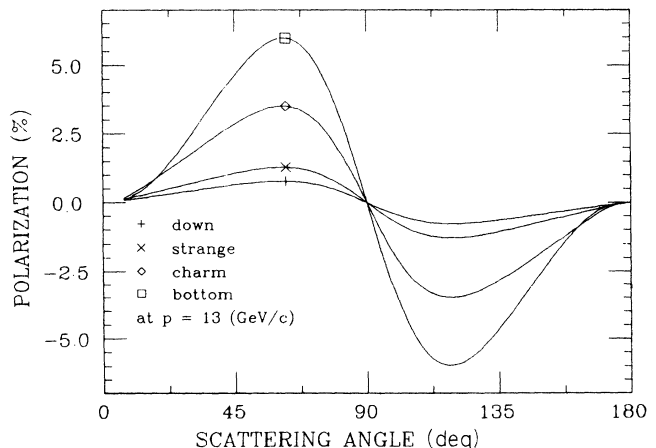


FIG. 3. Polarization of up, strange, charm, and bottom quarks at the subprocess c.m. momentum of $12 \text{ GeV}/c$. Parameters are identical to Fig. 2.

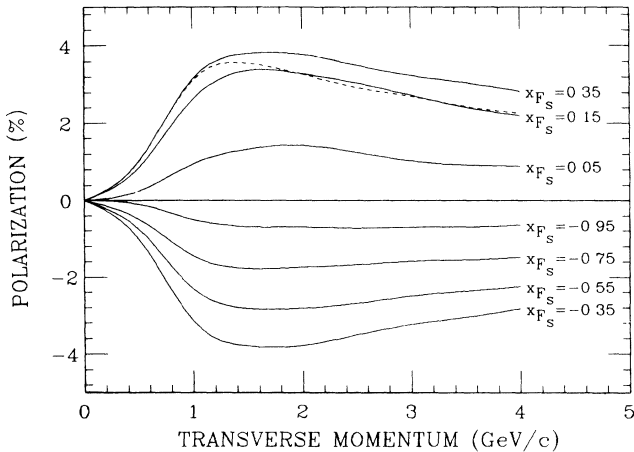


FIG. 4. Strange-quark polarization in the proton c.m. frame, $P_{c.m.} = 14$ GeV/c (400-GeV beam), after the convolution for the initial state gluons. x_F is the Feynman x for the strange quark. Dashed curve corresponds to $P_{c.m.} = 30.6$ GeV/c.

tion is given as a function of s -quark transverse momentum p_T at various values of Feynman x defined for the strange quark, x_{F_s} , the polarization reaches a peak, at $p_T \approx 1-1.5$ GeV/c, and then starts to drop very slowly. Only two regions of the full x_{F_s} scale are displayed in the figure, but the whole region can be read by using the symmetry, namely by changing the sign when going from the forward to the backward hemisphere. In the forward hemisphere, the polarization is positive and increases with x_{F_s} until $x_{F_s} \approx 0.25-0.35$ and then decreases. The difference in the backward hemisphere is only the sign. The dashed curve shows the polarization of the strange quark at the peak $x_{F_s} = 0.35$ for a beam of momentum 2000 GeV/c ($P_{c.m.} \approx 30.6$ GeV/c). There is no large energy dependence even at the quark level, which is very encouraging.

It is very important to realize that the nonlogarithmic terms in Eq. (1) are mainly responsible for the above properties, especially for the sign and for the p_T dependence (the plateau) of the polarization. Therefore, for the intermediate region of p_T ($1 < p_T < 5$ GeV/c) where the quark mass is significant, it is crucial to use the full expression for the polarization rather than its leading-logarithmic approximation. As another very interesting consequence of these calculations we note that the charm-quark polarization from gluon fusion in the protons' center-of-mass frame has very similar properties to the strange quark for the above values of parameters. The magnitude of the charmed-quark polarization at the peak is slightly larger than that of the strange quark. Also, the polarization reaches the peak at a larger p_T (~ 4 GeV/c) and then starts to drop very slowly similar to the s -quark polarization.

