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How to see the differences between quark and gluon jets

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Using hadronic jets in electron-positron annihilation, we suggest a simple and model-independent method to see the differences between quark and gluon jets. We define and analyze special energy-dependent moments of jets and choose those which are the most characteristic of the jet type. The main advantages of our method are that it handles the energy of a jet in an adequate way and studies a large part of the two- and three-jet events without strong cuts, thus reaching very high statistics. We clarify that the lack of these two features is the reason why studying this problem by some other methods led to inconclusive results.

The fundamental particles used in modern particle physics, particularly in QCD, are the quarks and gluons. On the other hand, nobody has seen any free quark or gluon. Our experiments produce hadrons. The detailed properties of hadronization processes are unknown. Nevertheless, we have a few, more or less successful phenomenological fragmentation models. Hadrons forming a jet are believed to mark the path of a quark or a gluon. This is the reason why we usually say that jets are the footprints of partons. To determine the momentum and the energy of a parton we use cluster algorithms and identify some properties of the cluster with those of the parton. There has been remarkable success in correlating the data for two- or three-jet variables with theoretical predictions based on partons. The next step to having a deeper insight into QCD is to see less inclusive processes. Thus we should determine the parton ancestor of a jet. At this point the most important question is to see the differences between the jets produced by a spin-half quark and a spin-one gluon.

It is even more important to see the differences between quark and gluon jets if we take into consideration that theory has definite predictions for these differences. Gluons carry a stronger color charge than quarks, and one expects this to produce a difference in their fragmentation, namely higher multiplicity, softer hadron spectrum, and broader p_t spectrum. Thus lattice gauge calculations show a gluon string tension $\frac{8}{3}$ times the corresponding quark string tension.[1] Due to the larger color charge, QCD predicts $r=\frac{9}{4}$ times larger multiplicity for gluon jets than for quark jets at the same infinite-limit energy range.[2] This ratio is valid for parton multiplicities. Second-order calculations, finite-energy corrections, and heavy-quark effects are intensively studied.[3] These corrections at present energies reduce the value of r to approximately 1.3. On the other hand, there are several theoretical papers to give methods by which the experimental data may be analyzed and the above-mentioned differences between quark and gluon jets can be seen.[4]

A large amount of data exists on fragmentation of quark and antiquark jets. Far less is known experimentally on fragmentation of high-energy gluon jets. The experimental problem in these studies is that in e^+e^- annihilation quark jets occur predominantly in the two-jet ($q\bar{q}$) topology, but gluon jets occur only in three- or more-jet topologies ($e^+e^- \to q\bar{q}g$, $\Upsilon \to ggg$). There is thus a large kinematic difference superimposed on any dynamical difference. In $p\bar{p}$ collisions two-jet topologies are a mixture of $q\bar{q}$, qg, and gg events. However, due to the parton density function most of the jets are gluon jets. The differences between quark and gluon jets are intensively studied in both processes.

Because of the simplicity of the initial state and the parton-level processes, a large part of our experimental knowledge on gluon-jet fragmentation comes from $e^+e^- \to q\bar{q}g$. However, most studies of gluon-jet fragmentation led to inconclusive results.

