Nonperturbative self-consistent unitary loop corrections to Skyrmion masses

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Quantum chromodynamics (QCD) effectively reduces to a nonlinear meson theory in the color-number $N_c \to \infty$ limit. The nucleon and Δ masses arising as solitons from such a theory usually turn out to be too large if phenomenological values are used for the meson parameters. Within a static-baryon approximation, however, we find that soft nonperturbative self-consistent unitary hadron-loop corrections (corresponding to quark-loop higher-$1/N_c$ orders in QCD) can lower these masses by the kind of magnitude needed to make them consistent with experiment, whereas a first-order $1/N_c$ correction is much smaller.

In 1960 Skyrme proposed a theory in which baryons appear as finite-energy soliton solutions of a nonlinear meson field theory, with an extra term involving multiple derivatives of the meson field to guarantee classical stability.[1] More recently, Witten pointed out that an effective meson theory of this type may be a good approximation to quantum chromodynamics (QCD) when the number of colors, N_c, becomes large.[2] With parameters taken from low-energy meson physics, however, one usually finds baryon/meson mass ratios which are too large compared with the experimental values;[3,4] a similar situation prevails for quenched quark-loop lattice calculations.[5]

Part of this discrepancy may arise from short-range gluon-exchange[4] effects. We shall see, however, that an important contribution can also come from soft (long-range) hadron loop diagrams. These effects, which arise from sea-quark loops in the underlying QCD theory, go to zero in the $N_c \to \infty$ limit; this is in fact the usual argument for neglecting them.[2] In Fig. 1, for example, the gluon loop (a) has a factor N_c, which is absent for the corresponding quark loop (b). But, on the other hand, (b) has a flavor-number factor N_f, which is absent for (a). The quark-loop suppression is therefore actually governed, not by $1/N_c$, but by N_f/N_c (Ref. 6). This is hardly small with $N_f \sim 2-3$ and $N_c = 3$, and suggests that a "perturbative" or iterative treatment of loops may give misleading results. This is true even if we have relatively narrow resonance widths; calculations show that, contrary to popular belief, such widths can be readily obtained even with fairly large loop corrections. We shall therefore use a self-consistent nonperturbative approach for dealing with loops. We shall nevertheless see, however, that, in a certain sense, the small-$1/N_c$ approximation continues to be valid and useful as an "input" for our calculation.

If we do take quark (q) loops into account we have,

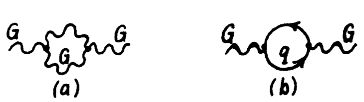

FIG. 1. (a) Gluon-loop and quark-loop contribution to the gluon (G) propagator.

e.g., the pion-nucleon $\pi N \to \pi N$ and $\pi N \to N\pi$ quark-loop sums of Fig. 2; we can also have $\bar{q} \to qq$ diquark loops and "cross terms" linking up alternating (a) and (b) subsums. These sums give, in turn, hadron-loop generalized infinite-ladder sums T of the form of Fig. 3, where the upper and lower "ladder" exchanges $A, B, \ldots$ should themselves be T sums.[6]

The low-mass ($\leq \hat{m}$) contributions L to the vertical-line exchanges $(a, \ldots), (b, \ldots), (b', \ldots), \ldots$ of Fig. 3 are related through crossing symmetry and self-consistency to the mass spectrum in the Mandelstam s channel (where $\sqrt{s}$ is the energy) which arises when we sum Fig. 3 or some similar set of graphs. They therefore implicitly take into account the quark loops of Fig. 3. The high-mass ($> \hat{m}$) contributions H to $(a, \ldots), \ldots$, on the other hand, should not include quark loops, since this would lead to double counting with Fig. 3(b), which explicitly contains such loops; similarly for $(b, \ldots), (b', \ldots), \ldots$ and the higher graphs of Fig. 3. Here $\hat{m}$ is the effective threshold above which Fig. 3(b) begins to give important mass-exchange contributions. [Actually we can always take a higher value of $\hat{m}$, but we must then also remove the corresponding low-mass ($\leq \hat{m}$) contribution from Fig. 3(b); similarly for $(b, \ldots), \ldots$ and the higher graphs of Fig. 3.]