All features discussed about the s -quark polarization are interesting and some of them can be matched with that of hyperons if the s quark with correct x_F is combined with the appropriate ud diquark. This suggests constructing a model for Λ production from the s quark based on its polarization. This work has begun, the results are

very encouraging, and a rough fit to data has been done.

According to SU(6) wave functions, the s quark has to be combined with a ud diquark in a zero-spin state to produce Λ . Hence, the polarization of Λ has to be associated with that of the s quark. If the polarized s quark is picked up by the ud diquark (valence) from the beam, then it is reasonable to assume that the transverse momentum of Λ is predominantly coming from that of the s quark. This gives the required p_T dependence. One might expect the strange quark produced from gluon fusion to dominate at smaller x_F , since gluons are highly concentrated at smaller x_{Bj} , and the valence diquark to carry a larger fraction ($\sim \frac{2}{3}$) of the proton's longitudinal momentum. With this picture, we argue that a hyperon produced in the forward hemisphere, at least for small x_F , is a recombination of a forward diquark with a backward s quark. This gives the correct sign for Λ polarization, but the explicit relation between x_{F_s} and x_F remains to be determined. An enhancement in magnitude is still needed. However, the recombination effects have not been incorporated and at the present it is not clear how these effects could increase the polarization. Perhaps, since the s quark has to be accelerated to join with the forward diquark, the polarization increases as suggested by the "Thomas-precession model."³

To approximate such recombination effects assume a linear relation of the form $x_F = a + bx_{F_s}$, and multiply the overall expression by a constant factor (A). We have compared our result for the polarization of Λ produced by 400-GeV protons on a proton target with the data^{10,11} for a beryllium target, neglecting the slight target dependence.¹¹ The results for $A = 6.3$, $a = 0.86$, and $b = 0.7$ are shown in Fig. 5. It should be pointed out that the parameters are adjusted only to fit data at $x_F = 0.3$ and curves for the other x_F values are plotted by using those parameters. Note that the data for 12-GeV protons on the Be target¹² are also included in the figure since the polarization is approximately independent of the beam energy. Of course, the values of a and b are not suitable for the whole region

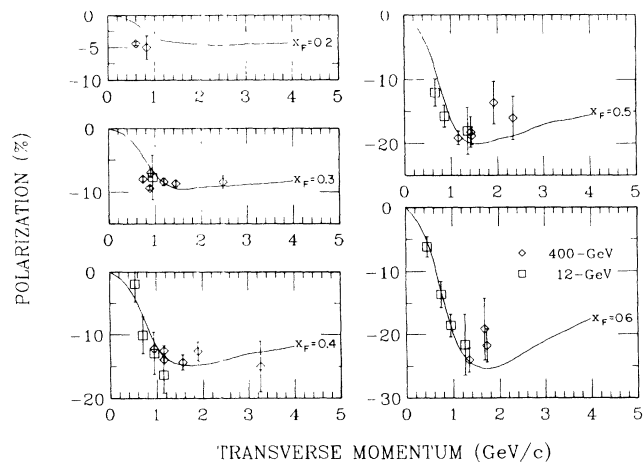


FIG. 5. Rough fit with the experimental data for Λ polarization on beryllium target. The data at 400 GeV (Refs. 10 and 11) and at 12 GeV (Ref. 12) are shown.

of x_F , unless there is some mechanism to cut off some regions of x_F . However, the result does not explain the data remarkably well. In particular, note that the striking kinematic dependence of the data is reproduced by the model.

Now recombination models in hadronic processes¹³ have been studied for many years. However, the effects of spin have not been included in these investigations. Surely the recombination (with spin incorporated) has to be done in a more systematic way, and the effect of other subprocesses has to be considered in the convolution. Nevertheless, based on the very encouraging results we have ob-

tained, we claim that the gluon fusion can be taken as a serious candidate for the seed of polarization.

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