The High Resolution Spectrometer (HRS) Collaboration has studied $e^+e^- \to q\bar{q}g$ at $W=29$ GeV.[5] They looked at nearly threefold symmetric events, selected by $110° < \phi_{ij} < 140°$ and $C_3 > 1.10$ for all jet pairs. (C_3 is the generalized sphericity.[6]) They have studied the multiplicities of gluon and quark jets. They have found for the ratio: $\langle n(g)\rangle/\langle n(q)\rangle = 1.29^{+0.20}_{-0.41}$. The Mark II Collaboration[7] has studied inclusive charged-particle distributions in terms of the fractional momentum $x_p = p_i/E_j$, where p_i is the momentum of the charged particle i and E_j is the energy of the jet to which it is assigned. They have taken advantage of the high statistics available to them in $e^+e^- \to$ hadrons. They have compared nearly symmetric three-jet events at 29 GeV and two-jet events at 19.2 GeV. Their study apparently favors a softer gluon fragmentation. On the other hand, the TASSO Collaboration[8] has repeated these studies at 35 GeV for symmetric three-jet and at 22 GeV for two-jet events. The result of the TASSO Collaboration does not confirm the above-mentioned Mark II result. The JADE Collaboration[9] has reported that particles coming from the least energetic jet (the most probable to be a gluon jet) in three-jet events tend to carry on the average larger p_t. In the studies of the CELLO Collaboration,[10] where they have compared the least energetic jet in three-jet events at $W=35$ GeV to two-jet events at $W=14$ GeV (thus they have compared a gluon-enriched sample of jets to a quark-enriched sample), they report $\langle p_{t3}(35)\rangle/\langle p_t(14)\rangle = 1.03 \pm 0.03 \pm 0.04$, not seeing any increase in p_t for gluon jets. Recently, the AMY Collaboration[11] at the KEK collider TRISTAN, using uncorrected data, has reported a significant difference in the rapidity of the leading particle between the two fastest jets and the slowest jet in their three-jet sample. However, the value of the rapidity for leading particles with high longitudinal momentum is strongly dependent on the jet axis determination.[12]

The study of fractional momentum distributions in symmetric three-jet and two-jet events is a very promising possibility to see the differences between quark and gluon jets, because in symmetric three-jet events where the jets are of nearly equal energy, a mistake in assigning a soft particle to a particular jet is irrelevant. Nevertheless, in order to have reasonable statistics, one has to study nearly symmetric three-jet events in a large angular range, e.g., $100° < \phi_{ij} < 140°$ for all jet pairs. (This is the choice of the TASSO Collaboration.[8]) It is easy to show that in this sample there are kinematic configurations where the energy of the most energetic jet is more than 40% higher than that of the less energetic jet. Thus the average energy of the gluon jets (the most probable to be a jet with the smallest energy) is far from being equal with the average energy of the quark jets. The kinematical differences are superimposed on the dynamical ones even in these type of symmetric-three-jet-events studies. Another serious problem is the statistics. The nearly symmetric three-jet selection criteria has reduced the multihadronic TASSO events from 45 852 to 396.

Thus, there is a basic necessity to reach better statistics and to take the energy of the jets into consideration. The main result of our work is that it shows how to do this.

The purpose of this paper is to introduce a method capable of seeing how an energy-dependent jet variable $A_{q/g}(E_j)$ (e.g., multiplicity, average p_t, rapidity, etc.) differs between quark (q) and gluon (g) jets at a given jet energy (E_j), studying two-jet and nonsymmetric three-jet events in e^+e^- annihilation.

For jets in three-jet events with different energies, a mistake in assigning a soft particle to a jet is no longer irrelevant. We have to study quantities in which the contributions of the "problematic" soft particles are suppressed. In this sense the $\sum p_t$ is a good variable, while the multiplicity is not. Second, studies of quantities such as jet-particle multiplicity, rapidity, or p_t with respect to the jet axis are much more dependent on the reconstructed jet direction than, e.g., the study of fractional momentum distributions in symmetric three-jet events. Thus, we will take care of the determination of the number of jets and their axis and one shall use an identical cluster algorithm for quark jets and gluon jets (analyzing two- and three-jet events, respectively).

The determination of the number, the energies and the directions of the jets can be done by using cluster algorithms (e.g., the d_{join} algorithm in the standard LUND 6.3 Monte Carlo program[13]). For simplicity, first we suppose that our cluster algorithm identifies n-parton events as n-jet events. Of course the jet number, as determined by the cluster algorithm, does not always coincide with the parton number. Events where it does not are called background. (Effects due to background will be studied later.)