In practice we can approximate H by its high-t Regge behavior

$$\operatorname{Im} H(s,t) = b_0(s)(t+\xi)^{\alpha_0(s)}\theta(t-\hat{m}^2) , \tag{1}$$

where ξ is independent of the Mandelstam momentum-

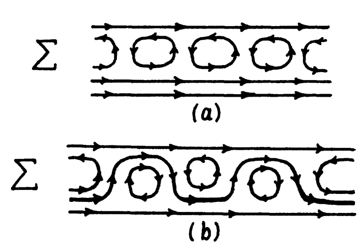

FIG. 2. Quark-loop contributions to πN scattering. The lines represent quarks and it is understood that gluon lines (not shown explicitly) must be added in.

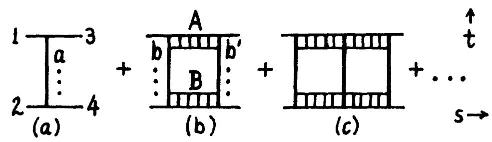

FIG. 3. Hadron-loop generalized infinite-ladder sums for the process $12 \to 34$ arising from quark-loop contributions such as Fig. 2. The lines represent hadrons, and $\sqrt{s}$ is the energy of 12 or 34 in the s channel.

transfer variable t, and $\alpha_0(s)$ is the leading Regge trajectory interpolating the s-channel mass spectrum in the absence of the internal quark loops of Fig. 2. In particular, the lowest state lying on the $\alpha_0(s)$ trajectory should then be the corresponding lowest classical Skyrme baryon, since this is what arises as a soliton from our effective tree-graph meson theory without any loop corrections involving baryons.

If we now make a static nucleon-mass $m_N \gg m_\pi, \omega$ approximation,[7] we find that Eq. (1) gives, when inserted into a fixed-s dispersion relation in t, a contribution

$$\phi h(\omega',\omega'',\omega)=\gamma_0(\hat{\omega}+\omega_0)/(\omega_0-\omega)(\hat{\omega}+\omega'+\omega''-\omega) \tag{2}$$

to the $\pi N \to N\pi$ off-shell P-wave amplitude $f(\omega',\omega'',\omega)$, at least if we drop nonpole and higher-ω pole contributions; Eq. (2) reduces to $h(\omega,\omega,\omega)=h(\omega)$ and f to $f(\omega,\omega,\omega)=f(\omega)=e^{i\delta}\sin\delta/q^3$ on shell, where δ is the (real) phase shift in the elastic-scattering region, $\omega=\sqrt{s}-m_N$, $q^2=\omega^2-m_\pi^2$, $\hat{\omega}=\hat{m}-m_N$, (ω_0+m_N) is the mass of our soliton, which is associated with the $\omega=\omega_0$ pole in Eq. (2), and γ_0 is related to b_0/α_0' at the same energy. The last denominator factor in Eq. (2) arises from the $t=\hat{m}^2$ threshold in Eq. (1). The low-mass N and $\Delta[=\Delta(1232)]L$ exchanges in $(a,\ldots)$, on the other hand, give a contribution to f of[7]

$$\phi\lambda=\gamma_N^x/(\omega_N+\omega'+\omega''-\omega)+\gamma_\Delta^x/(\omega_\Delta+\omega'+\omega''-\omega), \tag{3}$$

where $\omega_N=0$. Since N exchange dominates for isospin $(I)=\text{spin}(J)=\frac{3}{2}$, and Δ exchange for $I=J=\frac{1}{2}$, we will approximate Eq. (3) by

$$\phi\lambda(\omega',\omega'',\omega)\simeq\gamma^x/(\omega_x+\omega'+\omega''-\omega)\ , \tag{4}$$

where $\gamma^x=\gamma_N^x+\gamma_\Delta^x$ and $\omega_x\simeq\omega_N$ for $I=J=\frac{3}{2}$ and $\omega_x\simeq\omega_\Delta$ for $I=J=\frac{1}{2}$. We have dropped all meson exchange, which has been estimated to give a small contribution for low ω (Ref. 8).

