# $\tau$ decay in the massive-neutrino scenario

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This work analyzes the decay of the $\tau$, keeping all the final-state leptons on mass shell. Values of the branching ratios for leptonic and major hadronic modes are presented for nonzero values of the mass of the third-generation neutrino. The context of experimental $\nu_\tau$ mass limits, both the latest and earlier ones, and the hypothetical second massive $\tau$ neutrino are discussed.

Decay of the third-generation charged lepton $\tau$ has emerged as a focus of intensive research both theoretical and experimental.[1] In particular, branching ratios for the single-prong topologies retain their discrepancy. $\tau$ decay also provides an incisive tool to study the properties of its associated neutrino $(\nu_\tau)^2$. In the backdrop of possible neutrino oscillations and the astrophysical significance of interstellar dark matter,[3] the existence or absence of a finite neutrino mass acquires crucial importance.

In this work we have calculated all the major decay modes of the $\tau$, keeping the $\tau$ neutrino on mass shell, throughout. For the leptonic modes in particular, this is the first computation keeping all four leptons on mass shell. We are, therefore, able to extract more reliable constraints on the $\nu_\tau$ mass from $\tau$ decay experimental data than hitherto available. The sensitivity of the leptonic and major hadronic modes to the neutrino mass is demonstrated. It may be mentioned that the problem of a finite $\nu_\tau$ mass was theoretically studied[4] in the early stages of $\tau$-decay research. However their calculation of the leptonic modes keeping $\nu_\tau$ on mass shell puts the final-state charged lepton off mass shell. Since masses up to 70 MeV are considered, neglect of $m_\mu$ at least does not seem trivial in this context.

The $\eta$ connection has been explored through the rare $(\pi\eta)$ decay channel of an intermediate $\rho$. It has been found that this channel can contribute to order 0.2% to the branching ratio and is therefore compatible with current experimental limits.[5]

It is interesting to note that the structure of $\tau$ decay is insensitive to violations of the $\nu_\tau$ chirality to order $m_{\nu_\tau}$ in the rate. This arises from the cancellation of relevant $\nu_\tau$ mass terms from the matrix element in all cases. Imposition of mass-shell conditions on $\nu_\tau$ modifies the decay rates through the altered kinematical phase-space conditions and the structure, both to order $(m_{\nu_\tau})^2$.

The matrix elements for the leptonic and major hadronic decay modes corresponding to Figs. 1 to 4 can be written in the standard model:

$$\text{for } \tau \to \nu_\tau l \nu_l \ (l \text{ refers to } \mu \text{ or } e)\,,\quad M_1 = g_W^2[\bar{\psi}_l \Gamma^\alpha \psi_{\nu_l} F(W) \psi_{\nu_\tau} \Gamma_\beta \psi_\tau]\,. \tag{1}$$

$F(W)$ represents the $W$-boson propagator connecting the lepton pairs and $g_W$ is its coupling to the relevant leptonic pairs. (The same $g_W$ represents the $W$ coupling to quark pairs for the hadronic modes by universality of weak interactions.) The $\psi$ refer to the wave functions of the subscripted leptons and $\Gamma$ to the interaction at each vertex. Similarly for the process $\tau \to \pi\nu_\tau$, corresponding to Fig. 2 we have

$$M_{\pi\nu_\tau} = \Phi_\pi g_{\pi qq} g_W^2 [1/(\not{p}_u - m_u)]\gamma_5 \times [1/(\not{p}_d - m_d)]\Gamma^\alpha F(W) \psi_{\nu_\tau} \Gamma^\beta \psi_\tau \cos\theta \tag{2}$$

($\theta$ is the Cabibbo angle). For the Cabibbo-suppressed $K$ mode we have

$$M_{K\nu_\tau} = \Phi_K g_{Kqq} g_W^2 [1/(\not{p}_u - m_u)]\gamma_5 [1(\not{p}_s - m_s)] \times \Gamma^\alpha F(W) \psi_{\nu_\tau} \Gamma^\beta \psi_\tau \sin\theta\,. \tag{3}$$