First, one has to determine the energy dependence of the chosen variable for quark jets $[A_q(E_j)]$. This can be done by analyzing two-jet events at different jet energies (e.g., 22, 29, 35, 44, and 57 GeV). The jet energies in this case are half of the c.m. energies. $A_q(E_j)$ is simply the average of the jet variable:

$$A_q(E_j) = \frac{1}{N_2(E_j)} \sum_{i=1}^{N_2(E_j)} A^{(i)}(E_j) \,, \qquad D_q(E_j) = \frac{\sigma_q}{\sqrt{N_2(E_j)}} \,, \tag{1}$$

where $N_2(E_j) \gg 1$ is the number of jets in our two-jet sample at E_j jet energy, the summation runs over the jets in our sample, $\sigma_q^2(E_j)$ is the variance of the $A_q(E_j)$ variable, and $D_q(E_j)$ is the statistical error of our determination. $A^{(i)}(E_j)$ is the actual value of the variable for the ith jet having an energy E_j. To give the function of $A_q(E_j)$ for the energies between these fixed energies an interpolation must be used.[5]

If we know $A_q(E_j)$ it is easy to give $A_g(E_j)$. Let us denote $p_g(x_a | x_b, x_c)$ the probability that a jet (in a three-jet event) with a given $x_a = 2E_a/E_{\text{c.m.}}$ energy fraction is a gluon jet (the two other jets have x_b and x_c energy fractions). For instance, in the first order of α_s this probability is clearly

$$p_g(x_a | x_b, x_c) = \frac{x_b^2 + x_c^2}{(1-x_b)(1-x_c)} \left[\frac{x_a^2 + x_b^2}{(1-x_a)(1-x_b)} + \frac{x_b^2 + x_c^2}{(1-x_b)(1-x_c)} + \frac{x_c^2 + x_a^2}{(1-x_c)(1-x_a)} \right]^{-1} . \tag{2}$$

(The second-order formula is available as a FORTRAN code in the e^+e^- LUND Monte Carlo program.) The probability that this jet is a quark or an antiquark jet is

$$p_q(x_a | x_b, x_c) = 1 - p_g(x_a | x_b, x_c) \,. \tag{3}$$

We distribute the jets into different samples according to their jet energy. For these mixed quark-gluon samples we calculate the averages of the jet variable

$$A_{q+g}(E_j) = \frac{1}{N(E_j)} \sum_{i=1}^{N(E_j)} A^{(i)}(E_j) \,, \tag{4}$$

where $N(E_j)$ is the number of the jets in the sample containing jets with E_j jet energy. In these samples the expectation values for the number of gluons and quarks are

$$N_g(E_j) = \sum_{i=1}^{N(E_j)} p_g^{(i)} \,, \quad N_q = N(E_j) - N_g(E_j) \,, \tag{5}$$

where $p_g^{(i)}$ is the probability for the ith jet to be a gluon jet, given by the second-order form of Eq. (2). Using the value of $A_q(E_j)$ it is possible to give $A_g(E_j)$ and its variance $\sigma_g^2(E_j)$

$$A_g(E_j)=\frac{1}{N_g(E_j)}[N(E_j)A_{q+g}(E_j)-N_q(E_j)A_q(E_j)]\,,$$
$$\sigma_g^2(E_j)=\frac{1}{N_g(E_j)}\left\{\sum_{i=1}^{N(E_j)}A^{(i)2}(E_j)-N_q(E_j)[A_q^2(E_j)+\sigma_q^2(E_j)]\right\}-A_g^2(E_j)\,. \tag{6}$$

Determining both $A_q(E_j)$ and $A_g(E_j)$ the differences between quark and gluon jets can be seen as a function of the jet energy. The square of the statistical error of the determination of $A_g(E_j)$ is

$$D_g^2=\frac{1}{Np_g^2}\left[p_g\sigma_g^2+(1-p_g)\sigma_q^2+(1-p_g)^2N\frac{\sigma_q^2}{N_2}\right]. \tag{7}$$

(For simplicity we have not indicated the energy dependence.) Equation (7) is valid in the approximation where the $p^{(i)}$ probabilities are constant. For example, for symmetric three-jet events $p\approx\frac{1}{3}$.

To handle the large angle range and the kinematical problems mentioned above, this method should be used even in the nearly-symmetric-three-jet-event studies.