Figure 3 now gives the sum

$$f(\omega',\omega'',\omega)=\phi[\lambda(\omega',\omega'',\omega)+h(\omega',\omega'',\omega)] +\phi^2B(\omega',\omega'',\omega)+\cdots\ , \tag{5}$$

where[7]

$$\pi B(\omega',\omega'',\omega)=\int_{m_\pi}^{\Lambda}d\omega'''q'''^3[\lambda(\omega',\omega''',\omega)+h(\omega',\omega''',\omega)] \times[\lambda(\omega''',\omega'',\omega)+h(\omega''',\omega'',\omega)]/(\omega'''-\omega) \tag{6}$$

and where we have approximated the ladder exchanges $A,B,\ldots$ by simple N and π exchanges in Fig. 3(b), ...; we have introduced a sharp cutoff at $\omega'''=\Lambda$ to (roughly) take into account the Regge nature of the original $A,B,\ldots$ and the L-meson exchanges which we dropped in $(a,\ldots),(b,\ldots),\ldots$. These mesons can give contributions to Fig. 3(b) with fairly low thresholds. With our no-double-counting prescription, this is turn means that we must take

$$\hat{m}^2=(\omega_x+m_N)^2+1/2\alpha_x'\ , \tag{7}$$

which is half-way between our exchanged-state in Eq. (3) and the next state on the (approximately linear) Regge trajectory $\alpha_x(t)$ on which it lies.[6]

If we treat Eq. (5) as an expansion in the coupling-strength parameter ϕ that we are associating with each of the exchanges $(a,\ldots),(b,\ldots),\ldots$ in Fig. 3, and form its [1,1] Padé approximant, we obtain

$$f(\omega',\omega'',\omega)=\phi[\lambda(\omega',\omega'',\omega)+h(\omega',\omega'',\omega)]/\{1-\phi B(\omega',\omega'',\omega)/[\lambda(\omega',\omega'',\omega)+h(\omega',\omega'',\omega)]\}\ , \tag{8}$$

which is constructed so as to reproduce Eq. (5) up to order ϕ^2, if expanded in ϕ. Equation (8) satisfies elastic unitarity exactly below $\omega=\Lambda$ for $\omega'=\omega''=\omega$, and in fact reduces exactly to Eq. (5) for factorizable models. (See Appendix A.)

If γ_0/γ^x is small and $\omega_0 \gg \omega_x, \hat{\omega}$, as we shall confirm later, we can set

$$h(\omega,\omega''',\omega)/\lambda(\omega,\omega''',\omega)\simeq h(\omega)/\lambda(\omega) \tag{9}$$

within the integral of Eq. (6) for $\omega'=\omega''=\omega$, since the only region where h is then important is $\omega\approx\omega_0$, $\omega'''\gg\omega_x,\hat{\omega}$, where Eq. (9) should be reasonable. Indeed in Appendix A we argue that approximate h/λ universality may be valid more generally. Equation (8) then gives

$$f(\omega)=\phi\lambda(\omega)/d(\omega)\ , \tag{10}$$

where

$$d(\omega)=1-h(\omega)/[\lambda(\omega)+h(\omega)]-\phi I(\omega)/\lambda(\omega) \tag{11}$$

and

$$\pi I(\omega)=\int_{m_\pi}^{\Lambda}d\omega'''q'''^3[\lambda(\omega''')]^2/(\omega'''-\omega)\ . \tag{12}$$

A resonance or bound-state pole will then occur at $\omega=\omega_r$ if

$$d(\omega_r)=0\ , \tag{13}$$

since we then have $f(\omega)\simeq\gamma/(\omega_r-\omega)$ nearby, with coupling residue

$$\gamma = -\phi\lambda(\omega_r)/d'(\omega_r) \ . \tag{14}$$

Crossing symmetry then relates this to the γ^x of Eq. (4) through

$$\gamma^x_{IJ} = \sum_{I'} \sum_{J'} \alpha_{II'}\beta_{JJ'}\gamma_{I'J'} \ , \tag{15}$$

where the sums $\sum$ are over $I' = \frac{1}{2}, \frac{3}{2}$, and $J' = \frac{1}{2}, \frac{3}{2}$, and

$$\alpha = \beta = \begin{bmatrix} -\frac{1}{3} & \frac{4}{3} \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix} \ . \tag{16}$$

From a basic point of view, γ_0 and ω_0 would now be an input and the above equations would then be used to calculate γ and ω_r. But we can equally well do the reverse.