$g_{\pi qq}$ and $g_{Kqq}$ are the meson-quark coupling constants for pion and kaon.[6]

The vector-current-induced $\rho$ mode corresponding to Fig. 4 can be written

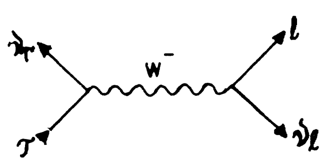


FIG. 1. Leptonic decay mode.

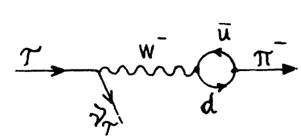


FIG. 2. Decay in pion channel.

$$M_{\rho\nu_\tau}=\Phi_\rho g_W^2 g_{\rho qq}[1/(\not{p}_u-m_u)]\gamma_\gamma[1/(\not{p}_d-m_d)]\Gamma^\alpha F(W)\bar{\psi}_{\nu_\tau}\Gamma^\beta\psi_\tau\cos\theta \; . \tag{4}$$

The quark loop can be collapsed to a point coupling constant for the hadronic sectors and the $W$-boson traversal shrunk to local $(V-A)$ coupling [since $W$-boson effects are of order $(m_\tau/m_W)^2$ and hence negligible].

The above expressions then simplify to

$$M_l=(G/\sqrt{2})(\bar{U}_{\nu_\tau}O^\alpha U_\tau \bar{U}_l O_\alpha U_{\nu_l}) \; , \tag{5}$$

where $O^\alpha=\gamma^\alpha(1+\gamma_5)$,

$$M_{\pi\nu_\tau}=(G/\sqrt{2})f_\pi p^\alpha \Phi_\pi \bar{U}_{\nu_\tau}O_\alpha U_\tau\cos\theta \; , \tag{6}$$

$$M_{K\nu_\tau}=(G/\sqrt{2})f_k p^\alpha \Phi_k \bar{U}_{\nu_\tau}O_\alpha U_\tau\sin\theta \; , \tag{7}$$

$$M_{\rho\nu_\tau}=(G/\sqrt{2})g_\rho \Phi_\rho^\alpha \bar{U}_{\nu_\tau}O_\alpha U_\tau\cos\theta \; , \tag{8}$$

where the scalar parameter $g_\rho$ of dimension $m^2$ can be obtained from the electromagnetic $\rho$ coupling. $f_\pi$ and $f_k$ are the standard decay constant for $\pi$ and $K$, and $\Phi_\pi$, $\Phi_k$ their wave functions. $p$ is the final-meson four-momentum and $\Phi_\rho^\alpha$ the $\rho$ wave function.

The structural dependence of the matrix elements on a nonzero $\nu_\tau$ mass can be readily studied by suitably defining the $\nu_\tau$ projection operators with appropriate mass terms. Retaining both $m_{\nu_\tau}$ and $m_{\nu_l}$ for the present, the squares of the matrix elements take the forms

$$|M_l|^2=(G^2/2)\mathrm{Tr}[(q_{\nu_\tau}+m_{\nu_\tau})O^\alpha(\not{p}_\tau+m_\tau)O_\beta(\not{p}_l+m_l)O_\alpha(\not{q}_{\nu_l}+m_{\nu_l})O^\beta] \; , \tag{9}$$

$$|M_\pi|^2=(G^2/2)f_\pi^2\Phi_\pi^2\cos^2\theta[m_\tau^2 T_\tau+m_{\nu_\tau}^2 T_{\nu_\tau}-m_{\nu_\tau}m_\tau(T_-+T_+)] \; , \tag{10}$$