In the following we will apply the method outlined above to determine special moments of quark and gluon jets. In our studies we have used the JETSET 6.3 LUND Monte Carlo program[13] in the form of second-order matrix elements, for finite jet-resolution parameters $y_{\min}$. Therefore in our treatment all partons are well separated so the fixed-order perturbative results are applicable in this region of the phase space. We have changed the QCD part of JETSET 6.3 to get rid of the Gutbrod-Kramer-Schierholz[14] approximation, by the method of the Mark J Collaboration.[15] The string fragmentation model has been used. We have analyzed the moments

$$M_{nm}(E_j)=\sum\left(\frac{p_t}{E_j}\right)^n\eta^m \tag{8}$$

at the previously mentioned 22, 29, 35, 44, and 57 GeV c.m. energies. 100000 hadronic events have been analyzed at each beam energy. The sum in (8) goes over the outgoing particles in a jet (η is the pseudorapidity, p_t is the transverse momentum of a definite particle, with respect to the jet axis). Similar moments were studied in Refs. 16–18. It is easy to see that M_{00} is the multiplicity of the jet. Clearly these moments are sensitive to the softer fragmentation, higher multiplicity, and broader p_t of the gluon jets as compared to the quark jets. There is a small dependence on the jet-finding algorithm parameter. However, varying it in wide ranges, the changes in M_{nm} are an order of magnitude smaller than the differences between the moments for quark and gluon jets. We have used the d_{join} algorithm of the standard LUND Monte Carlo program with the same d_{join} value for both the two- and three-jet events. We have chosen d_{join}, the parameter of the jet-finding algorithm, in such a way that we have approximately the same amount of background for two-jet events (from three- and four-parton events) and for three-jet events (from two- and four-parton events). Increasing the c.m. energy, one has to increase d_{join}. Thus, e.g., for $E_{\text{c.m.}}=29$ GeV, $d_{\text{join}}=1.7$ GeV; for $E_{\text{c.m.}}=44$ GeV, $d_{\text{join}}=1.9$ GeV; and for $E_{\text{c.m.}}=57$ GeV, $d_{\text{join}}=2.15$ GeV. Using these two- and three-jet events we have determined the M_{nm} moments for quark and gluon jets according to Eqs. (1) and (6), respectively.

To avoid the dangerous phase-space limit in three-jet events, where our fixed-order perturbative calculation is not valid, we have studied only those events where all the x_i energy fractions of the hadronic jets were smaller than 0.96. This cut reduces the background from two-jet events to a few percent depending on the c.m. energy. The small background does not affect the values of M_{nm}. The differences between the results gained from samples with few-percent two-jet background and without background are negligible. We have analyzed the sensitivity of our method to the background. Artificially enriching our sample with two-jet background events, we have seen that backgrounds above ~25% (which never occur at our energies and x_i cut) wash out the difference between the quark and gluon jets. We have studied the moments M_{nm}

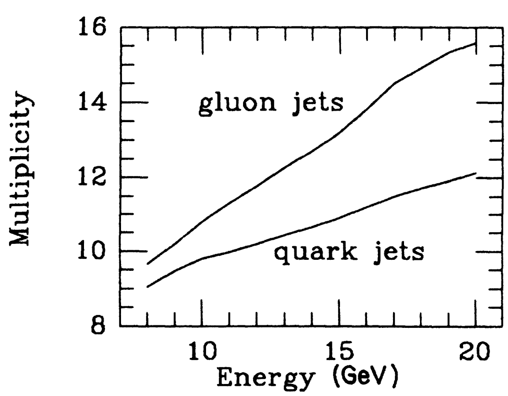

FIG. 1. Dependence of mean multiplicities (M_{00}) of quark and gluon jets on the jet energy.

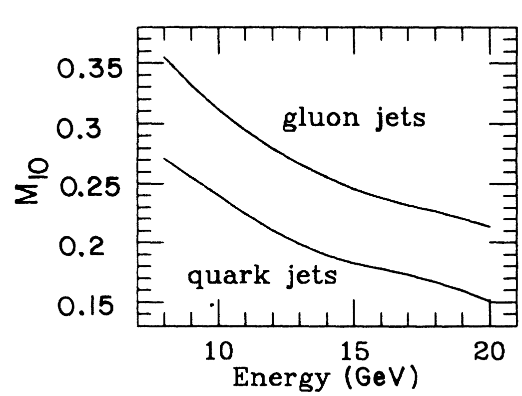

FIG. 2. Dependence of the average M_{10} value of quark and gluon jets on the jet energy.