We next assume that, at least for low ω, $I(\omega) \approx$const. This should work even better if higher-energy inelastic and meson-exchange effects are added to Eqs. (6) and (12), and gives $\gamma = \gamma^x$ in the $h \to 0$ limit, a result which also follows from superconvergence.[8] (See Appendix B.) Since we still want $\gamma = \gamma^x$ when $h \neq 0$, as required by the experimental πNN and $\pi N\Delta$ couplings, as well as by superconvergence,[8] we must also require

$$\frac{d}{d\omega}\left[\frac{h(\omega)}{\lambda(\omega)+h(\omega)}\right]_{\omega=\omega_r} = 0 \tag{17}$$

in Eq. (11). This gives

$$\omega_0 = 2(\hat{\omega}+\omega_r)/(1+\gamma_0/\gamma) - \hat{\omega} \ . \tag{18}$$

Using Eq. (7), which gives $\hat{\omega} - \omega_x \simeq 1/4m_N\alpha'_x \simeq 2m_\pi$ in the static $m_N \gg \hat{\omega}, \omega_x$ approximation, we then have soliton masses $m_N + \omega_0^{2I,2J}$ with

$$\omega_0^{33} - \omega_0^{11} = \omega_\Delta = m_\Delta - m_N \tag{19}$$

and

$$\omega_0^{11} = (\omega_\Delta + \tfrac{1}{4}m_N\alpha'_x)(\gamma^x - \gamma_0)/(\gamma^x + \gamma_0) \ . \tag{20}$$

In the limit of small γ_0/γ^x, Eq. (20) gives $\omega_0^{11} = \omega_\Delta + 1/4m_N\alpha'_x$. If we take the $I = J = \frac{1}{2}$ Skyrme mass of 1493–1519 MeV of Lacombe *et al.*,[4] for example, we then obtain $m_N = 933$–959 MeV, compared with the experimental value of 940 MeV. [A first-order $1/N_c$ correction, where the above external and exchanged baryon masses are replaced by the corresponding uncorrected solitons, only gives $m_N = 1265$–1291 MeV, with $\alpha'_0 \simeq \alpha'_x$]. Our m_N is somewhat sensitive to γ_0/γ^x, however, so the detailed agreement should not be taken too seriously (see below). Using Eq. (19) we also find that the mass difference of Lacombe *et al.*[4] between the $I = J = \frac{3}{2}$ and $I = J = \frac{1}{2}$ Skyrmions gives $m_\Delta - m_N = 279$–293 MeV, independent of γ_0/γ^x; this again is in good agreement with experiment.

To actually determine γ_0/γ^x we use Eqs. (11)–(13). Since we are assuming $I(\omega) \simeq$const, we will evaluate Eq. (12) at $\omega = m_\pi$, where our omitted higher-energy effects are expected to be least important and where the integral simplifies considerably. From Eq. (13) for $I = J = \frac{1}{2}, \frac{3}{2}$, where $\omega_r = 0, \omega_\Delta$, we obtain $\Lambda = 10.4m_\pi$ and $\gamma_0/\gamma^x = 0.16$, which is indeed small ($\ll 1$), as assumed above. However, it is large enough to shift our calculated m_N to 1091–1117 MeV, although $m_\Delta - m_N$ is of course unchanged. The agreement with experiment for m_N is not quite as good as it was for $\gamma_0 \to 0$, but we still have a considerable improvement over the uncorrected Skyrme model. Moreover, inelastic effects would have the effect of lowering Λ, γ_0/γ^x, and m_N, thereby improving the latter further.

In conclusion, we find that, because of important feedback effects which are usually not taken into account in other approaches, self-consistent nonperturbative unitary loop corrections can give significant mass reductions for the Skyrmions which arise from a classically stabilized effective nonlinear meson field theory. It is interesting to note that we continue having an approximate solution even when $\gamma_0/\gamma^x = 0$ exactly. This would correspond to a situation where we do not have a classically stable soliton solution of our meson field theory, but have, instead, a "quantum stabilization" of the type proposed recently by Jain, Schechter, and Sorkin.[9] Because of our static $m_N \gg m_\pi$ approximation we are not actually able to calculate the m_π/m_N ratio, as we would with a more accurate relativistic calculation. But it should be possible to relate the off-shell amplitude of Eqs. (5) and (8) to the "profile function" F assumed by Jain, Schechter, and Sorkin, permitting perhaps a way of actually calculating F.