where

$$\begin{aligned} T_\tau&=\mathrm{Tr}[(\not{p}_{\nu_\tau}+m_{\nu_\tau})(1-\gamma_5)(\not{p}_\tau+m_\tau)(1+\gamma_5)] \; , \\ T_{\nu_\tau}&=\mathrm{Tr}[(\not{p}_{\nu_\tau}+m_{\nu_\tau})(1+\gamma_5)(\not{p}_\tau+m_\tau)(1-\gamma_5)] \; , \end{aligned} \tag{11}$$

$$T\pm=\mathrm{Tr}[(\not{p}_{\nu_\tau}+m_{\nu_\tau})(1\mp\gamma_5)(\not{p}_l+m_l)(1\mp\gamma_5)] \; . \tag{12}$$

The rates for the $(2n\pi)$ and $(2n+1)\pi$ modes arising from the vector and axial-vector currents, respectively, can be written in an analogous manner. Simplifying and separating $m_{\nu_\tau}$ terms and carrying out the various trace calculations we have

$$|\bar{M}_l|^2=(G^2/2)64[(p_{\nu_\tau}\cdot p_l)(p_{\nu_\tau}\cdot p_\tau)-(p_{\nu_\tau}\cdot p_{\nu_l})(p_l p_\tau)] \; . \tag{13}$$

It may be noted that $m_{\nu_\tau}$ terms cancel out identically to all orders in the leptonic matrix elements so that the matrix element itself is insensitive to whether or not the neutrinos are on mass shell.

For the hadronic channels we obtain

$$|\bar{M}_{\pi\nu_\tau}|^2=4G^2f_\pi^2\cos^2\theta m_\tau^2\{(p_{\nu_\tau}\cdot p_\tau)+m_{\nu_\tau}^2[(p_{\nu_\tau}\cdot p_\tau)/m_\tau^2-2]\}\Phi_\pi^2 \; , \tag{14}$$

$$|\bar{M}_{k\nu_\tau}|^2=4G^2f_K^2\Phi_K^2\sin^2\theta m_\tau^2\{(p_{\nu_\tau}\cdot p_\tau)+m_{\nu_\tau}^2[(p_{\nu_\tau}\cdot p_\tau)/m_\tau^2-2]\} \; , \tag{15}$$

$$|\bar{M}_{\rho\nu_\tau}|^2=4G^2g_\rho^2\cos^2\theta m_\tau^2\{[(p_{\nu_\tau}\cdot p_\tau)/m_\rho^2](1+2m_\rho^2/m_\tau^2-2m_{\nu_\tau}^2/m_\tau^2)-(m_{\nu_\tau}^2/m_\rho^2)(1+m_\rho^2/m_\tau^2-m_{\nu_\tau}^2/m_\tau^2)\} \; . \tag{16}$$

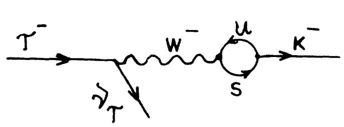


FIG. 3. Decay in kaon channel.

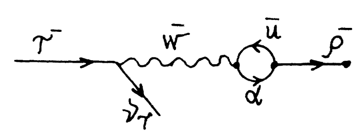


FIG. 4. Decay in $\rho$ channel.

For the phase-space integrations of the leptonic mode with three final-state leptons the decay rate can be written similarly to the muon decay case as

$$R_l=(2\pi)^2/(2\pi)^9\int |M_l|^2/M\omega_2\omega_1E_k d^3\mathbf{q}_1 d^3\mathbf{q}_2 d^3\mathbf{k}$$
$$\times\delta^4(P-q_1-q_2-k)\ , \qquad (17)$$

where $P$ is the initial four-momentum and $q_1$, $q_2$, $k$ those of the two neutrinos and final charged lepton. $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{k}$ are the corresponding three-momenta and $\omega_1,\omega_2$, $E$ the energies.