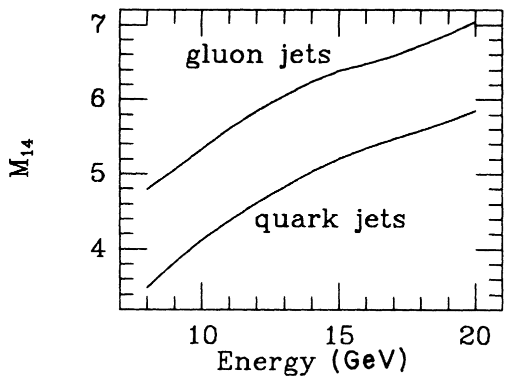

FIG. 3. Dependence of the average M_{14} value of quark and gluon jets on the jet energy.

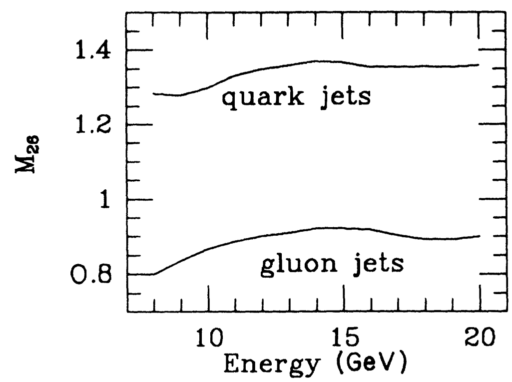

FIG. 4. Dependence of the average M_{26} value of quark and gluon jets on the jet energy.

(where $n,m=0,\ldots,10$) and concluded that M_{10}, M_{13}, M_{14}, M_{15}, M_{26}, and M_{27} are the most sensitive to the jet type. The jet-energy dependence of the average values of some of these moments (M_{10}, M_{14}, and M_{26}) as well as the jet-energy dependence of the mean jet multiplicity are shown in Figs. 1–4 for gluon and quark jets, respectively.

σ_q and σ_g (the square roots of the variances of the moments) have the same order of magnitude as the differences of the moments between quark and gluon jets:

$$|A_q(E_{\text{jet}})-A_g(E_{\text{jet}})|\sim\frac{\sigma_g+\sigma_q}{2}\,. \tag{9}$$

The best variable in our model calculation is M_{14}. For this moment the difference is 1.4 times larger than $(\sigma_g+\sigma_q)/2$ for 20-GeV jets. For the multiplicity this ratio is only 0.9. These very large variances [Eqs. (1) and (6)] give large statistical errors [Eq. (7)]. Thus for a few hundred, kinematically not clear, three-jet events (e.g., symmetric-three-jet-event studies in Ref. 8), there is no chance to have a conclusive result.

However, since the majority of the events are useful with the present method, the high statistics and the combined use of our moments may lead to much better results. The possibility of quark-gluon discrimination using these moments has been studied in Ref. 18, where a multidimensional discrimination analysis based on the LUND-model M_{nm} moment determination has given a 73% discrimination efficiency.

In this paper we have suggested a simple method to see the differences between quark and gluon jets. The full method is model independent in the sense that all the M_{nm} moments, or any other jet variable, should be determined from the experimental data. The differences between quark- and gluon-jet variables arise from the perturbative QCD prediction of Eq. (2) (more precisely its second-order version). We have tested our method to calculate special moments of quark and gluon jets in the LUND model. Remarkable differences can be seen between them. In this model calculation the square roots of the variances of the moments have the same order of magnitude as the difference between the moments for quark and gluon jets. Comparing our method to other methods (e.g., symmetric-three-jet-event studies), we can say that our method compares gluon jets with quark jets having exactly the same energy; thus there is no kinematical difference superimposed on the dynamical ones. The other advantage of our method is that it reaches high statistics. The statistics have an extremely important role because we want to see the difference between two quantities having very large variances.

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