Future calculations, with or without nonzero γ_0, might involve going beyond [1,1] Padé approximants and the static approximation. In our calculation above we used, in principle at least, $N_0N\pi$ and $\Delta_0N\pi$ couplings as basic H inputs in Eqs. (1) and (2). But, actually, the basic couplings arising from a Skyrme model would be $N_0N_0\pi$, $\Delta_0N_0\pi$, etc. By generalizing our $\pi N \to N\pi$ calculations to "processes" such as $\pi N \to N_0\pi$, however, we can relate $N_0N_0\pi, \ldots$ to $N_0N\pi, \ldots$ just as we related $N_0N\pi, \ldots$ to $NN\pi, \ldots$ in our calculations above.

Finally our self-consistent nonperturbative loop-correction techniques could be readily generalized and adapted to other approaches, such as lattice-QCD, bag and hadronic-string models and even potential calculations. Here again the quenched-loop approximation would be used to construct an H-type "input" into our loop calculations, but this time for mesons as well as baryons.

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APPENDIX A: FACTORIZABILITY AND h/λ UNIVERSALITY

Regge-resonance duality relates the couplings $\Gamma_a, \ldots$ of the resonances or bound states $a, \ldots$ of Fig. 3(a) to the couplings $b(s)$ of the leading Regge trajectory $\alpha(s)$ interpolating the s-channel mass spectrum arising from the sum of Fig. 3 (Ref. 10). It uses finite-energy sum rules (FESR's) derived from fixed-t dispersion relations

$$\int dt[\mathrm{Im}L(s,t)-b(s)v^{\alpha(s)}\theta(v)]\theta(\bar{t}-t)v^{\hat{n}-S_1-S_2}=0 \ , \tag{A1}$$

where the integer $\hat{n}=0$ for the lowest moment, $v=(t-u)/2+\zeta=t+(s-\sum m_i^2)/2+\zeta$ with t-independent ζ, $m_i=$mass, and $S_i=$spin of the external particle i $(=1,2,3,4)$ in Fig. 3, with $S_1+S_2\geq S_3+S_4$ ordering, $i=1,2,3,4$ in the sum $\sum$, and $\bar{t}$ is midway between a state such as a, which contributes $\Gamma_a\delta(t-m_a^2)$ to ImL, and the next state (or Regge recurrence) on the Regge trajectory $\alpha_a(t)$ on which it lies. We can combine such duality [between $a,\ldots;b,\ldots;b',\ldots;\ldots$ and factorizable $\alpha(s)$-Regge couplings] with the usual approximate contractibility of the $a,\ldots;b,\ldots;\ldots$ exchanges into kinematically factorizable "contact" interactions for (even moderately) high t in general graph theory.[11] This leads to the factorizable structure

$$f=\phi(\lambda+h)+\phi^2(\lambda'+h')\sum[\phi k(\lambda'''+h''')]^n k(\lambda''+h'') \ , \tag{A2}$$

for general angular momentum J, with $(\lambda'+h')(\lambda''+h'')=(\lambda+h)(\lambda'''+h''')$, k related to a loop integral, and $n=0,1,2,\ldots$ in the infinite n sum $\sum$. Equation (A2) gives an exact Eq. (8) and gives a J-plane Regge pole with factorizable residue

$$\beta'\beta''=\beta'K_{\alpha\alpha}\beta''+\beta'M_{\alpha\alpha_0}\beta_0''+\beta_0'N_{\alpha_0\alpha}\beta''+\beta_0'P_{\alpha_0\alpha_0}\beta_0'' \ , \tag{A3}$$

where β', β'' are 12α, 34α couplings, and $\beta_0'\beta_0''$ is the factorizable Regge residue one would have in the absence of loops.

The K,M,N,P are independent of the external lines within the above approximation. For $\beta''=\beta'$, Eq. (A2) can therefore be reduced to a quadratic equation for β_0'/β', whose solution is then also independent of the external lines. Since β'^2 is related by FESR duality to the couplings of the states $a,\ldots$ of Fig. 3(a), we conclude that h/λ also has a universality property, as in Eq. (9), even if we do not rely on the static-model Eqs. (2) and (4).