We have seen that placing neutrinos on mass shell does not alter $|M_l|^2$. So we can write, as for the massless case (which is analogous to the muon-decay problem),

$$R_l=4G^2/(2\pi)^5\int\frac{(Pq_2)(kq_1)}{M\omega_2E_k\omega_1}d^3\mathbf{q}_1 d^3\mathbf{q}_2 d^3\mathbf{k}$$
$$\times\delta^4(P-k-q_1-q_2)\ . \qquad (18)$$

The retention of mass terms for all final leptons makes the integrals more complicated than the muon decay case, and while keeping only one lepton on mass shell as in Ref. 4 would permit one to complete the integration analytically, keeping them all on mass shell requires numerical integration over the last variable.

We integrate over the $(k-q_1)$ system first in their center-of-mass frame using the four-dimensional $\delta$ function. After integrating over the remaining lepton direction, its energy has still to be integrated. We have

$$R_l=(G^2m_\tau^5/\pi^3)$$
$$\times\int x^2(1-2x+b_2-b_1-b_k)(E_k^2-m_3^2)^{1/2}/D\ , \qquad (19)$$

where

$$b_1=m_1^2/m_\tau^2,\quad b_2=m_3^2/m_\tau^2\ ,$$
$$b_k=m_k^2/m_\tau^2$$

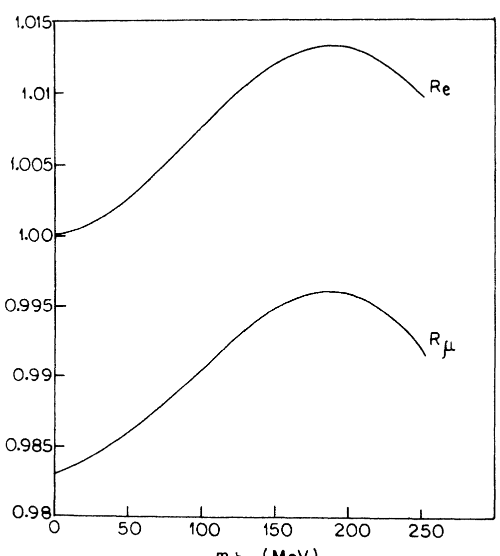


FIG. 5. Branching ratios for leptonic modes as a function of neutrino mass.

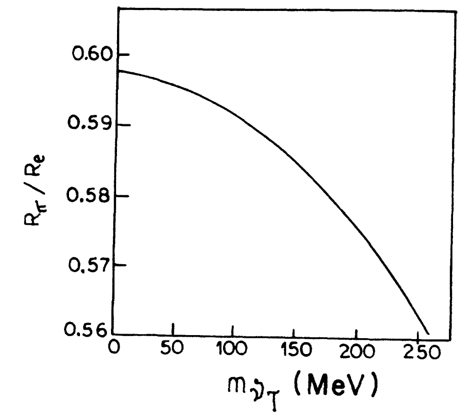


FIG. 6. Branching ratio of pion mode as compared to electron mode as a function of neutrino mass.

$$D=[(E_k^2-m_3^2+m_k^2+m_l^2)^{1/2}+E_k]\ ,$$
$$E_k=(m_\tau/2)[(1-x)^2+b_k-b_1]/(1-x)\ ,$$

and $x$ is the energy of the third lepton and $m_3$ its mass.

The integral has been evaluated numerically between the kinematically allowed limits and the branching ratios $R_l$ and $R_\mu$ for the electron and muon modes are displayed in Fig. 5 as a function of $\tau$ neutrino mass. The rate is insensitive to the value of the second neutrino mass up to 5.6 MeV. The variation in $R_l$ is of order $(m_{\nu_\tau}/m_\tau)^2$ and comes about from phase-space modification only in this case.