Finally, we note that, with $\mathrm{Im}L=\Gamma_a\delta(s-m_a^2)$, combining the $\hat{n}=0$ and $\hat{n}=1$ FESR of Eq. (A1) gives

$$\alpha(s)=S_1+S_2-1+v_a/(\bar{t}-m_a^2) \ . \tag{A4}$$

If $\zeta=$const and $\alpha'=\alpha_a'$, Eq. (A4) gives $\bar{t}=m_a^2+1/2\alpha'$, exactly halfway between the state a and its Regge recurrence.

APPENDIX B: STATIC-MODEL SUPERCONVERGENCE

The amplitude $f(\omega)$ obeys a useful "superconvergence" relation. We first note that Eq. (5) leads to an $f(\omega)$ which is analytic in ω except for poles and an $\omega\geq m_\pi$ "right-hand" cut in the physical-scattering region. An improved version of λ incorporating resonance width and background would also lead to an $\omega<-m_\pi$ "left-hand" cut, since the exact $f(\omega)$ satisfies the crossing relation[8]

$$f_{IJ}(\omega)=\sum_{I'}\sum_{J'}\alpha_{II'}\beta_{JJ'}f_{I'J'}(-\omega) \tag{B1}$$

as in Eqs. (15) and (16). Cauchy's theorem then leads to the dispersion relation

$$\pi f(\omega)=\int_{\omega_L}^{\infty}d\omega'[\mathrm{Im}f(\omega')/(\omega'-\omega)+\mathrm{Im}f(-\omega')/(\omega'+\omega)] \ , \tag{B2}$$

where bound-state poles are included by adding δ functions to Im$f(\omega')$ and at the same time extending the lower limit ω_L of the integral below $\omega=m_\pi$ to include them.

Since unitarity demands that $f(\omega)$ be bounded by ω^{-3} at infinity, as can be seen, e.g., from $f(\omega)=e^{i\delta}\sin\delta/q^3$, the ω^{-1} coefficient of Eq. (B2) must vanish at large ω, so we have the superconvergence relation[8]

$$\int_{\omega_L}^{\infty}[\mathrm{Im}f(\omega')-\mathrm{Im}f(-\omega')]=0 \ . \tag{B3}$$

If we assume that Im$f(\omega)$ is dominated by N and Δ, and make a (narrow-width) δ-function approximation for Δ, we obtain $\gamma=\gamma^x$, with γ^x given by Eq. (15). This result is obtained whether or not we take the $h\rightarrow 0$ limit.

The above static model ignores the relativistic meson-exchange effects of Fig. 2(a). Such effects have been estimated to be small for low-ω πN scattering, however.

[1] T. H. R. Skyrme, Proc. R. Soc. London **A260**, 127 (1961); A. P. Balachandran, V. P. Nair, S. G. Rajeev, and A. Stern, Phys. Rev. Lett. **49**, 1124 (1982).

[2] E. Witten, Nucl. Phys. **B223**, 422 (1983); **B223**, 433 (1983); G. Adkins, C. Nappi, and E. Witten, *ibid.* **B228**, 552 (1983).

[3] See, e.g., M. Mashaal, T. N. Pham, and Tran N. Truong, Phys. Rev. Lett. **56**, 436 (1986); Ulf-G. Meissner, A. Kaiser, A. Wirzba, and W. Weise, *ibid.* **57**, 1676 (1986).

[4] M. Lacombe, B. Loiseau, R. Vinh Mau, and W. N. Cottingham, Phys. Rev. D **38**, 1491 (1988).

[5] See, e.g., A. Patel, R. Gupta, G. W. Kilcup, and S. R. Sharpe, Phys. Lett. B **225**, 398 (1989).

[6] L. A. P. Balázs, Phys. Rev. D **34**, 2876 (1986); **39**, 1002 (1989).

[7] S. N. Biswas and L. Balázs, Phys. Rev. **156**, 1511 (1967).

[8] See, e.g., L. A. P. Balázs and J. M. Cornwall, Phys. Rev. **160**, 1313 (1967).

[9] P. Jain, J. Schechter, and R. Sorkin, Phys. Rev. D **39**, 998 (1989).

[10] See, e.g., P. H. Frampton, *Dual Resonance Models and Superstrings* (World Scientific, Singapore, 1986), pp. 22–28.

[11] R. Eden, P. Landshoff, D. Olive, and J. C. Polkinghorne, *The Analytic S-Matrix* (Cambridge University Press, Cambridge, England, 1980), pp. 132–136.