Both $R_\mu$ and $R_e$ increase slightly as one increases $m_{\nu_\tau}$ from zero to the present experimental upper limit of 35 MeV (Refs. 7 and 5). We extended this to 250 MeV, to partially include a hypothetical second $\tau$ neutrino that could exist at the 5% limit and also to allow for the earlier experimental upper limits[4] of 250 MeV for $m_{\nu_\tau}$. We observe an interesting change of gradient of the branching ratios in Fig. 5, as they pass through a broad peak around $m_{\nu_\tau}\sim 175$ MeV. For a larger value of $m_{\nu_\tau}$ the branching ratios are expected to fall rapidly. The ratio of $R_\mu/R_e$ is found to be practically constant for the studied range of $m_{\nu_\tau}$.

For the two-particle final-state hadronic modes the

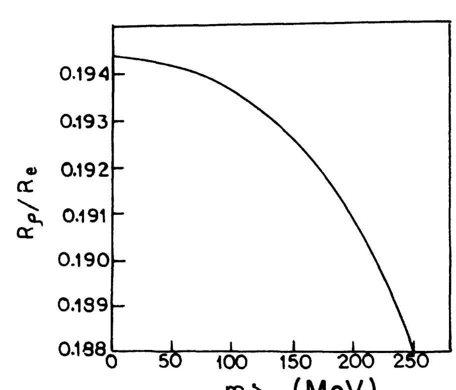


FIG. 7. Branching ratio of $\rho$ mode compared to electron mode as a function of neutrino mass.

phase-space integration is trivial, but is altered from the massless case by additional terms of order $(m_{\nu_\tau}/m_\tau)^2$. So in these modes the final rate differs from the corresponding massless cases by terms of this order, and they originate both from the structure of the matrix elements and the phase-space kinematics.

For the $(\pi-\nu_\tau)$ mode,

$$R_\pi = G^2 f_\pi^2 \cos^2\theta m_\tau^2 \sqrt{E_{\nu_\tau}^2 - m_\tau^2}\, D_\pi / 8\pi ,$$

where

$$D_\pi = [(1-h_\pi + h_{\nu_\tau})(1+h_{\nu_\tau}) - 4h_{\nu_\tau}]$$

with $h_{\nu_\tau} = (m_{\nu_\tau}/m_\tau)^2$ and $h_\pi = (m_\pi/m_\tau)^2$.

$E_{\nu_\tau}$ is expressible in terms of $h_{\nu_\tau}$ and $h_\pi$ so that numerical values of $R_\pi$ can be obtained without requiring further integrations. $R_\pi$ has been computed taking $\theta = 13°$, and $f_\pi = 130$ MeV. Figure 6 displays $(R_\pi/R_e)$ as a function of $m_{\nu_\tau}$.

The $\rho$ mode can be similarly computed, except that the $\rho$ spectral function $\gamma$, is also required.[8] Values of $(R_\rho/R_e)$ are plotted in Fig. 7 for $(\gamma^2/4\pi) = 2.6$.

The $(2n\pi)$, $(2n+1)\pi$ modes also give $(m_{\nu_\tau}/m_\tau)^2$ deviations from the massless case and the limits on $m_{\nu_\tau}$ are compatible with those derived from other modes.

For the $\eta$ connection, the $\rho \to \eta\pi$ decay can occur through a quark line rule violation due to the $s\bar{s}$ content of the final-state particle. This rare decay mode $(\rho\to\eta\pi)$ has a branching-ratio limit[5] of 0.8% (C.L. = 84%).

A limit on $\tau \to \eta\pi\rho$ can therefore be set by assuming the decay to proceed as

$$\tau \to \nu_\tau \rho$$
$$\quad\;\; \hookrightarrow \pi\eta ,$$

so the relevant branching-ratio fraction is $R' = R_\rho R_\eta$ where $R_\rho$ is branching ratio for $\tau \to \rho_{\nu_\tau}$ and $R_\eta$ that for $\rho \to \pi\eta$. Numerically this gives $R = 0.16\%$ and is compatible with the existing experimental upper limit for this branching fraction.[5]

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